On the impact of model-independent higher-order corrections to Higgs self-couplings

Based on [arXiv:2608.25705] with Wrishik Naskar and Georg Weiglein

Johannes Braathen (DESY)

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Slide credits and thanks to Wrishik Naskar!





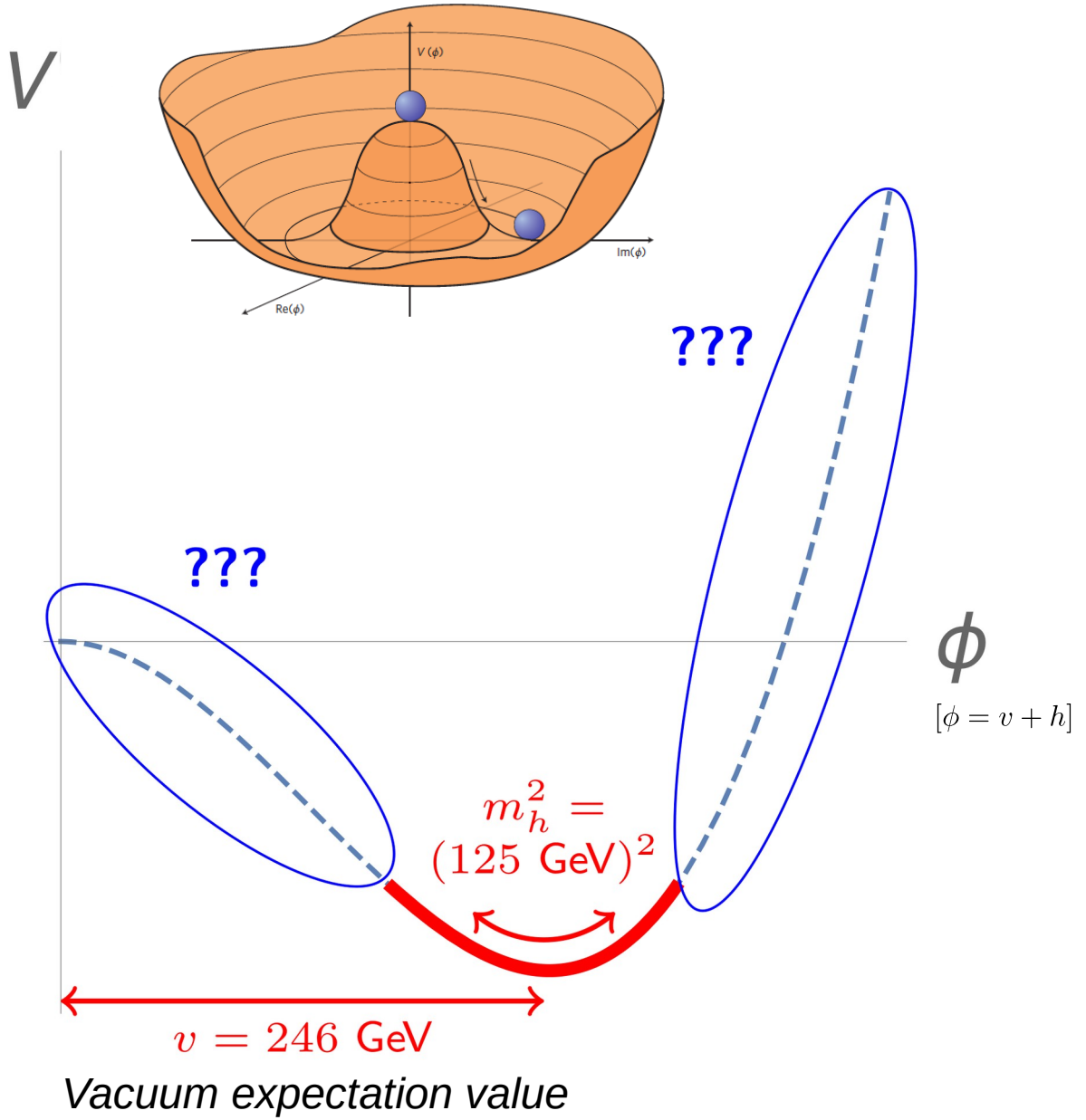
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DESY.

Form of the Higgs potential and trilinear Higgs coupling

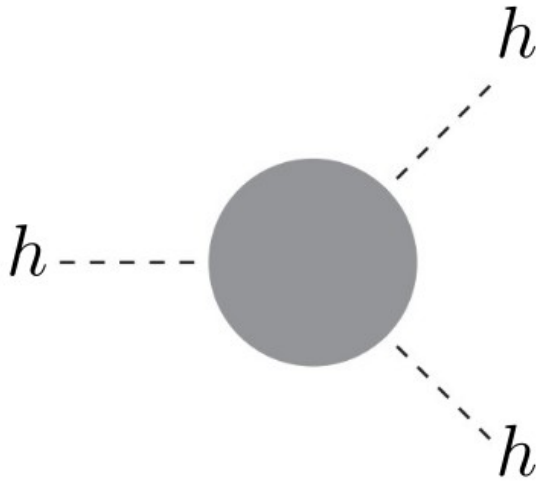
- Brout-Englert-Higgs mechanism = **origin of masses of elementary particles** ...
  … but very little known about the **Higgs potential** causing the **electroweak phase transition** (EWPT)

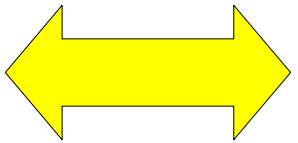


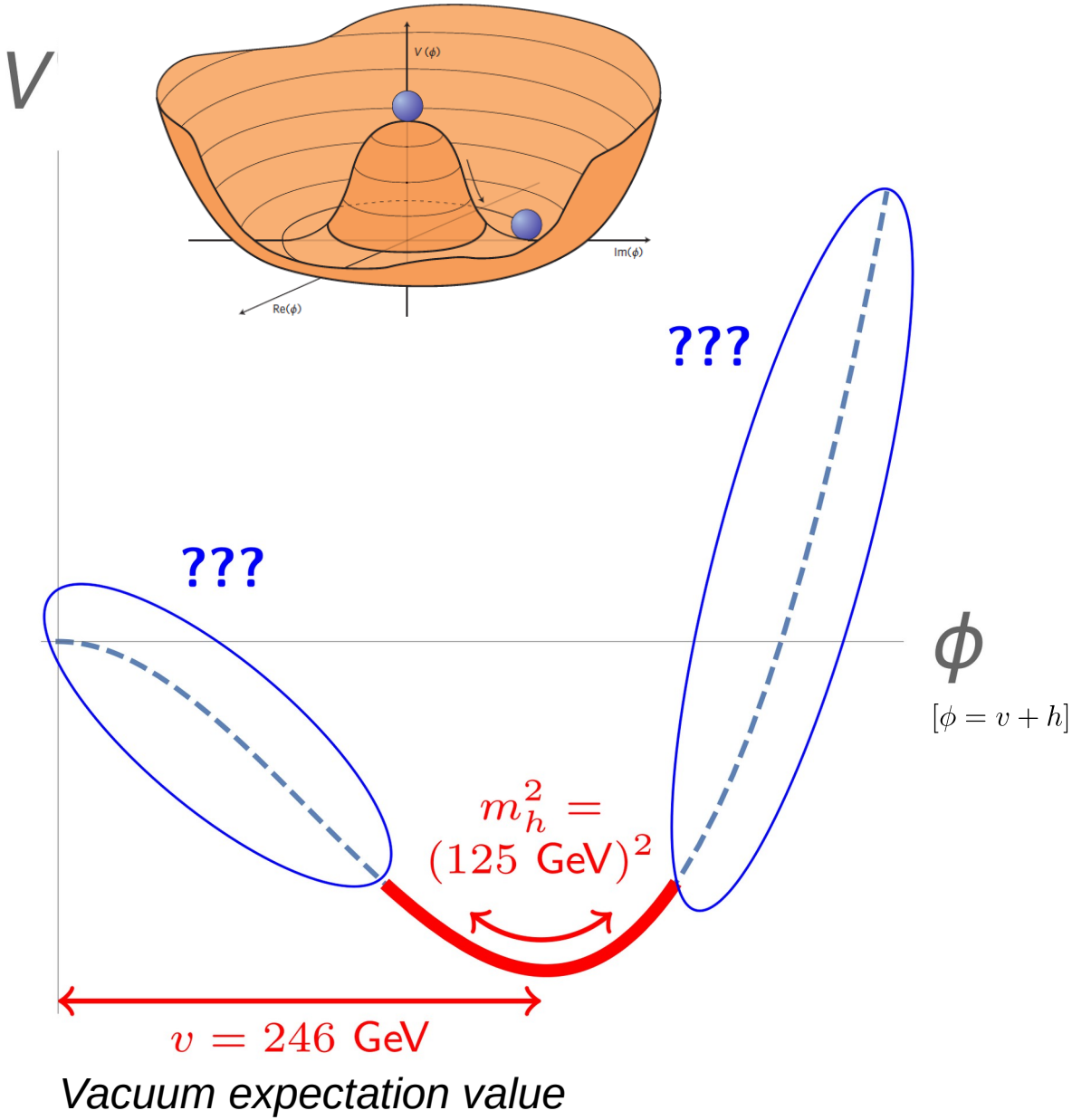
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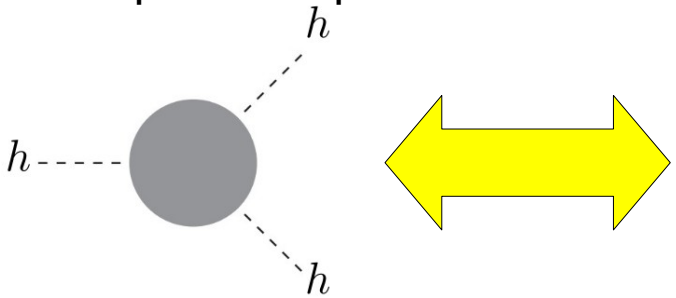




