Internal Alignment of VXD3

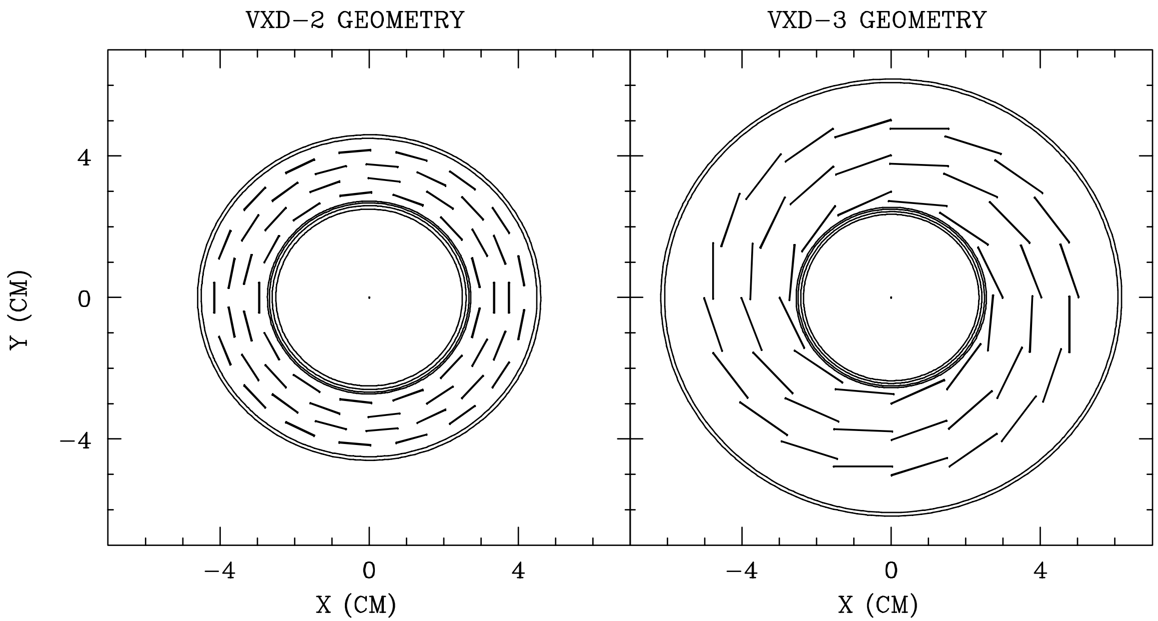
David Jackson

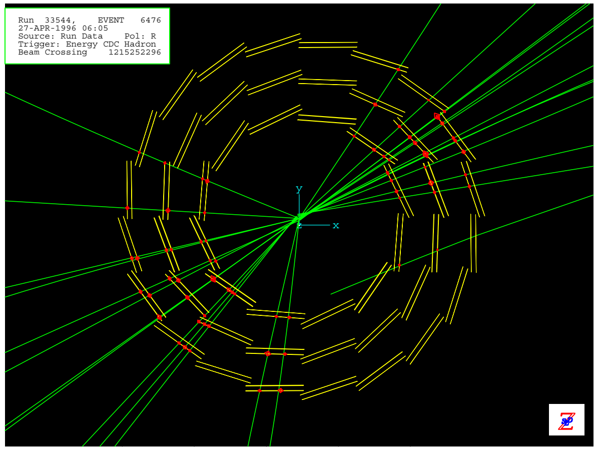
Oxford University / RAL

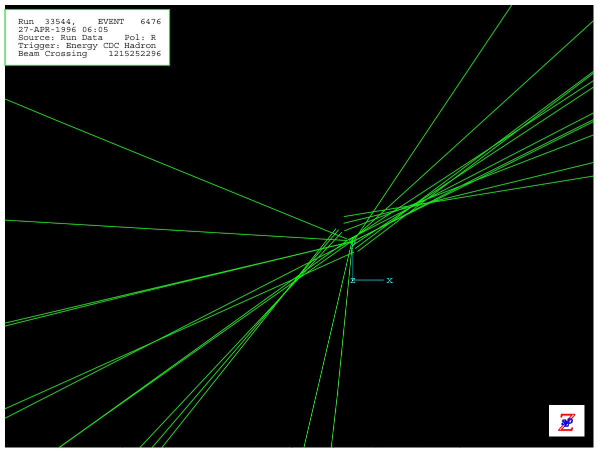
Overview

- VXD3 at SLD
- Observing misalignments with the track data
- Matrix technique to unfold alignment corrections
- Comments on SiD tracker alignment

