ILC, FCC-ee precision polarimetry with event fits

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Motivation: A_e at 10^{-4} needs the longitudinal polarization known to 10^{-4}

Measurement	Requirement on the polarization	Source
A_e at the Z pole, $\delta A_e/A_e = 10^{-4}$ (ILC 80/30)	$\delta P_-/P_- = 0.01\%$, $\delta P_+/P_+ = 0.09\%$	Madison [1], Table 3
σ_{ZH} at 250 GeV, 0.1% from P (ILC 80/30)	$\delta P_-/P_- = 0.15\%$, $\delta P_+/P_+ = 0.24\%$	Madison [1], Table 2
$\gamma\gamma$ luminosity at Z pole to 1 TeV, $\delta\mathcal{L}/\mathcal{L} = 10^{-4}$	$\delta P/\lvert P\rvert \approx 0.08\%$	Madison [1]
GigaZ, $\delta \sin^2\theta_{\text{eff}} = 1.3 \times 10^{-5}$ (Blondel scheme, 80/60)	$\delta P/P < 0.1\%$	Moortgat-Pick et al. [2]
BSM contact interactions (500 GeV, $\Lambda \approx 7 - 10$ TeV)	δP_T to set limits	Moortgat-Pick et al. [2]
SLD A_{LR} (achieved)	1.4% (A_{LR}), 0.5% polarimeter	SLD [3]

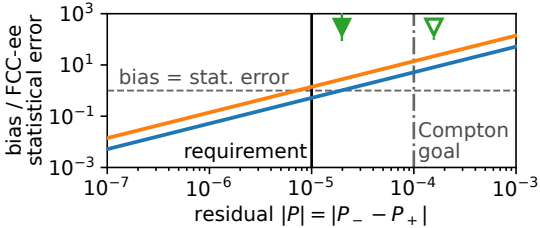
Our target: 10^{-4} relative on the luminosity weighted polarization.

FCC-ee: unpolarized collisions still need polarimetry

- **Colliding bunches can polarize by themselves:** the residual $P = P_- - P_+$ must be known to 10^{-5}.
- P acts like a shift of the initial state A_e: $\sin^2\theta_{\text{eff}}$ moves by 0.06 to 0.12 P (statistical target 1.2×10^{-6}).
- Z-pole $\mu\mu$ alone cannot separate P from $\sin^2\theta_{\text{eff}}$; with wide-angle Bhabha, event fits reach about 1.6×10^{-4} on the colliding bunches.
- So FCC-ee needs a polarimeter on the colliding bunches (10^{-4} systematic goal); event fits give an in situ cross-check.

Planned: a FeeBL [16] study of the polarization that FCC-ee colliding bunches, and outgoing τ leptons, pick up in the beam fields (timeline to be determined).

Bias on Z-pole measurements from a residual P



A_e from the τ polarization forward backward asymmetry

$\sin^2\theta_{\text{eff}}$ from $A_{FB}(\mu\mu)$

$\mu\mu$, couplings fixed: 2.0×10^{-5}

$\mu\mu$ + Bhabha, $\sin^2\theta_{\text{eff}}$ free: 1.6×10^{-4}

Born, pure Z, first order in P.
Event fits: Tera-Z, Asimov, statistics only.

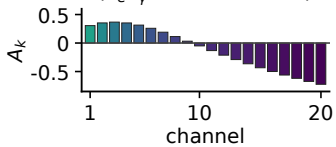
Our calculation: shift of $\sin^2\theta_{\text{eff}}$ (from $A_{FB}(\mu\mu)$) and of A_e (from τ polarization) caused by a residual $P_- - P_+$ of the colliding beams, in units of the FCC-ee statistical precision.

FCC FSR Vol. 1 [5], Tables 2 and 24; Vol. 2 [9], Secs. 1.7.4 and 1.7.5; EPOL support note [6].

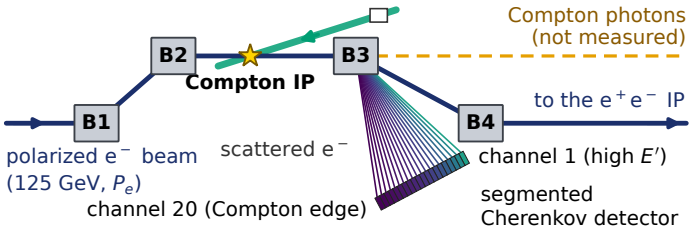
Existing methods: Compton polarimeter overview

B1 to B4: chicane dipoles; B3 bends the Compton electrons onto the detector

asymmetry per channel (P_eP_γ = 0.8 assumed)



laser 532 nm, circularly polarized, counter propagating; Pockels cell (white box) flips $\lambda = \pm 1$



Simplified upstream style sketch, not to scale; 20 channels, as in the ILC baseline gas Cherenkov detector (Beckmann et al. 2014)

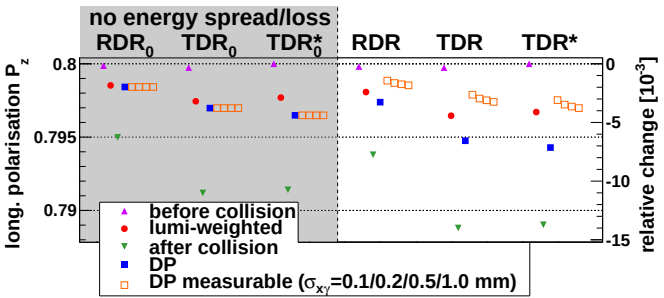
- Circularly polarized laser light scatters off the beam; a magnet spreads the scattered electrons by energy over channels k.
- Asymmetry per channel, laser helicity $\pm$:

$$A_k = \frac{N_k^+ - N_k^-}{N_k^+ + N_k^-} = P_e P_\gamma \, a_k$$

- a_k is known from QED and changes sign across the spectrum: each channel gives an estimate, with shared calibration uncertainties, and the pattern checks the calibration.

Inset: LO QED analyzing powers, 125 GeV beam, 532 nm laser, 20 channels of equal width in the linear dispersion coordinate. ILC: polarimeters upstream and downstream of the IP. ▸ Analyzing power ▸ Compton bridge

Polarization along the beam delivery system (Beckmann et al. 2014)



- Electron polarization before and after collision, luminosity weighted, and at the downstream polarimeter (DP), also as the laser measures it (open squares: DP measurable for laser spot sizes $\sigma_x = 0.1$, 0.2, 0.5 and 1.0 mm).
- The DP misses the luminosity weighted value by 0.07% (RDR) to 0.17 to 0.24% (TDR).

Beckmann et al. [4], Fig. 11(a); panel (b), the magnitude $|\vec{P}|$, shows the same pattern.

