

Top quark rare processes at future linear colliders

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Why rare top decays?

- In the SM, t → qγ, qZ, qg, qh (q = u, c) are loop- and GIM-suppressed
- Any observable rate is a direct probe of flavour, electroweak symmetry and heavy new physics
- The ordinary decay t → bW (B ≈ 1) provides a clean **tag** for the other top
- Tiny intrinsic rates ≠ background-free search: SM $t\bar{t}$ and reconstruction mistakes yield the same objects
- Linear colliders give millions of $t\bar{t}$ pairs, polarised beams and a clean initial state

$\sim 10^{-14}$

SM: B(t → cγ) ~ 5 × 10^{-14}, B(t → cZ) ~ 10^{-14}

$\sim 10^{-5}$

benchmark BR used to compare collider scenarios

$\sim 5 \times 10^{6}$

Born-level $t\bar{t}$ pairs at LCF550 (8 ab^{-1})

Durieux et al. (1412.7166), CLICdp (1807.02441), Abramowicz et al. (2503.19983)

Channels and scope of this work

Two-body

- $q\gamma$
- qZ
- qg
- qh

Leptonic

- $q\ell^-\ell^+$
- $qe^-\mu^+$ (CLFV)
- $q\nu\,\bar{\nu}$

Two gauge bosons

- $q\gamma\gamma$
- $q\gamma g$
- qgg
- $qZ\gamma$
- qZg
- qW^+W^-

q = u, c · B and L conserved · light new invisible states treated separately

1 Operators & public models

General-flavour Warsaw SMEFT

2 Validated decay widths

102 one-coefficient dim-6^2 contributions with SMEFTsim + MadGraph

3 Production × decay

Polarised Born $t\bar{t}$ from threshold to 3 TeV; ILC, CLIC, LCF scenarios and visibility estimates

Where do we stand?

I CMS trilepton CLFV search

- $B(t \to e\mu u) < 1.2\text{–}3.2 \times 10^{-8}$ (95% CL)
- $B(t \to e\mu c) < 2.2\text{–}5.0 \times 10^{-7}$
- Range: scalar → vector → tensor interactions

II CMS multilepton FCNC EFT fit (2026)

- 24 Wilson coefficients from same-sign dilepton and trilepton data
- Includes semileptonic FCNC contributions

III CLIC 380 GeV detector study

- Expected $B(t \to c\gamma) < 2.6 \times 10^{-5}$ for 1 ab^{-1} (BDT + CL_s)
- Also t → cH and charm + invisible

IV Phenomenology

- Collider + indirect observables with complex coefficients and CP
- Deep-learning strategies for e.g. tqγ at the LHC

Limits are interpretation-dependent: they cannot be transferred without matching flavour assumptions, scales and normalisations.

CMS (2312.03199), CMS (2608.19547), CLICdp (1807.02441), Kala & Nandi (2602.10201), Fuks et al. (2507.17807)

Lepton-collider precedents

1999

Contact interactions drive flavour-changing single top at e^+e^-

Bar-Shalom & Wudka (hep-ph/9905407)

2010

LC FCNC reach depends on the assumed operators

Guedes et al. (1007.2992)

2018

CLIC: $t \to c\gamma$, cH, c + invisible at 380 GeV

CLICdp (1807.02441)

2019

Two- and four-fermion FCNC in $e^+e^- \to t\bar{q} + \bar{t}q$

Shi & Zhang (1906.04573)

2026

Boosted $t \to cH$, cS at 1.5 TeV CLIC

Yang et al. (2607.09154)

Older projections vs the LHC are historical; mechanisms and operator lessons remain valid.

A gauge-invariant starting point: SMEFT

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \sum_i \frac{C_i}{\Lambda^2}\mathcal{O}_i + \sum_a \frac{D_a}{\Lambda^4}\mathcal{O}_a^{(8)} + \dots$$

Warsaw basis · general flavour · B and L conserved · C_i/Λ^2 quoted in TeV^{-2}

Y Dipoles

- $\mathcal{O}_{uB}^{ij} = (\bar{Q}_i\,\sigma^{\mu\nu}u_j)\,\tilde{\varphi}\,B_{\mu\nu}$
- $\mathcal{O}_{uW}^{ij} = (\bar{Q}_i\,\sigma^{\mu\nu}\tau^I u_j)\,\tilde{\varphi}\,W^I_{\mu\nu}$
- $\mathcal{O}_{uG}^{ij} = (\bar{Q}_i\,\sigma^{\mu\nu}T^A u_j)\,\tilde{\varphi}\,G^A_{\mu\nu}$

h Yukawa & currents

- $\mathcal{O}_{u\varphi}^{ij} = (\varphi^\dagger\varphi)(\bar{Q}_i u_j\tilde{\varphi})$
- $\mathcal{O}_{\varphi Q}^{(1),(3)\,ij} = (\varphi^\dagger i\overleftrightarrow{D}^{[I]}_\mu\varphi)(\bar{Q}_i\gamma^\mu[\tau^I]Q_j)$
- $\mathcal{O}_{\varphi u}^{ij} = (\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_i\gamma^\mu u_j)$