SiD Tracking Meeting
SLAC
25th March 2005


VXD−2 GEOMETRY
VXD−3 GEOMETRY
Y (CM)
X (CM)
X (CM)


Run 33544, EVENT 6476
27-APR-1996 06:05
Source: Run Data Pol: R
Trigger: Energy CDC Hadron
Beam Crossing 1215252296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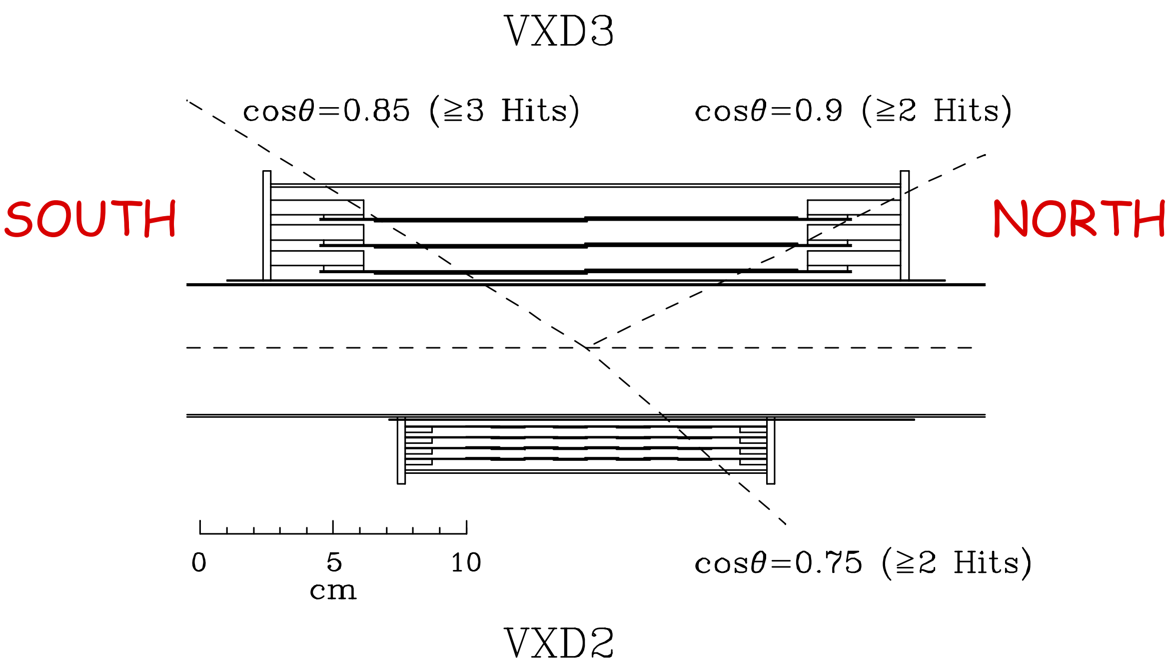

VXD3
cosθ=0.85 (≧3 Hits)
cosθ=0.9 (≧2 Hits)
SOUTH
NORTH
cm
cosθ=0.75 (≧2 Hits)
VXD2

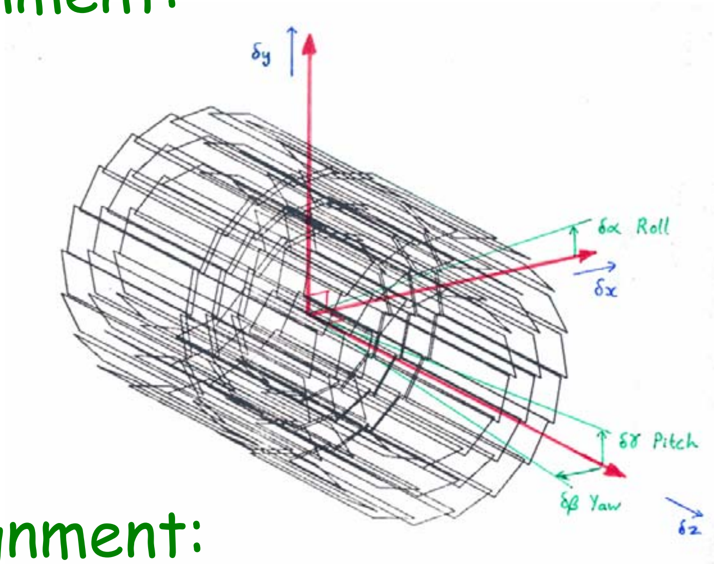
Rigid Body Alignment in 3D:

3 translation + 3 rotation parameters

Global Alignment:

(align to CDC)

1 x



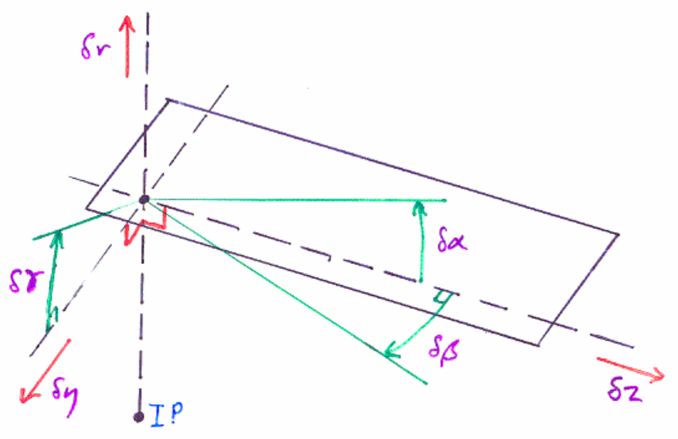
= 6 parameters

CCD single hit resolution < 5μm, Optical survey precision ~ 10μm

Internal Alignment:

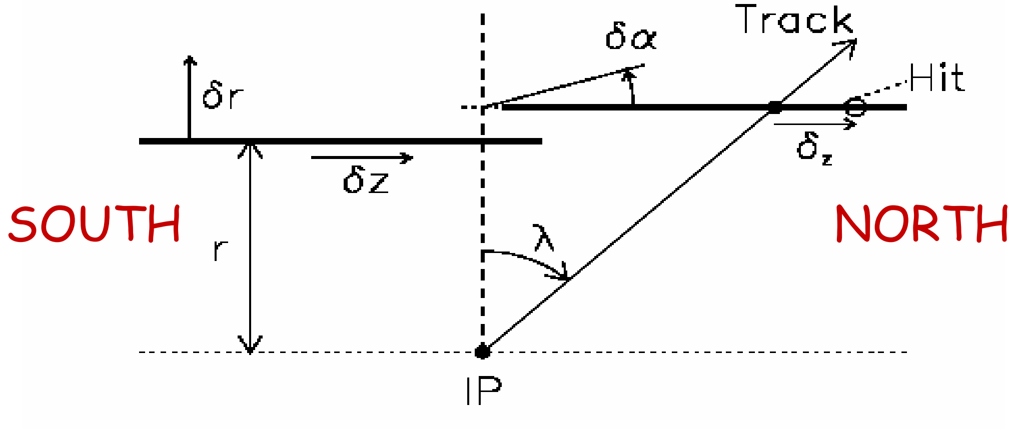
(mainly internal to VXD3)

96 x

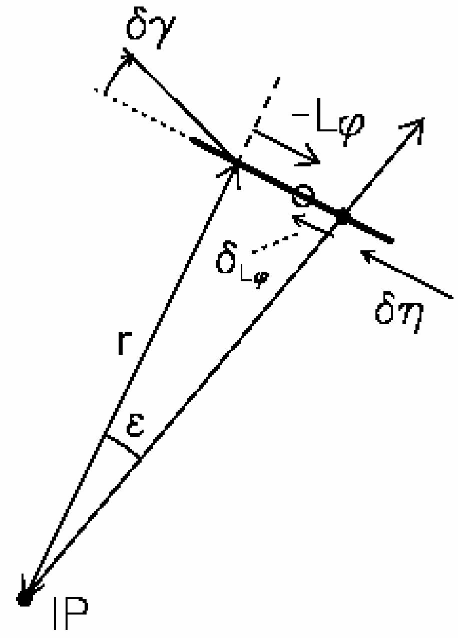


= 576 parameters

Apparent hit position on a CCD due to misalignment.