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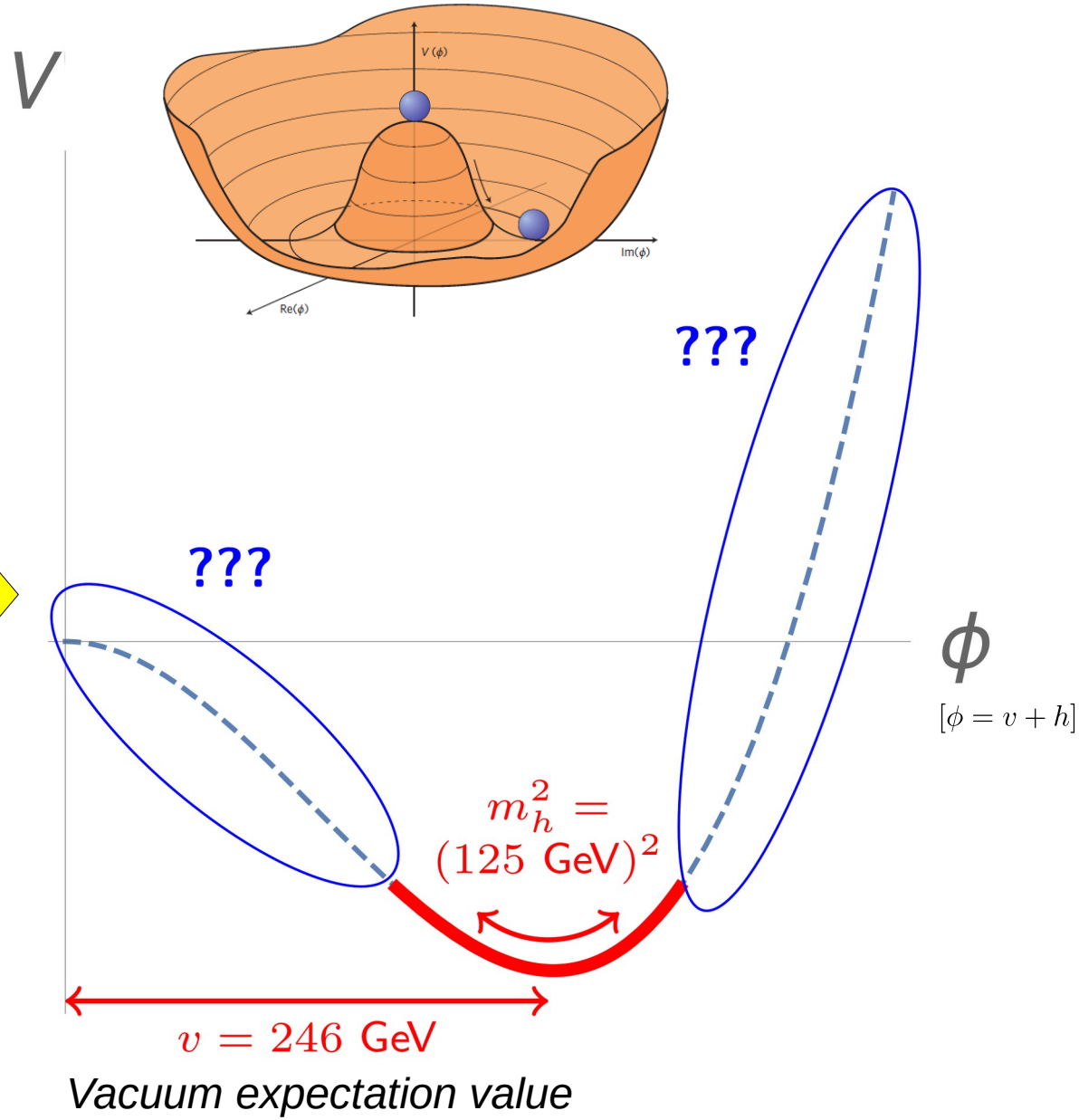
- **Trilinear and quartic Higgs self-coupling, λ_{hhh} and λ_{hhhh}**, crucial to understand the shape of the potential



In the SM: $V^{(0)}_{\rm SM} = \frac{1}{2} m_h^2 h^2 + \frac{1}{3!} \underbrace{\left(\frac{3m_h^2}{v}\right)}_{\equiv (\lambda^{(0)}_{hhh})^{\rm SM}} h^3 + \frac{1}{4!}\left(\frac{3m_h^2}{v^2}\right) h^4 + \cdots$

In general: $V^{(0)} = \frac{1}{2} m_h^2 h^2 + \frac{1}{3!} \overbrace{\kappa_\lambda \left(\frac{3m_h^2}{v}\right)}^{\equiv \lambda_{hhh}} h^3 + \frac{1}{4!} \kappa_4 \left(\frac{3m_h^2}{v^2}\right) h^4 + \cdots$

with $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda^{(0)}_{hhh})^{\rm SM}$ and $\kappa_4 \equiv \lambda_{hhhh}/(\lambda^{(0)}_{hhhh})^{\rm SM}$



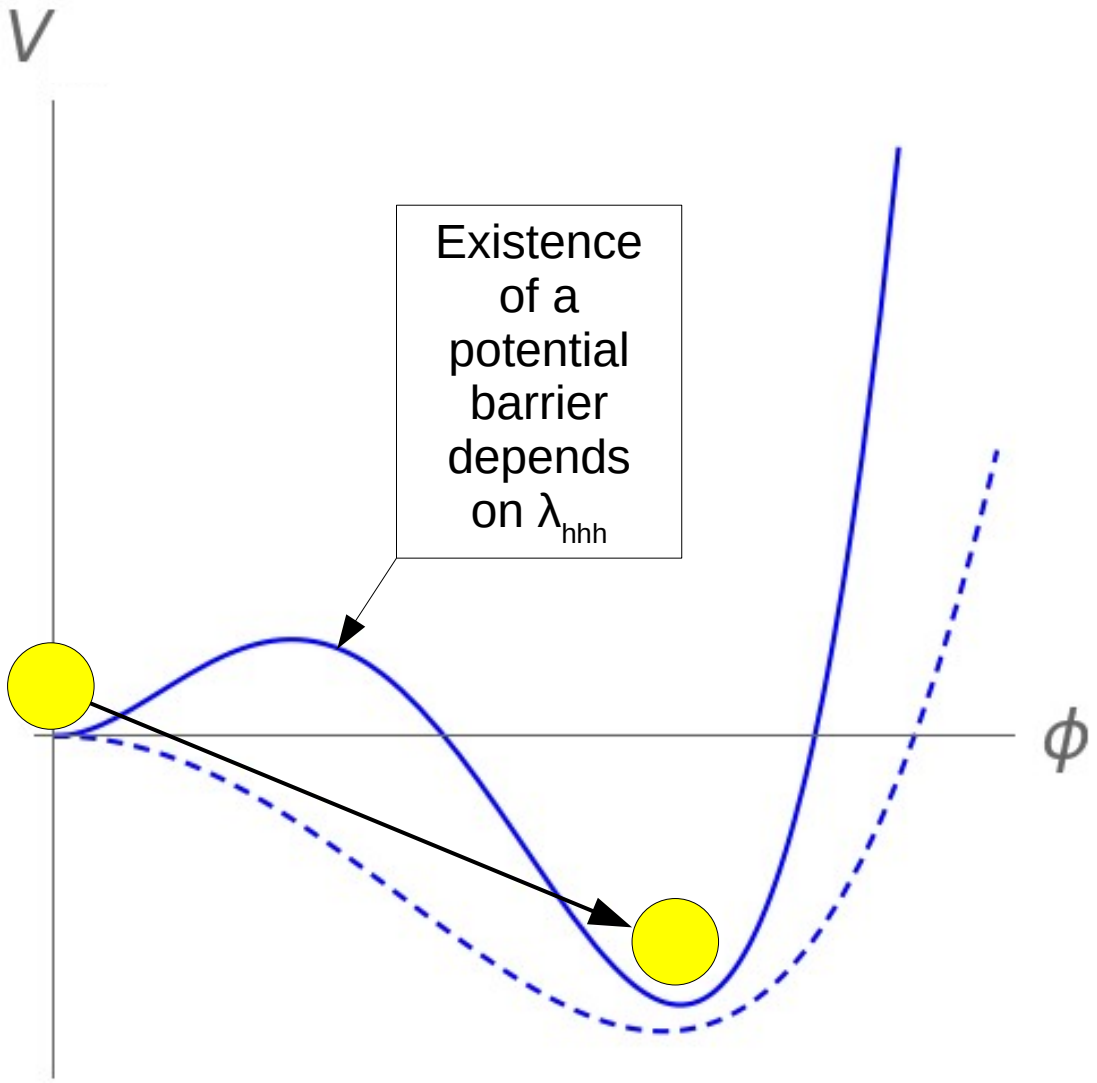
Form of the Higgs potential and baryon asymmetry

- Brout-Englert-Higgs mechanism = **origin of masses of elementary particles** ...
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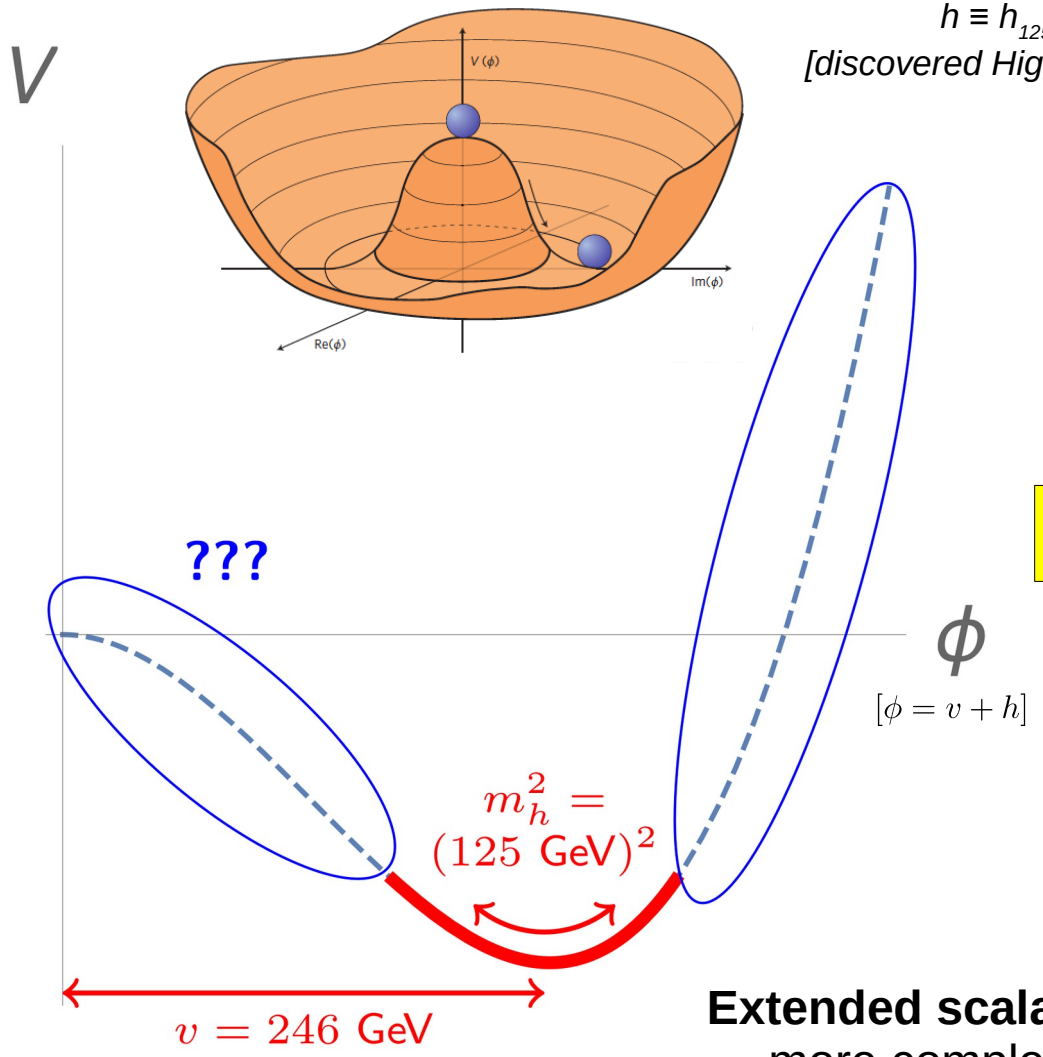
- **Trilinear and quartic Higgs self-coupling, λ_{hhh} and λ_{hhhh},** crucial to understand the shape of the potential