▸ Upstream, IP, downstream

Depolarization inside the bunch crossing (FeeBL beam viewer, ILC 250)

Interval L (integrated)	5.13557e+31 m⁻² / crossing
Fraction of total L	0.00445
L below / above W bins	1.279e+29 / 0.000
Beam 1 ⟨E⟩L	124.943699 GeV
Beam 2 ⟨E⟩L	124.928176 GeV
Configured P_1 / P_2 [1]	0.800000 / 0.300000
Bunch P_1 / P_2 [1], snapshot 15	0.799971 / 0.299989
Interval ⟨P_1⟩L / ⟨P_2⟩L [1]	0.799868 / 0.299944
Polarization L weight	5.13557e+31 m⁻² / crossing
Peak selection	W ≥ 244.99400 GeV (98% design Wnom)
Peak L weight	5.12065e+31 m⁻² / crossing

step 15 of 62

Interval L (integrated)	3.91403e+31 m⁻² / crossing
Fraction of total L	0.00339
L below / above W bins	1.071e+31 / 0.000
Beam 1 ⟨E⟩L	121.246165 GeV
Beam 2 ⟨E⟩L	121.329217 GeV
Configured P_1 / P_2 [1]	0.800000 / 0.300000
Bunch P_1 / P_2 [1], snapshot 45	0.790861 / 0.296594
Interval ⟨P_1⟩L / ⟨P_2⟩L [1]	0.787259 / 0.295184
Polarization L weight	3.91403e+31 m⁻² / crossing
Peak selection	W ≥ 244.99400 GeV (98% design Wnom)
Peak L weight	1.95915e+31 m⁻² / crossing

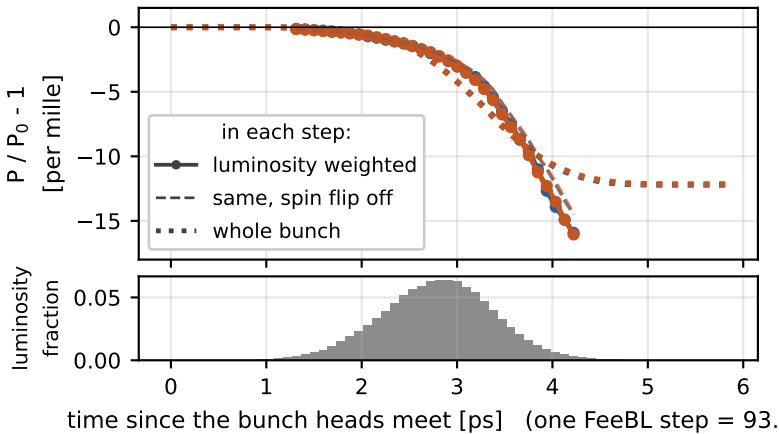
step 45 of 62

▸ Slice helicity

FeeBL's beam viewer [16], ILC 250; $P/P_0 - 1$ in per mille:

	lumi. weighted		whole bunch	
Step	e^-	e^+	e^-	e^+
15	-0.16	-0.19	-0.04	-0.04
31	-2.62	-2.56	-3.61	-3.62
45	-15.93	-16.05	-11.42	-11.35

Helicity during the bunch crossing: luminosity weighted against whole bunch

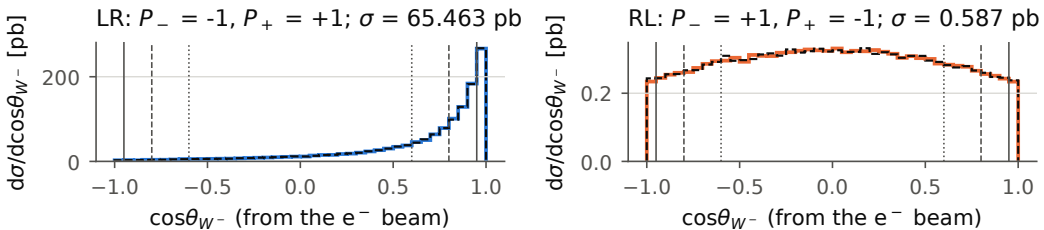


Whole crossing (e^- / e^+)
luminosity weighted:
-3.2 / -3.3 per mille
whole bunch after:
-12.2 / -12.2 per mille

FeeBL [16], ILC 250, $P_0 = 0.8$ (e^-), 0.3 (e^+). Solid: luminosity weighted helicity of the collisions in each step; dotted: whole bunch. Bottom: luminosity per step. ▸ CAIN, GUINEA-PIG++ ▸ Channel P

How event fits reconstruct the beam polarization

W^- polar angle spectra, opposite helicity corners (LL, RR: $\propto m_e^2$, negligible)



WHIZARD 3.1.8 LO, W^+W^- on shell, $\sqrt{s}$ = 250 GeV, G_μ scheme; no systematics.

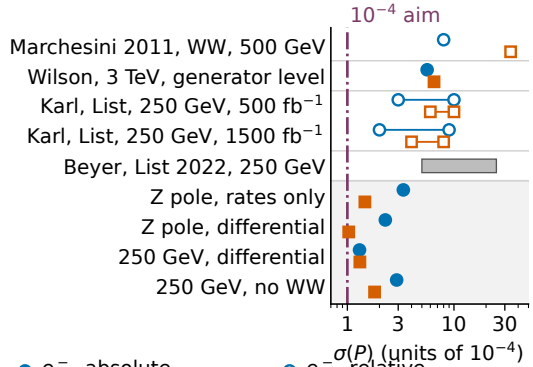
Each helicity corner h has its own shape $f_h(z)$ (LR peaks forward, RL is much less forward peaked). Each setting sees the mixture $dN/dz = \mathcal{L}\sum_h w_h\,\sigma_h f_h(z)$, $w_h = (1 + h_-P_- + h_+P_+ + h_-h_+C)/4$; fitting it to the whole sample gives P_-, P_+ and C (rates alone: Blondel [10]). ▸ Estimands ▸ QED rank ▸ All corners ▸ Corner gates

Existing methods: event fits reach a few 10^{-4} to 10^{-3}

Study	$\sigma(P_-)$, $\sigma(P_+)$
Marchesini [11]	0.08%, 0.34% rel. (WW, full simulation, 500 GeV, 80/30, 50% same sign)
Wilson [12]	5.6, 6.5×10^{-4} abs. (single boson channels only: single γ, Z and W; CLIC 3 TeV; generator level, WHIZARD)
Karl, List [13]	0.1% (500 fb^{-1}); 0.09%, or 0.07% with a 0.25% polarimeter constraint (1.5 ab^{-1}); rel.
Beyer, List [14]	0.05 to 0.25% ($\mu\mu$, $\mu\nu jj$)
Madison [15]	1.30, 1.31×10^{-4} abs.

Statistical only.

Best statistical projections: a few 10^{-4} to 10^{-3}; none includes full systematics.



▸ Sources ▸ Claims ▸ WW with ISR ▸ Differential fits ▸ Fitted $|P_\pm|$ ▸ 50 mrad smearing

Existing limits: Compton budgets are a few 10^{-3}; EW fits tie δP to physics

Source of uncertainty	$\delta\mathcal{P}_z/\mathcal{P}_z$
Detector analysing power	$0.15 - 0.2\,\%$
Detector linearity	$0.1\,\%$
Laser polarisation	$0.1\,\%$
Electronic noise and beam jitter	$0.05\,\%$
Total	$0.25\,\%$

Spin transport, 250 GeV, no collisions	10^{-3}
Beam and polarization alignment	0.72
Random misalignments, orbit corrected	0.35
Beam parameters (10% emittance)	0.03
Precession in detector magnets	0.01
Bunch rotation (crossing angle)	< 0.01
Synchrotron radiation	0.005
Total	0.80

Beckmann et al. [4], Tables 6 and 10.