(a) rz View

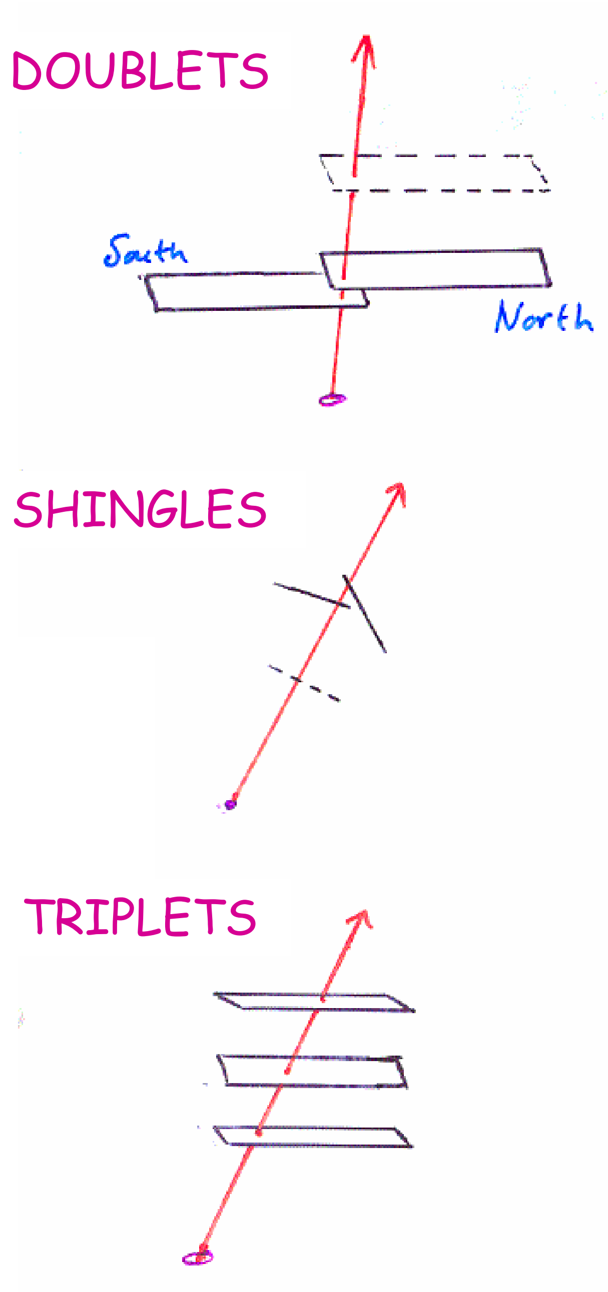


(b) rφ View

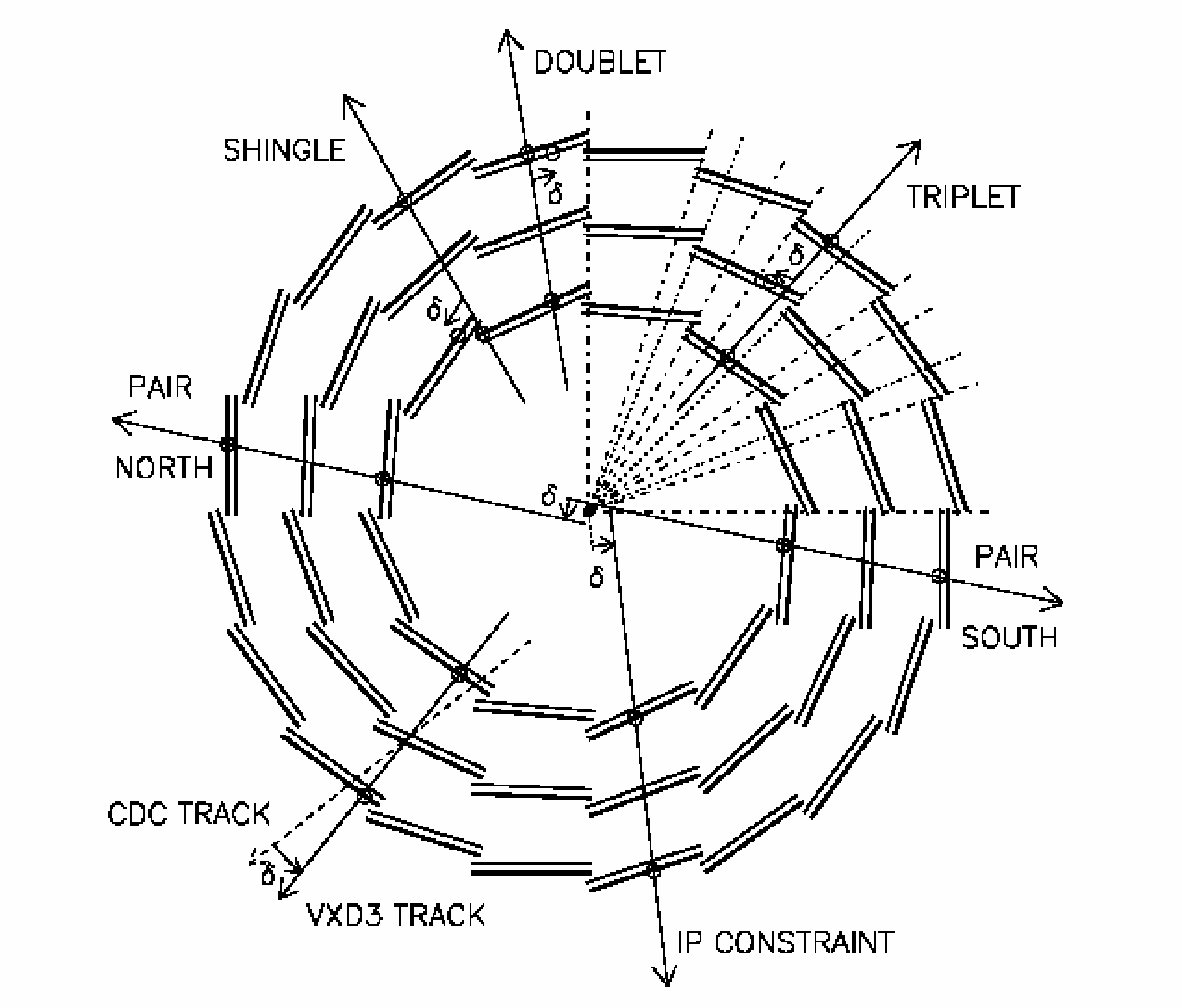
$$\delta_z = -\delta z + \delta r \tan\lambda + \delta\alpha\, r \tan^2\lambda + \delta\gamma L_\phi \tan\lambda + \delta\beta L_\phi$$

$$\delta_{L\phi} = -\delta\eta + \frac{\delta r}{r} L_\phi + \frac{\delta\gamma}{r} L_\phi^2 + \delta\alpha L_\phi \tan\lambda - \delta\beta\, r \tan\lambda$$

- The CCDs themselves provide the most precise measurements of the track trajectory
