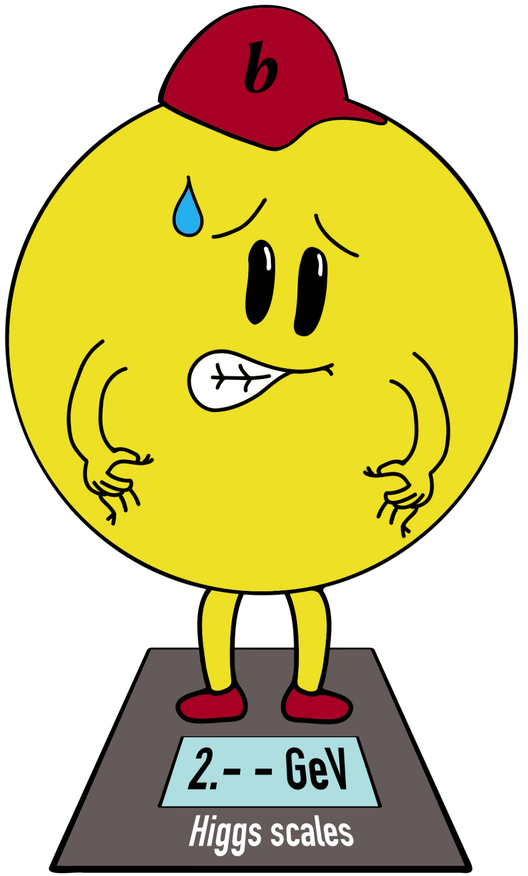
The bottom quark mass at high scale

Juan Ramírez Alfaro, María Moreno Llácer & Marcel Vos

Instituto de Física Corpuscular (IFIC -CSIC/UV)

International Workshop on Future Linear Colliders 2026

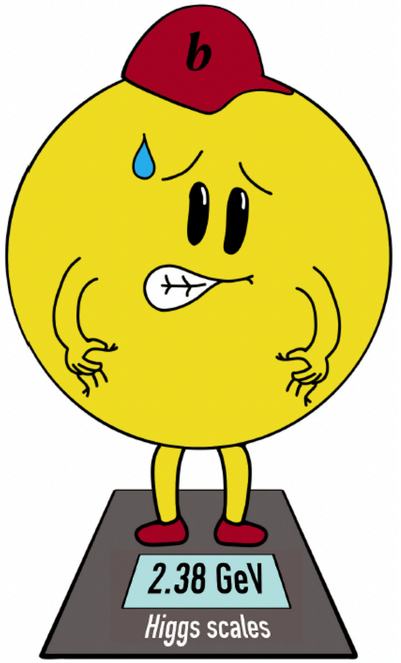
KEK, Tsukuba, 12 Nov 2026



This work is supported by FPU24/02210 PhD grant

Last LCWS…

- **Scale evolution** of QCD parameters is **experimentally testable**.
- **Updated measurement** of bottom quark mass at high scale → $m_b(m_H) = 2.38\,^{+0.24}_{-0.21}$ GeV
  - Full Correlation matrix allows to **significantly decrease uncertainty** (**33%**) via **combination**.
  - Provides **proof for running of quark masses**; **no running ruled out by** 9σ.
- **Future Colliders** have a **high potential for improvement**, particularly **Linear** Facilities: **experimental uncs. projected to reach the level of today's theoretical ones!**
- **Near Future:**
  - **Integrate within ATLAS** analysis groups to further develop & refine the measurement
  - Address the **elephant in the room**: we inevitably **assume SM for Higgs decays**
    - Simultaneous measurement of b-Yukawa & m_b
    - Design scenarios for different effects of new physics
  - **Refine the prediction for LCF**, reproduce "covariance matrix treatment" (Junping Tian & Jorge De Blas).

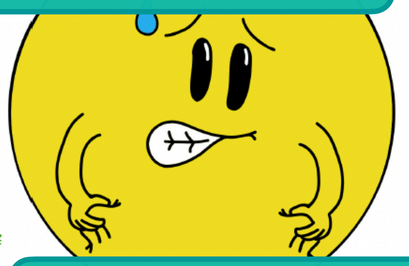


Last LCWS…

- **Scale evolution** of QCD parameters is **experimentally testable**.

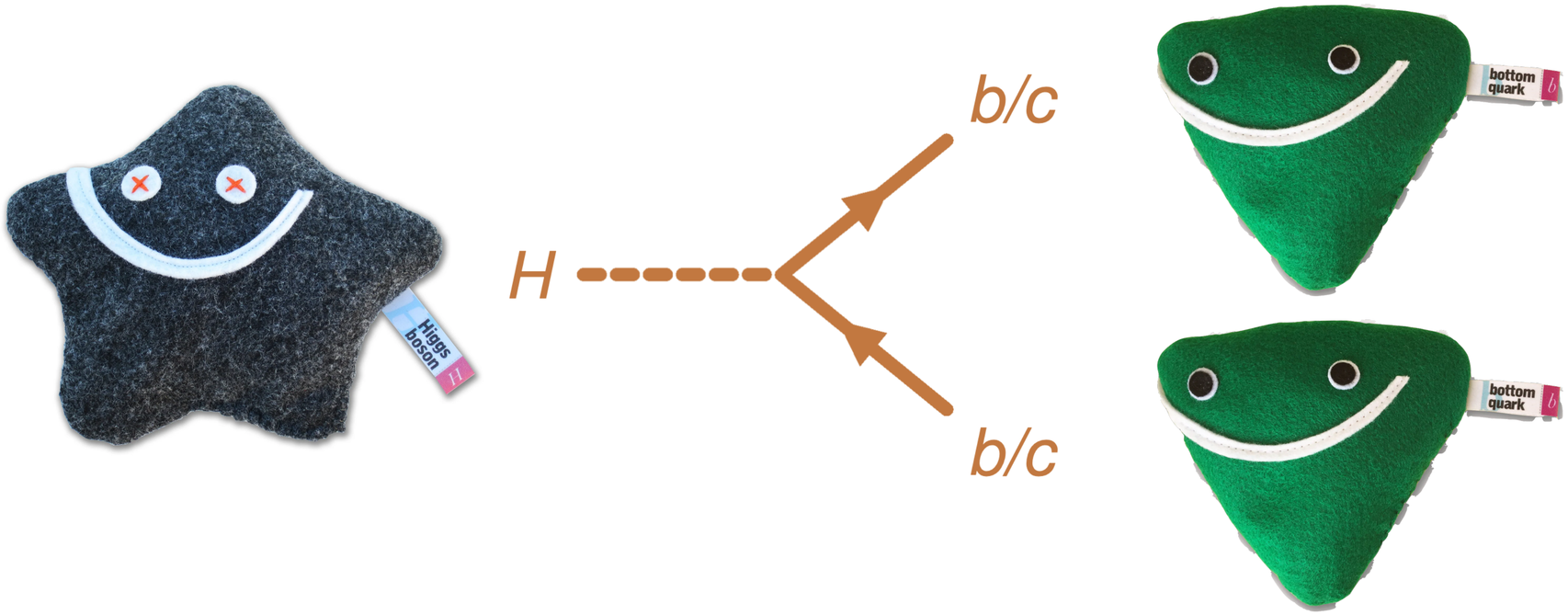
Updated Methodology for $m_b(m_H)$ extraction using full ATLAS power!

- **Future Colliders** have a **high potential for improvement**, particularly **Linear** Facilities: **experimental uncs. projected to reach the level of today's theoretical ones!**
- **Near Future:**
  - ✅ ~~**Integrate within ATLAS** analysis groups to further develop & refine the measurement~~
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    - Simultaneous measurement of b-Yukawa & m_b
    - Design scenarios for different effects of new physics
  - **Refine the prediction for LCF**, reproduce "covariance matrix treatment" (Junping Tian & Jorge De Blas).



Proposal & (VERY) preliminary estimates

Introduction & Motivation



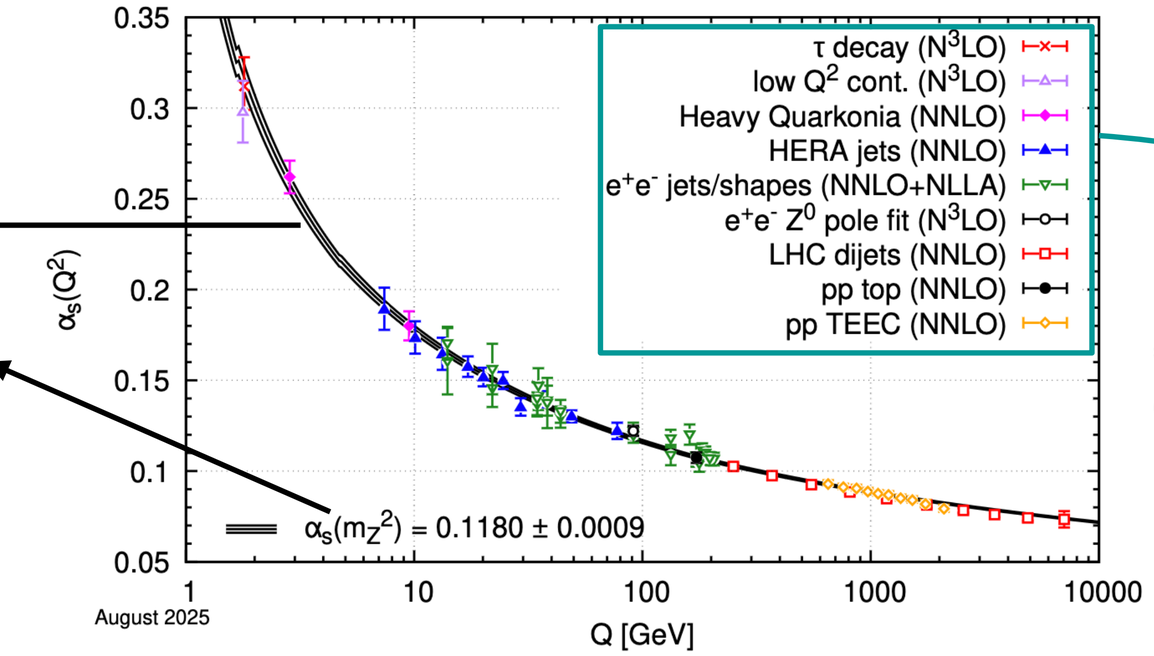
Running parameters

- Standard Model (SM) and Quantum Chromodynamics (QCD) precisely predict the **scale evolution** i.e. "**running**" of its (renormalized*) free parameters: **strong coupling** & **quark masses**.

Reference input value at some energy μ_0 is evolved to any other scale thanks to **Renormalization Group Equations** (RGEs)

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \beta(\alpha_s)$$

QCD β function: strong coupling running



F. Takahashi *et al.* (Particle Data Group), Int. J. Mod. Phys. A 41, 2630011 (2026)

Measurements over a broad energy range can be compared to the RGEs predictions

Experimental test of the running!

(*): Scheme considered in this work: *modified minimal subtraction* or $\overline{MS}$ renormalization scheme ($m_b \equiv m_b^{\overline{MS}}$)

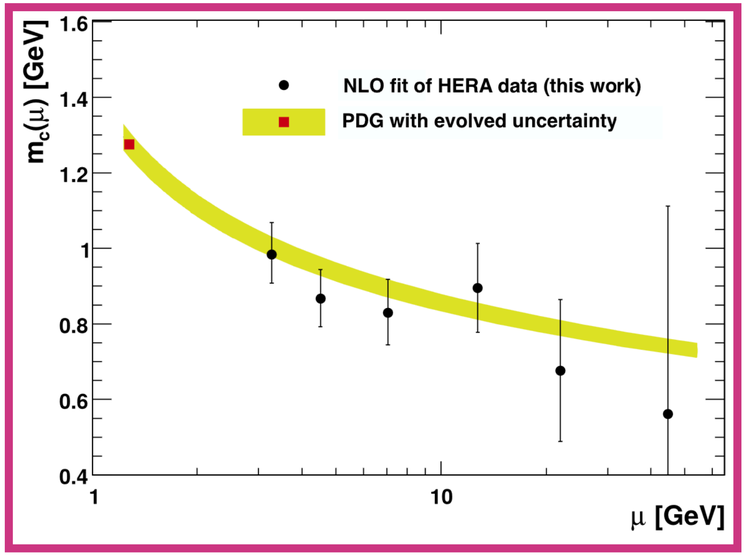
Running parameters

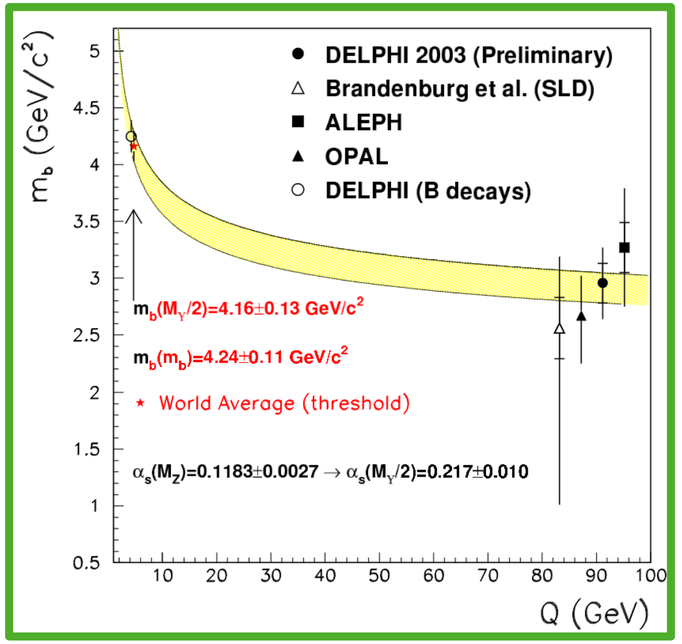
- Standard Model (SM) and Quantum Chromodynamics (QCD) precisely predict the **scale evolution** i.e. "**running**" of its (renormalized*) free parameters: **strong coupling** & **quark masses**.