- Among **Sakharov conditions** necessary to explain **baryon asymmetry of the Universe via electroweak phase transition** (= *electroweak baryogenesis*):
  - ***Strong first-order EWPT (SFOEWPT)***
    → barrier in Higgs potential
    → O(30-100%) deviation in λ_{hhh} from SM prediction
    if SFOEWPT along Higgs field direction

  see e.g. [Kanemura, Okada, Senaha, '05], [Grojean, Servant, Wells, '06], [Biekötter et al., '22] (2HDM), or [JB, Heinemeyer, Pulido Boatella, Verduras Schaeidt '25] (RxSM)

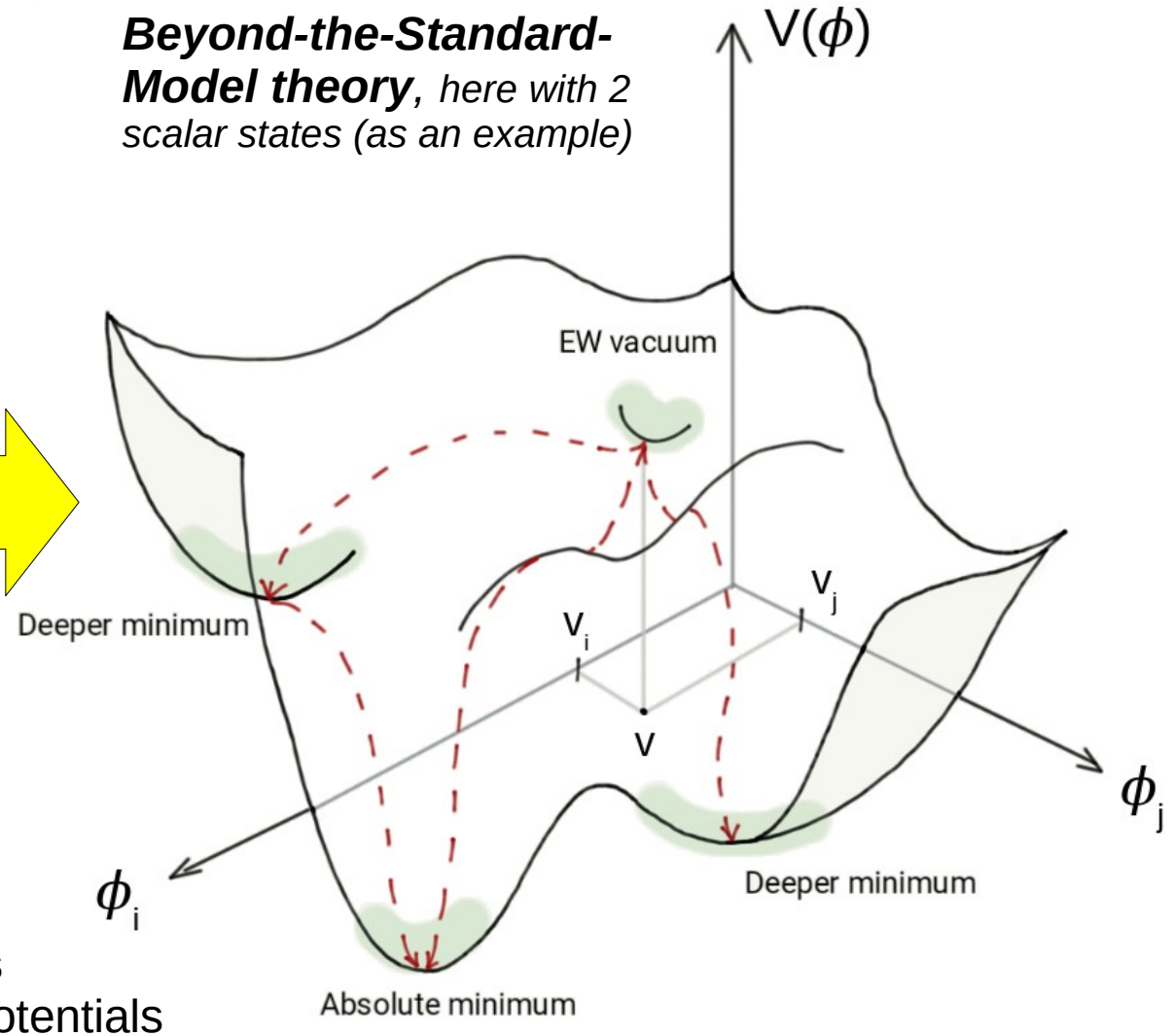


Form of the Higgs potential and trilinear scalar couplings



$h \equiv h_{125}$
[discovered Higgs boson]

***Beyond-the-Standard-Model theory**, here with 2 scalar states (as an example)*



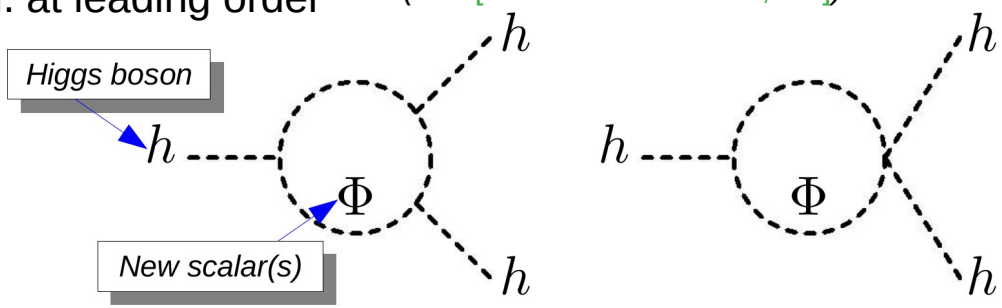
Extended scalar sectors
→ more complex Higgs potentials
→ more trilinear scalar couplings needed to reconstruct potential

Figure by [K. Radchenko Serdula '24]

Probing New Physics with the trilinear Higgs coupling

[Bahl, JB, Weiglein **Phys. Rev. Lett.** '22]

- **Large effects from New Physics possible in λ_{hhh}** due to radiative corrections from extra scalars, e.g. at leading order *(see [Kanemura et al. '02, '04])*



→ **"mass splitting effects"** $g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2}$

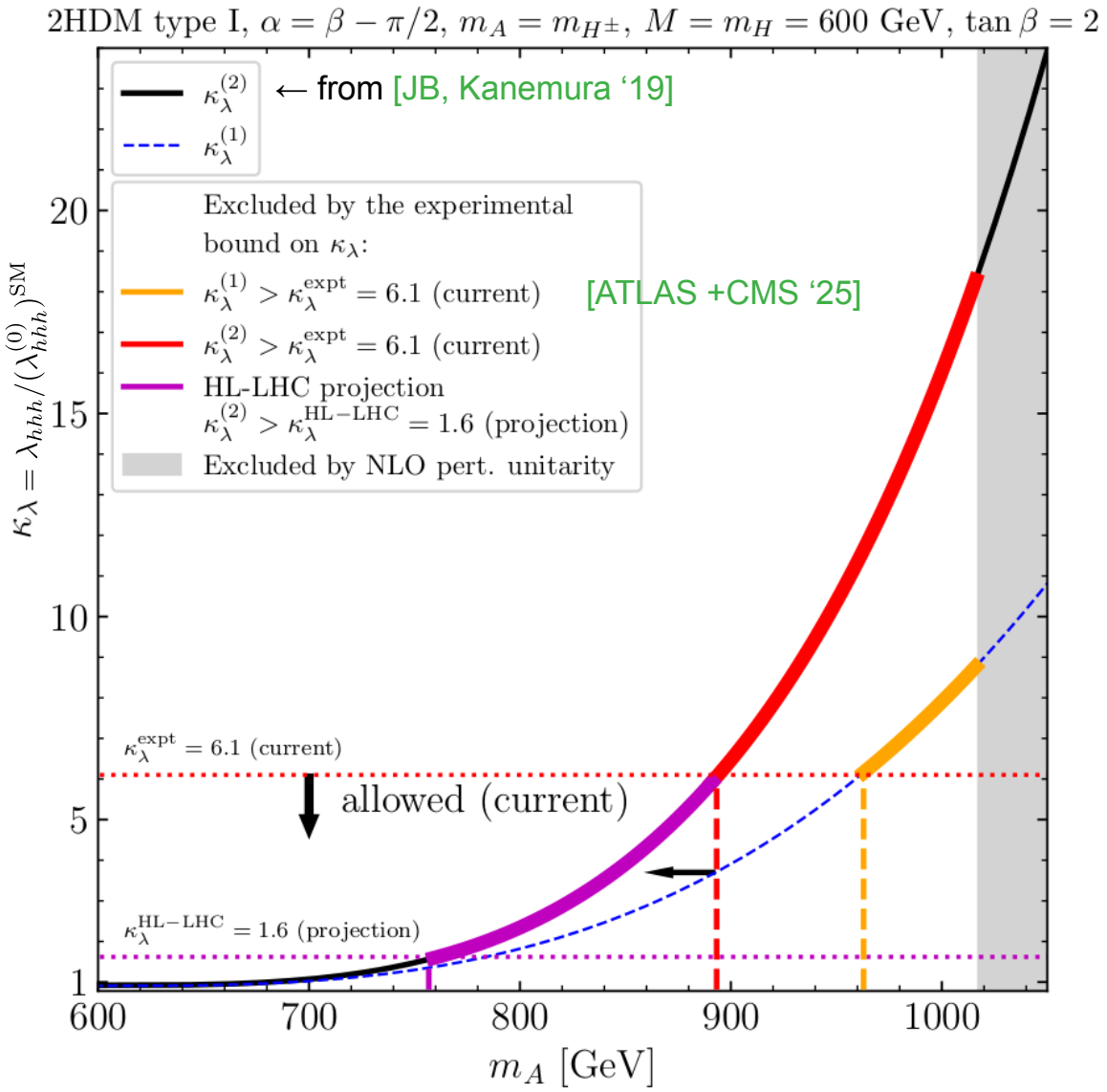
- Comparing latest exp. bounds

$$-0.71 < \kappa_\lambda = \frac{\lambda_{hhh}}{(\lambda_{hhh}^{(0)})^{\rm SM}} < 6.1$$ [ATLAS & CMS Run 2 combination '25]

with precise theory predictions for λ_{hhh} provides a **powerful new tool to constrain BSM models**