ij Flavour indices

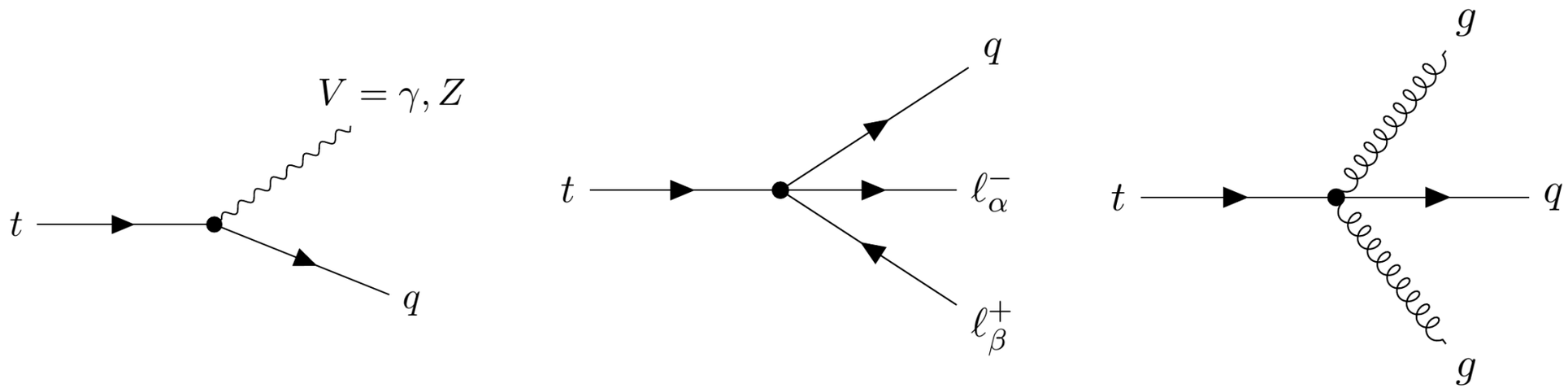
- ij = (a3) or (3a), a = 1, 2 → q = u, c
- a3 and 3a are **independent** chiral structures
- Up-quark mass basis; CKM in lower doublet components

Grzadkowski et al. (1008.4884), Aguilar-Saavedra (0811.3842), Aguilar-Saavedra et al. (1802.07237)

Electroweak symmetry ties the channels together

$$C_{u\gamma} = c_W\, C_{uB} + s_W\, C_{uW}, \qquad C_{uZ} = -\, s_W\, C_{uB} + c_W\, C_{uW}$$

- A single C_{uB} or C_{uW} produces **both** neutral dipoles → qγ and qZ are correlated
- A photon-only dipole is a tuned combination and still carries charged-current effects
- Z currents: left-handed probes $C_{\phi Q}^{(1)} - C_{\phi Q}^{(3)}$, right-handed $C_{\phi u}$
- Independent tqγ, tqZ anomalous couplings obscure these correlations



Neutral-boson emission · semileptonic contact · non-Abelian chromodipole contact (dots = EFT insertions)

Durieux et al. (1412.7166), Aguilar-Saavedra (0811.3842), Aguilar-Saavedra (0904.2387)

Semileptonic operators and lepton-flavour violation

V Vector structures

- $\mathcal{O}_{\ell Q}^{(1),(3)} = (\bar{L}_\alpha \gamma_\mu [\tau^I] L_\beta)(\bar{Q}_i \gamma^\mu [\tau^I] Q_j)$
- $\mathcal{O}_{\ell u}$, $\mathcal{O}_{eQ}$, $\mathcal{O}_{eu}$: right-handed analogues

T Scalar & tensor

- $\mathcal{O}_{\ell equ}^{(1)} = (\bar{L}_\alpha e_\beta)\, \varepsilon\, (\bar{Q}_i u_j)$
- $\mathcal{O}_{\ell equ}^{(3)} = (\bar{L}_\alpha \sigma_{\mu\nu} e_\beta)\, \varepsilon\, (\bar{Q}_i \sigma^{\mu\nu} u_j)$

- Charged leptons select $C_{\ell Q}^{(1)} - C_{\ell Q}^{(3)}$; neutrinos select the **sum**
- → t → qνν̄, charged currents and low-energy flavour probe correlated directions
- CLFV: each charge assignment ($qe^-\mu^+$ vs $q\mu^-e^+$) is an independent coefficient
- Low-energy $SU(3)\times U(1)_{em}$ analyses need matching before comparison

Davidson et al. (1507.07163), Zhang (1404.1264), Durieux et al. (1412.7166)

Channel map at one dimension-six insertion

Decay	Leading dimension-six sources	Main interpretation issue
$q\gamma$	uB, uW dipoles	Correlated qZ and charged currents
qZ	Dipoles and Higgs–quark currents	Multiple Lorentz structures
qh	$u\phi$, after mass diagonalisation	Use physical Yukawa couplings
qg	uG	QCD mixing and jet backgrounds
$q\ell^-_\alpha\ell^+_\alpha$	γ^*, $Z^{(*)}$, $h^{(*)}$ and contact terms	Pole regions, interference, lepton masses
$q\ell^-_\alpha\ell^+_\beta$, $\alpha \neq \beta$	Semileptonic contact terms	Explicit lepton flavour and charge assignment
$q\nu_\alpha\bar{\nu}_\beta$	$Z^{(*)}$ and left-lepton contacts	Missing energy; electroweak correlations

Tree level in the EFT amplitude; loop contributions enlarge this map. Higgs-pole dileptons are Yukawa-suppressed for light leptons.