- **Anomalous mass dimension**: QCD predicted running of quark masses

$$\mu^2 \frac{\partial m_q(\mu)}{\partial \mu^2} = \gamma_m \left[\alpha_s(\mu)\right] m_q(\mu)$$

- **Experimentally confirmed** for:
  - Charm quark mass by HERA [Ghizko et al., PLB775 (2017)]
  - Top quark mass by CMS [PLB803 (2020)] (see also Catani et al., JHEP08 (2020))
  - **Bottom quark mass** by DELPHI, SLD, ALEPH, OPAL; see cf. Kluth, ROPP69 (2006) → $m_b(m_b)$ & $m_b(m_Z)$

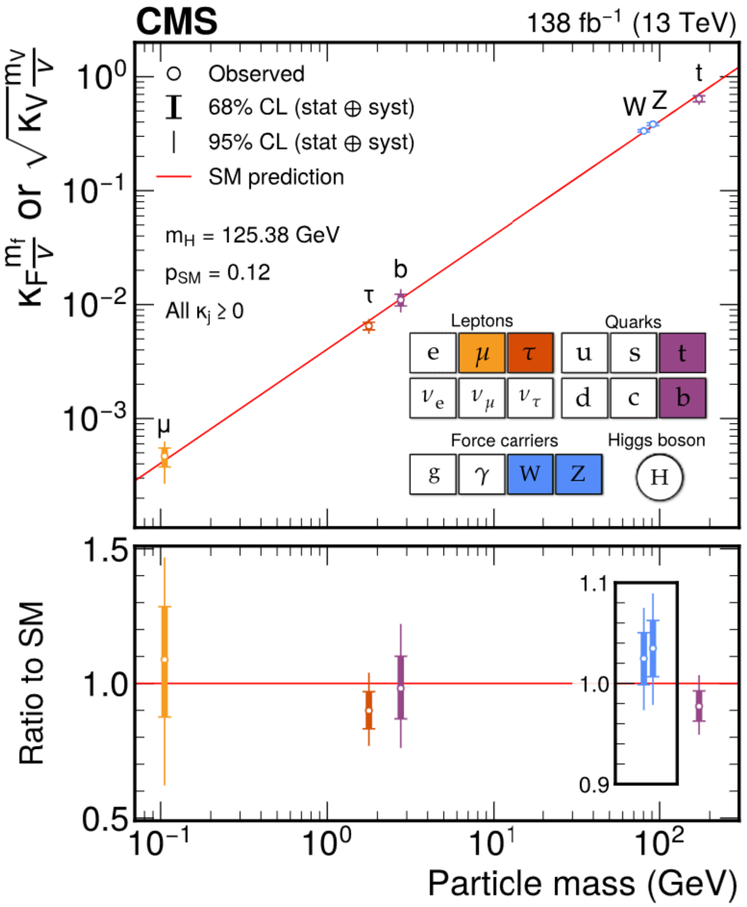




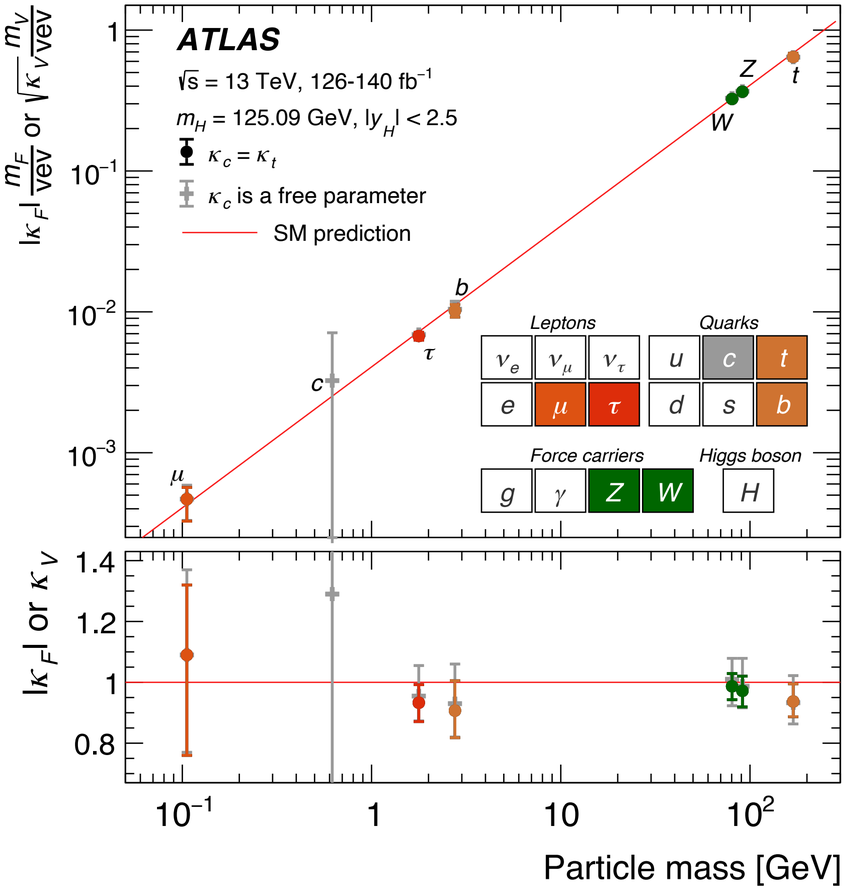
(*): Scheme considered in this work: *modified minimal subtraction* or $\overline{MS}$ renormalization scheme ($m_b \equiv m_b^{\overline{MS}}$)

Higgs couplings and fermion masses

- Since 2012, ATLAS & CMS at LHC have measured **couplings of the Higgs boson** to the SM particles:



CMS, ROPP89 (2026)

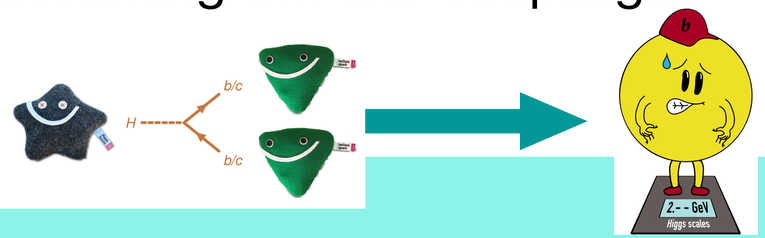


ATLAS, arXiv:2608.07332 (submitted to ROPP)

- In the SM, Higgs couplings are **proportional to the particle mass.**
- Higgs boson **decay rates** into fermions f **depend on** m_f:

$$\Gamma(H \to f\bar{f}) = \frac{1}{32\pi} \frac{g^2 m_f^2}{m_W^2} N_C^f m_H \left(1 - \frac{4m_f^2}{m_H^2}\right)^{3/2}$$

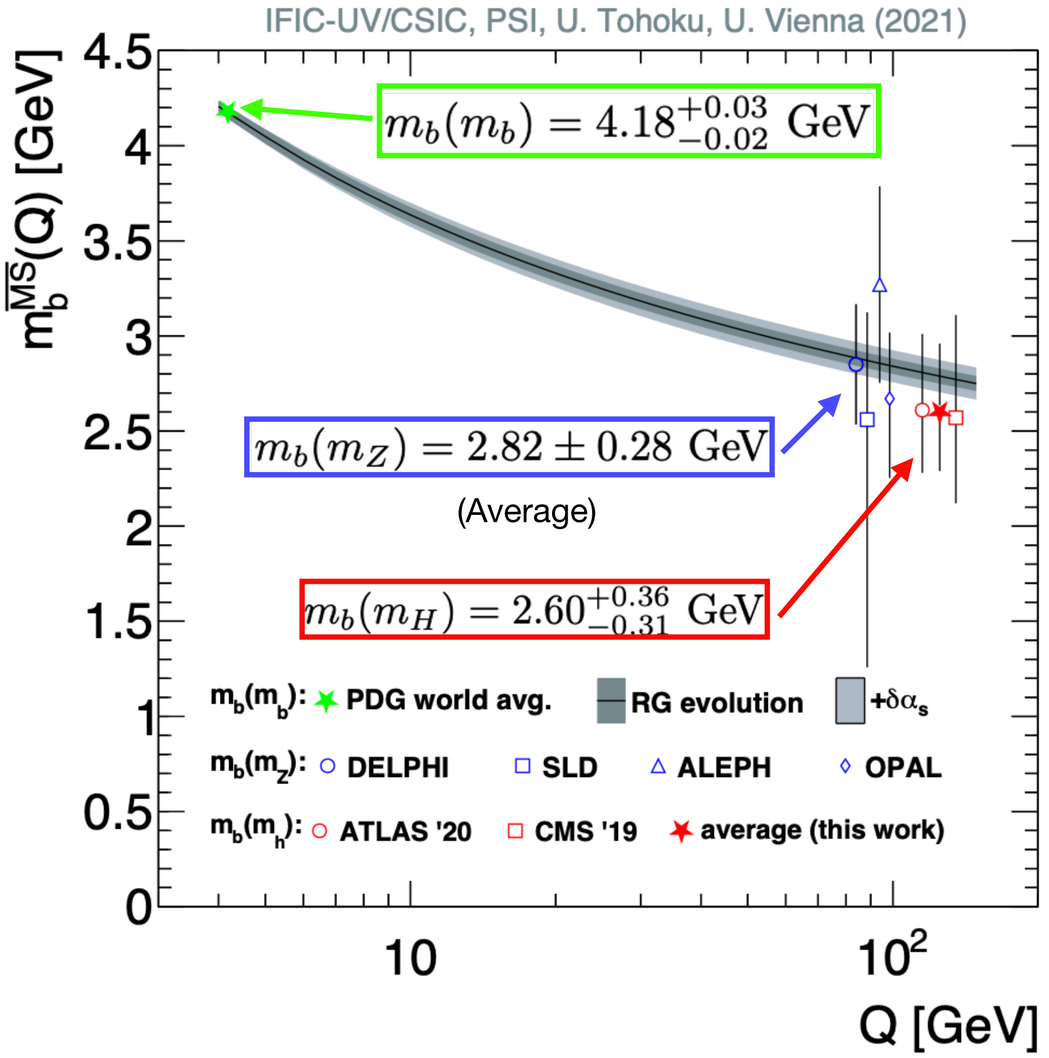
- **Fermion masses at high scales can be measured via Higgs <u>precision</u> measurements,** assuming SM-like couplings.



$m_b(m_H)$ and running

- $H \to b\bar{b}$ measurements allow to extract $m_b(m_H)$: m_b at the energy scale of the Higgs boson mass m_H (characteristic scale of the process!)
- **First measurement**: Aparisi et al., PRL128 (2022), $m_b(m_H) = 2.60^{+0.36}_{-0.30}$ GeV (ATLAS and CMS data)
- $m_b(m_H)$ **further extends the running test** up to $Q = m_H$. **No-running ruled out** by nearly 7σ, while evolution remains compatible with SM within 1σ.

$m_b(m_H)$ measurement and m_b running test have the potential to become a precision test of SM and QCD, especially at new collider facilities.



RG Evolution: REvolver (*Comput. Phys. Commun.* 270, 108145 (2022), arXiv: 2102.01085 [hep-ph])

$m_b(m_H)$ at LHC from ATLAS Run 2 measurements



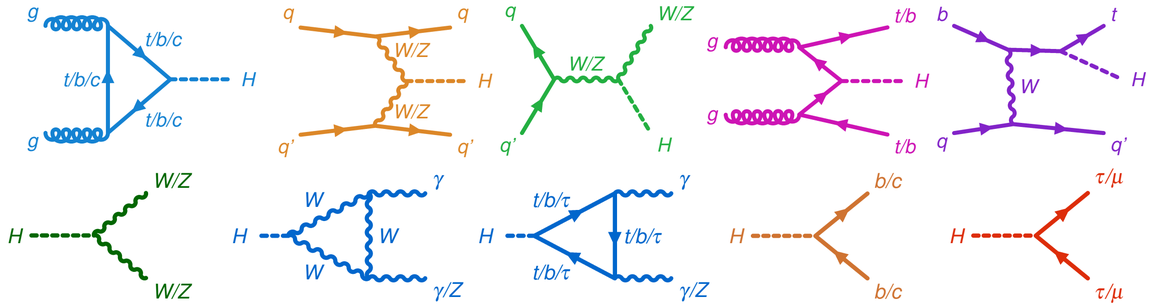
NEW: arXiv 2608.07332 [hep-ex]

$m_b(m_H)$ from ATLAS Coupling Measurements

- **Inputs**: 26 analysis of individual Higgs boson production σ_i & decay B_f channels ($\sigma_i \times B_f$) using up to 140 fb^{-1} pp collision data collected from ATLAS at $\sqrt{s} = 13$ TeV (**Run 2 output**).

σ_i (single H): ggF, VBF, VH, ttH, bbH, tH

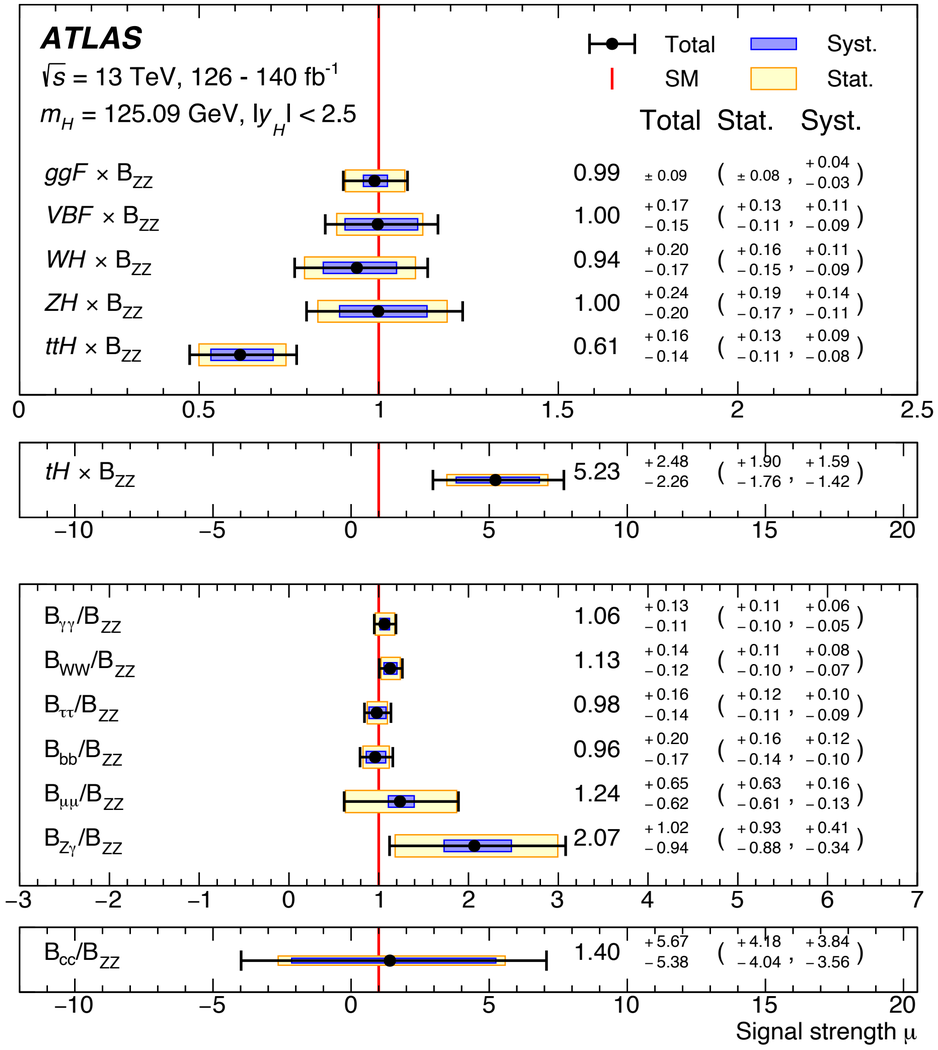
B_f : WW, ZZ, $\gamma\gamma$, γZ, bb, cc, $\tau\tau$, $\mu\mu$



- **Combined likelihood model** to simultaneously **measure** prod. xsecs in ZZ decay, $\sigma_i \times B_{ZZ}$, and BR ratios B_f/B_{ZZ}.