- Construct internal constraints with track fixed to two CCD hits and measure ‘residual’ to the third
- All CCDs in for each residual type contribute to the residual in proportion to a lever-arm weight
- In ‘overlap’ regions only 2 CCDs contribute a significant weight
- VXD3 ‘doublets’, ‘shingles’ and ‘triplets’ connected the North/South halves, CCDs within each layer and the three layers of the detector respectively.



…three further residual types were added

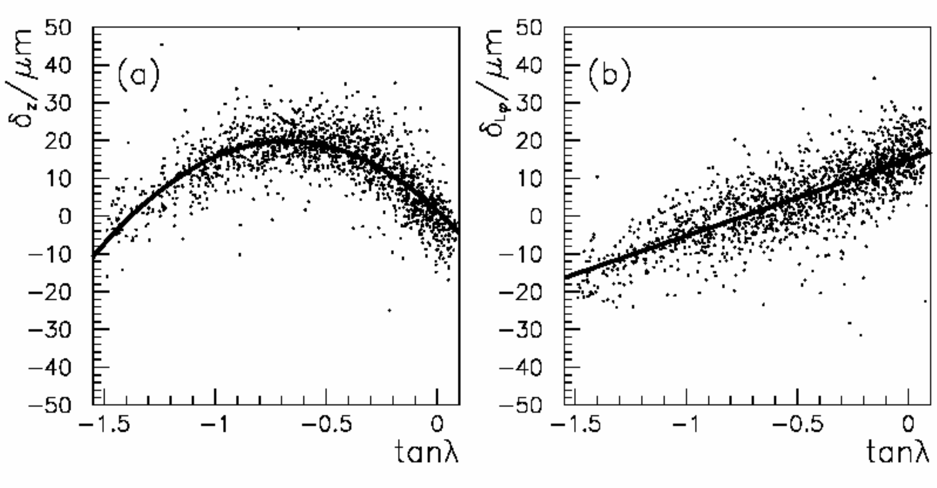


Functional forms of residual distributions for 3D rigid body misalignments:

Type	Functional Form	N_I	N_C
Shingles	$\delta_z = s_1^{\parallel} + s_2^{\parallel}\tan\lambda + s_3^{\parallel}\tan^2\lambda$	96	288
	$\delta_{L\phi} = s_1^{\perp} + s_2^{\perp}\tan\lambda$	96	192
Doublets	$\delta_z = d_1^{\parallel} + d_2^{\parallel}L_\phi$	48	96
	$\delta_{L\phi} = d_1^{\perp} + d_2^{\perp}L_\phi + d_3^{\perp}L_\phi^2$	48	144
Triplets	$\delta_z = t_1^{\parallel} + t_2^{\parallel}\tan\lambda + t_3^{\parallel}\tan^2\lambda + t_4^{\parallel}L_\phi\tan\lambda + t_5^{\parallel}L_\phi$	80	400
	$\delta_{L\phi} = t_1^{\perp} + t_2^{\perp}L_\phi + t_3^{\perp}L_\phi^2 + t_4^{\perp}L_\phi\tan\lambda + t_5^{\perp}\tan\lambda$	80	400
Pairs	$\delta_{rz} = p_1^{\parallel} + p_2^{\parallel}\tan\lambda + p_3^{\parallel}\tan^2\lambda$	28	84
	$\delta_{r\phi} = p_1^{\perp} + p_2^{\perp}\tan\lambda$	28	56
	$\delta_\phi = p_1^{\phi} + p_2^{\phi}\tan\lambda$	28	56
CDC Angle	$\delta_\lambda = c_1^{\lambda} + c_2^{\lambda}\tan\lambda + c_3^{\lambda}\tan^2\lambda$	56	168
	$\delta_\phi = c_1^{\phi} + c_2^{\phi}\tan\lambda$	56	112
IP Constraint	$\delta_{r\phi} = i_1^{\perp} + i_2^{\perp}\tan\lambda$	56	112
	Total	700	2108