Durieux et al. (1412.7166), Aguilar-Saavedra (0811.3842), Davidson et al. (1507.07163)

Widths and the EFT expansion

$$\Gamma(t \to q\gamma) = \frac{m_t^3 v^2}{8\pi\Lambda^4}\left(|C_{u\gamma}^{a3}|^2 + |C_{u\gamma}^{3a}|^2\right)$$

fixes the dipole normalisation; scales as C^2 and Λ^{-4}

$$|\mathcal{M}|^2 = |\mathcal{M}_{\rm SM}|^2 + \frac{2\,\mathrm{Re}(\mathcal{M}_{\rm SM}^{*}\mathcal{M}_6)}{\Lambda^2} + \frac{|\mathcal{M}_6|^2}{\Lambda^4} + \ldots$$

+ $2\mathrm{Re}(\mathcal{M}_{SM}{}^{*}\mathcal{M}_8)/\Lambda^4$, absent here

I Squared dim-6 is the leading signal

- For FCNC / CLFV the SM amplitude is tiny or absent
- Dropping the quadratic term would remove the signal
- Not complete at Λ^{-4}: assumes dim-8 and double insertions are controlled
- Keep all interference among retained dim-6 operators

II Dileptons are not a pole product

- $\mathcal{M}_{q\ell\ell} = \mathcal{M}_{\gamma^*} + \mathcal{M}_{Z(*)} + \mathcal{M}_{h(*)} + \mathcal{M}_{4f}$ interfere
- $B(t \to qZ)\, B(Z \to \ell\ell)$ is only the pole approximation
- Define Z-pole and off-pole regions and a lower $m_{\ell\ell}$ cut
- Hard scale ~m_t in decay vs $\sqrt{s}$ in crossed production

Zhang (1404.1264), Brivio & Trott (1706.08945), Aguilar-Saavedra et al. (1802.07237)