- Only B_{bb}/B_{ZZ} depends on $m_b(m_H)$: **parametrized in $m_b(m_H)$ terms** using **HDECAY 6.61** predictions (N4LO QCD, NLO EW).

$$\frac{\left(\Gamma_{bb}/\Gamma_{ZZ}\right)_{obs}}{\left(\Gamma_{bb}/\Gamma_{ZZ}\right)_{SM}} = 0.96^{+0.20}_{-0.17} \longrightarrow \frac{\left(\Gamma_{bb}/\Gamma_{ZZ}\right)_{obs}}{\left(\Gamma_{bb}/\Gamma_{ZZ}\right)_{SM}} = 1 + \sum_{n=1}^{5} a_n \left[\left(\frac{m_b(m_H)}{m_b^{ref}(m_H)}\right)^n - 1\right]$$



NEW: arXiv 2608.07332 [hep-ex]

$m_b(m_H)$ measurement

RG Evolution: REvolver (*Comput. Phys. Commun.* 270, 108145 (2022), arXiv: 2102.01085 [hep-ph])

- **First determination of $m_b(m_H)$ by any LHC experiment!**

$$m_b(m_H) = 2.73\,^{+0.27}_{-0.25}\ \text{GeV}$$
$$= 2.73\,^{+0.22}_{-0.20}\,(\text{stat.})\,^{+0.15}_{-0.14}\,(\text{syst.})\,^{+0.07}_{-0.06}\,(\text{theo.})\ \text{GeV}$$

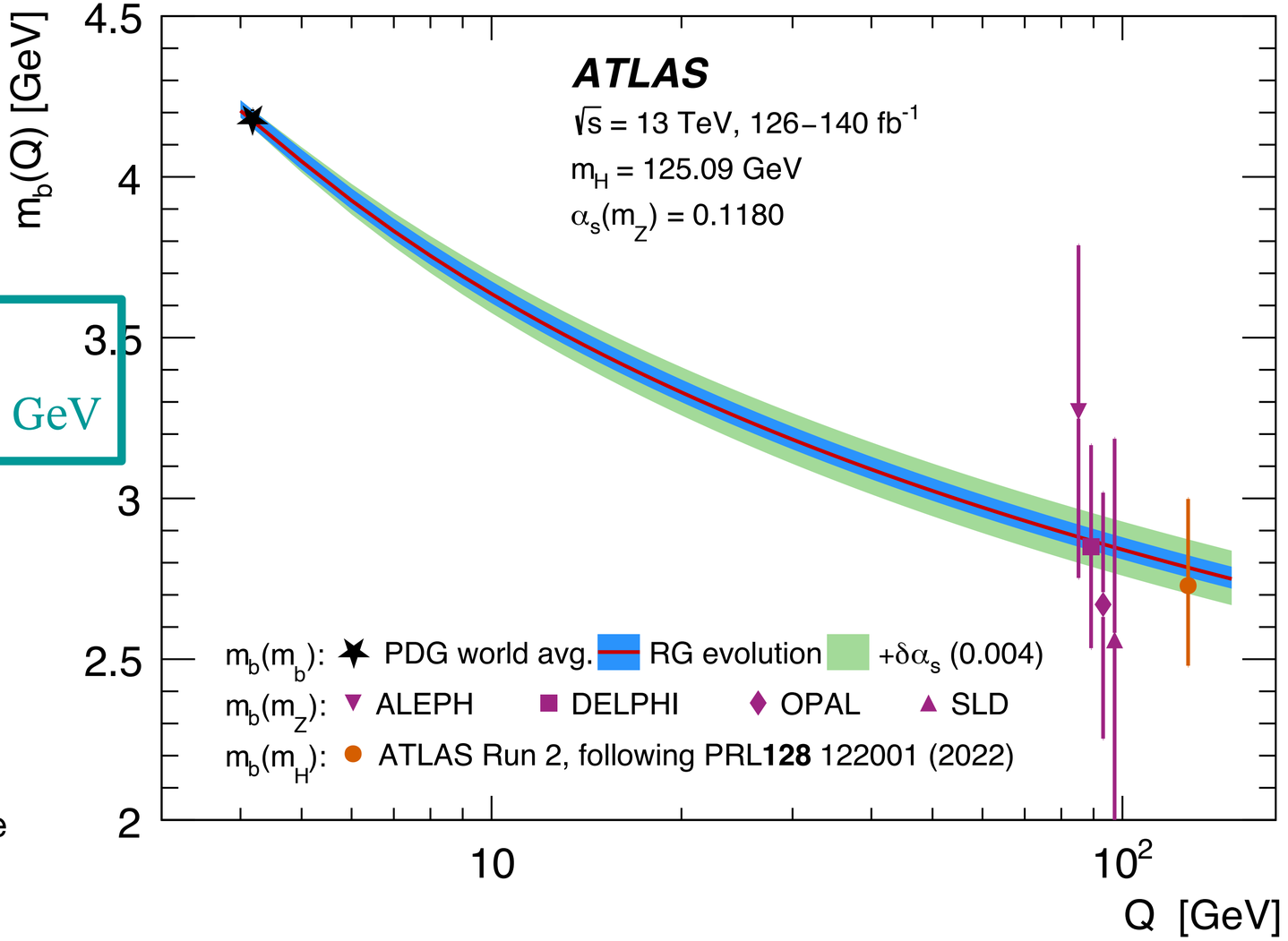
- Note: CMS provide its Legacy Run 2 result on Γ_{bb}/Γ_{ZZ} (ROPP86, 2026). We (the authors, NOT ATLAS) interpret it as a mass:

$$\frac{(\Gamma_{bb}/\Gamma_{ZZ})_{obs}}{(\Gamma_{bb}/\Gamma_{ZZ})_{SM}} = 0.66\,^{+0.17}_{-0.14}$$

$$\downarrow$$

$$m_b(m_H) = 2.26\,^{+0.29}_{-0.24}\ \text{GeV}$$

Low value driven by ttH(bb)



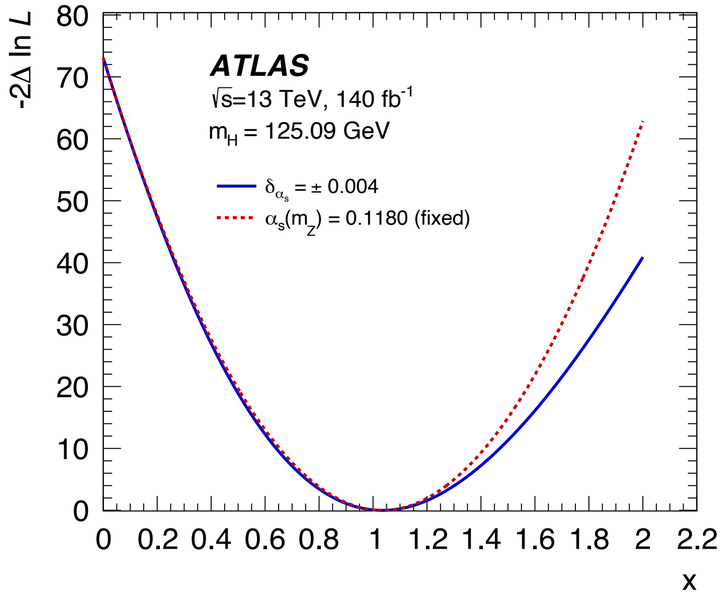
NEW: arXiv 2608.07332 [hep-ex]

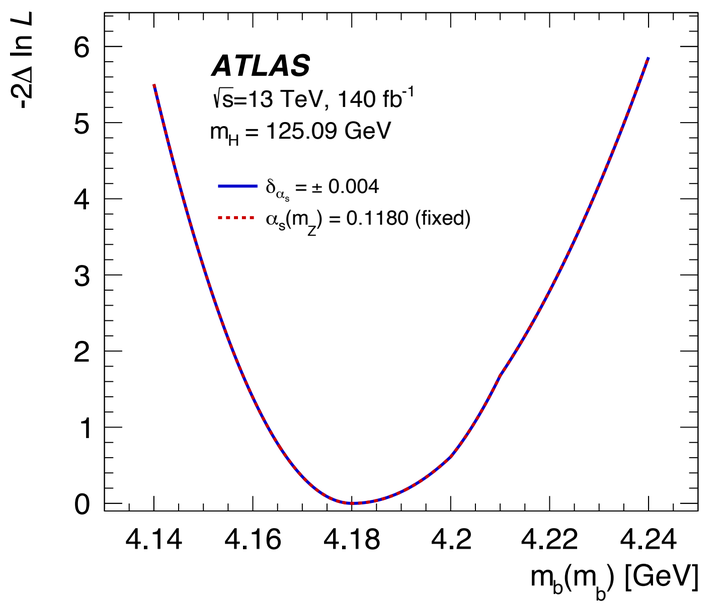
$m_b(m_H)$ and m_b running

- m_b **running tested** using parametrization

$$m_b(\mu; x, m_b(m_b)) = x\left[m_b^{SM}(\mu; m_b(m_b)) - m_b(m_b)\right] + m_b(m_b)$$

- m_b^{SM} is RGE prediction; $m_b(m_b)$ is the reference point & x **interpolates** b/w running scenarios.
- Likelihood fit to $m_b(m_b)$ PDG average, $m_b(m_Z)$ from LEP & SLD, and the extracted $m_b(m_H)$.





- Best-fit results:

$$m_b(m_b) = 4.18 \pm 0.02\ \text{GeV}$$

$$x = 1.04 \pm 0.12\ (\text{exp.}) \pm 0.05\ (\alpha_s)$$

- **Agreement with SM** ($x = 1$) while **excluding** the **no-running scenario** ($x = 0$) by **more than** 8σ.

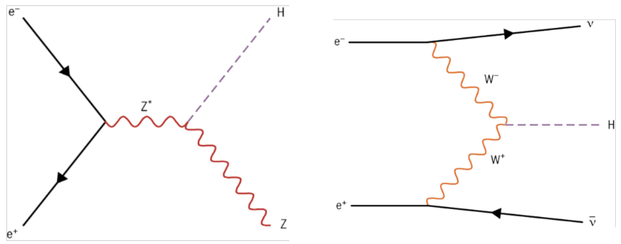
NEW: arXiv 2608.07332 [hep-ex]

Prospects for Future Colliders



Life after LHC

- Immediate upgrade: **High-Luminosity LHC** (HL-LHC). Projected operation at $\sqrt{s} = 14$ TeV, ~x20 more integrated lumi L than in Run 2! ($L_{HL-LHC} = 3$ ab^{-1}).
  - **Preliminary estimates** by **scaling** with luminosity (Scenario 2: statistical nature, require further treatment of systematics & theory).
- Highest-priority next collider: e^+e^- **Higgs factory** for precision measurements. CERN Council endorsed **FCC-ee** this year; one alternative is **LCF**. While different in design, physics projections are similar between the two.
  - **Higgs couplings** expected to be measured with **unprecedented precision**. Best prospect for $m_b(m_H)$ is B_{bb}/B_{WW}, expected **sub-% sensitivity**.



Left: ZH production, dominant mode for LCF250 & FCCee240/365
Right: WW fusion, relevant at LCF550

Collider	$\sqrt{s}$ [GeV]	$\int \mathcal{L}$ [ab^{-1}]
FCC-ee	240	5
	365	1.5
LCF	250	3
	550	8

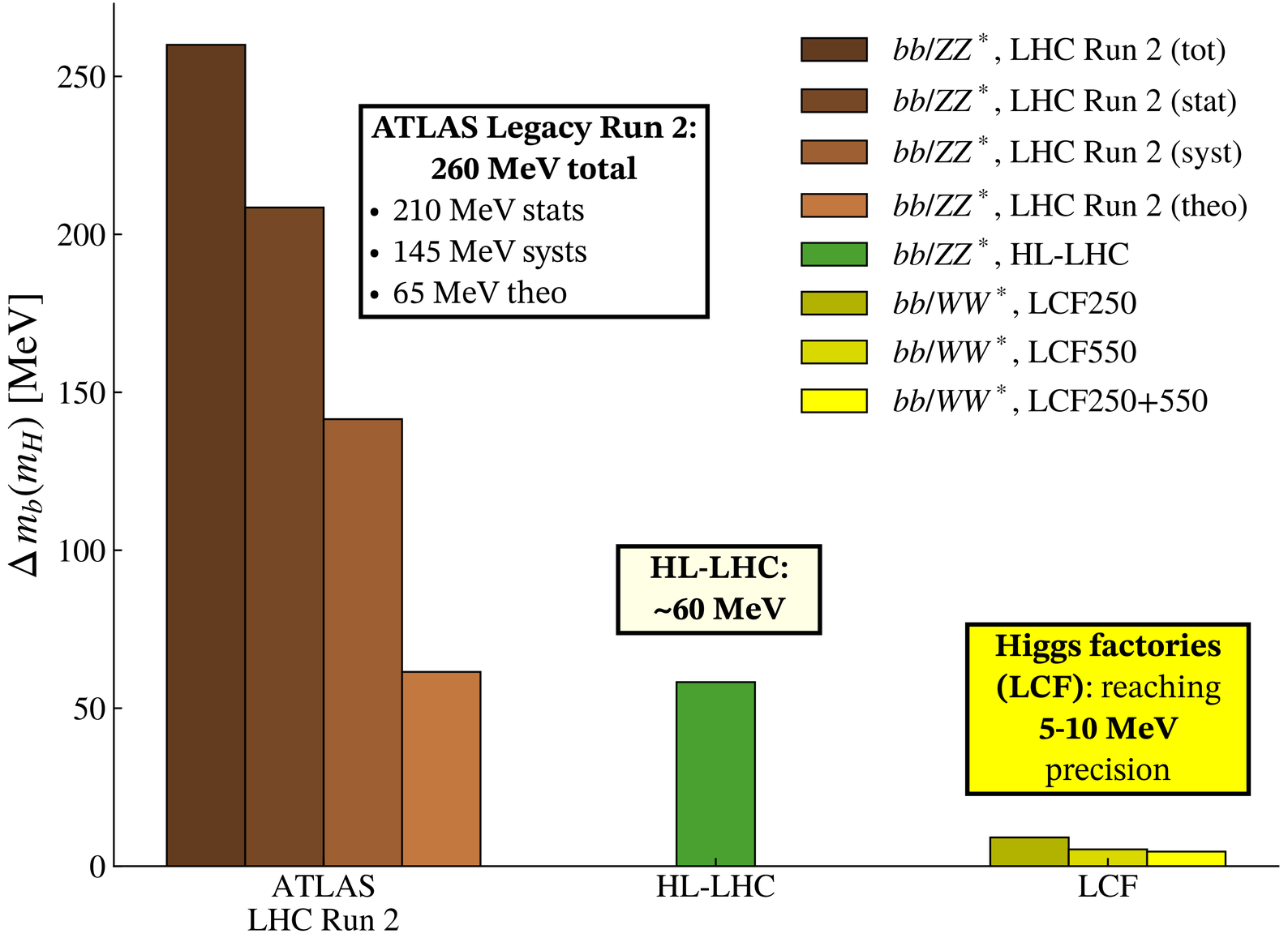
Baseline programs for FCC-ee & LCF

Operation	Relative precision on $BR(H \to bb)/BR(H \to WW^*)$
LCF250	0.65 %
LCF550	0.38 %
LCF250+550	0.33 %