- BSM deviations in κ_λ also **motivated from the point of view of early-Universe evolution** (e.g. SFOEWPT)

- Quartic Higgs coupling is less constrained, but receives even larger radiative corrections, see e.g. [JB, Kanemura '19]



Experimental constraints and theoretical interpretations

Experimental constraints

$$\kappa_\lambda = \frac{\lambda_{hhh}}{\lambda_{hhh}^{\mathrm{SM},(0)}}, \qquad \kappa_4 = \frac{\lambda_{hhhh}}{\lambda_{hhhh}^{\mathrm{SM},(0)}}$$

- Current bounds:

$$\kappa_\lambda \in (-0.71, 6.1), \ \kappa_4 \in (-177, 185)$$

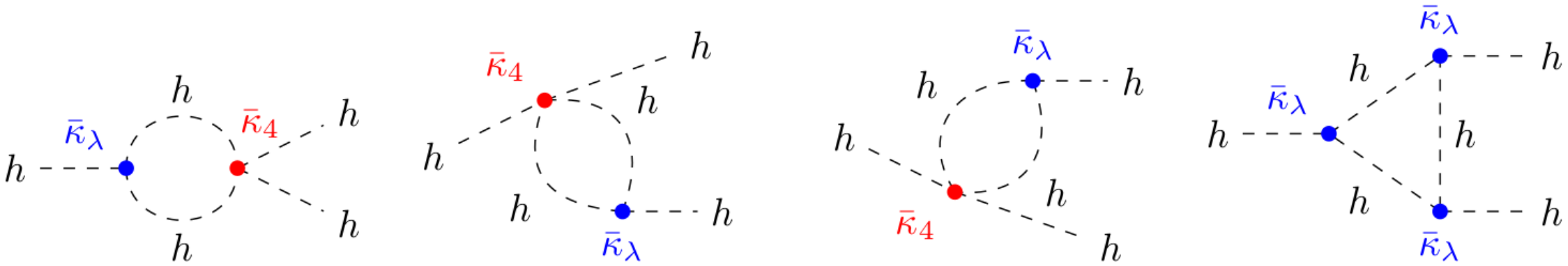
Limits from di-Higgs: [ATLAS+CMS Run 2 comb '26]
Limits from triple-Higgs: [CMS '26]

Theoretical limitations

- Predictions typically rely on one-loop BSM corrections, with higher orders included where available.
- Large scalar-sector corrections can make higher-order contributions numerically important, even when formally subleading.
- Loop-corrected self-couplings can enter further SM-like loop diagrams, generating additional contributions beyond the usual fixed-order prediction.

A reliable interpretation of experimental bounds requires all numerically relevant contributions to κ_λ and κ_4 to be included.

One-loop example



- New physics effects captured effectively in $\bar{\kappa}_\lambda$, $\bar{\kappa}_4$ coupling modifiers.
- κ_λ predictions now also depend on $\bar{\kappa}_4$, and higher powers of $\bar{\kappa}_\lambda$.

$$\kappa_\lambda^{(1)} = \underbrace{\bar{\kappa}_\lambda}_{\text{tree-level}} + \underbrace{f_1(\bar{\kappa}_\lambda, \bar{\kappa}_4) + f_2(\bar{\kappa}_\lambda^3)}_{\text{one-loop}}$$

Rectangular experimental bounds on the Higgs 3-point interaction cannot be interpreted as a direct bound on the underlying self-coupling.

Effective potential calculation of Higgs self-couplings

Higgs self-interactions parameterised through two independent inputs, $\bar{\kappa}_\lambda$ and $\bar{\kappa}_4$ (similar to a HEFTy description): *see e.g.* [Brivio et al. '14, '16]

$$V^{(0)}(h) = t_h h + \frac{m_h^2}{2} h^2 + \frac{\bar{\kappa}_\lambda}{6} \frac{3(m_h^2 - t_h/v)}{v} h^3 + \frac{\bar{\kappa}_4}{24} \frac{3(m_h^2 - t_h/v)}{v^2} h^4.$$

Field-dependent Higgs mass and trilinear coupling:

$$\hat{m}_h^2(h) = m_h^2 + \bar{\kappa}_\lambda \frac{3m_h^2}{v} h + \bar{\kappa}_4 \frac{3m_h^2}{2v^2} h^2, \quad \hat{\lambda}_{hhh}(h) = \frac{3m_h^2}{v} \left(\bar{\kappa}_\lambda + \bar{\kappa}_4 \frac{h}{v} \right).$$

$$V_{\text{eff}}(h) = V^{(0)}(h) + \overbrace{\frac{1}{16\pi^2} \mathcal{F}\left(\hat{m}_h^2(h)\right)}^{V^{(1)}(h)} \overbrace{- \frac{\hat{\lambda}_{hhh}^2(h)}{3072\pi^4} I(\hat{m}_h^2, \hat{m}_h^2, \hat{m}_h^2) + \frac{3m_h^2 \bar{\kappa}_4}{2048\pi^4 v^2} A_0^2(\hat{m}_h^2)}^{V^{(2)}(h)}$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{\Delta V_{\text{eff}}}$$

$\mathcal{F}$ and A_0 are one-loop functions; I is the (finite-part) two-loop vacuum integral.

[Ford, Jack, Jones '92], [Martin '01], [Weiglein '94]

Effective potential calculation of Higgs self-couplings

For the on-shell tadpole and Higgs-mass (M_h) prescription

[Kanemura, (Kiyoura,) Okada, Senaha, Yuan '02, '04], [JB, Kanemura '19, '19], [Bahl, JB, Gabelmann, Paßehr '25]

$$\kappa_\lambda = \bar{\kappa}_\lambda + \left(\frac{3M_h^2}{v}\right)^{-1} \mathcal{D}_3 \Delta V_{\text{eff}}\big|_{h=0}, \qquad \kappa_4 = \bar{\kappa}_4 + \left(\frac{3M_h^2}{v^2}\right)^{-1} \mathcal{D}_4 \Delta V_{\text{eff}}\big|_{h=0}$$

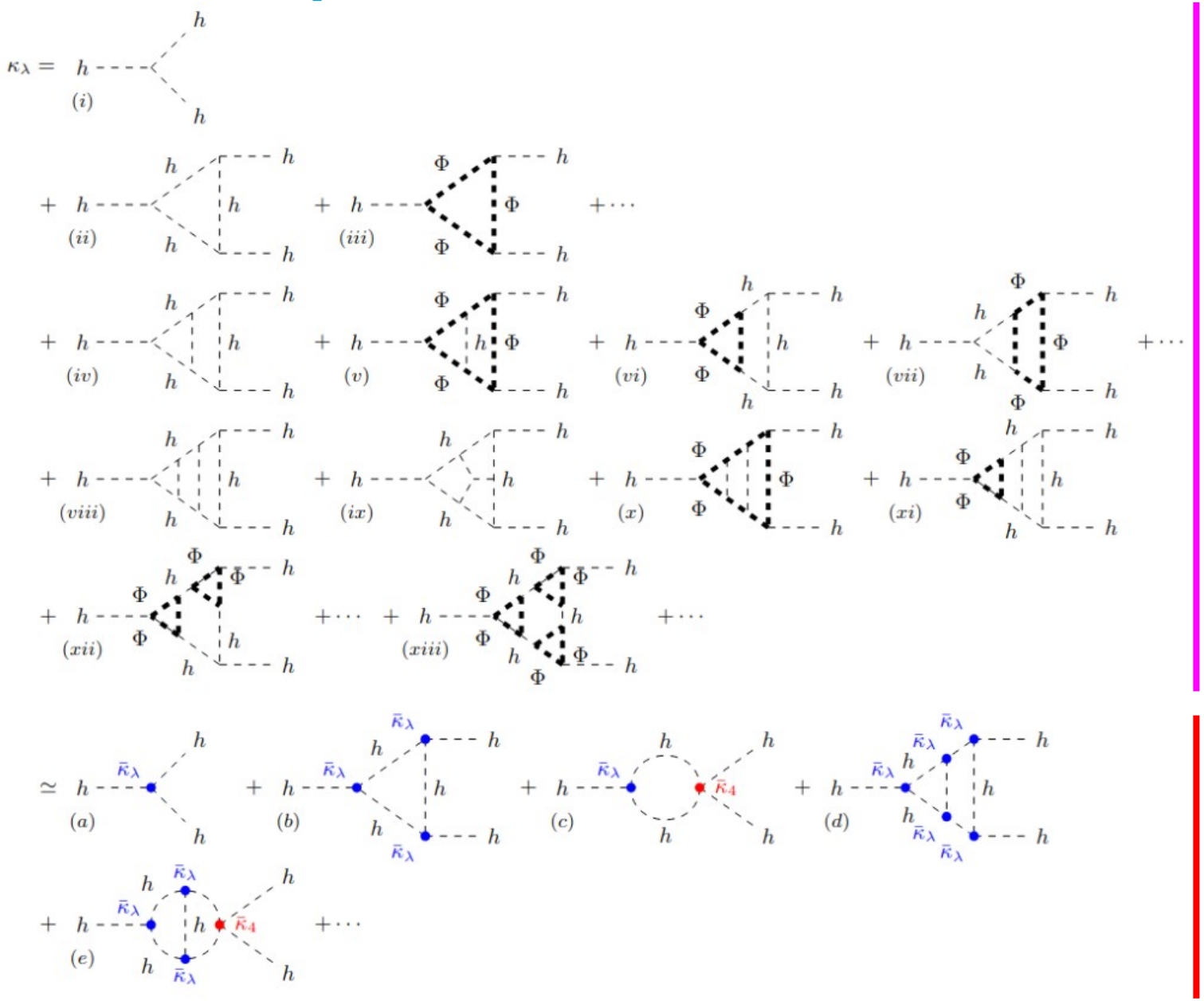
$$\mathcal{D}_3 = \frac{\partial^3}{\partial h^3} - \frac{3}{v}\left(\frac{\partial^2}{\partial h^2} - \frac{1}{v}\frac{\partial}{\partial h}\right), \quad \mathcal{D}_4 = \frac{\partial^4}{\partial h^4} - \frac{3}{v^2}\left(\frac{\partial^2}{\partial h^2} - \frac{1}{v}\frac{\partial}{\partial h}\right).$$

Result (only κ_λ)

$$\kappa_\lambda^{(1)} = \bar{\kappa}_\lambda + \frac{3M_h^2}{32\pi^2 v^2}\left[3\bar{\kappa}_\lambda^3 - \bar{\kappa}_\lambda + \bar{\kappa}_4 + (\bar{\kappa}_4 - \bar{\kappa}_\lambda)(3\bar{\kappa}_\lambda - 1)\ln\frac{M_h^2}{Q^2}\right], \quad \kappa_\lambda^{(2)} = \kappa_\lambda^{(1)} + \mathcal{O}\left(\frac{M_h^4}{(16\pi^2 v^2)^2}\right).$$

The two-loop term is included analytically (see Backup); predictions retain renormalisation-scale dependence through the $\overline{\text{MS}}$ field and effective-coupling renormalisation.