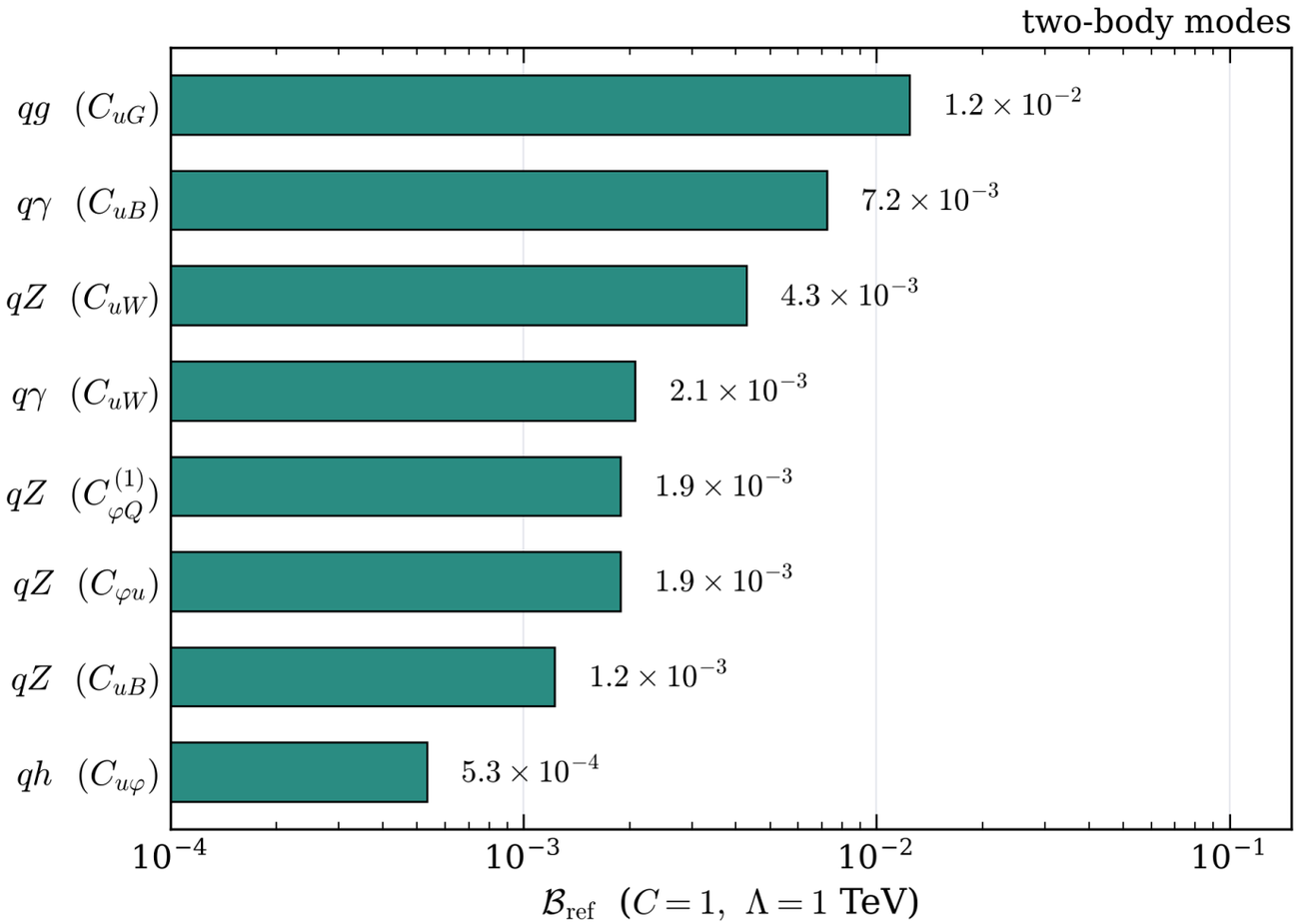
Two gauge bosons in a top decay

Final state	On-shell threshold	Status
$q\gamma\gamma$, $q\gamma g$, qgg	m_q	Open; resolved photons/jets
$qZ\gamma$, qZg	$m_q + m_Z$	Open, ~80 GeV above threshold
qW^+W^-	**$m_q + 2m_W$**	**Open, only ~12 GeV of phase space**
qZZ	$m_q + 2m_Z > 182$ GeV	Closed on shell; qZZ^* possible
$bW^+\gamma$, bW^+g	$m_b + m_W$	Open charged-current radiative
bW^+Z	≃ 176 GeV	Closed fully on shell

- Dim-6 is enough: one dipole insertion + radiation gives $q\gamma\gamma$, $q\gamma g$; non-Abelian pieces of O_{uG}, O_{uW} give $tqgg$ and $tqWW$ contacts
- Independent dim-8: O_{uBB}, O_{uWW}, $O_{uGG} = (\bar{Q}u\phi)XX$ — not supplied by dim-6 UFOs
- Higgs pole: $t \to qh$, $h \to \gamma\gamma$ measures $B(qh)\cdot B(h \to \gamma\gamma)$; keep a separate mass window
- Massless radiation needs explicit isolation / jet definitions

Jenkins (hep-ph/9612211), d'Enterria & Le (2312.11211), Eilam et al. (hep-ph/0601151), Bradshaw & Chang (2304.06063)

Two-body reference branching fractions



- Chromodipole C_{uG} gives the largest rate (α_s enhancement)
- A unit C_{uB} gives **both** $q\gamma$ (7.2×10^{-3}) and qZ (1.2×10^{-3}): not independent anomalous couplings
- $C^{(1)}_{\phi Q}$ and $C_{\phi u}$ give identical qZ widths at this order
- qH smallest; u and c rows agree in the massless restriction

Rates scale as $|C|^2(1\text{ TeV}/\Lambda)^4$; checked against analytic widths. Zhang (1404.1264), Durieux et al. (1412.7166)

Leptonic modes, including CLFV

Final state	Coefficient	B_{ref}
qe^-e^+	C_{uB}	6.59×10^{-5}
qe^-e^+	C_{uW}	1.48×10^{-4}
qe^-e^+	$C_{\phi Q}^{(1)} = C_{\phi u}$	6.22×10^{-5}
qe^-e^+	$C_{lq}^{(1)11a3}$	2.40×10^{-6}
$qe^-\mu^+$	$C_{lq}^{(1)12a3}$ (vector)	2.41×10^{-6}
$qe^-\mu^+$	$C_{lequ}^{(1)12a3}$ (scalar)	6.03×10^{-7}
$qe^-\mu^+$	**$C_{lequ}^{(3)12a3}$ (tensor)**	**2.89×10^{-5}**
$q\nu_e\bar{\nu}_e$	C_{uW}	2.78×10^{-4}
$q\nu_e\bar{\nu}_e$	$C_{\phi Q}^{(1)} = C_{\phi u}$	1.23×10^{-4}