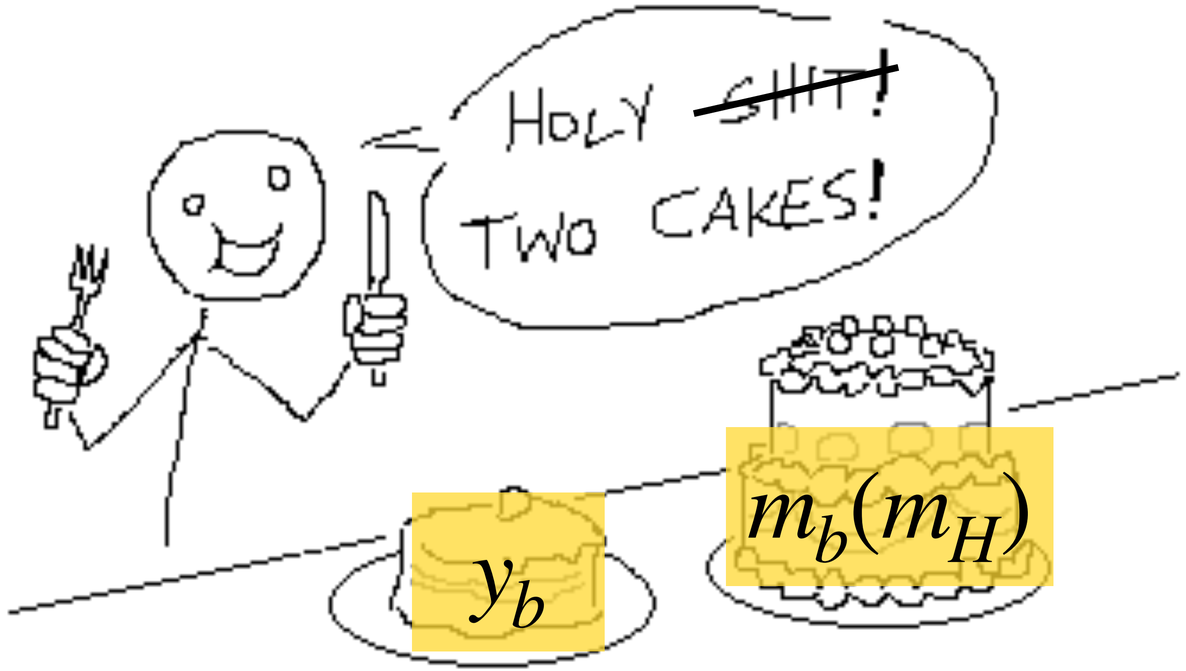
Preliminary LCF estimates, kindly provided by Junping Tian (Tokyo U., ICEPP)

Projected future uncertainties

- At **HL-LHC**, ~75% improvement**,** as far as **60 MeV**.
- Even better for a **future Higgs factory (LCF)**, below **10 MeV**
  - >95% improvement
  - Potential for being the **most precise measurement of m_b!!**
- Note that **theory & systematics** should be further addressed to estimate their impact.



Simultaneous measurement of m_b and y_b



Simultaneous measurement of m_b & y_b

- While well-motivated, ATLAS $m_b(m_H)$ extraction **assumes agreement with SM**: $y_b \propto m_b$.
- A more **model-independent extraction** could be achieved if y_b and m_b were **separated**.
  - Achievable by adding a **SMEFT operator that modifies** y_b: $C_{d\phi}[33]$ coefficient

$$y_b(m_b, C_{d\phi}[33]) = y_b^{SM}(m_b) + \delta y_b(C_{d\phi}[33]) = y_b^{SM}(m_b) - \frac{v^2}{\Lambda^2} C_{d\phi}[33] + \mathcal{O}(1/\Lambda^4)$$

- With **independent** y_b and m_b, it is possible to **extract them simultaneously**:
  - **Inclusive rate** Γ_{bb} still depends quadratically on the **coupling** y_b.
  - **Mass effects** enhanced in differential observables: **3-jet rate decay width** (LEP extraction of $m_b(m_Z)$)

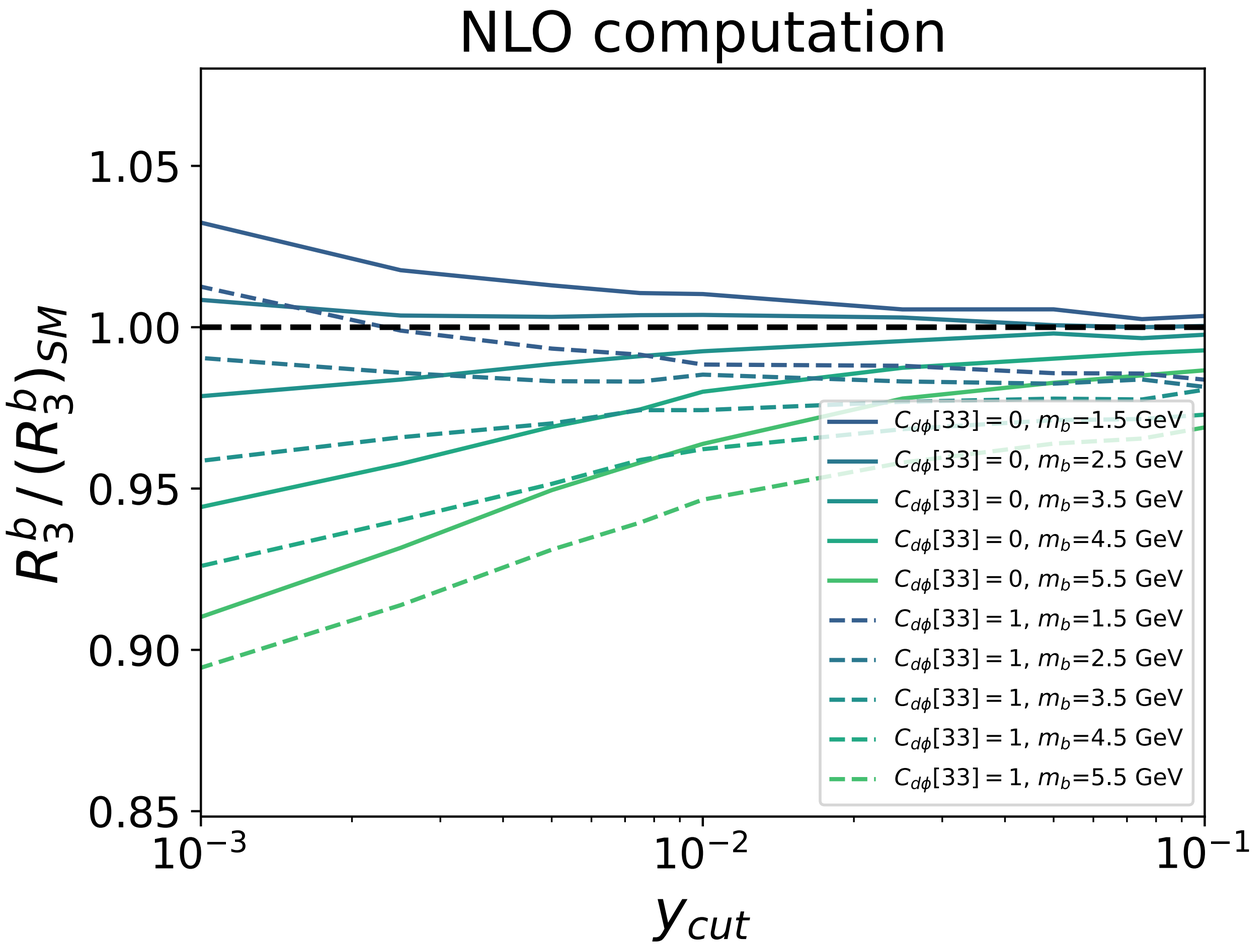
$$R_3^b(y_{cut}) = \frac{\Gamma_{bb,3j}(y_{cut})}{\Gamma_{bb}}$$

Depend on jet resolution variable y_{cut}

Measuring R_3^b in $H \to b\bar{b}$ can extract m_b (shape) and y_b (normalization) simultaneously

Simultaneous measurement of m_b & y_b

- **Feasibility study** of simultaneous extraction together with theorists P.P.Giardino (IFT) & M.Forslund (Princeton).
- Use of their **code "NEWiSH"** (https://arxiv.org/pdf/2508.14966).
  - 2- & 3-body Higgs decays at NLO QCD and EW corrections + all contributions from dim.6 SMEFT.
  - Produced NLO differential predictions for Γ_{bb} and $\Gamma_{bb,3j}(y_{cut})$ (private comm.) with Durham y_{cut} as functions of m_b and $C_{d\phi}[33]$ (linear dependence on SMEFT operator).
- Build R_3^b and its normalized version by its SM value ($m_b = m_b^{ref}(m_H)$, $C_{d\phi}[33] = 0$)



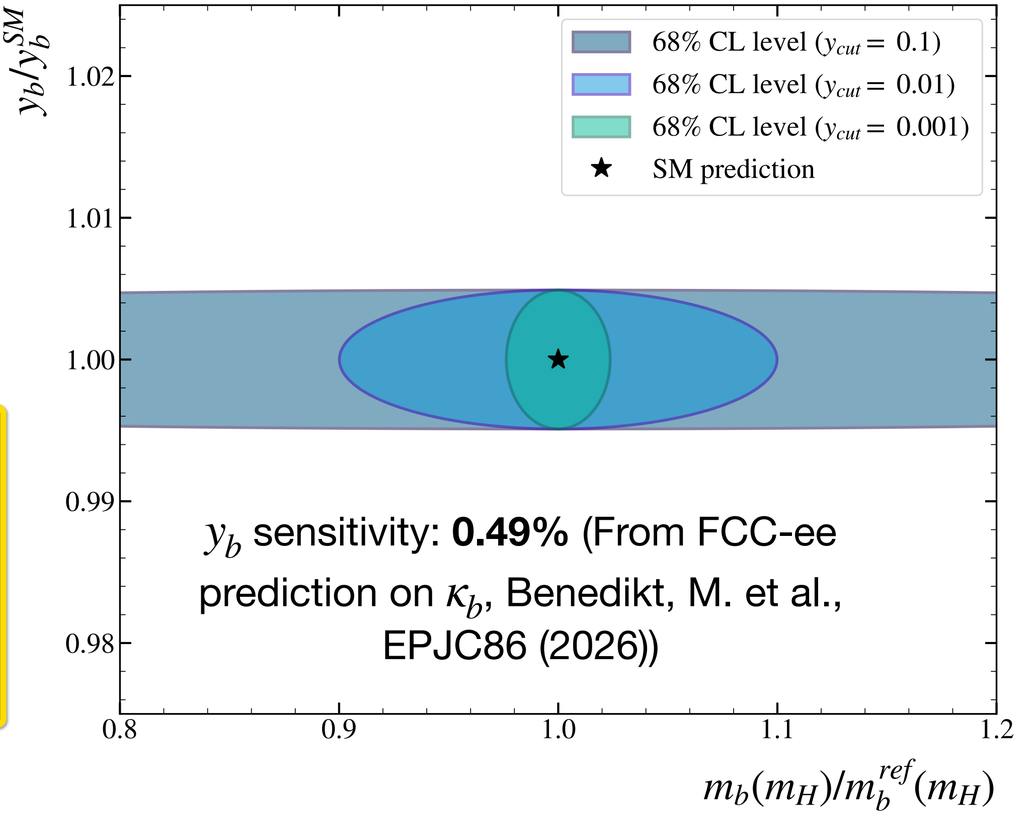
Simultaneous measurement of m_b & y_b

- Predictions confirm that at NLO y_b mainly affects normalization while m_b influences the shape.
- **Mass sensitivity preliminarily estimated** for FCC-ee ZH production at $\sqrt{s} = 240$ GeV, **fixing** $C_{d\phi}[33] = 0$ to estimate $R_3^b = f(m_b)$ parametrization.