Compton polarimeters

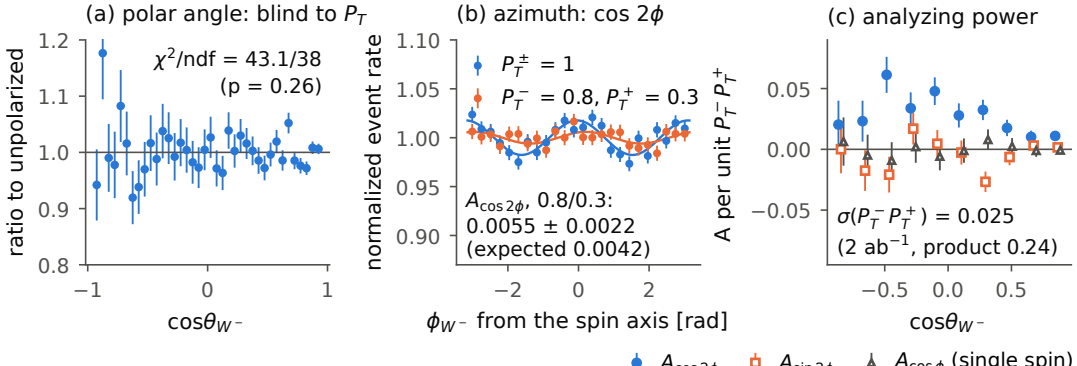
0.25% per polarimeter (design), 0.5% at SLD. With beamstrahlung the downstream value misses the luminosity weighted P by 0.07% (RDR) to 0.17 to 0.24% (TDR): "for percent level, but not for per mille level precision".

Vector boson event fits

"The same processes as used for physics analyses" (Karl and List [13]): they rest on SM chiral structure; NLO EW changes the polarization dependence by 10^{-3} to 10^{-2} (Madison [1]). Single bosons and boson pairs are themselves precision SM and BSM channels with hard theory, so we do not use them.

W pairs with both beams transversely polarized along the same axis

Transverse polarization in W^+W^-: both beams transversely polarized along the same axis



WHIZARD 3.1.8 LO, W^+W^- on shell, $\sqrt{s}$ = 250 GeV, G_μ scheme; no systematics.

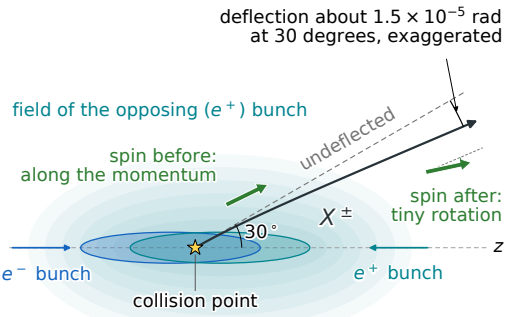
WW cannot give a useful transverse polarization measurement: only $P_T^- P_T^+$ enters, and only to ± 0.025.

Statistical; ILC 250, 2 ab^{-1}, $P_T^- = 0.8$, $P_T^+ = 0.3$. Single beam terms need a helicity flip, $O(m_e/\sqrt{s})$; Conditional projection; analyzing power MC uncertainty not propagated. ▸ Transverse kernels

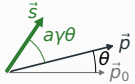
Outgoing leptons: depolarization during deflection by the opposing bunch

Outgoing wide-angle leptons: deflection and spin in the beam field

Top view; schematic, not to scale; angles exaggerated



Thomas-BMT precession in the beam field



momentum turns by θ,
spin by $(1 + a\gamma)\,\theta$;
helicity loss
$1 - h = (a\gamma\theta)^2/2$;
$a \approx \alpha/(2\pi)$ for both e and μ;
at equal energy $\gamma_\mu = \gamma_e/207$

helicity loss about 9×10^{-6} (electron), 2×10^{-10} (muon) at 30 degrees; rigid round bunch estimate

Numbers: section 4.5 estimate (ILC 250, 125 GeV leptons)

Spin: the fits sum over final state spins, so the spin turn alone changes no counted rate (helicity loss about 9×10^{-6} for electrons, 2×10^{-10} for muons). Deflection: $\Delta\theta \approx 1.5 \times 10^{-5}$ rad at 30° (FeeBL [16] scaled estimate), comparable to the tracker polar angle resolution (fast det.: 1.4×10^{-5} rad, 125 GeV muons); its effect on acceptance and fit shapes is not yet propagated through the fit. ▸ Curves ▸ Spin transport

The question: which event types have Z-pole statistics and simple theory?

Requirements: high statistics also at the Z pole; theory dominated by QED with known weak terms; spin dependence in the shape; inside an ILD-like acceptance.

Event type	Z pole	250 GeV	Theory	Role
$\mu^+\mu^-$ (2-body)	2.05 nb	1.73 pb	simple	fitted here
e^+e^- (Bhabha)	6.14 nb	539 pb	t channel	fitted here
$\gamma\gamma$	60.9 pb	8.11 pb	pure QED	fitted here
$\mu^+\mu^-\gamma$	48.5 pb	480 fb	simple	fitted here
$\mu^+\mu^-\gamma$ (LumiCal)	788 fb	111 fb	as above	added here (GLIP LumiCal photon)
$e^+e^-\gamma$	75.6 pb	6.01 pb	moderate	fitted here
$e^+e^-\gamma$ (LumiCal)	17.3 pb	1.95 pb	as above	added here (GLIP LumiCal photon)
$\gamma\gamma\gamma$	1.34 pb	349 fb	pure QED	fitted here
$\mu^+\mu^-\mu^+\mu^-$	50.5 fb	2.27 fb	moderate	fitted here
$e^+e^-\mu^+\mu^-$	128 fb	13.2 fb	involved	fitted here
W^+W^-	below threshold	16.5 pb	intensive	used in previous studies (Marchesini 2011; Karl and List 2017; Beyer and List 2022)
ZZ	below threshold	1.1 pb	intensive	used in previous studies (single and four fermion channels, Karl and List 2017)
$q\bar{q}$	41.1 nb	11.1 pb	QCD	not studied here (hadronization)
$\tau^+\tau^-$	2.05 nb	1.73 pb	tau decays	not studied here (tau decays)
$\nu\bar{\nu}\gamma$	785 fb	3.26 pb	W exchange	not studied here (one visible photon)
ZH	below threshold	0.2 to 0.3 pb	signal	not studied here (signal channel)
e^+e^-X (two photon)	order nb	order nb	soft QCD	not studied here (forward)

Unpolarized LO cross sections in each selection; LumiCal rows: $E_\gamma > 0.1\,E_{\mathrm{beam}}$; W^+W^- and ZZ on shell.

