# Multi-channel phase-space integration with Feynman-diagram-gauge amplitudes for high-energy lepton colliders

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on

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# Precision simulation at multi-TeV lepton colliders

Lepton colliders provide clean final states over a wide angular range.

Heavy particles in the central region can accompany leptons that escape close to the beam.

FD-gauge (Feynman-diagram-gauge) amplitudes and dedicated phase-space maps let us calculate five-body cross sections up to 100 TeV.

- Numerical stability in the forward region
- Efficient integration on CPUs and GPUs

Kaoru Hagiwara, Junichi Kanzaki, Fabio Maltoni, Kentarou Mawatari, Ya-Juan Zheng

"Multi-channel phase space with Feynman-diagram-gauge amplitudes"

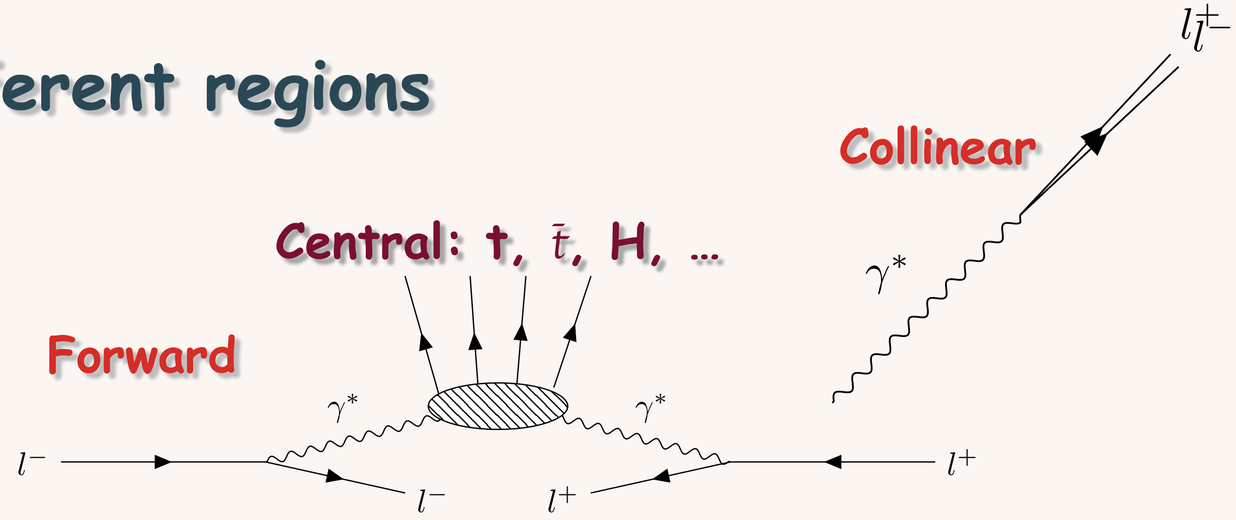
Phys. Rev. D 114, 036002 (2026), arXiv:2603.01139 [hep-ph]

Presented by J. Kanzaki at LCWS2026 in Nov. 10, 2026

# Why the phase-space integral is difficult

## Different diagrams peak in different regions

Collinear lepton pairs and forward photon exchange produce narrow peaks.




## The forward region matters

Even leptons emitted too close to the beam to be detected can affect the cross section of the observed final state.

## The full amplitude must be integrated

Sum the amplitudes before squaring to retain interference. Use a phase-space map suited to each peak.