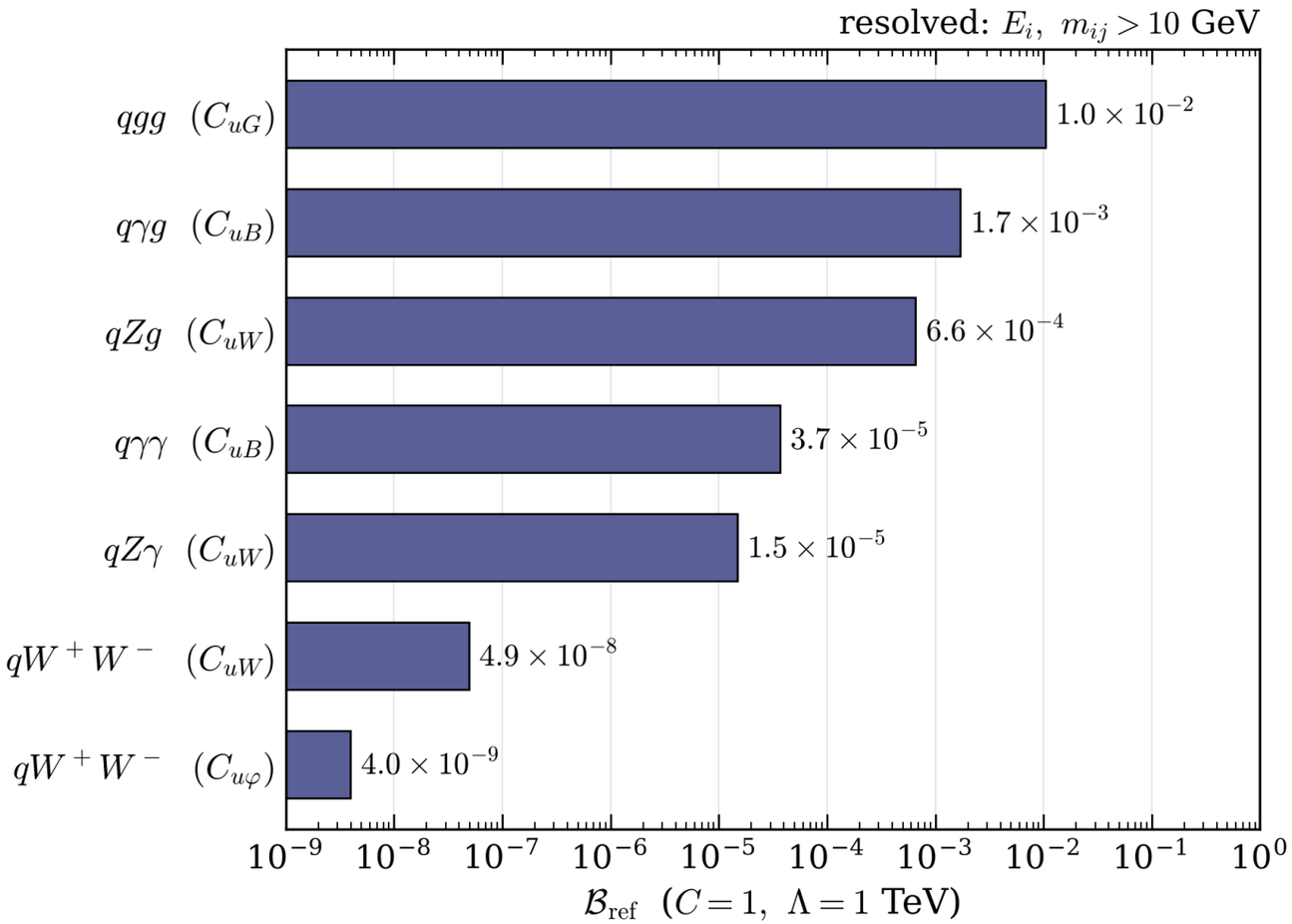
eμ **CLFV contact widths**

- $\Gamma_V = \dfrac{m_t^5}{1536\,\pi^3\Lambda^4}$
- $\Gamma_S = \Gamma_V/4, \qquad \Gamma_T = 12\,\Gamma_V$

- qμμ, qττ: same neutral-current rows; contacts relabelled 22a3, 33a3
- $qe^-\mu^+$ only: $q\mu^-e^+$ needs its own coefficient (no factor 2)
- Neutrinos: one species; ×3 only for pure neutral current

$m_{\ell\ell} > 10$ GeV for charged dileptons. Davidson et al. (1507.07163), Zhang (1404.1264)

Multiboson modes and Higgs-pole estimates



Higgs pole ($C_{u\varphi}$ = 1)	$B_{ref}(t \to qh \to qVV)$
h → γγ	1.26 × 10⁻⁶
h → Zγ	8.14 × 10⁻⁷
h → gg	2.35 × 10⁻⁵

- qW⁺W⁻: only 11.7 GeV of phase space → B ≲ 10⁻⁷; more energetic tops do not help
- h → gg pole is ~2× low: LO form factor gives B(h → gg) = 4.4% vs ≈ 8% in the SM
- Off-shell qWW*, qZZ* need multi-fermion final states (not yet included)

Resolved cuts E_i, $m_{i\,j}$ > 10 GeV. Jenkins (hep-ph/9612211), Eilam et al. (hep-ph/0601151), d'Enterria & Le (2312.11211)