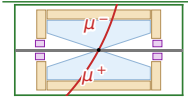
Quick LO integrals in these selections (on shell G_μ); the fits use other selections. All fit inputs use the effective leptonic mixing angle. ▸ Types ▸ QED null ▸ Four leptons

Example event types: used, added (GLIP) and not used

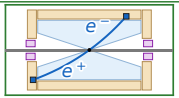
Event topologies: used, added and not used

GLIP

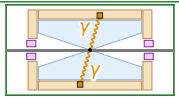
Used



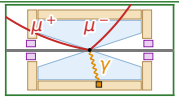
(1) $\mu^+\mu^-$
2-body, back to back



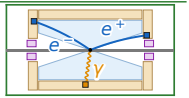
(2) wide-angle
Bhabha e^+e^-



(3) $\gamma\gamma$

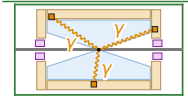


(4) $\mu\mu\gamma$,
photon in ECAL

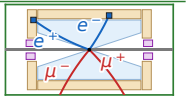


(5) $ee\gamma$,
photon in ECAL

Used (continued)

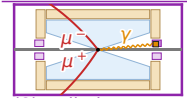


(6) $\gamma\gamma\gamma$

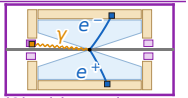


(7) four leptons:
$ee\mu\mu$ (drawn), $\mu\mu\mu\mu$

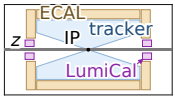
added: GLIP LumiCal photon, $E_\gamma > 0.1\,E_{\text{beam}}$



(8) radiative return
$\mu\mu\gamma$, γ in LumiCal

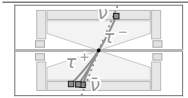


(9) wide-angle $ee\gamma$,
γ in LumiCal

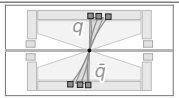


ILD-like side view,
tracker $|\cos\theta| < 0.95$

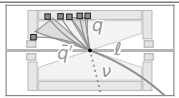
Not used



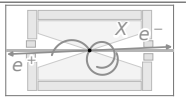
(10) $\tau^+\tau^-$



(11) $q\bar{q}$, two jets



(12) $W^+W^- \to q\bar{q}'\ell\nu$
not for calibration



(13) two photon
$e^+e^- \to e^+e^-X$
forward, mostly lost

Line styles

- $\mu^\pm$ track
- $e^\pm$ track
- photon
- jet
- ν, missing
- calorimeter hit

Original schematic, not to scale;
not an event display from simulation.

Kinematic variables: calculation form and motivation

Key variables per channel (L longitudinal, T transverse)

Channel	Definition	Motivation	L/T
all channels	$N_s = \mathcal{L}_s\,\epsilon \sum_h w_h \sigma_h$, $w_h = (1 + h_- P_- + h_+ P_+ + h_- h_+ C)/4$	mixture likelihood	L
2-body $\mu\mu$	$\cos\theta$ of the μ^- to the e^- beam; $d\sigma/d\cos\theta \propto (1 + c^2) + 2Ac$	Production angle	L
3-body $\ell\ell\gamma$	$x_\gamma = 2E_\gamma/\sqrt{Q^2}$ in the Q rest frame	Photon energy fraction	L
3-body $\ell\ell\gamma$	$m^2_{\ell\ell}/Q^2 = (p_{\ell-} + p_{\ell+})^2/Q^2$	Pair mass fraction	L
3-body $\ell\ell\gamma$	$\cos\theta^*$ of the ℓ^- in the pair rest frame to a Collins-Soper-like axis, $\hat{z} \propto \hat{p}_{e^-} - \hat{p}_{e^+}$ (boosted)	Lepton angle in the pair rest frame against a Collins-Soper-like axis	L
$\gamma\gamma\gamma$	$\hat{b} \cdot (\hat{k}_1 \times \hat{k}_2)$, photons ordered in energy, Q rest frame (naive T odd)	Orientation of the three photon plane (P odd, naive T odd)	no
transverse	$\cos(2\phi - \phi_- - \phi_+)$; ϕ: azimuth of the μ^- (or forward photon) about the e^- beam	Double transverse spin correlation	T
4-body $ee\mu\mu$	$\cos\theta_1$ of the negative lepton of pair 1 against $-\vec{q}_2$, pair 1 rest frame	Decay angle in pair 1 frame	L
4-body $ee\mu\mu$	$\epsilon(p_1^-, p_1^+, p_2^-, p_2^+)/(Q^2)^2$: covariant triple product (naive T odd)	Covariant triple product (naive T odd)	L

Q: sum of all final state four vectors; $q_{\ell\ell} = p_{\ell-} + p_{\ell+}$. Photon energy fractions use the Q rest frame, the lepton angle the $q_{\ell\ell}$ rest frame; $\hat{b}$: the e^- beam direction. Polar angles and charge separate A_{LR} from A_{FB}; azimuthal harmonics carry transverse spin.

▸ Notation ▸ All variables

Resolving power: the shortlist, generator against detector level

Four metric panels; compare only within a panel

Channel	Variables	Par.	Generator	Detector (fast det.)	det/gen
A. Shape Fisher information per selected event, ILC 250 $(-0.8, +0.3, -0.24)$, single features (largest per channel and parameter)					
$\mu\mu\gamma$	x_γ, $m^2_{\ell\ell}/Q^2$, $\cos\theta^*$ (Collins-Soper-like)	P_-	0.00244	0.00246	1.011
$\mu\mu\gamma$	x_γ, $m^2_{\ell\ell}/Q^2$, $\cos\theta^*$ (Collins-Soper-like)	C	0.00192	0.00194	1.011
$ee\gamma$	x_γ, $m^2_{\ell\ell}/Q^2$	P_-	0.000779	0.000767	0.9836
$ee\gamma$	x_γ, $m^2_{\ell\ell}/Q^2$, $\cos\theta^*$ (Collins-Soper-like)	C	0.0105	0.0105	0.9983
$\mu\mu\mu\mu$	charge weighted $\cos\theta$, angle set, masses and charge	Δ	0.00581 ± 0.0002	to do	
$ee\mu\mu$	m_1, m_2, $(m_1 - m_2)/(m_1 + m_2)$	Δ	0.00555 ± 0.0002	to do	
B. $\mu\mu$ signed angle: information on one beam magnitude (others fixed), 4 runs at 1 pb^{-1}, ILC 250, fast det. tracker					
$\mu\mu$	signed $\cos\theta$	m_L, m_R	0.00378, 0.00547	0.00378, 0.00547	0.9995, 0.9997
$\mu\mu$	signed $\cos\theta$	p_L, p_R	0.000655, 0.000793	0.000654, 0.000792	0.9996, 0.9996
C. $\gamma\gamma$ forward photon azimuth harmonics: $I(X, X)$ per accepted event (Y equal), analytic Born kernel					
$\gamma\gamma$ (ILC 250)	azimuth harmonics	X	0.0673	0.0675	1.003
$\gamma\gamma$ (FCC-ee Z 91.2)	azimuth harmonics	X	0.104	0.104	1.003
$\gamma\gamma$ (FCC-ee 240)	azimuth harmonics	X	0.104	0.104	1.003
D. $\mu\mu\gamma$ transverse: information on (X, Y) per event, ILC 250, generator level					
$\mu\mu\gamma$	matrix element observable	X, Y	0.0418	to do	
$\mu\mu\gamma$	$\phi(\mu^-)$ harmonics	X, Y	0.0015	to do	

Rule fixed before ranking (notes); ranked on the validation split, so a held out check is still needed. Detector: an ILD-like fast detector simulation. det/gen near 1 within the finite template precision, not an information gain.