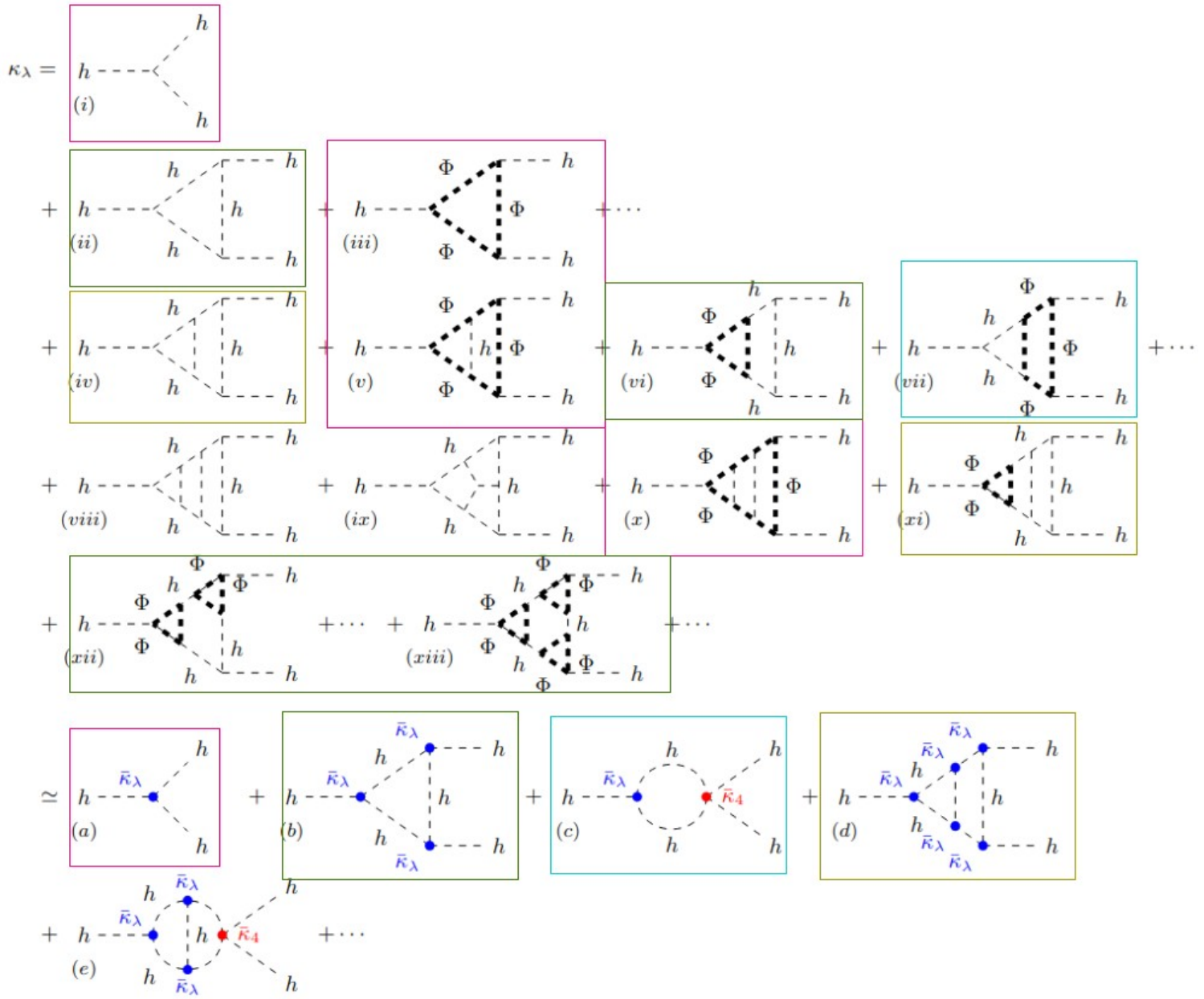
Higher-order effects captured



In some BSM model (thick lines = BSM scalar Φ)

In our effective setup

Higher-order effects captured



In some BSM model (thick lines = BSM scalar Φ)

In our effective setup

Theoretical considerations

> **Tree-level Perturbative Unitarity:** $hh \to hh$ scattering. [Di Luzio et al. '17], [Stylianou, Weiglein '23]

> **SMEFT Correlations:** *See e.g.* [Brivio et al. '19], [Aebischer et al. '25], [Dittmaier et al. '26]

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}^{d=4} + \frac{C_6}{\Lambda^2}\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^3 + \frac{C_8}{\Lambda^4}\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^4$$

$$\bar{\kappa}_\lambda = 1 + \frac{2C_6 v^4}{m_h^2\Lambda^2}\,,\ \bar{\kappa}_4 = 1 + 12\frac{C_6 v^4}{m_h^2\Lambda^2} + 8\frac{C_8 v^6}{m_h^2\Lambda^4}$$

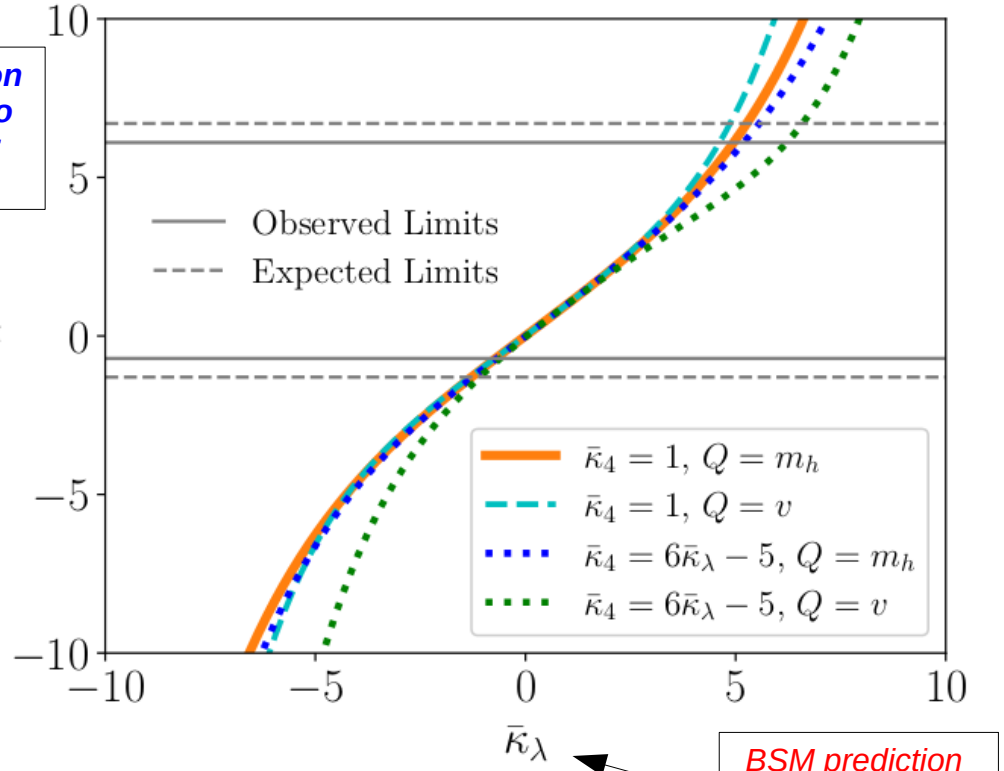
- **Dimension-6 correlation:** $\bar{\kappa}_4 - 1 = 6(\bar{\kappa}_\lambda - 1) \Rightarrow \boxed{\bar{\kappa}_4 = 6\bar{\kappa}_\lambda - 5}$
- **Convergence of SMEFT expansion** (dim-6 > dim-8)

$$\boxed{|\bar{\kappa}_4 - 1 + 6(\bar{\kappa}_\lambda - 1)| < 6|\bar{\kappa}_\lambda - 1|}$$

[Stylianou, Weiglein '23]

1D results using two-loop calculation of κ_λ

BSM prediction including also higher order effects



BSM prediction at one or two loops

Q	$\bar{\kappa}_4$	Bounds from $\kappa_\lambda^{(2)}$
m_h	1	$-0.71 < \bar{\kappa}_\lambda < 4.95$
m_h	$6\bar{\kappa}_\lambda - 5$	$-0.69 < \bar{\kappa}_\lambda < 5.20$
v	1	$-0.73 < \bar{\kappa}_\lambda < 4.62$
v	$6\bar{\kappa}_\lambda - 5$	$-0.61 < \bar{\kappa}_\lambda < 6.22$

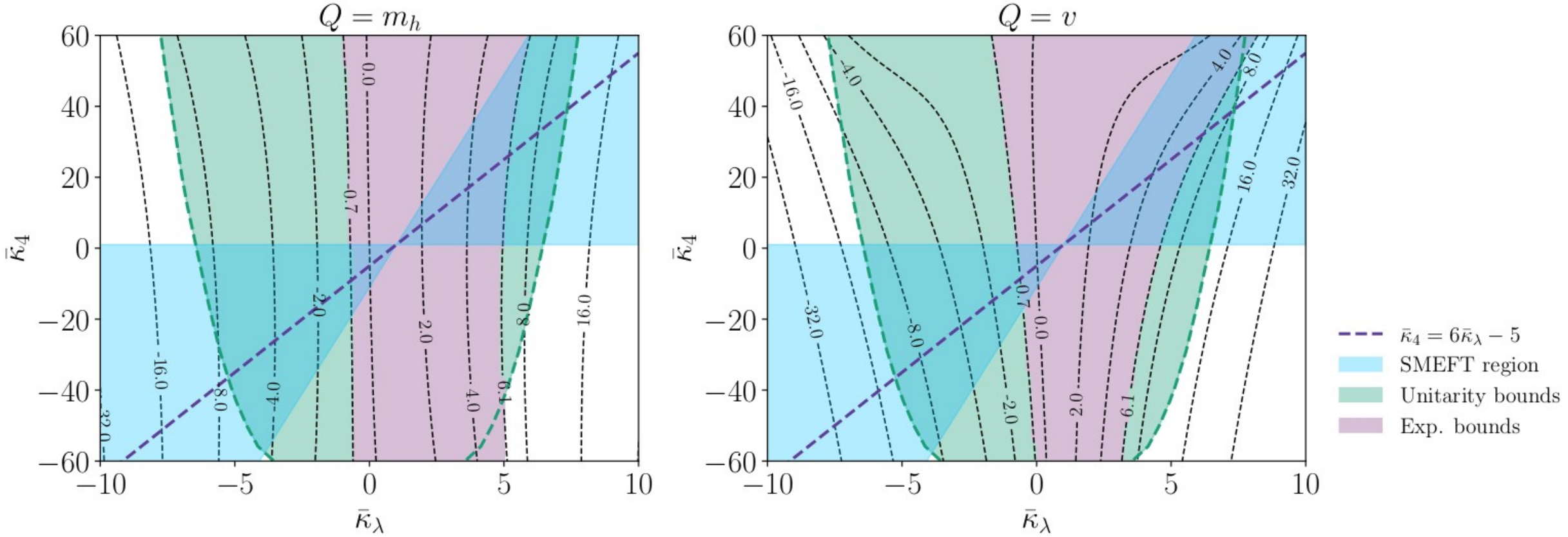
> Experimental limits: $\kappa_\lambda \in (-0.71, 6.1)$

[ATLAS+CMS Run 2 combination '26]

> Significant numerical difference in the upper bounds when including 2-loop effects.