# Single-diagram-enhanced multi-channel integration

Start with the squared coherent sum of all diagrams.

n: number of diagrams

Weight: $w_j$, $\sum w_j = 1$

$$\int d\Phi \left|\sum_{i=1}^{n} \mathcal{M}_i\right|^2 = \sum_{j=1}^{n} \int d\Phi \frac{|\mathcal{M}_j|^2}{\sum_k |\mathcal{M}_k|^2} \left|\sum_{i=1}^{n} \mathcal{M}_i\right|^2$$

Singular behaviors

$$= \sum_{j=1}^{n} \int d\Phi \, |\mathcal{M}_j|^2 \frac{\left|\sum_i \mathcal{M}_i\right|^2}{\sum_k |\mathcal{M}_k|^2}$$

F = O(1) from interference in the FD gauge

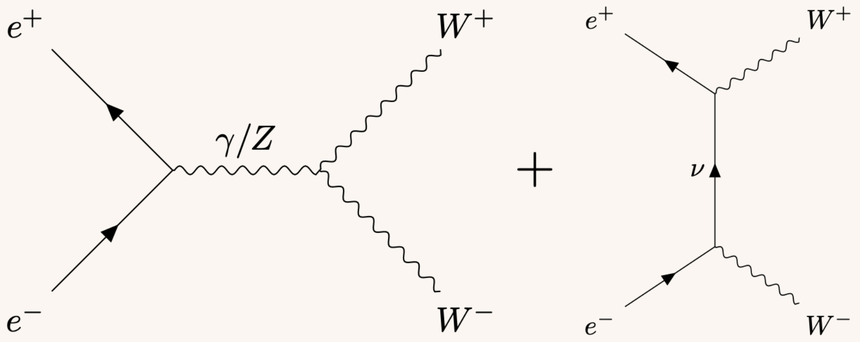
Integrate each channel with its own map

Use phase-space variables adapted to the propagators of diagram j, together with the corresponding Jacobian.

Maltoni and Stelzer, JHEP 02 (2003) 027
Hagiwara et al., Phys. Rev. D 114, 036002 (2026), arXiv:2603.01139

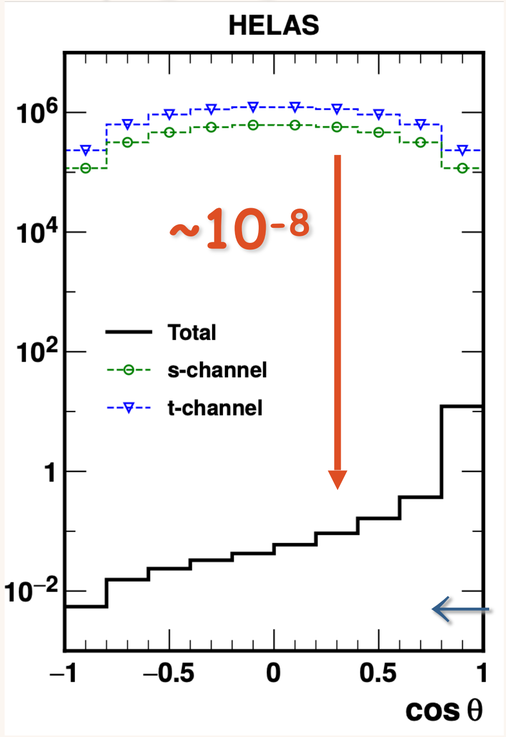
# FD gauge and gauge cancellations

$$e^+e^- \to W^+W^-$$




@ 10 TeV

## Unitary gauge




$$\text{s-channel: } |\mathcal{M}_\gamma|^2 + |\mathcal{M}_Z|^2$$

$$\text{t-channel: } |\mathcal{M}_\nu|^2$$

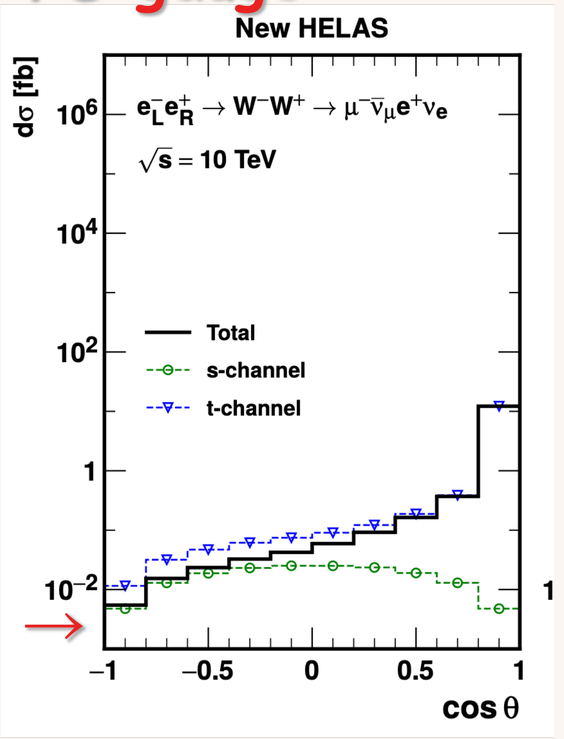
$$\text{total: } |\mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_\nu|^2$$