Top production across collision energies

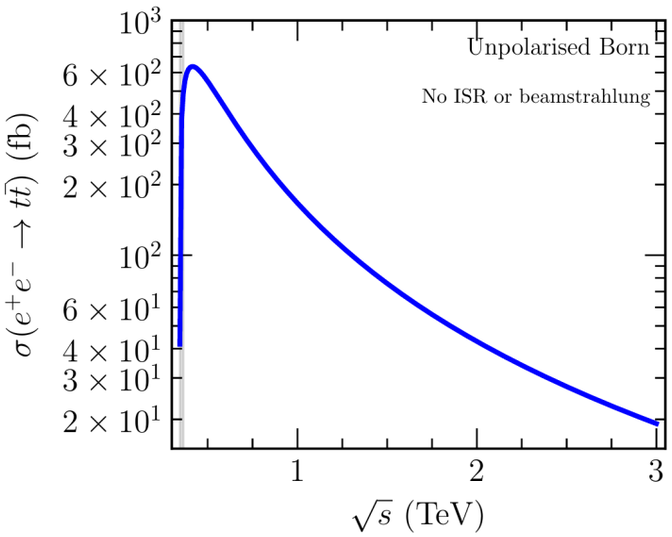
250 GeV	345–365 GeV	380–550 GeV	1–1.5 TeV	3 TeV
$t\bar{t}$ closed; off-shell single top; EFT-induced $t\bar{q}$	Threshold $\gamma^*, Z^* \to t\bar{t}$	Continuum $t\bar{t}$ $t\bar{t}Z, t\bar{t}H$ open	Boosted $t\bar{t}, t\bar{t}H$, single top, $t\bar{t}\nu\bar{\nu}$	$\sigma \propto 1/s$; single top & fusion grow
No tag-and-probe sample	**Needs resummation, Γ_t, beam spectrum**	**Large, well-separated pair sample**	**Boosted reconstruction harder**	**Energy alone does not raise a fixed BR**

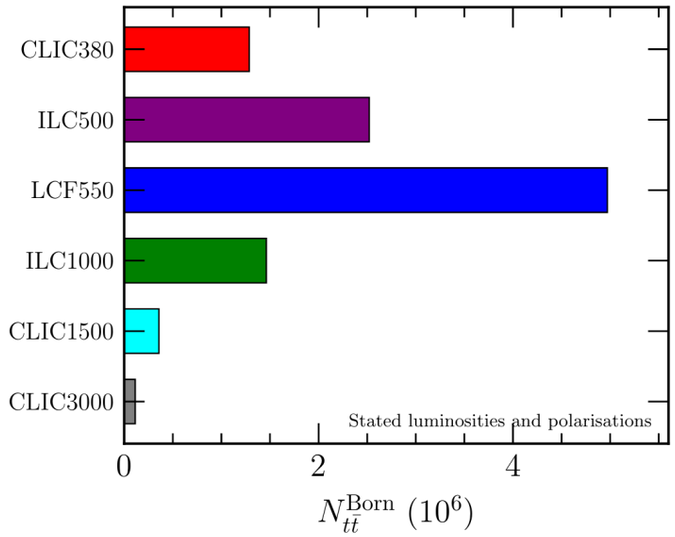
e **Direct FCNC production $e^+e^- \to t\bar{q} + \bar{t}q$ is a separate measurement**

- Open below threshold; electron-flavour contacts grow with energy. Leading yield is $L\sigma_{FCNC}B(t \to bW)\varepsilon$, **not** × another rare BR
- Here: rare decays in SM $t\bar{t}$ only. A 1 TeV decay scale is not a valid 3 TeV production cutoff

CLICdp (1807.02441), Escamilla et al. (1712.02763), Abramowicz et al. (2503.19983), Shi & Zhang (1906.04573)

Polarised Born tt̄ production





Left: unpolarised Born σ(e⁺e⁻ → tt̄). Right: tt̄ pairs for the stated luminosities and polarisations.

√s [GeV]	σ_0 [fb]	σ_{LR} [fb]	σ_{RL} [fb]
380	584.7	1665.7	673.2
500	550.8	1542.2	660.9
1000	166.7	458.9	208.0
3000	19.2	52.5	24.2

$$\sigma = \frac{1}{4}\left[(1-P_-)(1+P_+)\,\sigma_{LR} + (1+P_-)(1-P_+)\,\sigma_{RL}\right]$$

- $e_L^- e_R^+$ enhances $t\bar{t}$ by ~2.5× over $e_R^- e_L^+$ $e^-_R e^+_L$ — and charged-current backgrounds too
- Threshold points (350, 365 GeV) are kinematic references only
- At 3 TeV, beam spectra matter: full CLIC simulation ≈ 310k pairs vs Born ≈ 113k in 5 ab⁻¹

Alwall et al. (1405.0301), CLICdp (1807.02441)