▸ Features ▸ Four leptons ▸ Detector

Cross sections in the frozen selections

LO, generator level; selections differ by channel

	unpolarized			ILC $(-0.8, +0.3, -0.24)$
Process	91.2 GeV	240 GeV	250 GeV	250 GeV
$\mu\mu$	2.05 nb	1.88 pb	1.73 pb	2.35 pb
ee (Bhabha)	6.14 nb	585 pb	539 pb	549 pb
$\gamma\gamma$	60.9 pb	8.79 pb	8.11 pb	10.1 pb
$\mu\mu\gamma$	48.5 pb	523 fb	480 fb	665 fb
$\mu\mu\gamma$ (fiducial)	89.8 pb†	434 fb†	397 fb	560 fb
$\mu\mu\gamma$ (fiducial, effective angle)	91.3 pb†	n/e	n/e	n/e
$ee\gamma$	75.6 pb	6.37 pb	6.01 pb	6.2 pb
$ee\gamma$ (fiducial)	137 pb†	6.3 pb†	5.8 pb	5.94 pb
$ee\gamma$ (fiducial, effective angle)	142 pb†	n/e	n/e	n/e
$\gamma\gamma\gamma$	1.34 pb	371 fb	349 fb	434 fb
$\mu\mu\mu\mu$	50.5 fb	2.41 fb	2.27 fb	3.52 fb
$ee\mu\mu$	128 fb	13.9 fb	13.2 fb	15.7 fb
$q\bar{q}$	41.1 nb	to do	11.1 pb	20 pb
$\nu\bar{\nu}\gamma$	785 fb	to do	3.26 pb	6.15 pb
WW	below threshold	17.2 pb	16.5 pb	38.3 pb
ZZ	below threshold	1.16 pb	1.1 pb	1.87 pb

Selections: two-body particles and photons 10 to 170 degrees; 250 GeV three- and four-body leptons $|\cos\theta| < 0.95$. †Samples with cuts scaled by $\sqrt{s}/250$ ($E_\gamma > 3.648$ and 9.6 GeV); other rows $E_\gamma > 10$ GeV. n/e: not evaluated.

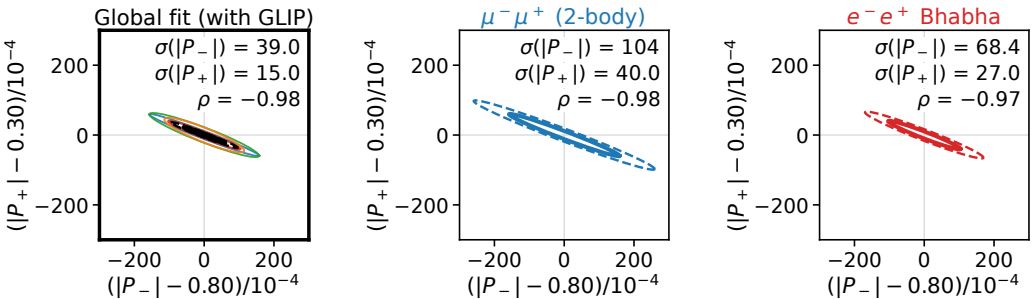
Quick LO integrals in these selections (WHIZARD, on shell G_μ); the fits use other selections. All fit inputs use the effective leptonic mixing angle. ▸ Bhabha hybrid ▸ Bhabha $O(\alpha)$ ▸ WHIZARD references

Fits per event class and global, longitudinal: ILC 250

ILC 250 (2 ab^{-1}, H20, 80 % / 30 %), including the GLIP LumiCal radiative return additions

global fit · e^-e^+ Bhabha · $e^-e^+\gamma$ · solid: 68.3 % (2D) · dashed: 95.4 % (2D)

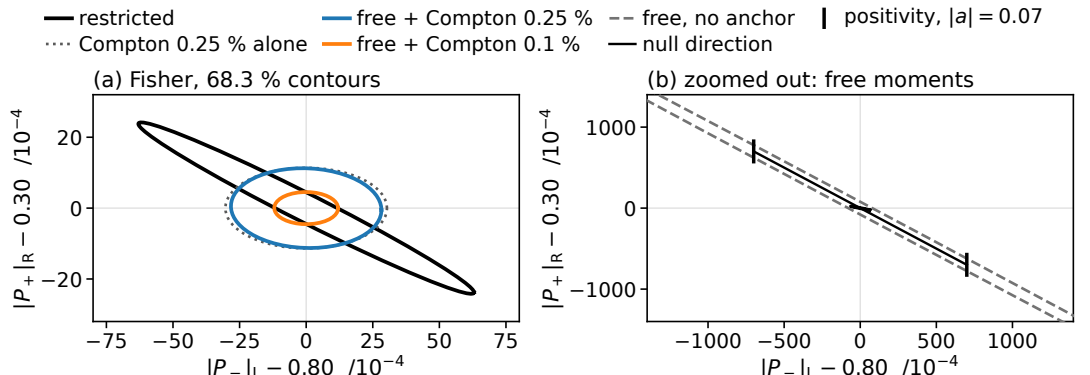
$\mu^-\mu^+$ (2-body) · $\mu^-\mu^+\gamma$



Asimov, statistics only, LO, generator level; luminosity and SM cross sections known. Units 10^{-4}.
Restricted fit: tied magnitudes, perfect reversal, factorized C. Four-lepton contours lie outside.
Global fit: best combination $\sigma = 2.8$ along (0.35, 0.94) $\approx |P_-||P_+|$. GLIP: plugin information estimate.