$$\mathcal{F} = \frac{|\mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_\nu|^2}{|\mathcal{M}_\gamma|^2 + |\mathcal{M}_Z|^2 + |\mathcal{M}_\nu|^2}$$

← F ≪ 1

F = O(1) →

## FD gauge




From J.C, K.H, J.K., K.M.: Eur.Phys.J.C 83 (2023) 10, 922, arXiv:2203.10440 [hep-ph]

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# Phase-space maps from diagram topologies

$$\mu^-\mu^+ \to \nu_\mu \bar{\nu}_\mu t\bar{t}H$$

## 89 diagrams, 12 topologies(*)

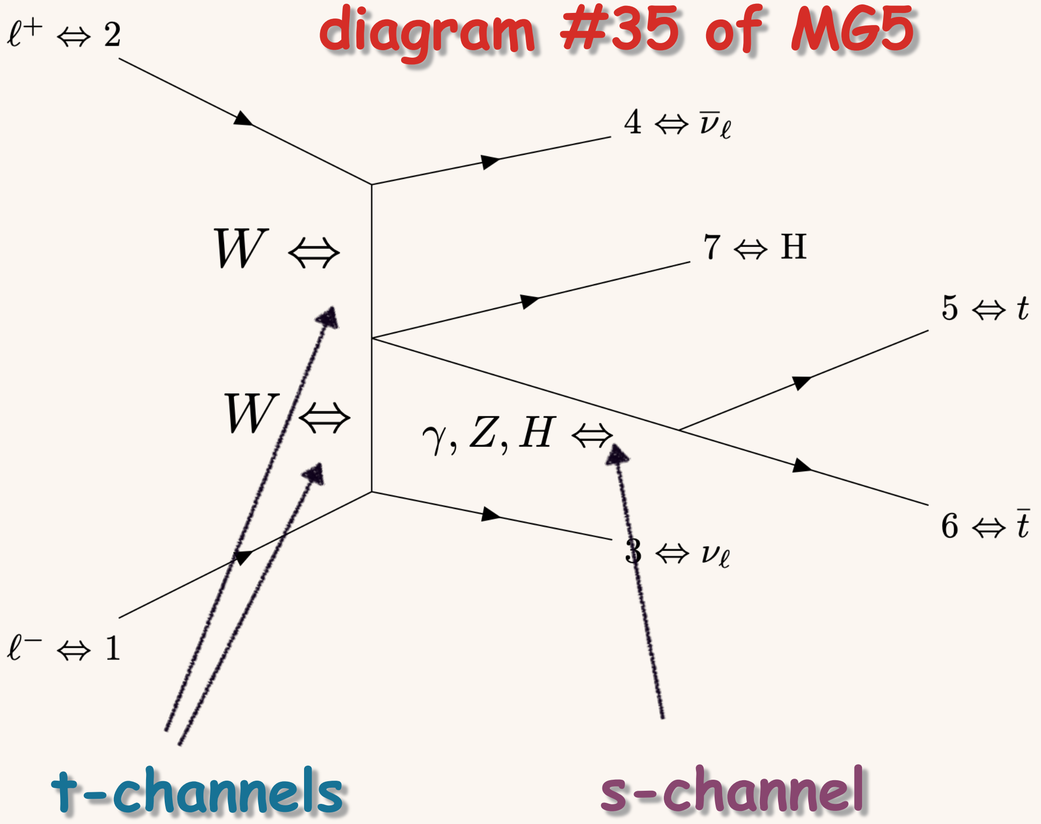
The SM final state contains a neutrino pair, a top-antitop pair and a Higgs boson.