> **Bounds dependent on renormalisation-scale choice!**

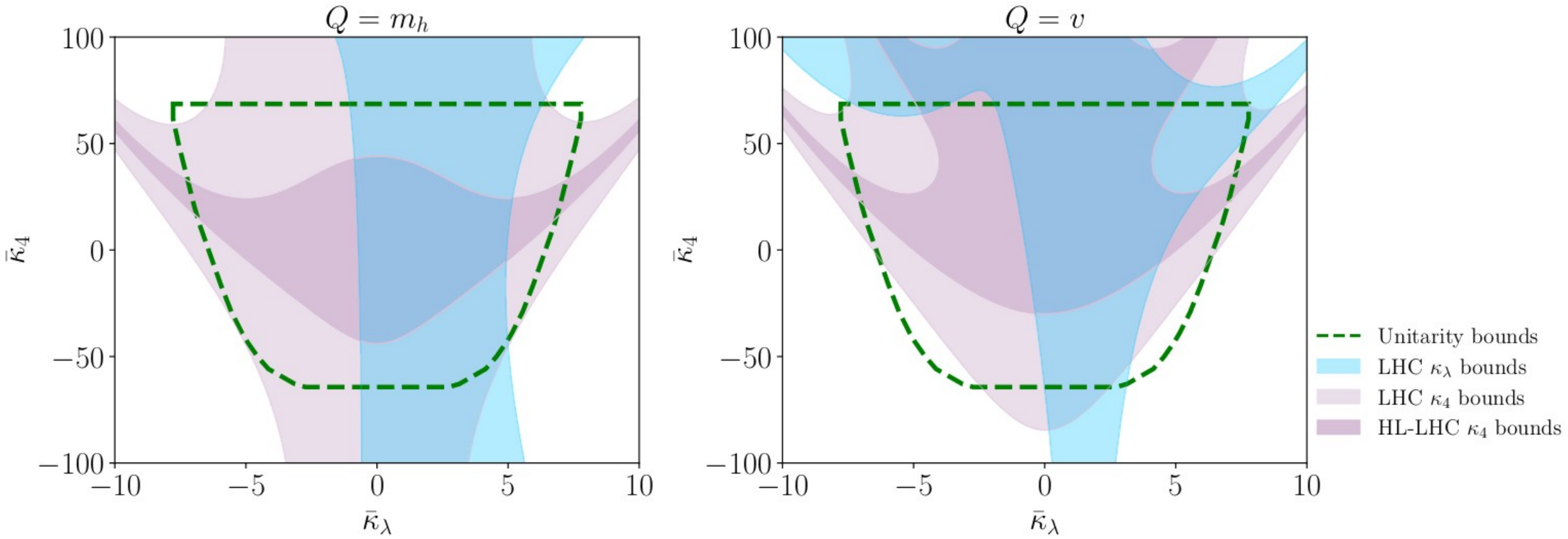
2D results using two-loop calculation of κ_λ



- **Tree-level Unitarity bounds:** $hh \to hh$ scattering.
- **SMEFT region:** $|\bar{\kappa}_4 - 1 + 6(\bar{\kappa}_\lambda - 1)| < 6|\bar{\kappa}_\lambda - 1|$

[Stylianou, Weiglein '23]

2D results using two-loop calculation of κ_4



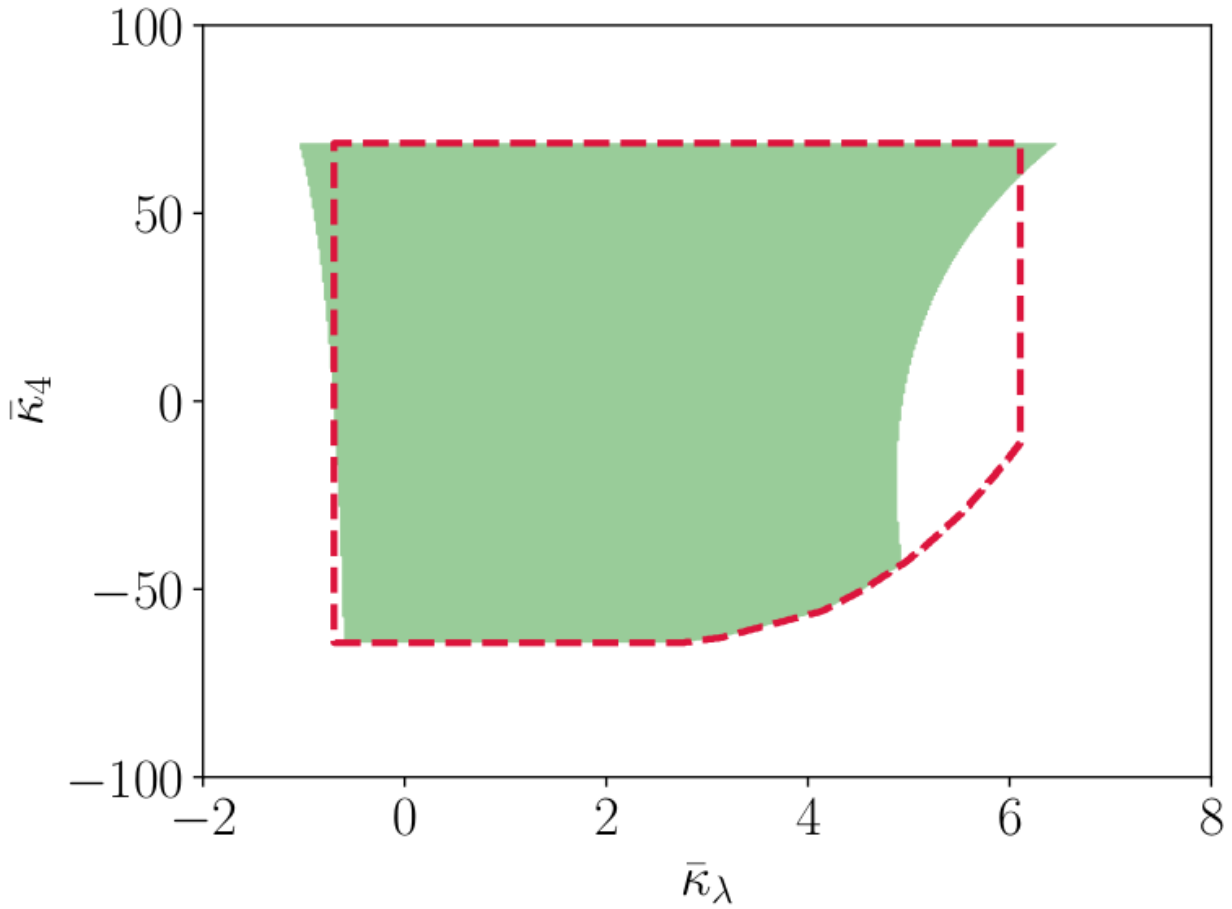
- **Tree-level Unitarity bounds:** $hh \to hh$ scattering.
- HL-LHC κ_4 bounds assume $\bar{\kappa}_4 = 1$ is realised in Nature.

[Stylianou, Weiglein '23]

Combined effect on current limits at LHC

Combining limits from searches for di- and triple-Higgs production and from perturbative unitarity

[JB, Naskar, Weiglein ‘26]