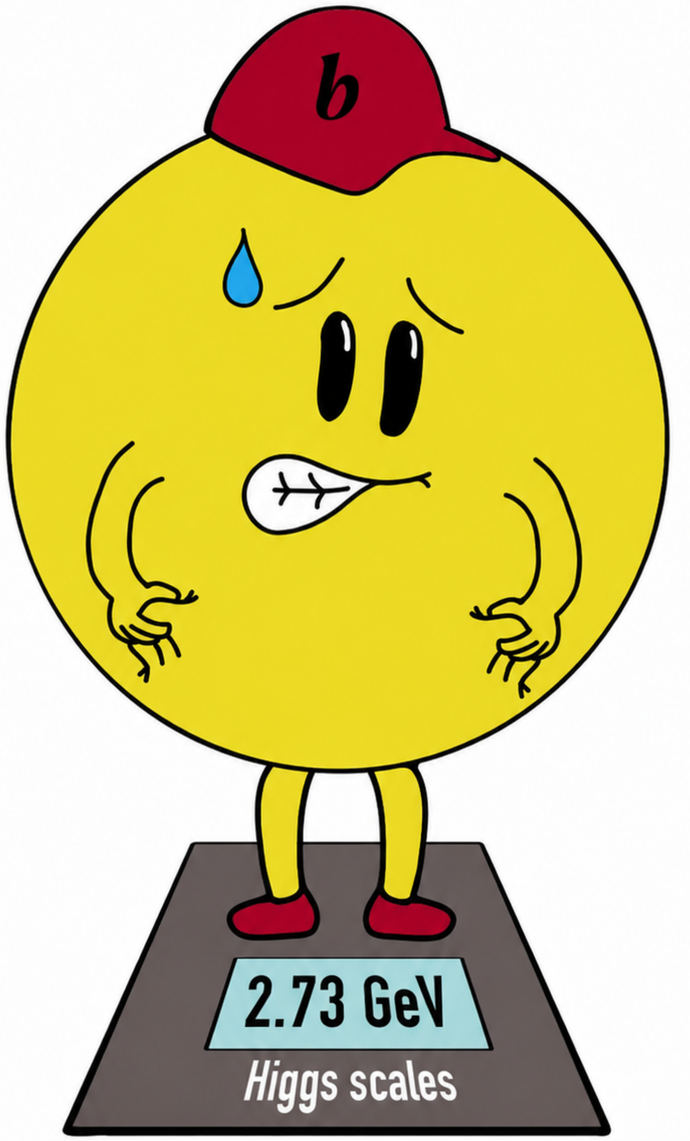
y_{cut}	N_{bb}	N_{bbj}	$\delta R_3^b / R_3^b$ [%]	$\delta m_b / m_b$ [%]
0.001	663800 ± 800	414400 ± 600	0.20	~2.4
0.01		164800 ± 400	0.28	~10
0.1		24830 ± 160	0.65	~50-80

- **Work in progress**:
  - Estimate sensitivity for both m_b and y_b in a **simultaneous fit** to Γ_{bb} and R_3^b.
  - Address **theory and systematic uncertainties**.
  - Produce **proper projections** (full simulation of realistic processes).





Conclusion & Outlook



Conclusion & Outlook

- The **bottom quark mass at high scale** can be **measured** to **good precision with Higgs data.** ATLAS achieved **first-ever determination by any LHC experiment**:

$$m_b(m_H) = 2.73\,^{+0.22}_{-0.20}\,(\text{stat.})\,^{+0.15}_{-0.14}\,(\text{syst.})\,^{+0.07}_{-0.06}\,(\text{theo.})\ \text{GeV} = 2.73\,^{+0.27}_{-0.25}\ \text{GeV}$$

  - Limited by current experimental uncertainties: **excellent prospects for improvement** at HL-LHC and Higgs factories.

- **Demonstrates** the **rejection of the no-running scenario** for quark masses, even assuming conservative uncertainties.

- **Potential** for improvement in a **LEP-like extraction** simultaneously with Yukawa coupling.

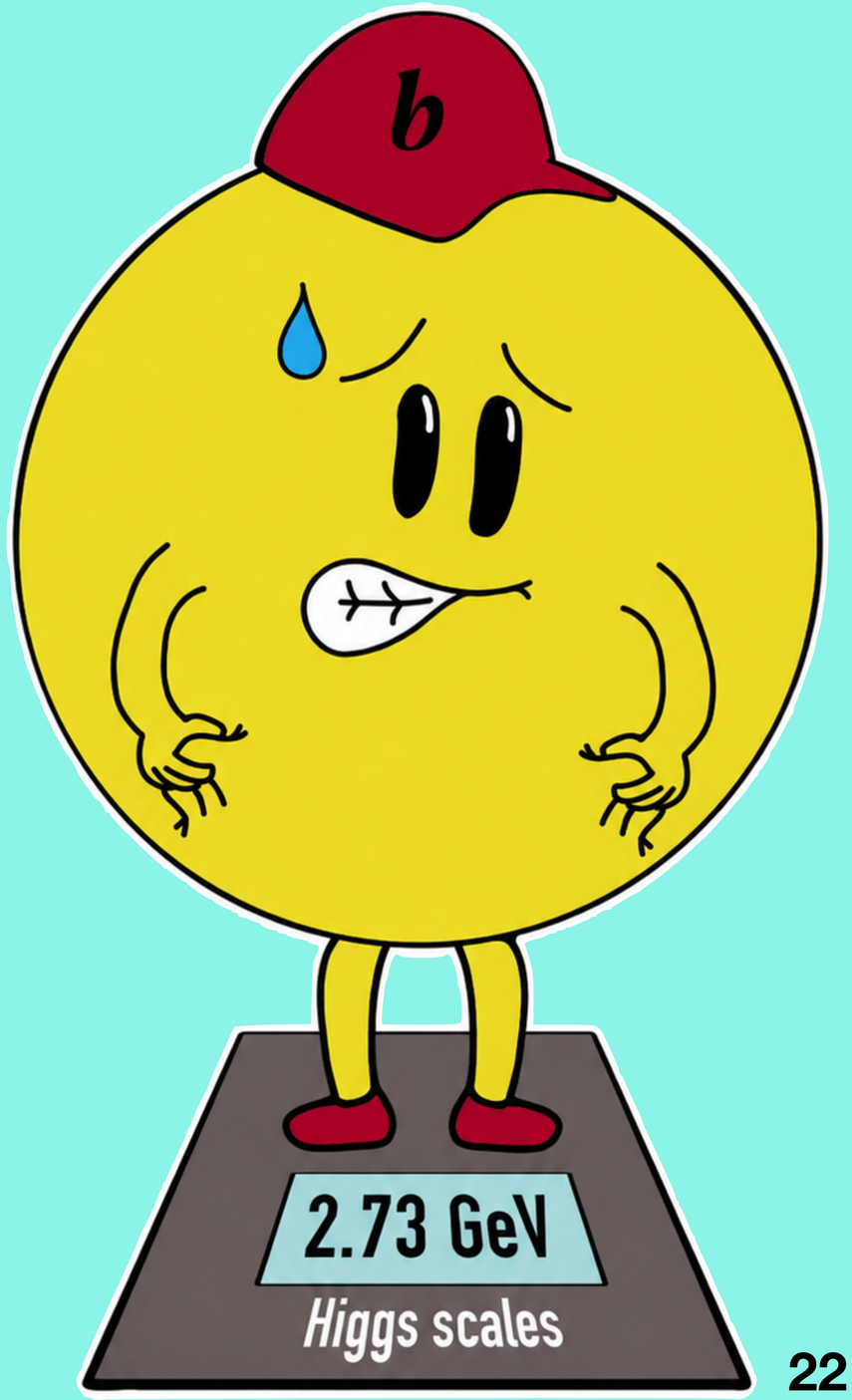
  - At Higgs factory, **similar sensitivity to ATLAS extraction** (10%) but more model-independent.

  - Still **preliminary work**. More detailed studies are required to determine actual feasibility.

Thank you for your attention!

Contacts (✉️): juan.ramirez.alfaro@cern.ch, maria.moreno.llacer@cern.ch, marcel.vos@cern.ch

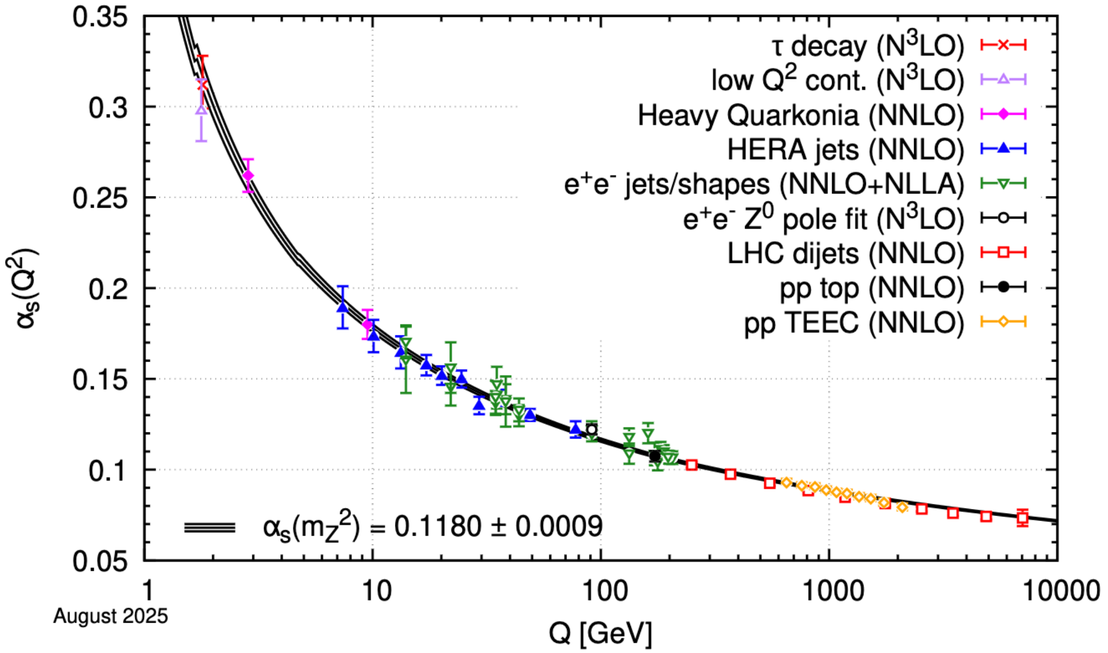
This work is supported by FPU24/02210 PhD grant



Backup

Backup: The SM and running parameters

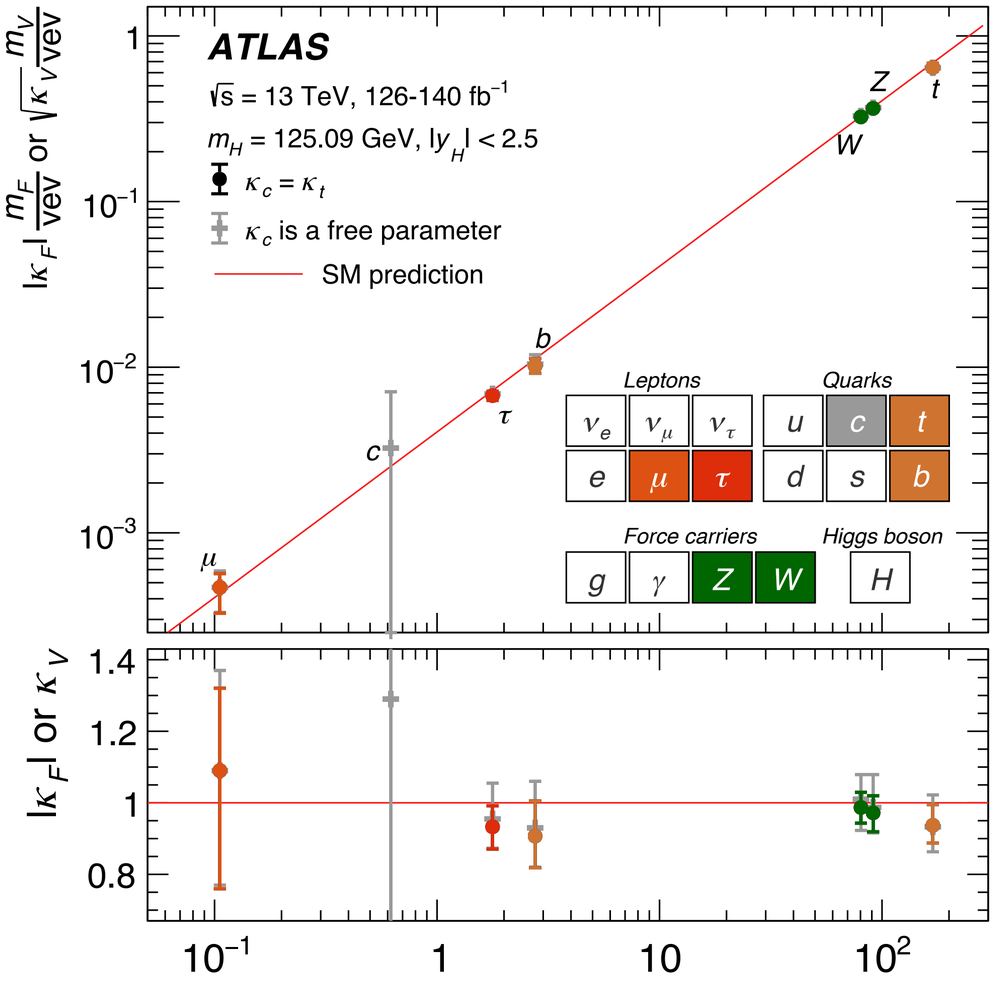
- Standard Model (SM): best description of particle physics up to date. Theory framework: some SM quantities (masses, couplings) are treated as free parameters, typically renormalized; two key properties:
  - Values must be determined experimentally at a given renormalization energy scale μ (or Q).
  - A reference measurement at energy μ_0 can be evolved (Renormalization Group Equations) to find the value at any other μ i.e. the parameter evolves/“runs” with the scale: running constants.
- Precise predictions for QCD parameters’ running (strong coupling $\alpha_s(\mu)$ and quark masses $m_q(\mu)$).



F. Takahashi *et al.* (Particle Data Group), Int. J. Mod. Phys. A 41, 2630011 (2026)

Backup: Why do we assume $(y_b)_{SM}$?

- So far, there has been **no evidence of the opposite**!
- Higgs coupling **measurements** up to date **align with the Standard Model** prediction: <u>the coupling of the Higgs to SM-particles depends linearly on the particle's mass</u>.
- If there are any new physics hiding around, they have a subleading effect.