## Reuse the same map

Assign the external momenta and internal propagator species for each diagram.

dshat for subsystem masses
ph2s and ph2t for two-body splittings

diagram #35 of MG5




* Our calculation is based on the FD gauge output from MadGraph5 (MG5): http://madgraph.phys.ucl.ac.be/

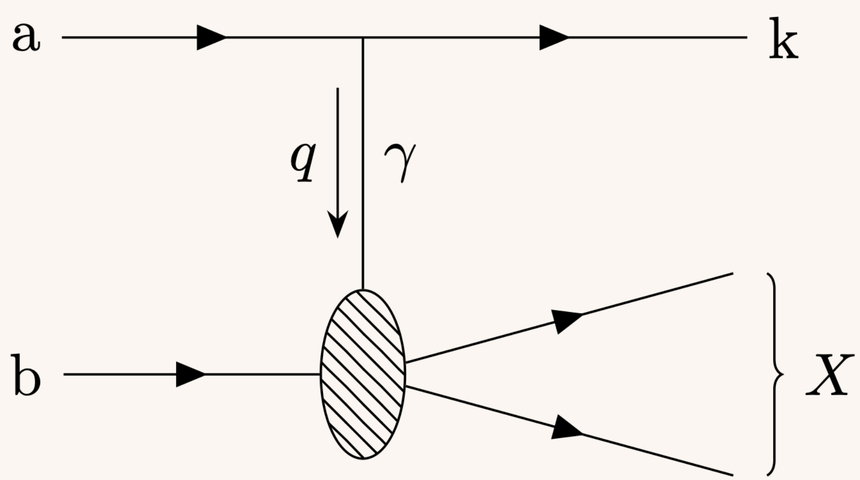
See Hagiwara et al., Phys. Rev. D 114, 036002 (2026), arXiv:2603.01139, Sec. III

# Sampling the very forward region

## Resolve small photon virtualities

$Q^2 \equiv -q^2 > 0, \quad Q^2 \geq Q_{min}^2$

Evaluate the minimum stably; it depends on lepton mass and kinematics.




## Sample in log $Q^2$

The Jacobian cancels one power of the photon propagator denominator.

## Keep the interference in a direct calculation

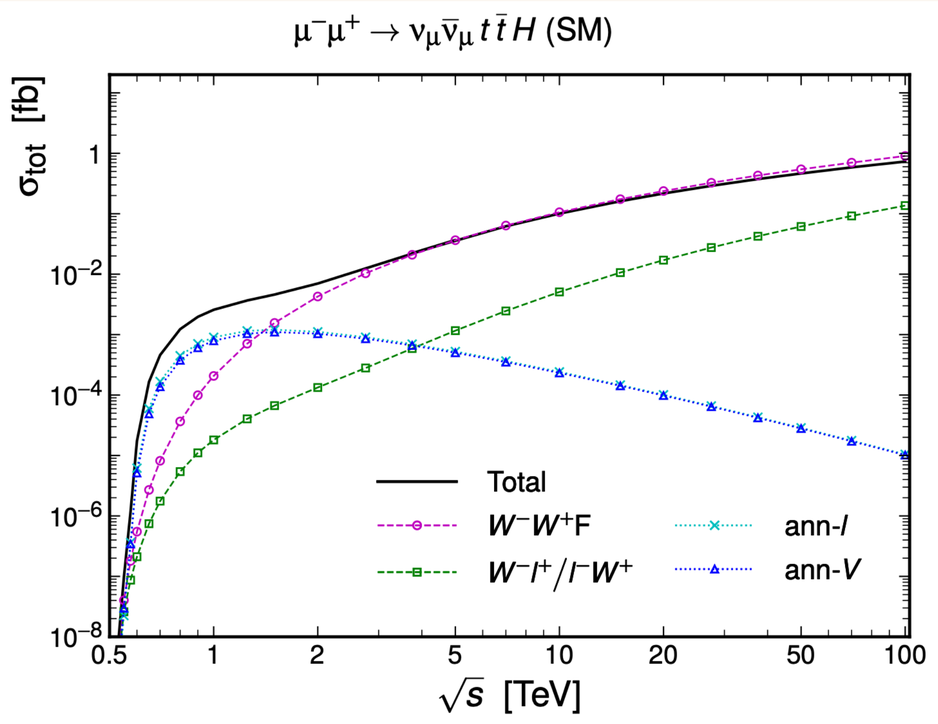
FD-gauge diagrams expose the physical singular behaviors.
We retain γ–Z and transverse–longitudinal interference.