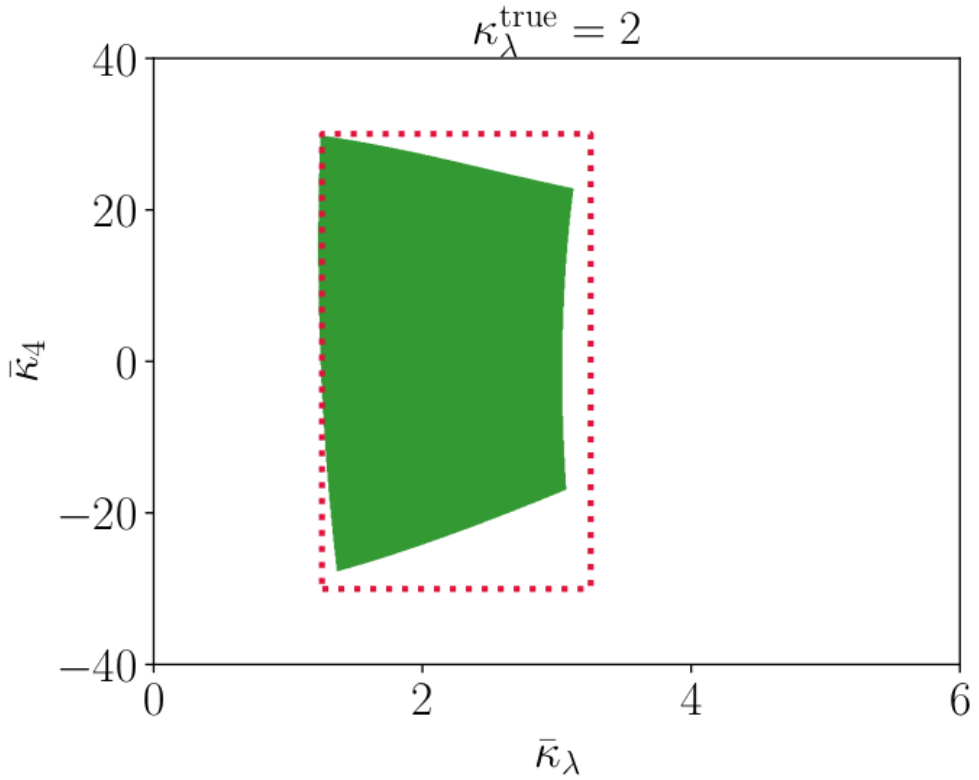


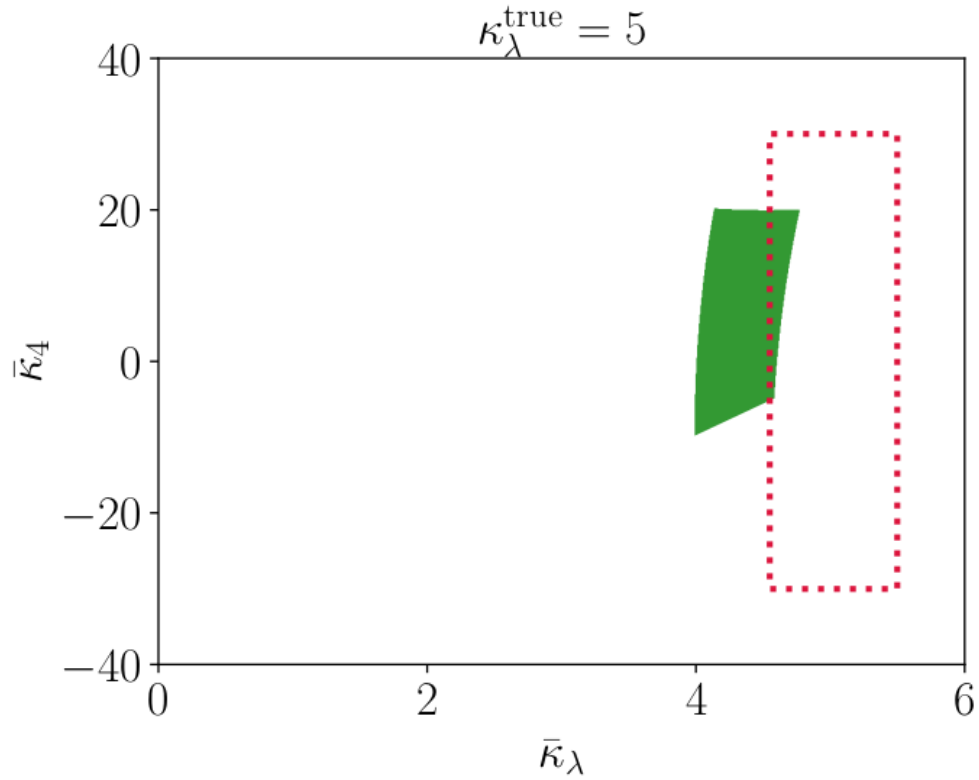
- $Q = m_h$
- **Red-dashed line:** Rectangular constraints on κ_λ, κ_4 + unitarity.
- **Green region:** Modification of allowed region including higher order effects.
- **Perturbative unitarity** still provides strongest bounds along κ_4 direction.

Bounds on κ_λ: [ATLAS+CMS Run 2 combination ‘26]
Bounds on κ_4: [CMS ‘26]
Perturbative unitarity: [Stylianou, Weiglein ‘26]

Expected bounds at HL-LHC

Combining limits from searches for di- and triple-Higgs production





Rectangular regions may (will) get smaller with improved analysis techniques, detector improvements at HL-LHC.

Prospect limits on κ_λ (for varying κ_λ) from di-Higgs searches: [ATLAS+CMS HL-LHC highlights '25]

Prospect limits on κ_λ and κ_4 (only available for κ_4=1) from triple-Higgs searches: [Stylianou, Weiglein '23] + [ATLAS '25]

Implementation in public code: `check_kappas`

`check_kappas`

Implementation can be found in: `https://gitlab.com/wrishik/check_kappas.git`

- **Input:**
  ```
  ./check_kappas.py --kl 5.0 --k4 8.0 --Q 125.09 --loop 2 --mode kl
  ```
- **Output:**
  ```
  Passed perturbative unitarity constraints!
  kappa_lambda limit check: False
  ```
- Current (future) bounds can be updated by hand in the code.
- Efficient for command-line scans over $\bar{\kappa}_\lambda$, $\bar{\kappa}_4$ values.

What should be provided as the input: n-loop predictions for κ_λ, κ_4 from your favourite model (check our paper to see EXACTLY what kind of contributions are needed for the input!).

Summary

- The trilinear and quartic Higgs self-couplings can receive **large corrections from BSM physics**, even for points allowed by state-of-the-art theoretical and experimental constraints
- In our work, we **pointed out and calculated a well-defined class of higher-order BSM contributions, which can lead to sizeable modifications of κ_λ and κ_4**
- **Reliable interpretations of double- and triple-Higgs searches must account for all numerically relevant higher-order contributions to constrain the BSM parameter space**
- The residual renormalisation-scale dependence highlights the need for a consistent assessment of **theoretical uncertainties** at fixed physical input observables, and for careful interpretation of limits from experiments in terms of underlying parameter space of the BSM models

What lies ahead?

- Moving from rectangular bounds, to including correlations between double- and triple-Higgs production in the determination of κ_λ, κ_4 → public tool `check_kappas`
- Connection with realistic models (consistently)

Thank you very much for your attention!

Contact

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Backup slides

Numerical differences in κ_λ: one- and two-loop results

$\bar{\kappa}_\lambda$	$\bar{\kappa}_4$	Q	$\kappa_\lambda^{(1)} - \bar{\kappa}_\lambda$	$\kappa_\lambda^{(2)} - \bar{\kappa}_\lambda$
1.5	1	m_h	0.018	0.019
1.5	$6\bar{\kappa}_\lambda - 5$	m_h	0.036	0.029
1.5	1	v	0.026	0.024
1.5	$6\bar{\kappa}_\lambda - 5$	v	−0.004	−0.007
2	1	m_h	0.047	0.050
2	$6\bar{\kappa}_\lambda - 5$	m_h	0.062	0.056
2	1	v	0.064	0.071
2	$6\bar{\kappa}_\lambda - 5$	v	−0.021	−0.036
5	1	m_h	0.852	1.188
5	$6\bar{\kappa}_\lambda - 5$	m_h	0.911	0.799
5	1	v	1.038	2.035
5	$6\bar{\kappa}_\lambda - 5$	v	−0.019	−0.354

> **Significant loop-effects** for larger values of $\bar{\kappa}_\lambda$, can drastically change the interpretation of the experimental upper-bounds.

Full one- and two-loop expressions for κ_λ in the OS scheme

i.e. OS scheme for m_h and t_h

$$\kappa_\lambda^{(1)} = \bar{\kappa}_\lambda + \frac{3M_h^2}{32\pi^2 v^2}\left\{3\bar{\kappa}_\lambda^3 - \bar{\kappa}_\lambda + \bar{\kappa}_4 + (\bar{\kappa}_4 - \bar{\kappa}_\lambda)(3\bar{\kappa}_\lambda - 1)\ln\frac{M_h^2}{Q^2}\right\}$$

$$\begin{aligned}\kappa_\lambda^{(2)} =& \kappa_\lambda^{(1)} + \frac{9M_h^4}{2048\pi^4 v^4}\Big\{3\bar{\kappa}_\lambda^3[1 - c_{xxx} + 4\bar{\kappa}_\lambda + 12\bar{\kappa}_\lambda^2] + \bar{\kappa}_\lambda\bar{\kappa}_4[10 - 2c_{xxx} - 15\bar{\kappa}_\lambda + 15c_{xxx}\bar{\kappa}_\lambda - 42\bar{\kappa}_\lambda^2] \\ & - 2\bar{\kappa}_4^2[5 - c_{xxx} - 6\bar{\kappa}_\lambda + 6c_{xxx}\bar{\kappa}_\lambda] + \Big[\bar{\kappa}_\lambda\bar{\kappa}_4(4 - 15\bar{\kappa}_\lambda - 18\bar{\kappa}_\lambda^2) + 3\bar{\kappa}_\lambda^3 + \bar{\kappa}_4^2(12\bar{\kappa}_\lambda - 4)\Big]\ln^2\frac{M_h^2}{Q^2} \\ & - 2\Big[\bar{\kappa}_4^2(9\bar{\kappa}_\lambda - 5) + \bar{\kappa}_\lambda\bar{\kappa}_4(5 - 12\bar{\kappa}_\lambda - 21\bar{\kappa}_\lambda^2) + 3\bar{\kappa}_\lambda^3(1 + 2\bar{\kappa}_\lambda + 6\bar{\kappa}_\lambda^2)\Big]\ln\frac{M_h^2}{Q^2}\Big\}.\end{aligned}$$

with $$c_{xxx} \equiv -\frac{i}{\sqrt{3}}\left[\frac{\pi^2}{9} - 4\,\mathrm{Li}_2\left(\frac{1 - i\sqrt{3}}{2}\right)\right] \approx 2.3439\,.$$

Full one- and two-loop expressions for κ_4 in the OS scheme

i.e. OS scheme for m_h and t_h

$$\kappa_4^{(1)} = \bar{\kappa}_4 + \frac{3M_h^2}{32\pi^2 v^2}\left\{\bar{\kappa}_4 - \bar{\kappa}_\lambda + 18\bar{\kappa}_4\bar{\kappa}_\lambda^2 - 9\bar{\kappa}_\lambda^4 + \left[3\bar{\kappa}_4^2 + \bar{\kappa}_\lambda - \bar{\kappa}_4 - 3\bar{\kappa}_\lambda^2\right]\ln\frac{M_h^2}{Q^2}\right\}$$

$$\begin{aligned}
\kappa_4^{(2)} = \kappa_4^{(1)} + \frac{9M_h^4}{2048\pi^4 v^4}\Big\{ & 12(1-c_{xxx})\bar{\kappa}_4^3 - 2\bar{\kappa}_4^2\left(5 - c_{xxx} + 3\bar{\kappa}_\lambda + 81\bar{\kappa}_\lambda^2\right) + 3\bar{\kappa}_\lambda^3\left(1 - c_{xxx} + 6\bar{\kappa}_\lambda + 6\bar{\kappa}_\lambda^2 - 90\bar{\kappa}_\lambda^3\right) \\
& + \bar{\kappa}_\lambda\bar{\kappa}_4\left[10 - 9\bar{\kappa}_\lambda - 36\bar{\kappa}_\lambda^2 + 504\bar{\kappa}_\lambda^3 + c_{xxx}(15\bar{\kappa}_\lambda - 2)\right] \\
& + \left[12\bar{\kappa}_4^3 - 4\bar{\kappa}_4^2 + \bar{\kappa}_\lambda\bar{\kappa}_4(4 - 21\bar{\kappa}_\lambda) + 3\bar{\kappa}_\lambda^3 - 2\bar{\kappa}_4^2(2 - 3\bar{\kappa}_\lambda + 9\bar{\kappa}_\lambda^2)\right]\ln^2\frac{M_h^2}{Q^2} \\
& - 2\left[9\bar{\kappa}_4^3 - \bar{\kappa}_4^2(5 + 126\bar{\kappa}_\lambda^2) + 3\bar{\kappa}_\lambda^3(1 + 3\bar{\kappa}_\lambda + 3\bar{\kappa}_\lambda^2 - 27\bar{\kappa}_\lambda^3) + \bar{\kappa}_4\bar{\kappa}_\lambda(5 - 12\bar{\kappa}_\lambda - 18\bar{\kappa}_\lambda^2 + 189\bar{\kappa}_\lambda^3)\right]\ln\frac{M_h^2}{Q^2}\Big\}\,.
\end{aligned}$$

with $$c_{xxx} \equiv -\frac{i}{\sqrt{3}}\left[\frac{\pi^2}{9} - 4\,\mathrm{Li}_2\left(\frac{1-i\sqrt{3}}{2}\right)\right] \approx 2.3439\,.$$