ATLAS, arXiv:2608.07332 (submitted to ROPP)

Backup: Check that m_b from $H \to b\bar{b}$ is indeed $m_b(m_H)$

- The point can be illustrated by considering the **convergence of the perturbative series**. When taking $\mu = m_H$ as the renormalization scale for both m_b and α_s, the leading QCD correction series for $H \to b\bar{b}$ when expanding in m_b^2/m_H^2 takes the form:

$$1 + \delta_{\mathrm{QCD}} = 1 + 0.2030 + 0.0374 + 0.0019 - 0.0014$$

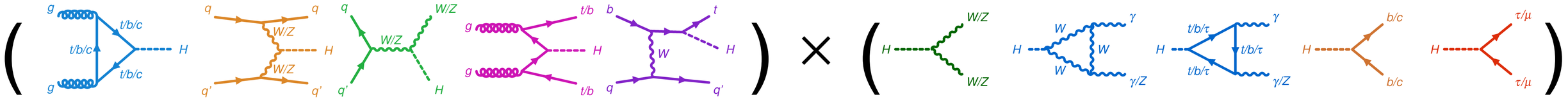
- However, when taking $\mu = m_b$ the convergence is much poorer and presents large perturbative uncertainties due to logarithmic uncertainties $\ln(m_H/m_b)$, which are otherwise resumed to all orders in $\mu = m_H$:

$$1 + \delta_{\mathrm{QCD}} = 1 - 0.5665 + 0.0586 + 0.1475 - 0.1274$$

- Renormalization scales can also be chosen independently for m_b and α_s. However, the **best results** for the convergence are obtained when **both scales** are in the **order of** m_H**.**

Backup: ATLAS Run 2 Coupling Measurements

- **Inputs**: Higgs boson production & decay channels (σ_i & B_f) measurements from **LHC Run 2** output → up to 140 fb^{-1} of pp collision data at $\sqrt{s} = 13$ TeV.



ggF, VBF, VH, ttH, bbH, tH

WW, ZZ, $\gamma\gamma$, γZ, bb, cc, $\tau\tau$, $\mu\mu$

+ DiHiggs production channels (not considered for $m_b(m_H)$ analysis)

- **Combination of 26 analyses** targeting individual production and decay processes. Measurements compared with state-of-the-art theory predictions.
- **Systematic uncertainties treatment**: identical sources (consistent definitions/calibration/ modelling schemes) are correlated across analyses.
- **Combined likelihood model** from inputs, results extracted from **profile likelihood ratio** test, $-2\ln L$.

663 POIs & **~6300** NPs !!!

NEW: arXiv 2608.07332 [hep-ex]

Backup: ATLAS uncertainty treatment (1)

- **Statistical Uncertainties**: Signal regions of input analyses non-overlapping by construction.
  - Some CRs for background processes overlap with SRs from other analyses, but the effect is negligible w.r.t. the total uncertainty.
- **Systematic Theory Uncertainties**:
  - Uncertainties on Higgs boson production x-secs and BRs applied to SM predictions
  - From missing higher-order terms and PDFs; treated as correlated for identical sources (e.g. analyses using same PDF set).
  - α_s uncertainty, correlated across PDF and BR schemes.

Backup: ATLAS uncertainty treatment (2)

- **Systematic Experimental Uncertainties**:
  - Luminosity and pile-up modelling, correlated across input analyses.
  - Trigger efficiencies, correlated across analyses using same algorithms & configurations
  - Reconstruction, ID & calibration of γ, e, μ, τ, jets & MET, correlated when compatible schemes are used.
  - Hadronic jet definitions, JES & MET uncertainties described by a common set of sources (while schemes may vary across analyses), so correlated in most cases.
  - Large-radius jet resolution & mass scale uncertainties included in boosted analyses, assumed uncorrelated with the small-radius jet uncertainties.
  - Hadronic jet flavour tagging uncertainties, uncorrelated since input analyses often use different algorithms/WPs. Only correlated when those choices are compatible (only for on-shell and off-shell $H \to ZZ^*$, $H \to WW^*$ measurements).

Backup: ATLAS Combination Procedure

- **Combined likelihood model** built from input analyses' likelihoods.
  - Measurement parameters (**POIs**): signal rates in analysis regions. Up to **663**.
    - Can be Higgs boson production xsec, decay BRs, signal-strength parameters (ratio observed rate / SM expectation). The latter **includes uncertainties in the SM expectation**.
    - Quantities can also be expressed in terms of parameters of specific models e.g. $m_b(m_H)$!
  - Syst. uncs: enter as **NPs** (correlated uncs - same NP), introduce a **constraint** in the likelihood. About **6300**.
- Fitted for each POI using **profile likelihood ratio**, $-2\ln L$. NPs free to vary (baseline fit).
- **POI uncertainties decomposed** by either:
  - *Conditional uncertainty*: uncertainty from a set of NPs obtained by fixing them to their best-fit value and subtracting in quadrature the POI uncertainty in this configuration from the baseline one (Method used for $m_b(m_H)$ extraction).
  - *Shifted auxiliary observable*: not relevant for $m_b(m_H)$. For details, check paper (arXiv 2608.07332 [hep-ex]).

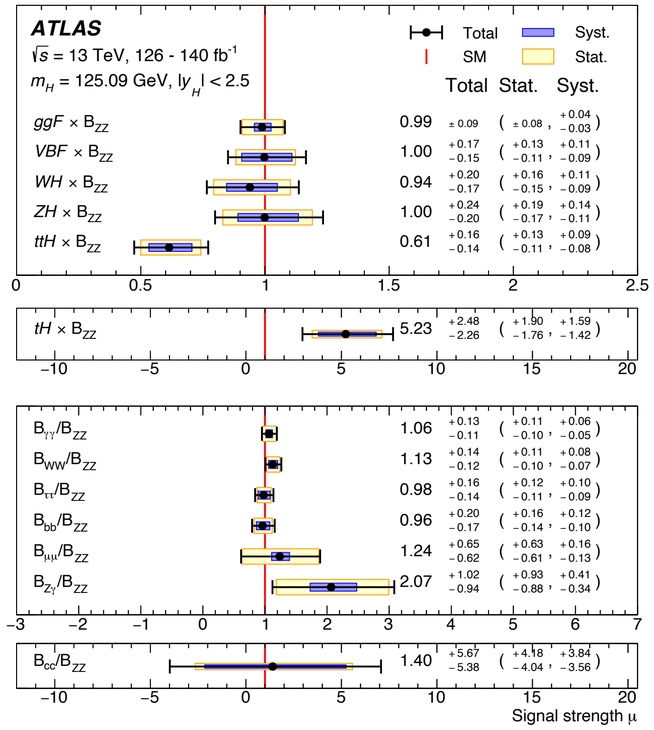
Backup: $m_b(m_H)$ extraction (1)

- Same combination of analyses as the one used for $\sigma_i \times B_f \equiv \sigma_i^f$ measurements.
- Under the **narrow width approximation**, σ_i^f can be **parametrized** using prod. x-secs in $H \to ZZ^*$ final state, σ_i^{ZZ}, and ratios of partial widths, Γ_f/Γ_{ZZ}, as

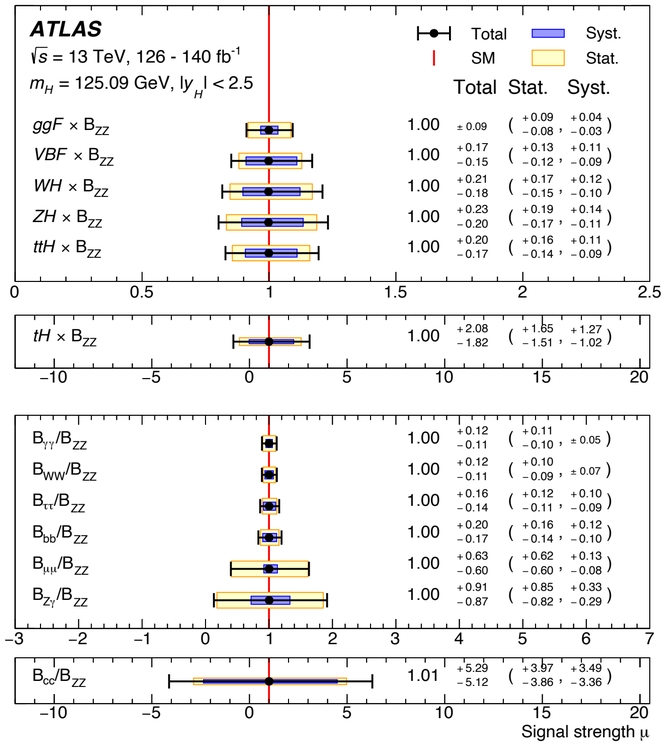
$$\sigma_i^f = \sigma_i^{ZZ}\left(\frac{\Gamma_f}{\Gamma_{ZZ}}\right)$$

- **POIs considered**: ggF, VBF, WH, ZH, ttH, tH (production) and bb, WW*, $\tau\tau$, cc, $\gamma\gamma$, $Z\gamma$, $\mu\mu$ (decays). Rest of channels are assumed to match its SM prediction.
  - Simultaneous fit of production & decay modes reduces sensitivity to potential deviations from SM predictions.
  - Use of "normalized" Γ_f/Γ_{ZZ} further allows convenient cancellations (width effects, correlated uncertainties…)

Observed



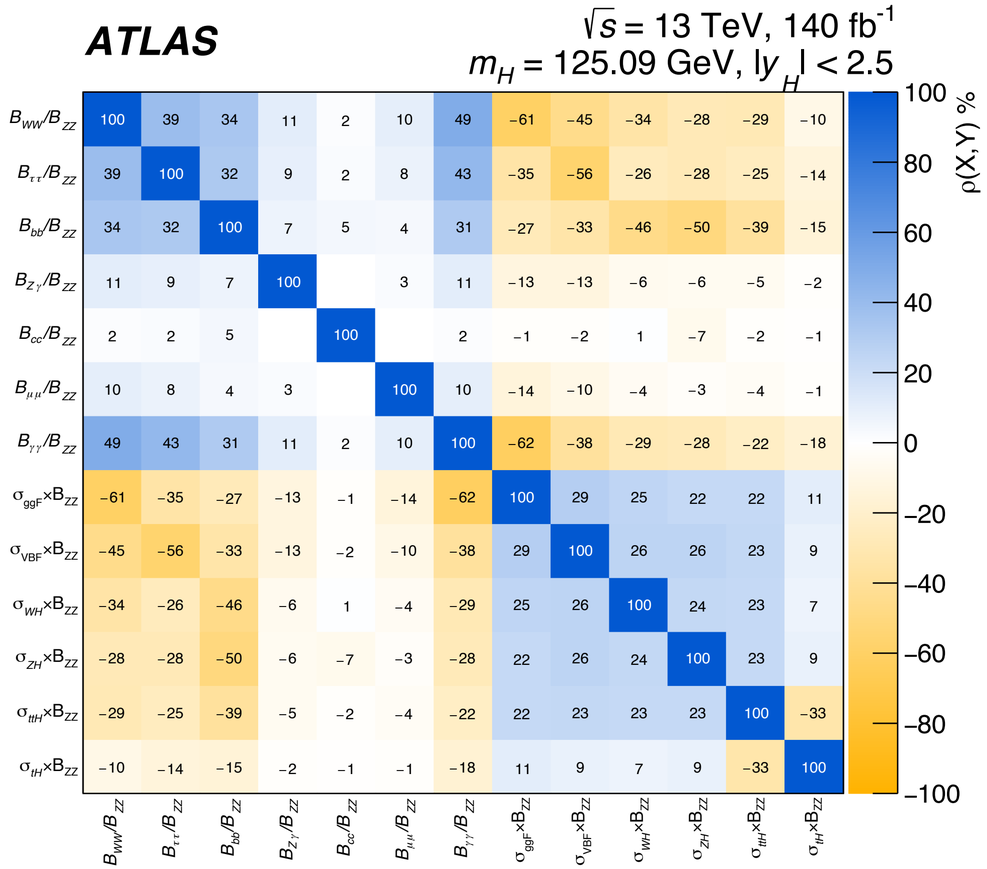
Expected (Asimov)



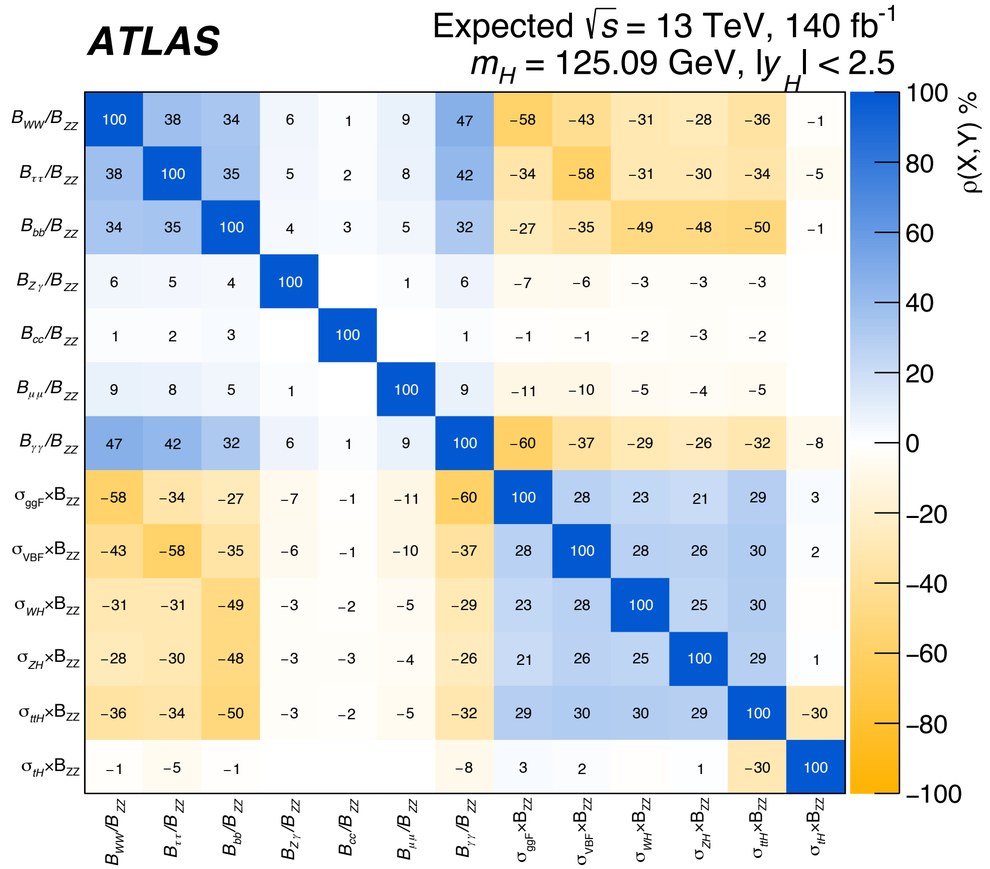
ATLAS, arXiv:2608.07332 (submitted to ROPP)

Backup: $m_b(m_H)$ extraction (1)