Ordinary incoherent PDF approximations omit these terms.

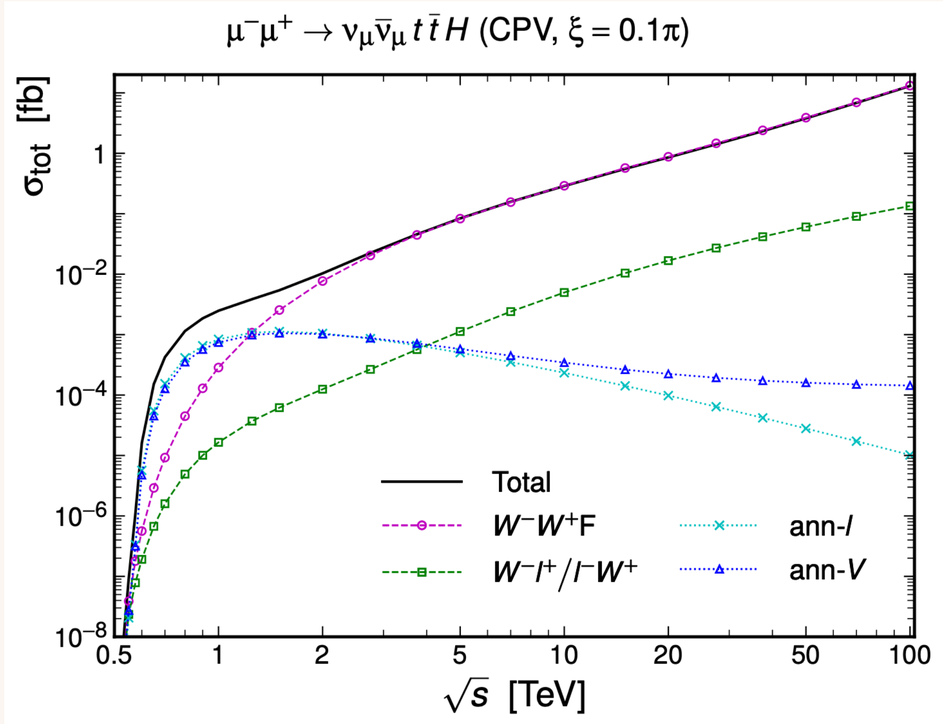
Hagiwara et al., Phys. Rev. D 114, 036002 (2026), arXiv:2603.01139, Secs. II–III