Linear-collider operating scenarios

Scenario	√s [GeV]	L [ab^{-1}]	Polarisation (e^-, e^+)	$\bar{\sigma}_{\text{Born}}$ [fb]	$N_{t\bar{t}}$ [10^6]	S at B = 10^{-5}
CLIC380	380	2.2	±0.8, 0	584.7	1.29	5.2
ILC500	500	4	±0.8, ±0.3	630.1	2.52	10.1
LCF550	**550**	**8**	**±0.8, ±0.6**	**621.5**	**4.97**	**19.9**
ILC1000	1000	8	±0.8, ±0.2	182.7	1.46	5.9
CLIC1500	1500	4	±0.8, 0	89.3	0.36	1.4
CLIC3000	3000	5	±0.8, 0	22.6	0.11	0.5

$$N_{t\bar{t}} = 1000\, L\, \bar{\sigma}$$

L in ab^{-1}, $\bar{\sigma}$ in fb, polarisation-weighted

$$S = 2N_{t\bar{t}}\, B\, \varepsilon$$

with B = 10^{-5}, ε = 0.20 in the last column

Proposed scenarios, not guaranteed datasets. Born level: no ISR, beamstrahlung or QCD corrections.

Abramowicz et al. (2503.19983), CLICdp (1807.02441)

From top pairs to rare-decay signals

$$S_i = 2N_{t\bar{t}}\, B_i\, B(t \to bW)\, f_{\text{tag}}\, f_{\text{daughter},i}\, \varepsilon_{\text{rec},i}$$

factor 2: either top decays rarely; tag-side factors counted once

$$Z_{A,0} = \sqrt{2\left[(S+B)\ln\left(1+\frac{S}{B}\right) - S\right]}$$

Asimov median discovery significance

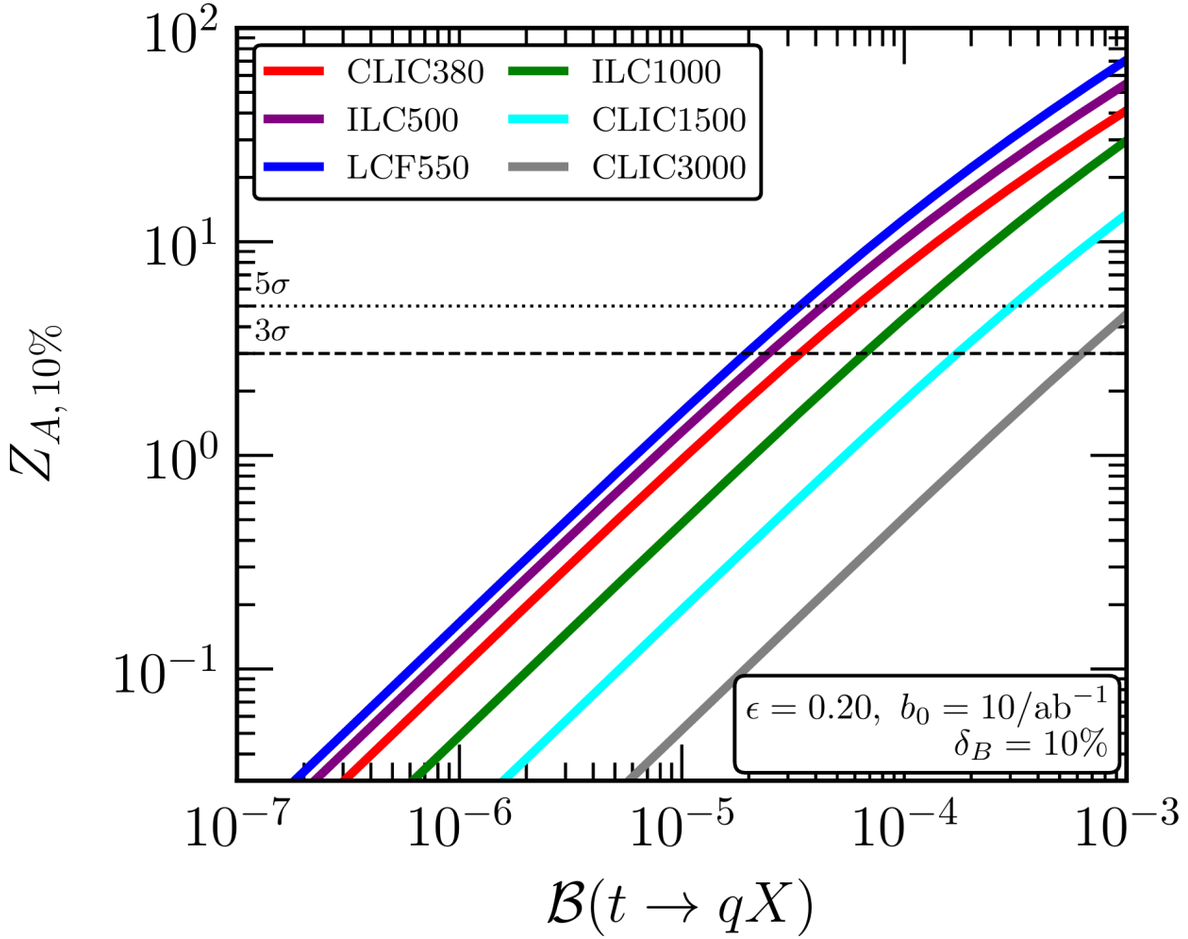
- Single-top sample: $S_i = N_t B_i \varepsilon_i$ (multiplicity 1)
- No extra factor 2 for q = u, c unless both coefficients are on
- $t\bar{t}\nu_e\bar{\nu}_e$ at 3 TeV: 28 000 events → only 0.11 signal events at B = 10^{-5}
- Spin correlations change acceptance, not inclusive rates