Observed



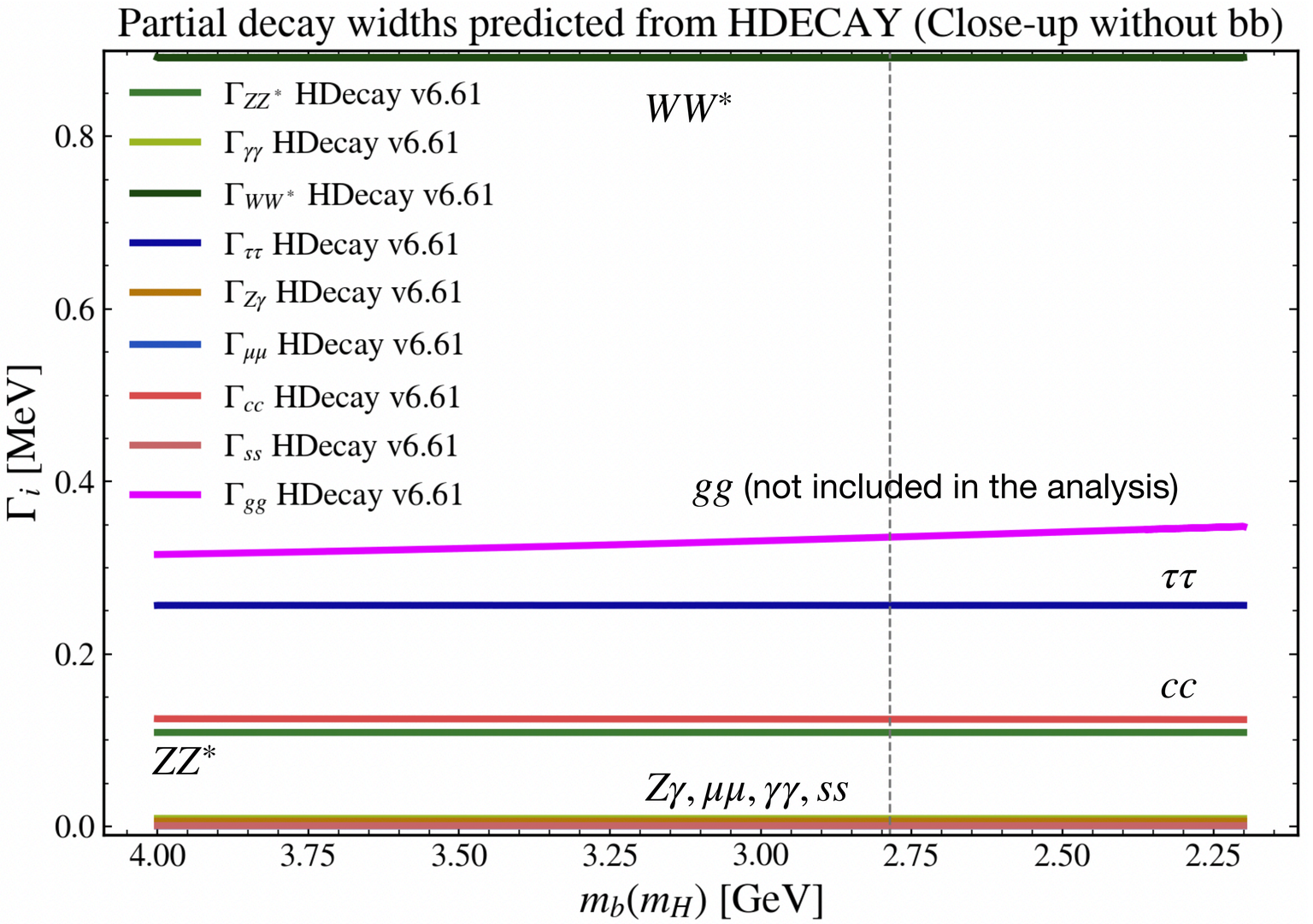
Expected (Asimov)



ATLAS, arXiv:2608.07332 (submitted to ROPP)

Backup: $m_b(m_H)$ extraction (2)

- **Only Γ_{bb}/Γ_{ZZ} is assumed to depend on $m_b(m_H)$, and so is expressed in terms of it.**
- The rest of x-secs and partial widths depend only weakly and are left free to vary in the fit (reducing sensitivity to potential deviations from SM).
- Note about the plot: ZZ and WW don't correspond to YR4 predictions, since it uses Prophecy4f results. Nevertheless, take-home message is that no dependence on $m_b(m_H)$ is seen on the relevant channels.



Backup: $m_b(m_H)$ extraction (3)

- Γ_{bb}/Γ_{ZZ} **parametrized as a function on** $m_b(m_H)$ following:

$$\frac{\left(\Gamma_{bb}/\Gamma_{ZZ}\right)_{obs}}{\left(\Gamma_{bb}/\Gamma_{ZZ}\right)_{SM}} = 1 + \sum_{n=1}^{5} a_n \left[\left(\frac{m_b(m_H)}{m_b^{ref}(m_H)}\right)^n - 1\right]$$

$\left(\frac{\Gamma_{bb}}{\Gamma_{ZZ}}\right)_{\mathrm{SM}}$	a_1	a_2	a_3	a_4	a_5
22.00	−0.07086	1.144	−0.1361	0.06038	−0.01167

- **Fifth-order polynomial**: empirical choice allowing for flexible interpolation.
  - Leading dependence is quadratic (at SM, $\Gamma_{bb} \propto m_b^2$), but higher order terms needed for subleading effects.
  - Stable results for polynomial orders between O(3) and O(6): corresponding changes in $m_b(m_H)$ negligible when compared to exp. uncertainties.

	mu; mb(mH)_SM	Diff w/ O(3)	Diff w/ O(4)	Diff W/ O(5)	Diff W/ O(6)
O(3)	1,0000712	0	-0,000015	-0,000020	-0,000073
O(4)	1,0000713	0,000015	0	-0,000005	-0,000058
O(5)	1,0000714	0,000020	0,000005	0	-0,000053
O(6)	1,0000719	0,000073	0,000058	0,000053	0

Comparison on the value of $\mu_{bb/ZZ} = f(m_b^{ref}(m_H))$ for different polynomial orders

- O(3)-O(5): 2.0e-5% rel. diff.
- O(4)-O(5): 4.5e-6% rel. diff.
- O(6)-O(5): 5.3e-5% rel. diff.

Backup: $m_b(m_H)$ extraction (4)

- Expression obtained from **HDECAY 6.61** (HD6.61). Offers Γ_{bb} computation at N4LO in QCD and NLO in EW corrections, and can use $m_b(m_H)$ as an input → **[2.2, 4.0] GeV scan with 0.5 MeV step**.
- **Corrections** applied to HDECAY predictions at $m_b(m_H) = m_b^{ref}(m_H) = 2.787$ GeV in order to match Yellow Report 4 result, assumed in the rest of the ATLAS work.
  - **Reason**: YR4 uses 1) an older HDECAY version for all rates and 2) Prophecy4f. **Differences**:
    - **bb and cc** partial widths differ b/w YR4 and HD6.61 by **0.8%** and **5%** respectively.
    - **ss** decay channel contributes in HD6.61 but NOT in YR4.
    - **WW and ZZ** values are obtained **from Prophecy4f**, which computes $H \to ZZ^*/WW^* \to 4f$. Thus, Prophecy4f also provides an **interference term** between ZZ and WW.
    - The use of Prophecy4f also induces a **change** in the **Total Width** of the Higgs boson. **Prescribed by YR4** as $\Gamma_H = \Gamma_H^{HD} - \Gamma_{WW}^{HD} - \Gamma_{ZZ}^{HD} + \Gamma_{4f}^{Proph} = \Gamma_H^{HD} - \Gamma_{WW}^{HD} - \Gamma_{ZZ}^{HD} + \left(\Gamma_{WW}^{Proph} + \Gamma_{ZZ}^{Proph} + \Gamma_{int}^{Proph}\right)$
  - **Rest of decay channels** agree b/w HD6.61 and YR4 with **below per mille level differences**.

Backup: $m_b(m_H)$ extraction (4)

- **Corrections applied** to the level of **partial widths**, the same for all $m_b(m_H)$ input values:
  - **Multiplicative correction** to **bb and cc,** $\kappa_b = 1.0082, \kappa_c = 0.9500$, such that $\Gamma_{qq}^{YR4} = \kappa_q \Gamma_{qq}^{HD,SM}$.
  - **WW and ZZ** HD6.61 values **replaced by Prophecy4f results**.
  - **Prophecy4f interference term** estimated from YR4 SM-values as $\Gamma_{int}^{Proph} = \Gamma_H - \sum \Gamma_i^{YR4}$.

$$\Gamma_H = \{\Gamma_H^{HD} - \Gamma_{WW}^{HD} - \Gamma_{ZZ}^{HD} + \Gamma_{WW}^{Proph} + \Gamma_{ZZ}^{Proph}\} + \Gamma_{int}^{Proph} \equiv \sum_i \Gamma_i^{YR4} + \Gamma_{int}^{Proph}$$

  - **Total Width** for each $m_b(m_H)$ value is modified according to YR4 prescription + changes in bb/cc/ss:
    - First apply quark corrections to HD6.61 width (Γ_H^{HD}): $\Gamma_H^{HD,q-corr} = \Gamma_H^{HD} + (\kappa^b - 1)\Gamma_{bb}^{HD} + (\kappa^c - 1)\Gamma_{cc}^{HD} - \Gamma_{ss}^{HD}$
    - From Γ_{int}^{Proph} estimation, compute Prophecy4f correction $\Gamma_{4f}^{Proph} = 0.9915$ MeV
    - Follow the YR4 prescription to recompute the width, $\Gamma_H^{HD,corr} = \Gamma_H^{HD,q-corr} - \Gamma_{WW}^{HD} - \Gamma_{ZZ}^{HD} + \Gamma_{4f}^{Proph}$

$\Gamma_H^{HD,corr}$ for $m_b(m_H) = m_b^{ref}(m_H)$ differs from YR4 value by less than **0.0002%**!

Backup: Running test set up

- Testing parametrization $m_b(\mu; x, m_b(m_b)) = x\left[m_b^{SM}(\mu; m_b(m_b)) - m_b(m_b)\right] + m_b(m_b)$ for running. Two parameters, $m_b(m_b)$ (reference mass for running) and x (characterizes running).
  - Impact of $\alpha_s(m_Z)$ uncertainty on the RGE evolution m_b^{SM} is also tested → introduced as **NP**.
- From **REvolver** software, compute a **3d grid** $m_b(m_b) \times \mu \times \alpha_s(m_Z)$ of m_b^{SM} points & interpolate them to get $m_b^{SM}(\mu; m_b(m_b), \alpha_S(m_Z))$ function. Then fit the experimental points to this function.
- **Likelihood fit**: approximate the experimental points $(\mu_i, m_b^{i\ +\sigma_i^+}_{\ -\sigma_i^-})$ term by a split gaussian likelihood (we don't have access to all the points' likelihoods). For the $\alpha_s(m_Z)$ uncertainty on the running, introduce a gaussian constraint term with mean $\alpha_s(m_Z)^{nom} = 0.1180$ and std. dev. $\sigma_{\alpha_s(m_Z)} = \delta_{\alpha_s} = 0.004$.

$$-2\ln L(m_b(m_b), x, \alpha_s(mZ)) = \sum_i \frac{\left(m_b^i - m_b(\mu_i; m_b(m_b), x, \alpha_s(mZ))\right)^2}{\sigma_i^2} + \frac{\left(\alpha_s(m_Z) - \alpha_s(m_Z)^{nom}\right)^2}{\sigma^2_{\alpha_s(m_Z)}}$$

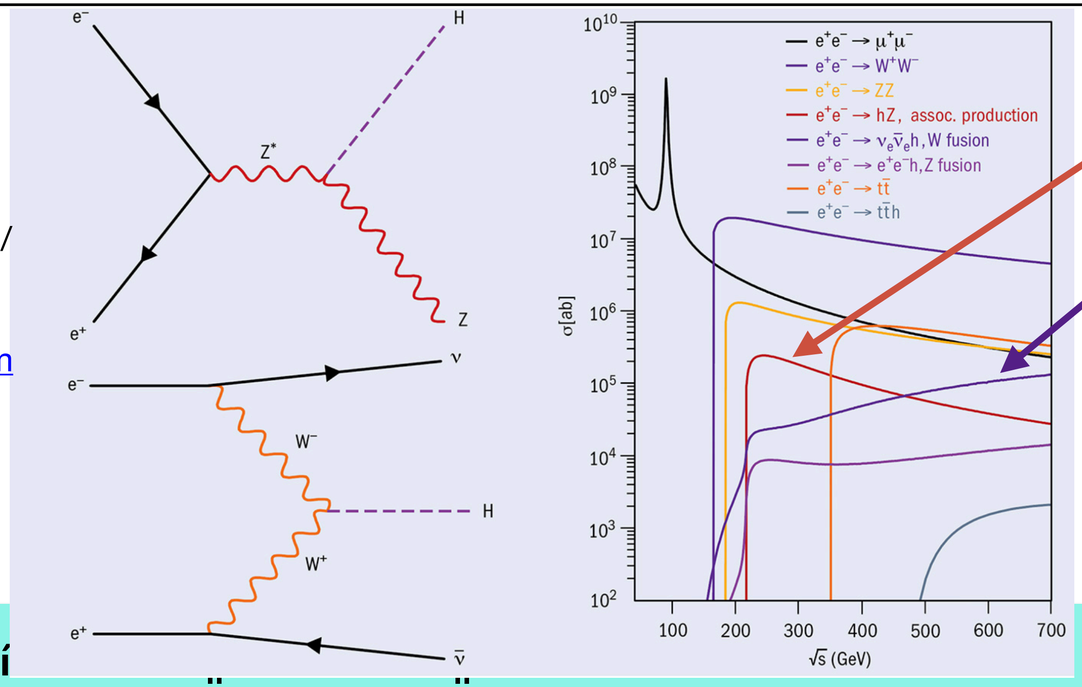
$$\sigma_i = \sigma_i^+ \text{ for } m_b^i < m_b(\mu_i; m_b(m_b), x)$$

$$\sigma_i = \sigma_i^- \text{ for } m_b^i > m_b(\mu_i; m_b(m_b), x)$$

Backup: e^+e^- colliders as Higgs factories

- From arXiv:2503.19983v3 [hep-ex]:

> An e^+e^- collider is the ideal machine for precision measurements. Because electron and positrons are elementary particles, their reactions are simple and display the structure of the underlying interactions directly. Backgrounds are dominated by electroweak processes, and these are also simple and — more importantly — precisely understood at the part-per-mil level. The low event rates relative to proton collisions allow the construction of low-material-budget, high-precision detectors and of trigger-less data taking. All of these features minimize systematic uncertainties. This makes it possible to measure small deviations from the SM with high confidence and credibility.



Credit: R. K. Ellis; R. K. Ellis / Durham. Taken from https://cerncourier.com/a/targeting-a-higgs-factory/

HZ associated production; dominant at ~[200, 450] GeV

WW fusion ($\nu_e\bar{\nu}_e H$); dominant above 450 GeV

- **LCF250 & FCCee240** are mostly dominated by HZ prod.
- **FCCee365** still dominated by HZ, but WW fusion is now relevant.
- **LCF550** explores the region where WW fusion is dominant.