# Five-body cross sections up to 100 TeV

$$\mu^-\mu^+ \to \nu_\mu \bar{\nu}_\mu t\bar{t}H$$




Standard Model




CP-violating SMEFT

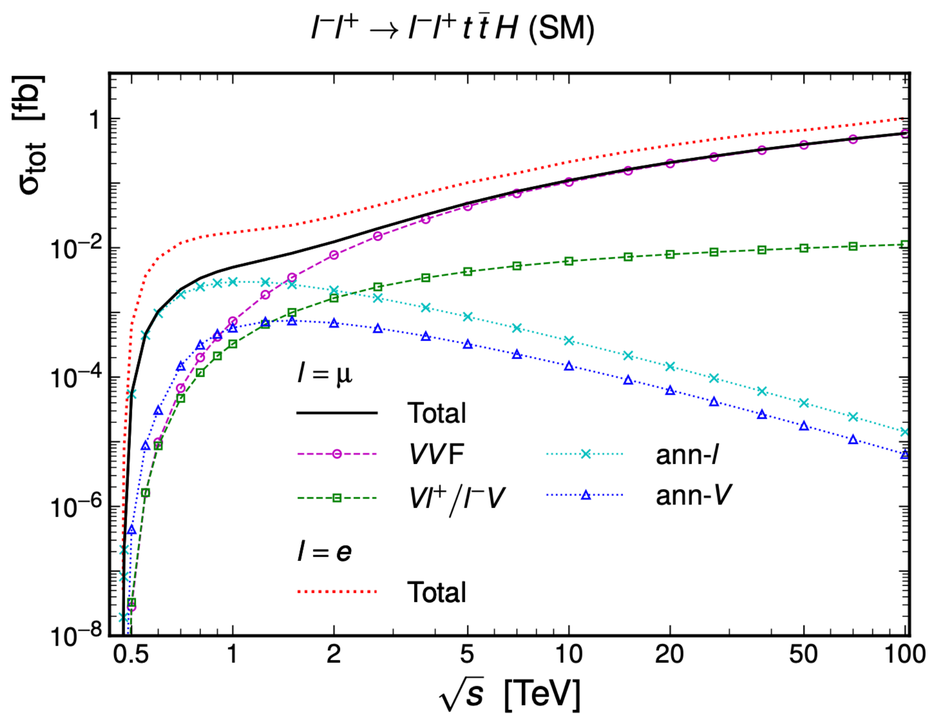
FD gauge makes subprocess groups informative where interference is small.

See also Phys. Rev. D 114, 036002 (2026), arXiv:2603.01139, Fig. 10

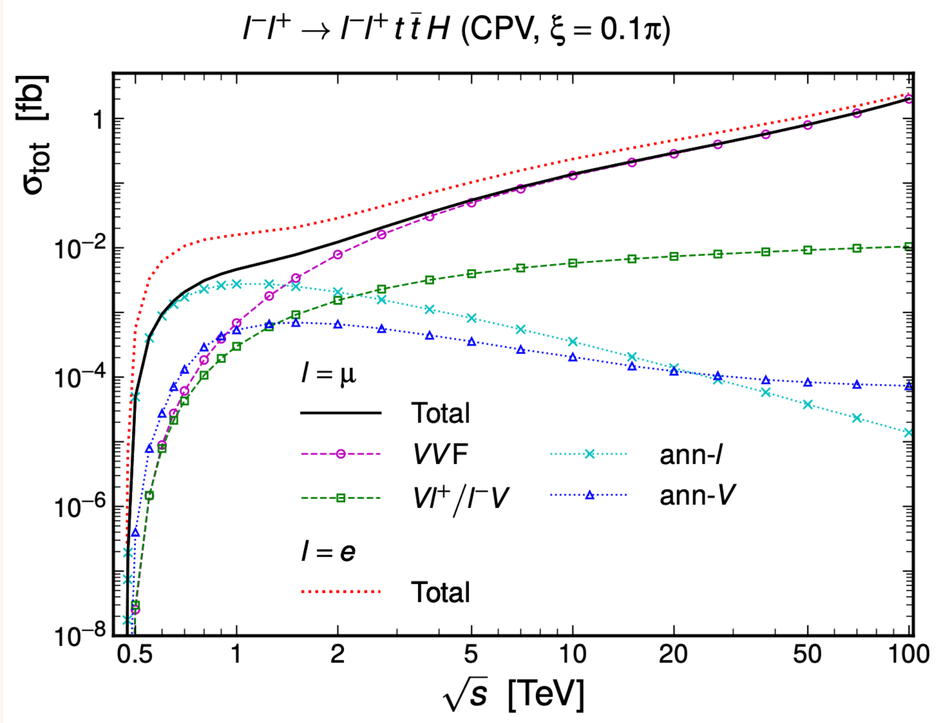
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# Five-body cross sections up to 100 TeV