All classes · Fast det. · GLIP · GLIP fits

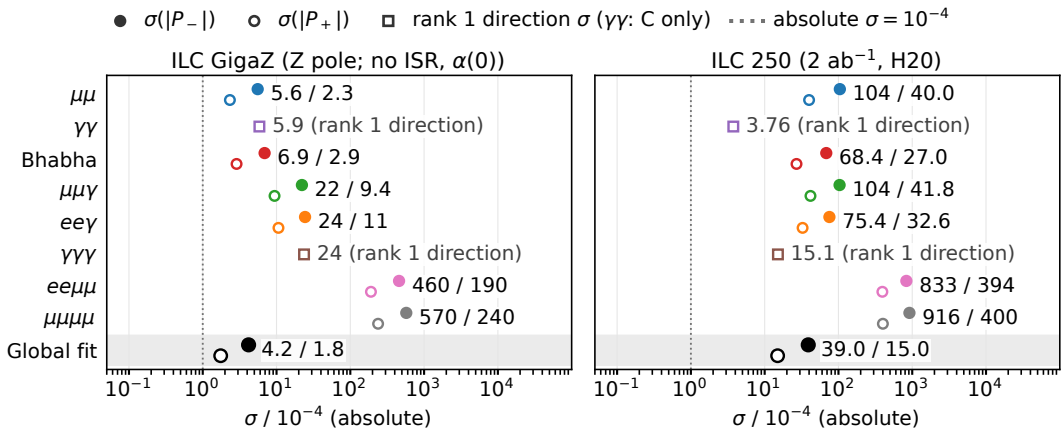
Free moments need a Compton anchor for the common helicity scale



ILC 250, 10^{-4}	restricted	Compton 0.25%	restricted + Compton 0.25%	restricted + Compton 0.1%	free + Compton 0.25%	free + Compton 0.1%
$\sigma(\lvert P_-\rvert)$ L, R	41.6	20	10.3	4.8	18.6, 18.2	7.9, 7.9
$\sigma(\lvert P_+\rvert)$ L, R	16.0	7.5	3.9	1.8	7.4, 7.4	3.0, 3.0

ILC 250, 2 ab^{-1}, H20; Asimov, statistics only, LO, generator level. Restricted: tied magnitudes, perfect helicity reversal, $C = P_- P_+$. Free: P_-, P_+ and C fitted separately in each setting; null coordinate a: $|P_\pm|_L$ up, $|P_\pm|_R$ down by a, every C_r fixed. Anchor: one Compton measurement per beam and helicity state, at 0.25% (one polarimeter's design precision) or 0.1%; upstream and downstream are not combined. Without the GLIP additions (their information estimate is not reliable along a): restricted 41.6 / 16.0 here, while the global on the previous page (39.0 / 15.0) includes them.

Run scenarios: ILC GigaZ and ILC 250

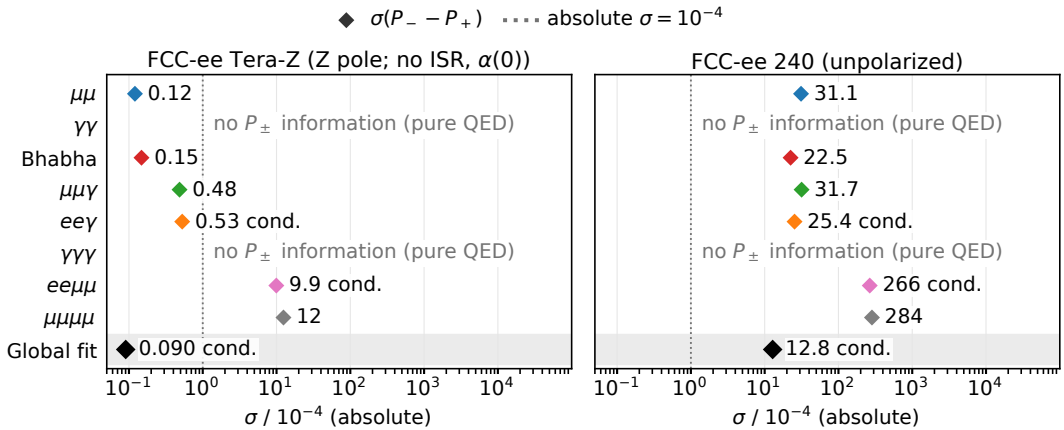


Global fit at ILC 250 includes the GLIP LumiCal radiative return additions; none at the Z pole.
Asimov, statistics only, LO, generator level; luminosity known; GLIP: plugin information estimate.

Dotted line: absolute 10^{-4}; the target is 10^{-4} relative. With a free luminosity per setting, $\gamma\gamma$ rates alone cannot separate C from $\mathcal{L}$. The Z pole scenarios have no GLIP addition.

▸ GigaZ ▸ Four scenarios ▸ All scenarios ▸ GLIP

Run scenarios: FCC-ee Tera-Z and FCC-ee 240

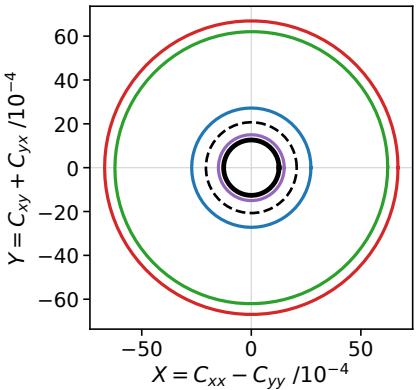


Global fit at FCC-ee 240 includes the GLIP LumiCal radiative return additions; none at the Z pole.
Asimov, statistics only, LO, generator level; cond.: $P_- + P_+$ held fixed; GLIP: plugin information estimate.

Residual polarization about 0. Absolute luminosity and SM cross sections assumed known (about 63% of the Tera-Z $\mu\mu$ information comes from the rate). ▸ Tera-Z ▸ FCC-ee 240 ▸ GLIP

Residual transverse spin correlations per event class: ILC 250

ILC 250, without the GLIP additions: residual transverse spin correlations, all classes on one scale
No useful separate transverse beam mean constraint from these channels at the assumed luminosities



$\sigma(X)$ in units of 10^{-4}
solid: 68.3 % (2D)

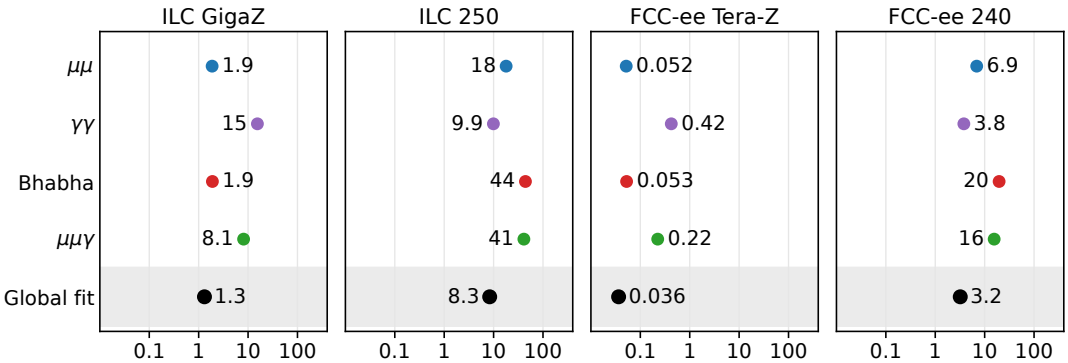
- $\mu^-\mu^+$ (2-body): $\sigma(X)$ = 17.9
- $\gamma\gamma$ (2-body): $\sigma(X)$ = 9.91
- e^-e^+ Bhabha: $\sigma(X)$ = 44.1
- $\mu^-\mu^+\gamma$: $\sigma(X)$ = 41.0
- global fit: $\sigma(X)$ = 8.34
- global fit, 95.4 % (2D)

▸ All classes

Asimov, statistics only, LO, generator level. X, Y: transverse spin correlations of the two beams.