σ Background uncertainty and low counts

- Gaussian constraint on b with $\sigma_B = \delta_B B$; we use $\delta_B = 10\%$
- Large B: $Z \simeq S/\sqrt{(B + (\delta_B B)^2)}$ → systematic saturation
- Low counts: exact Poisson diagnostic; no infinite Z at zero background
- Zero observed → $S_{95} = 2.996$ is a limit, not a 3σ discovery

Cowan et al. (1007.1727)

How far can a counting search go?



Scenario	5σ BR (b_0 = 10/ab^{-1})
CLIC380	6.1×10^{-5}
ILC500	4.3×10^{-5}
LCF550	**3.4×10^{-5}**
ILC1000	1.2×10^{-4}
CLIC1500	3.1×10^{-4}
CLIC3000	1.1×10^{-3}

- Common ε = 0.20, δ_B = 10%: exposes the top-count dependence only
- Statistics-limited: BR reach ∝ $L^{-1/2}$, coefficient ∝ $L^{-1/4}$
- Background-free: L^{-1} and $L^{-1/2}$

Conditional counting estimates, not machine reach projections. Cowan et al. (1007.1727)

Channel by channel: rate- or background-limited?

Mode (coefficient)	B at C = 0.1	S_{380}	Z_{380}	S_{550}	Z_{550}
qγ (C_{uB})	**7.2×10^{-5}**	**37.2**	**1.48**	**143.9**	**1.83**
qg (C_{uG})	1.2×10^{-4}	31.9	0.05	123.4	0.05
qgg (C_{uG})	1.0×10^{-4}	13.4	0.02	51.8	0.02
qγg (C_{uB})	1.7×10^{-5}	4.4	0.06	17.0	0.07
qZ ($C_{\phi Q}^{(1)}$)	1.9×10^{-5}	0.65	0.06	2.5	0.09
$qe^-\mu^+$ ($C_{lequ}^{(3)}$)	2.9×10^{-7}	0.15	0.10	0.58	0.19
qW^+W^- (C_{uW})	4.9×10^{-10}	< 0.01	< 0.01	< 0.01	< 0.01

CLIC380 (2.2 ab^{-1}) and LCF550 (8 ab^{-1}); Z includes a 10% background nuisance

B **Background-limited**

qg, qgg, qγg: large S, but b_0 ~ 300–3000 / ab^{-1}

S **Rate-limited**

qeμ: $b_0 \approx 1$ / ab^{-1}, but too few events at C = 0.1

Y **Best of the set**

qγ: Z ≈ 1.5–1.8; still below 3σ at C = 0.1

Efficiencies and backgrounds are not obtained from simulation. Coefficient reach: $|C_i/\Lambda^2| = \sqrt{(B_{i,Z} / B_{i,ref})}$ TeV^{-2}

Reality check: the CLIC t → cγ detector study

340

selected signal events for B = 10^{-3}, 1 ab^{-1}

91

selected background (26 six-fermion + 65 four-fermion)

2.6×10^{-5}

published expected 95% CL limit for 1 ab^{-1} (BDT shape + CL_s)

L [ab^{-1}]	B(t → cγ)	S	B	Z_0	$Z_{10\%}$
2.2	2.6×10^{-5}	19.5	200.2	1.35	0.78
2.2	1.0×10^{-4}	74.8	200.2	5.00	2.92
4.3	2.6×10^{-5}	38.0	391.3	1.89	0.86
4.3	1.0×10^{-4}	146.2	391.3	6.99	3.23

Cut-count rescaling of the published selection, $S = 340\ (B/10^{-3})(L/1\ ab^{-1})$

- Shape-based exclusion ≠ cut-count discovery: a smaller Z at the limit BR is consistent
- Calibration already includes the selection: do not multiply by the toy ε again
- Charm-specific: not transferable to t → uγ
- CLIC numbers assume unpolarised beams and a fully hadronic spectator top

CLICdp (1807.02441), Cowan et al. (1007.1727)

Summary and analysis priorities

Framework	One coefficient convention links two-body FCNC, semileptonic/CLFV and gauge-completed multiboson amplitudes
Decays	102 validated dim-6^2 contributions; $q\gamma$ and qZ, and qg and qgg, are correlated by gauge symmetry
Production	$B = 10^{-5}$, $\varepsilon = 0.2$: ~5 events at CLIC380, ~20 at LCF550 → $Z \approx 0.95$ and 1.6 ($b_0 = 10/ab^{-1}$, $\delta_B = 10\%$)
Priorities	$q\gamma$, CLFV $qe\mu$ and resolved $q\gamma\gamma$; qZ, qh for correlated operator information
Next	Beam spectra, detector simulation, background programme, multi-coefficient fit with consistent width