$$l^-l^+ \to l^-l^+t\bar{t}H$$




Standard Model




CP-violating SMEFT

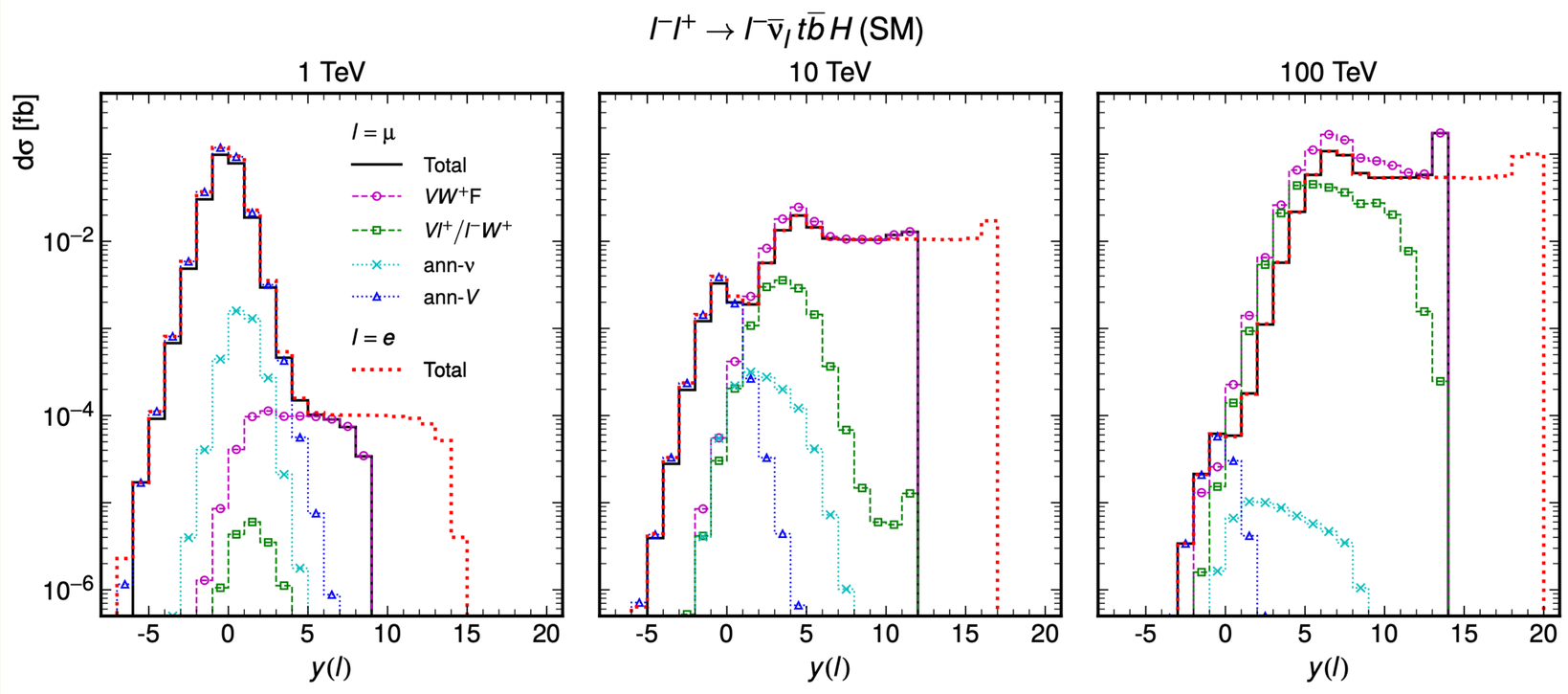
No transverse-momentum cuts.