$X = C_{xx} - C_{yy}$ and $Y = C_{xy} + C_{yx}$: the two beam transverse spin correlations that the cross sections depend on (via $\cos 2\phi$ and $\sin 2\phi$); for independent beams $X + iY = P_T^- P_T^+ e^{i(\phi_- + \phi_+)}$. With m_e neglected, helicity is conserved at the beam vertices, so $C_{xx} + C_{yy}$ and $C_{xy} - C_{yx}$ do not enter any of our channels: only X and Y are measurable.

Residual transverse spin correlations per scenario



$\sigma(X)$ / 10^{-4} of the transverse spin correlation $X = C_{xx} - C_{yy}$ ($\sigma(Y)$ within 1 %)

Asimov, statistics only, LO, generator level. Couplings: effective angle.
e e γ, $\gamma\gamma\gamma$ and four-lepton classes: to do. Z pole two-body rates: no ISR, $\alpha(0)$.

Bhabha matches $\mu\mu$ at the Z pole. Global $\sigma(X)$ with all four channels, in 10^{-4}: 1.31 (GigaZ), 8.34 (ILC 250), 0.036 (Tera-Z), 3.21 (FCC-ee 240); transverse spin correlations of the two beams, not separate beam means.

Recap: what is established, and what is outstanding

What the calculations establish

- Polarimetry requirements for ILC 250 , FCC-ee 240 are plausible to achieve
- Polarimetry requirements for ILC GigaZ need additional work to be robust

Not established

- Need to handle bias across the methods, Compton polarimetry and event fit polarimetry with respect to luminosity weighted.
- Flesh out Compton+Events global fit
- Go beyond detector level and include reconstruction
- Include other promising processes, variables and LumiCal (if relevant).
- Include other runs, collider scenarios.

▸ Main approximations ▸ Outlook ▸ Limits ▸ Status ▸ Tools

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Backup slides ahead, quality not guaranteed

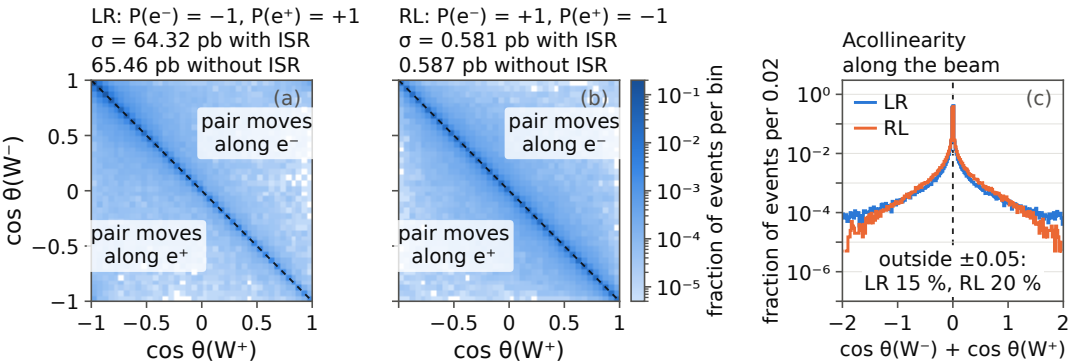
Backup: main approximations and assumptions

1. Leading order, generator level: no backgrounds and no radiative corrections beyond the LO matrix elements, except the ISR studies.
2. Expected (Asimov) precision, statistics only: no systematics and no uncertainty from finite simulated template statistics.
3. Fixed electroweak couplings (effective angle). At the Z pole a residual P is degenerate with A_e, so fixed couplings make the P constraint optimistic (FCC-ee: 2.0×10^{-5} with couplings fixed against 1.6×10^{-4} with $\sin^2\theta_{\text{eff}}$ free).
4. Luminosity and cross section normalizations known per helicity setting; floating them weakens several constraints.
5. Spin model: the ILC headline uses tied magnitudes, perfect helicity reversal and factorized $C = P_- P_+$. The free moment model leaves a common helicity shift that needs a helicity resolved Compton anchor.
6. FCC-ee results for channels without an exact Σ null are conditional on $\Sigma = P_- + P_+$ held fixed.
7. Detector: ideal acceptance and resolution apart from the fast detector simulation tracker benchmark and the parametrized GLIP LumiCal response; perfect bremsstrahlung recovery, no material effects.
8. Beam effects: the luminosity spectrum and beam-beam depolarization are not folded into the event kernels; the channel dependent effective P (2 to 8×10^{-4} at ILC 250) is not converged in the horizontal mesh.
9. Outgoing deflection: spin only argument; the acceptance and shape bias is not propagated through the fit.
10. GLIP additions: plugin information estimate with uncorrected finite template bias, which tends to be optimistic; 31 to 77 mrad acceptance at both colliders.
11. Transverse: X and Y assumed common across helicity settings and channels; ideal azimuthal efficiency (no $\cos 2\phi$ detector harmonics).
12. Compton anchor: independent Gaussian helicity resolved measurements (0.25% or 0.1%), transferred perfectly to the luminosity weighted IP polarization.

back

W pairs with ISR: the pair leaves the back to back line

WHIZARD 3.1.8, $e^+e^- \to W^+W^-$ at 250 GeV, LO with ISR structure function; generator level



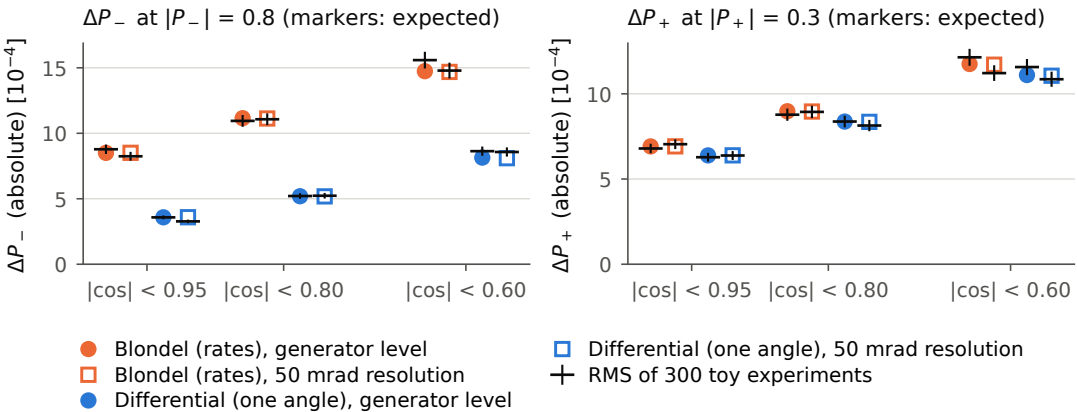
Dashed: no radiation. Angles from the e^- beam, laboratory frame. No FSR from the W bosons.