Expressions for κ_λ and κ_4 in the $\overline{\text{MS}}$ scheme

i.e. $\overline{\text{MS}}$ scheme for m_h and t_h

$$\kappa_\lambda|^{\overline{\text{MS}}} = \left(\frac{3m_h^2}{v}\right)^{-1} \left.\frac{\partial^3 V_{\text{eff}}}{\partial h^3}\right|_{h=0}, \quad \kappa_4|^{\overline{\text{MS}}} = \left(\frac{3m_h^2}{v^2}\right)^{-1} \left.\frac{\partial^4 V_{\text{eff}}}{\partial h^4}\right|_{h=0}.$$

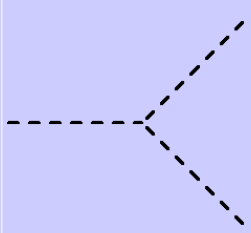
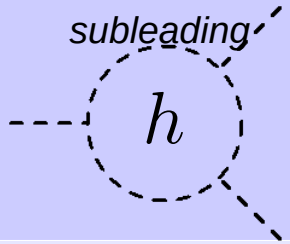
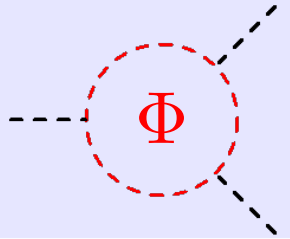
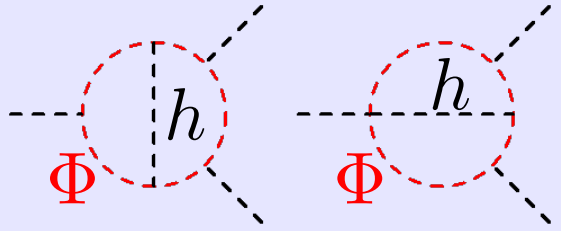
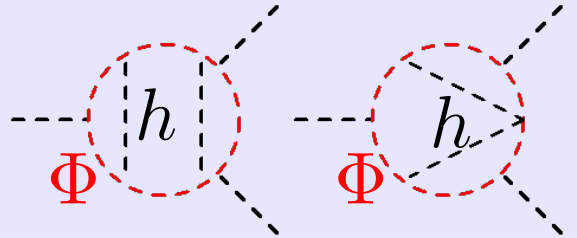
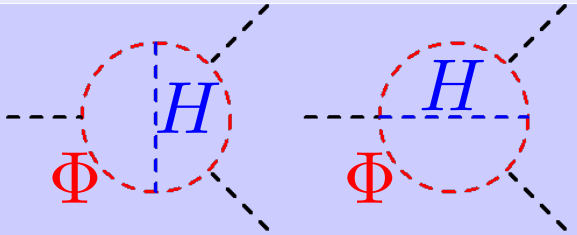
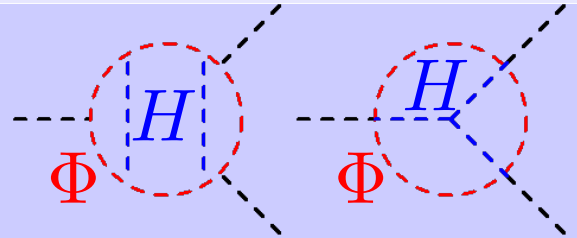
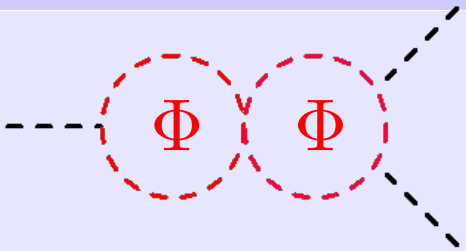
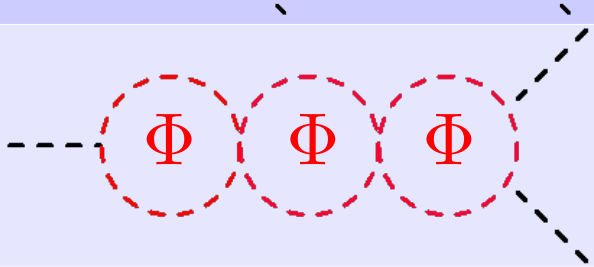
$$\kappa_\lambda^{(1)}|^{\overline{\text{MS}}} = \bar{\kappa}_\lambda + \frac{9m_h^2}{32\pi^2 v^2}\bar{\kappa}_\lambda\left(\bar{\kappa}_\lambda^2 + \bar{\kappa}_4 \ln\frac{m_h^2}{Q^2}\right), \quad \kappa_4^{(1)}|^{\overline{\text{MS}}} = \bar{\kappa}_4 + \frac{9m_h^2}{32\pi^2 v^2}\left(6\bar{\kappa}_4\bar{\kappa}_\lambda^2 - 3\bar{\kappa}_\lambda^4 + \bar{\kappa}_4^2 \ln\frac{m_h^2}{Q^2}\right).$$

$$\kappa_\lambda^{(2)}|^{\overline{\text{MS}}} = \kappa_\lambda^{(1)}|^{\overline{\text{MS}}} + \frac{27m_h^4}{1024\pi^4 v^4}\bar{\kappa}_\lambda\left\{6\bar{\kappa}_\lambda^4 - 8\bar{\kappa}_\lambda^2\bar{\kappa}_4 + (1-2c_{xxx})\bar{\kappa}_4^2 - [3\bar{\kappa}_\lambda^4 - 11\bar{\kappa}_\lambda^2\bar{\kappa}_4 + 3\bar{\kappa}_4^2]\ln\frac{m_h^2}{Q^2} + 3\bar{\kappa}_4^2\ln^2\frac{m_h^2}{Q^2}\right\}$$

$$\kappa_4^{(2)}|^{\overline{\text{MS}}} = \kappa_4^{(1)}|^{\overline{\text{MS}}} + \frac{27m_h^4}{1024\pi^4 v^4}\left\{(1-2c_{xxx})\bar{\kappa}_4^3 - 33\bar{\kappa}_4^2\bar{\kappa}_\lambda^2 + 87\bar{\kappa}_4\bar{\kappa}_\lambda^4 - 45\bar{\kappa}_\lambda^6 - 3(\bar{\kappa}_4^3 - 17\bar{\kappa}_4^2\bar{\kappa}_\lambda^2 + 16\bar{\kappa}_4\bar{\kappa}_\lambda^4 - 6\bar{\kappa}_\lambda^6)\ln\frac{m_h^2}{Q^2} + 3\bar{\kappa}_4^3\ln^2\frac{m_h^2}{Q^2}\right\}$$

Examples of scalar contributions to λ_{hhh} in aligned 2HDM

BSM scalars:
$\Phi \in \{H, A, H^\pm\}$
$m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$

Coupling/Order	0L	1L	2L	3L
g_{hhhh}		*subleading* 	*subleading*	*subleading*
$g_{(h)h\Phi\Phi}$ $\left[g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2}\right]$	-			
$g_{(h)H\Phi\Phi'}$ [$g_{(h)G\Phi\Phi'}$ case similar]	-	-		
$g_{\Phi\Phi\Phi'\Phi'}$ [2 BSM scalars of species Φ, 2 of species Φ']	-	-		

[NB: 1 h can be replaced by a VEV]

→ *no further type of coupling entering after 2L*

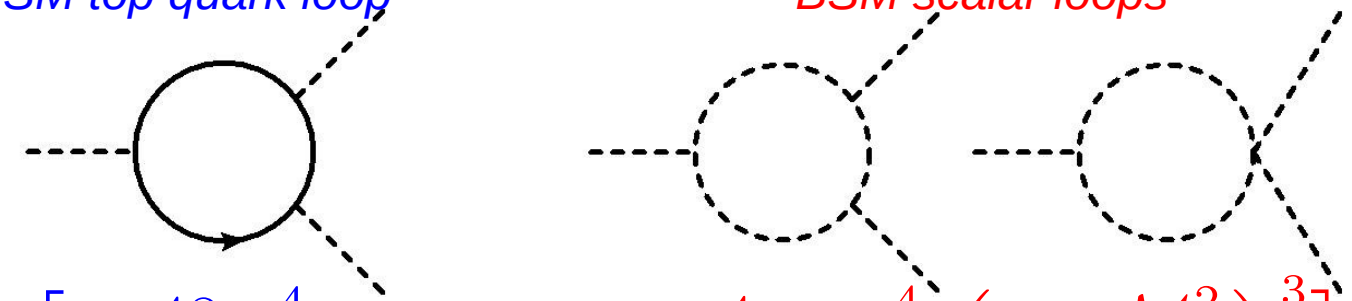
→ *for each class of diagrams, perturbative convergence can be verified!*

e.g. in [Bahl, JB, Weiglein PRL '22]

One-loop mass-splitting effects

- **Leading one-loop** corrections to λ_{hhh} in models with extended sectors (like 2HDM):

SM top quark loop *BSM scalar loops*



$$\delta^{(1)}\lambda_{hhh} \supset \frac{1}{16\pi^2}\left[-\frac{48m_t^4}{v^3} + \sum_{\Phi}\frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3\right]$$

First found in 2HDM: [Kanemura, Kiyoura, Okada, Senaha, Yuan '02]

$\mathcal{M}$: **BSM mass scale**, e.g. soft breaking scale M of Z_2 symmetry in 2HDM
n_Φ : # of d.o.f of field Φ

- Size of new effects depends on how the BSM scalars acquire their mass: $m_\Phi^2 \sim \mathcal{M}^2 + \tilde{\lambda}v^2$

$$\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3 \longrightarrow \begin{cases} 0, \text{ for } \mathcal{M}^2 \gg \tilde{\lambda}v^2 \\ 1, \text{ for } \mathcal{M}^2 \ll \tilde{\lambda}v^2 \end{cases} \longrightarrow \textbf{Huge BSM effects possible!}$$