See also Phys. Rev. D 114, 036002 (2026), arXiv:2603.01139, Fig. 26

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# Forward leptons from t-channel photon exchange

$$l^- l^+ \to l^- \overline{\nu}_l t \bar{b} H$$




At higher energies, the forward region is dominated by channels containing t-channel photon exchange.

No transverse-momentum cuts.

See also Phys. Rev. D 114, 036002 (2026), arXiv:2603.01139, Fig. 18

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# FDMC: Fortran release in preparation

## Scope of the first release

FD-gauge amplitudes from MG5, modular phase-space maps, and external Cuba*/VEGAS integration.

Initial examples: the three five-body lepton-collider processes.

The Cuba library -
a library for multidimensional numerical integration: https://feynarts.de/cuba/

## Run specifications in each process directory

```
fdmc-init single runs/demo
fdmc-run create runs/demo/runs/run-001 \
    --spec runs/demo/run.json
```

Snapshot inputs, execute diagram channels in parallel and collect results.

* Planned repository: github.com/jkanzaki105/fdmc

# C++/CUDA implementation and validation

## Native C++/CUDA integrand

Translate the Fortran phase-space and MG5/HELAS routines.
Evaluate the full amplitude and MCPS weight in double precision.

## Checks at 1, 10 and 100 TeV

Compare all 89 maps and all helicities against Fortran.
Check complex amplitudes at identical momenta and complete integrands.

## GPU integration based on my 2011 study (*)

Keep a common Fortran BASES driver on the CPU.
Evaluate batches of integrands on the GPU.

GPU: A100-PCIE-40GB, CUDA 13.0.88. CPU reference: Core i9-9900K.

* J. Kanzaki, Eur. Phys. J. C 71 (2011) 1559, arXiv:1010.2107

# GPU integrand throughput (preliminary)

**Fixed-point benchmark:** $\mu^-\mu^+ \to \nu_\mu \bar{\nu}_\mu t\bar{t}H$

**1 TeV, channel for diagram #35, 4096 points × 5 repeats, double precision.**
**Phase space + all 89 diagrams × all helicities + MCPS weights.**

| Implementation / hardware | Time / 4096 points | Speedup vs Fortran |
|---|---|---|
| Fortran / Core i9, one thread | 3104.946 ms | 1.00× |
| C++ / Core i9, one thread | 1794.178 ms | 1.73× |
| **CUDA kernels / A100** | **16.090 ms** | **192.97×** |

**A100: about 193× vs Fortran and 112× vs C++**

GPU kernel time excludes transfers and CPU random-number generation.
Core i9-9900K (1 thread) and A100-PCIE-40GB runs use different machines.

# Adaptive BASES integration (preliminary)

**One-channel benchmark:** $\mu^-\mu^+ \to \nu_\mu \bar{\nu}_\mu t\bar{t}H$

1 TeV, channel for diagram #35, NCALL = 10,000, 5 + 5 iterations.
8192 evaluations/iteration, 81,920 total, double precision.
Integral and error agree at printed precision across implementations.

| Integrand / hardware | Integration wall time | Speedup vs Fortran |
|---|---|---|
| Fortran / Core i9, one thread | 61.5198 s | 1.00× |
| C++ / Core i9, one thread | 33.4647 s | 1.84× |
| CUDA / A100 + EPYC host | 0.521627 s | 117.94× |

**A100: about 118× vs Fortran and 64× vs C++**

Includes CPU sampling, adaptation and GPU transfers. One channel only.
Cross-machine comparison at fixed budget, with convergence study pending.

# Summary and prospects

## Five-body integration up to 100 TeV

FD-gauge amplitudes and diagram-based maps handle resonances and very forward photon exchange while retaining interference.

## Fortran release in preparation

Package the current lepton-collider examples with Cuba/VEGAS and a documented run/scan workflow.

## Five-body C++/CUDA pilot

A100 gives about 118× faster one-channel BASES integration than Core i9 Fortran (one thread), after numerical checks.

Next: wider validation, more processes and automated diagram-to-map generation.