Backup: Higgs boson couplings at Higgs factories

- Higgs couplings are expected to be **measured at the Higgs factories with unprecedented precision** [7]:
  - $H \to bb/\tau\tau/cc$ projected to reach **sub-% relative precision**
  - $H \to WW^*/gg$ will reach **high precision** as well. However, $H \to ZZ^*$ **won't be that competitive** (smaller BR in comparison).
  - Rare decays like $H \to \gamma\gamma$ are also **limited by their small BRs** and won't be measured with the same precision as in the HL-LHC.
- **Best prospect** for $m_b(m_H)$ analysis: bb/WW^* **ratio**, expected to achieve **sub-% precision**:

Operation	Relative precision on $BR(H \to bb)/BR(H \to WW^*)$
LCF250	0.65 %
LCF550	0.38 %
LCF250+550	0.33 %

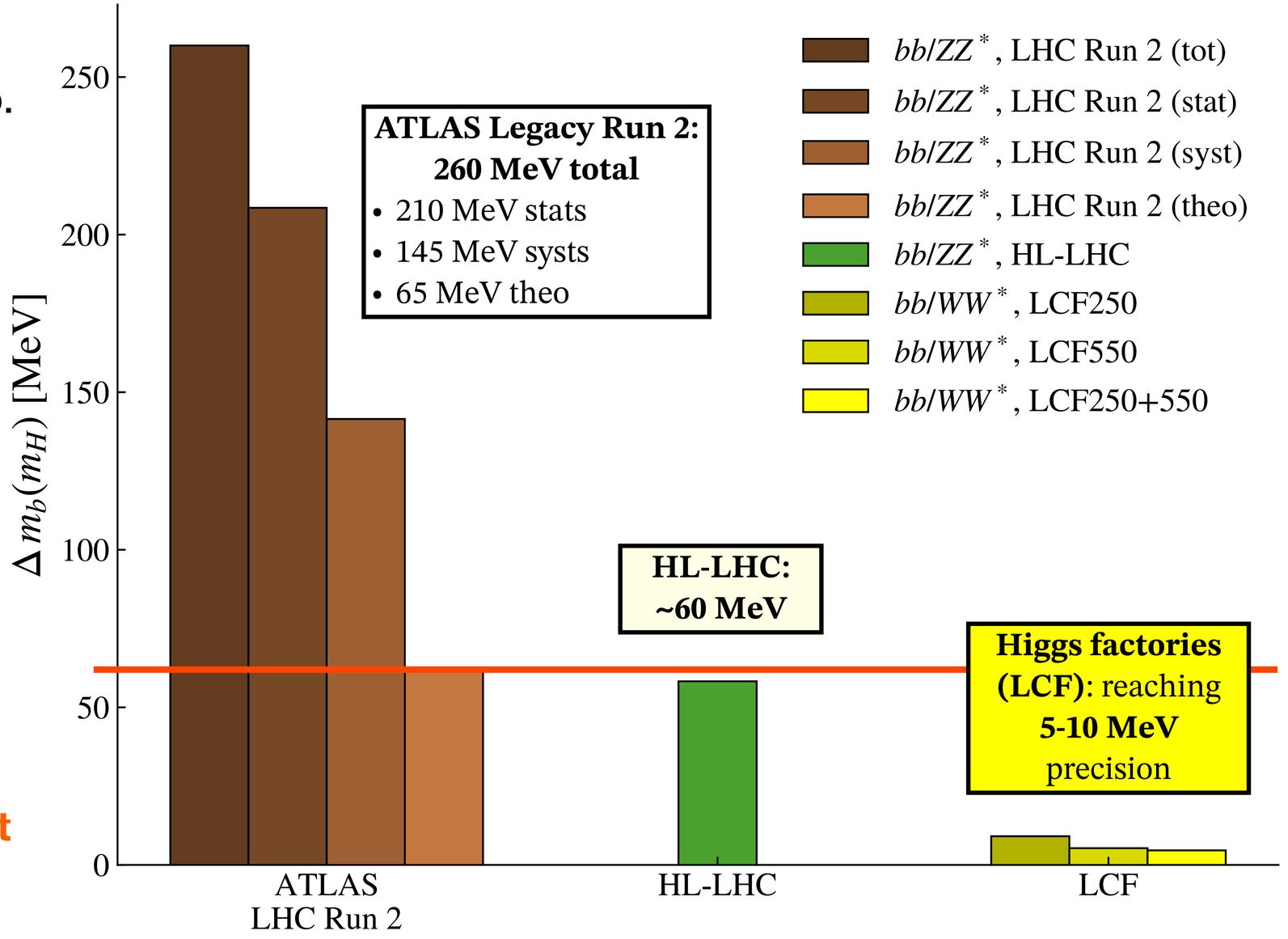
Preliminary LCF estimates, kindly provided by Junping Tian (Tokyo U., ICEPP)

Backup: Projected uncertainties for Future Facilities

- **HL-LHC estimates** (preliminarily) obtained by scaling the ATLAS ’26 exp. uncs. with the integrated lumi L as $1/\sqrt{L}$ (Scenario 2). NOTE: no Theo. Uncs. considered!
- **Higgs factories (LCF) estimates** come from bb/WW^* estimates.

Operation	Relative precision on $BR(H \to bb)/BR(H \to WW^*)$
LCF250	0.65 %
LCF550	0.38 %
LCF250+550	0.33 %

- At Higgs factories, $m_b(m_H)$ has the **potential to compete** with $m_b(m_b)$ (expected to go **below 10 MeV**) for being the **most precise measurement of** m_b.



Backup: Motivation for using R_3^b in simultaneous measurement

In the Standard Model (SM), the partial decay width of a Higgs boson into a pair of bottom quarks is given - at leading order (LO) - by the following expression:

$$\Gamma\left(H \to b\bar{b}\right) = \frac{3\,m_H}{16\,\pi}\,y_b^2\,\left(1 - 4\frac{m_b^2}{m_H^2}\right)^{3/2}, \qquad (5)$$

where y_b is the Yukawa coupling between the Higgs boson and the bottom-antibottom pair, and m_H and m_b are the masses of the Higgs boson and the bottom quark, re-

The dependence on y_b is quadratic, a feature that is retained even after introducing higher-order quantum chromodynamics (QCD) and electroweak (EW) corrections to the decay width. There is a much weaker dependence on the bottom quark mass. In practice, the factor $(1 - \frac{m_b^2}{m_H^2})^{3/2} \gtrsim 0.999$ is smaller than a per mille of 1, and therefore numerically negligible at the current precision.

Pure mass effects can be enhanced in differential predictions, such as for example, the gluon emission rate of bottom quarks, that was used to measure $m_b(m_Z)$ at LEP [18]. These take advantange of the suppression of collinear gluon radiation of the bottom quark, known as the "dead cone" effect, to define a mass-sensitive observable R_3^b based on the three-jet rate in $Z \to b\bar{b}$ events.

The enhanced sensitivity to mass effects can be understood by considering that, while inclusive observables depend on the quark mass m_q and the energy of the process Q as m_q^2/Q^2, the dependence for differential observables changes to $m_q^2/(y_{cut}\,Q^2)$. Here, y_{cut} is the jet resolution variable, a parameter of jet-clustering algorithms that defines the jet multiplicity [19] and is generally much smaller than unity, crucially increasing the size of these mass terms.

Applying this logic to $H \to b\bar{b}$ decays (for which $Q^2 = m_H^2$), the three-jet decay width has a sensitivity to the bottom quark mass that can be estimated at LO by modifying the mass-dependent factor from the inclusive rate (5) to $(1 - \frac{m_b^2}{y_{cut}\,m_H^2})^{3/2}$. For y_{cut} values between 0.05 and 0.01, the sensibility lies from 3% to 9% and clearly improves the sub-per mille level estimations of the inclusive rate. Therefore, a well-motivated choice for the mass-sensitive observable in $H \to b\bar{b}$ events is

$$R_3^b(y_{cut}) = \frac{\Gamma_{3j}^b(y_{cut})}{\Gamma^b}, \qquad (6)$$

Backup: Technicalities of NEWiSH predictions

This contains results for $H \to b\bar{b}$ and $H \to b\bar{b}g$ computed using NEWiSH for various values of $m_b(\mu)$, and Durham parameter y_{cut} in the SM and for the coefficient $C_{d\phi}[33]$ which generates a deviation in the bottom Yukawa coupling:

$$y_b = y_b^{\rm SM} + \delta y_b = y_b^{\rm SM} - \frac{v^2}{\Lambda^2} C_{d\phi}[33]$$

We work at $\mathcal{O}(1/\Lambda^2)$ and set $\Lambda = 1$ TeV and $C_{d\phi}[33] = 1$. Since we are working at linear order, different values of Λ or $C_{d\phi}[33]$ can be obtained by simple rescaling.

We use Durham clustering defined by

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)}{Q^2}(1 - \cos\theta_{ij})$$

where θ_{ij} is the angle between p_i and p_j and $Q^2 = m_H^2$ since we are always in the rest frame of the Higgs.

Both parameters $C_{d\phi}[33](\mu)$ and $m_b(\mu)$ are in the $\overline{\rm MS}$ scheme and taken as inputs defined at the scale μ. We have results at $\mu = [m_H/2, m_H, 2m_H]$, with corresponding $\alpha_S = [0.1251, 0.1126, 0.1024]$ determined from RunDec.

Note that no RG running is used for m_b or $C_{d\phi}[33]$ since it would require the full SMEFT RGE's, and so in our scale variation we are taking our input m_b, $C_{d\phi}[33]$ to be at the varied scale rather than at $\mu = m_H$.

The directory "results/" contains the processed datafile for inclusive $H \to b\bar{b}$ and $H \to b\bar{b}g$ results in a self-explanatory format. The "plotter.nb" shows how

From his numbers, we define:

$$\Gamma_3^b(y_{cut}) = \frac{H \to b\bar{b}g(y_{cut})}{H \to b\bar{b}}$$

Where the numerator can either be computed at LO or NLO, and the result depends on mb(mH) and yb (mimicking mb(mZ) extraction at DELPHI, https://arxiv.org/pdf/hep-ex/0603046

This observable is further normalized by its SM predicted value (mb(mH)=2.787 GeV, yb=yb^{SM} <--> $C_{d\phi}[33]$=0)

Values generated for

- mb = [1.5, 5.5] GeV, 0.05 GeV step
- $C_{d\phi}[33]$=0 (SM) and $C_{d\phi}[33]$=1

Backup: Estimation of R_3^b statistical uncertainties

To study the feasibility of the simultaneous extraction proposed in this work, we have estimated the sensitivity of the observables by simulating an experimental measurement of $H \to b\bar{b}X$ events at the FCC-ee [27, 28], the future flagship installation at CERN for Higgs physics studies. We consider the FCC-ee $\sqrt{s} = 240$ GeV operation point and the Higgstrahlung $e^+e^- \to ZH$ production process, since it is the dominant Higgs boson production mode at that energy (*HZ point* for short) [27].

The production cross-section of this process is given by

$$\sigma_{ZH} = \frac{G_F^2 m_Z^2}{96\pi s} \left(v_e^2 + a_e^2\right) \lambda^{1/2} \frac{\lambda + 12 m_Z^2/s}{\left(1 - m_Z^2/s\right)^2}, \quad (9)$$

where $a_e = -1$, $v_e = -1 + 4\sin^2\theta_w$ and $\lambda = \left[1 - \left(m_H + m_Z\right)^2/s\right]\left[1 - \left(m_H - m_Z\right)^2/s\right]$. Substituting the inputs by its PDG values [1] allows us to predict $\sigma_{ZH} = 238.13$ fb at $\sqrt{s} = 240$ GeV.

This result can be further used to estimate the number of $H \to b\bar{b}X$ events N_{bbX} that would be recorded at the FCC-ee for the considered Higgs production mechanism and energy. Such number can be computed as $N_{bbX} = \mathcal{L} \cdot \sigma\left(e^+e^- \to ZH(\to b\bar{b}X)\right)$, where $\mathcal{L}$ is the recorded integrated luminosity and $\sigma\left(e^+e^- \to ZH(\to b\bar{b})\right)$ is the cross-section of the interest Higgs boson process. Under the well-justified narrow width approximation, the process cross-section can be factorized into its production and decay contributions, and N_{bbX} can be expressed as

$$N_{bbX} = \mathcal{L} \cdot \sigma_{ZH} \cdot BR(H \to b\bar{b}X), \quad (10)$$

where $BR(H \to b\bar{b}X)$ is the branching fraction of the corresponding decay mode.

We assume $\mathcal{L} = 5$ ab^{-1} since this is the physics goal of the FCC-ee HZ operation point [27]. For estimating the branching ratio, we consider a SM decay of the Higgs boson, so that we can use our NEWiSH predictions of Γ^b (at NLO) and $\Gamma^b_{3j}(y_{cut})$ for $m_b = 2.787$ GeV, $C_{d\phi}[33] = 0$ and $\mu = m_H$. To further transform them into branching fractions, we take the SM value of the total Higgs decay width to be $\Gamma_H = 4.0879$ MeV. This has been

These estimates on $N_{bb(j)}$ and their uncertainties can be further used to provide a statistical uncertainty on Γ^b and Γ^b_{3j} as $\left(\delta\Gamma^b_{(3j)}/\Gamma^b_{(3j)}\right)_{stat} = \delta N_{bb(j)}/N_{bb(j)} = 1/\sqrt{N_{bb(j)}}$; which in turn can be propagated to provide an estimate of the statistical uncertainty on R_3^b as

$$\left(\delta R_3^b/R_3^b\right)_{stat} = \sqrt{(1/N_{bb}) + (1/N_{bbj})}. \quad (11)$$

Finally, to provide an estimated mass sensitivity, we can parametrize our predictions of $R_3^b/(R_3^b)_{SM}$ in terms of the $\overline{\text{MS}}$ b-quark mass m_b, for each fixed y_{cut} value and with $C_{d\phi}[33] = 0$, $\mu = m_H$. An order 4 polynomial is chosen for each parametrization; different orders have also been tested, and they provide the same results. To propagate the R_3^b uncertainties, we have assumed a central value of $R_3^b/(R_3^b)_{SM} = 1$, since the statistical uncertainties are estimated under the SM scenario. The resulting

Backup: Estimation of R_3^b statistical uncertainties

Our estimations of N_{bb} and N_{bbj} for addressing statistical uncertainties

y_{cut}	Γ^b [MeV]	Γ^b_{3j} [MeV]	N_{bb}	N_{bbj}
0.001	2.279	1.423	663800 ± 800	414400 ± 600
0.01		0.566		164800 ± 400
0.1		0.085		24830 ± 160