15% (LR) and 20% (RL) of the events leave the back to back line by more than 0.05; 2D fit gain not yet quantified.

back

Differential fits beat rates; tighter polar angle cuts cost precision

Precision on $|P_-|$ and $|P_+|$ vs polar angle cut: ILC 250, 2 ab^{-1}, H20, $W^+W^- \to q\bar{q}'\ell\nu$

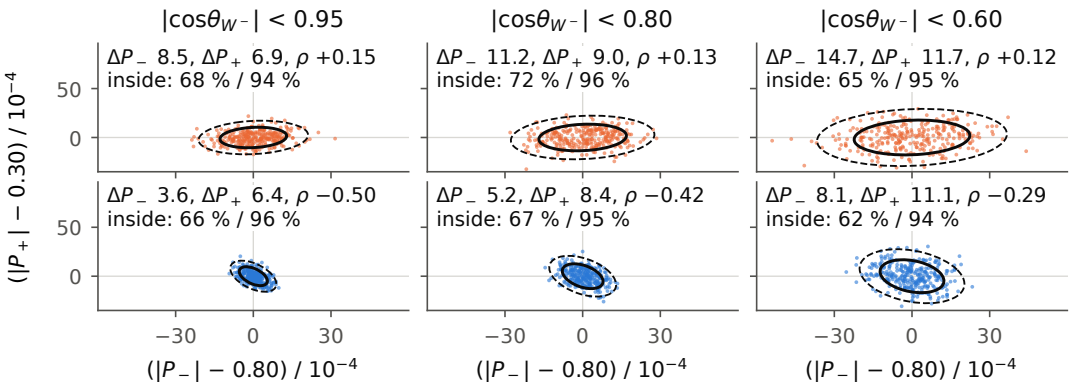


WHIZARD 3.1.8 LO, W^+W^- on shell, $\sqrt{s}$ = 250 GeV, G_μ scheme; no systematics.

Truth W^- production angle scaled by the branching ratio; tied magnitudes, perfect reversal, factorized C (free C leaves an exact null direction); templates without ISR. ‣ WHIZARD references ‣ back

Fitted $|P_+|$ against $|P_-|$ for 80% and 30% polarization

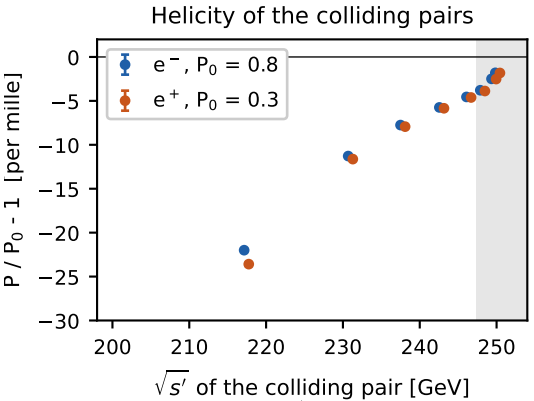
Fitted residuals, true $|P_-| = 0.8$, $|P_+| = 0.3$; ILC H20, 2 ab^{-1}; generator level

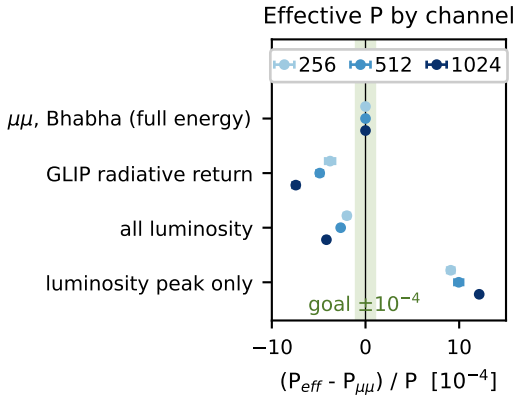


WHIZARD 3.1.8 LO, W^+W^- on shell, $\sqrt{s} = 250$ GeV, G_μ scheme; no systematics.

Tied magnitudes, perfect reversal, factorized C; generator level, LO; templates without ISR. ▸ 50 mrad smearing ◂ back

Each channel sees its own effective polarization





FeeBL, ILC 250. Gray band: $\sqrt{s'} \geq 0.99\sqrt{s}$, 68 % of the luminosity. Left: mesh 1024x2048.

Right: mean of both beams for 256, 512, 1024 cells in x (light to dark); bars: seed or n_m spread.

GLIP radiative return events see a beam polarization lower than full energy $\mu\mu$ and Bhabha by 4 to 7×10^{-4} (relative), growing as the horizontal mesh of the beam simulation is refined and not yet converged; on every mesh it stays above the 10^{-4} goal. 3.8, 4.9, 7.4×10^{-4} at 256, 512, 1024 cells in x; every mesh, seed and setting: 2 to 8×10^{-4}. GigaZ: about 10^{-5}. A combined fit needs the effective P per channel. FeeBL [16], ILC 250 [17]; horizontal mesh not converged.

back

A granular LumiCal (GLIP) adds radiative returns to $\mu\mu$ and Bhabha

Worth including: GLIP tagged radiative return dimuons improve the $\mu\mu$ channel by about 12% (ILC 250: $\sigma(|P_-|)$ 104→92×10^{-4}; FCC-ee 240: $\sigma(P_- - P_+)$ 31.1→27.3×10^{-4}) and the global fit by 4 to 6%. Uncorrected plugin estimate; effective mixing angle couplings; template and nuisance effects not propagated.

Selection: charged pair in the tracker ($|\cos\theta| < 0.95$); one LumiCal photon with $E_\gamma > 0.1\,E_{\rm beam}$ in 31 to 77 mrad (ILD LumiCal window), no other photon cut. For the Z return subset the LumiCal photon carries about 108 GeV (103 GeV at 240 GeV). Electrons: perfect bremsstrahlung recovery assumed. Full selection and resolution: backup.

		σ [fb]							
Process	$\sqrt{s}$	LR	RL	LL	RR	unpol.	ILC point	Events	/2-body
$\mu^+\mu^-\gamma$	250	261	182	9.79×10^{-7}	9.77×10^{-7}	111	159	2.64×10^{5}	0.064
Z peak*		163	105	7.97×10^{-7}	7.94×10^{-7}	67	99		
$\mu^+\mu^-\gamma$	240	292	202	1.18×10^{-6}	1.18×10^{-6}	124	178	1.33×10^{6}	0.0657
Z peak*		188	121	9.58×10^{-7}	9.66×10^{-7}	77.1	114		
$e^+e^-\gamma$	250	1985	1853	1990	1980	1950	1981	3.89×10^{6}	3.6×10^{-3}
Z peak*		171	115	42.8	42.7	92.7	120		
$e^+e^-\gamma$	240	2146	1996	2190	2180	2128	2157	2.3×10^{7}	3.6×10^{-3}
Z peak*		198	134	51.9	51.7	109	140		

*$|m_{\ell\ell} - m_Z| < 10$ GeV. ILC point $(-0.8, +0.3, -0.24)$; events: ILC 250 2 ab^{-1} (H20), FCC-ee 240 10.8 ab^{-1}; /2-body: ratio to the two-body rate; cross sections: on shell G_μ quick integrals. LO, no ISR structure function, no beamstrahlung.

GLIP: Madison [18]. ▸ GLIP fits ◂ back