A total of 700 polynomial fits to residual distributions like…

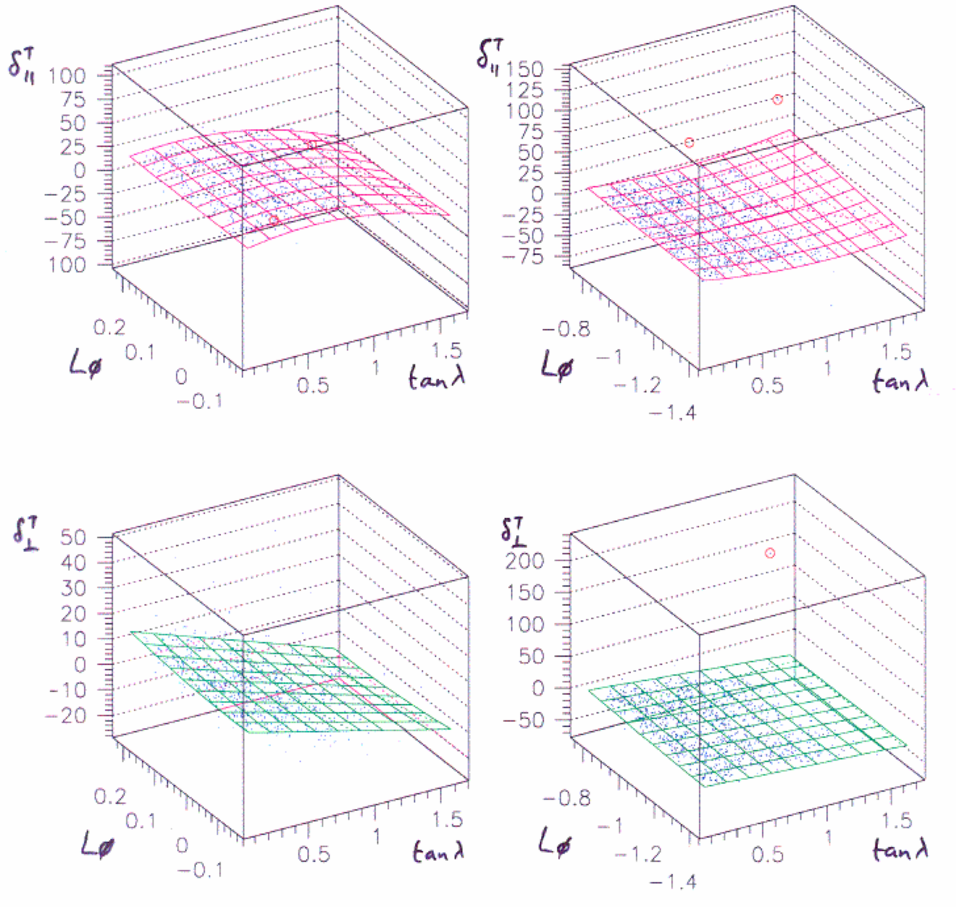
The two fits to one shingle region



- The above shingle conforms very well to the predicted functional forms
- Vertical scatter is due to the intrinsic spatial hit resolution of the CCDs
- Removal of outlayers is shown by the red circles on the triplet fits

The two fits to each of two triplet regions (one triple on left, the other on right)

$$\delta_{\parallel}^{T} = t_1^{\parallel} + t_2^{\parallel} \tan\lambda + t_3^{\parallel} \tan^2\lambda + t_4^{\parallel} \tan\lambda L\phi + t_5^{\parallel} L\phi$$



$$\delta_{\perp}^{T} = t_1^{\perp} + t_2^{\perp} L\phi + t_3^{\perp} L\phi^2 + t_4^{\perp} \tan\lambda L\phi + t_5^{\perp} \tan\lambda$$

Internal Alignment Matrix Equation I

The residual fits involve a large number of related simultaneous linear equations in the unknown alignment parameters; these are organised into a single matrix equation

$$
\begin{pmatrix}
\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\
\ldots\ldots\mathbf{A}(w_i, r_i, a_i, b_i, f_i)\ldots\ldots \\
\vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots
\end{pmatrix}
\begin{pmatrix}
\delta \mathrm{z}_1 \\ \vdots \\ \delta \mathrm{z}_{96} \\ \delta\eta_j \\ \delta \mathrm{r}_j \\ \delta\alpha_j \\ \delta\beta_j \\ \delta\gamma_j \\ \delta \mathrm{x} \\ \delta \mathrm{y}
\end{pmatrix}
=
\begin{pmatrix}
s_1^{\parallel}(96) \\ s_2^{\parallel}(96) \\ s_3^{\parallel}(96) \\ s_1^{\perp}(96) \\ s_2^{\perp}(96) \\ d_1^{\parallel}(48) \\ \vdots \\ t_1^{\parallel}(80) \\ \vdots \\ \vdots \\ p_1^{\parallel}(28) \\ \vdots \\ c_1^{\lambda}(56) \\ \vdots \\ i_1^{\perp}(56) \\ i_2^{\perp}(56)
\end{pmatrix}
$$

Weight matrix – determined to an extremely good approximation from the known ideal geometry

Alignment corrections to be determined

Coefficients measured in residual fits

Singular Value Decomposition

$$\underset{m \times n}{\begin{pmatrix} \mathbf{A} \end{pmatrix}} = \underset{\substack{m \times m \\ \text{othogonal}}}{\begin{pmatrix} \mathbf{U} \end{pmatrix}} \underset{\substack{m \times n \\ \text{diag.}}}{\begin{pmatrix} s_1 & & & \\ & s_2 & & \\ & & \ddots & \\ & & & s_r \end{pmatrix}} \underset{\substack{n \times n \\ \text{orthogonal}}}{\begin{pmatrix} \mathbf{V}^T \end{pmatrix}}$$

$s_1 \ldots s_r$ are called the 'singular values' of matrix **A**; $s_i \sim 0$ corresponds to a singularity of **A**

Here's the SVD trick:

define the inverse $\mathbf{A}^+ = \mathbf{V}\mathbf{S}^+\mathbf{U}^T$ with $\mathbf{S}^+ = \begin{pmatrix} 1/s_1 & & & \\ & 1/s_2 & & \\ & & \ddots & \\ & & & 1/s_r \end{pmatrix}$ with $1/s_i = 0$ if $s_i \sim 0$

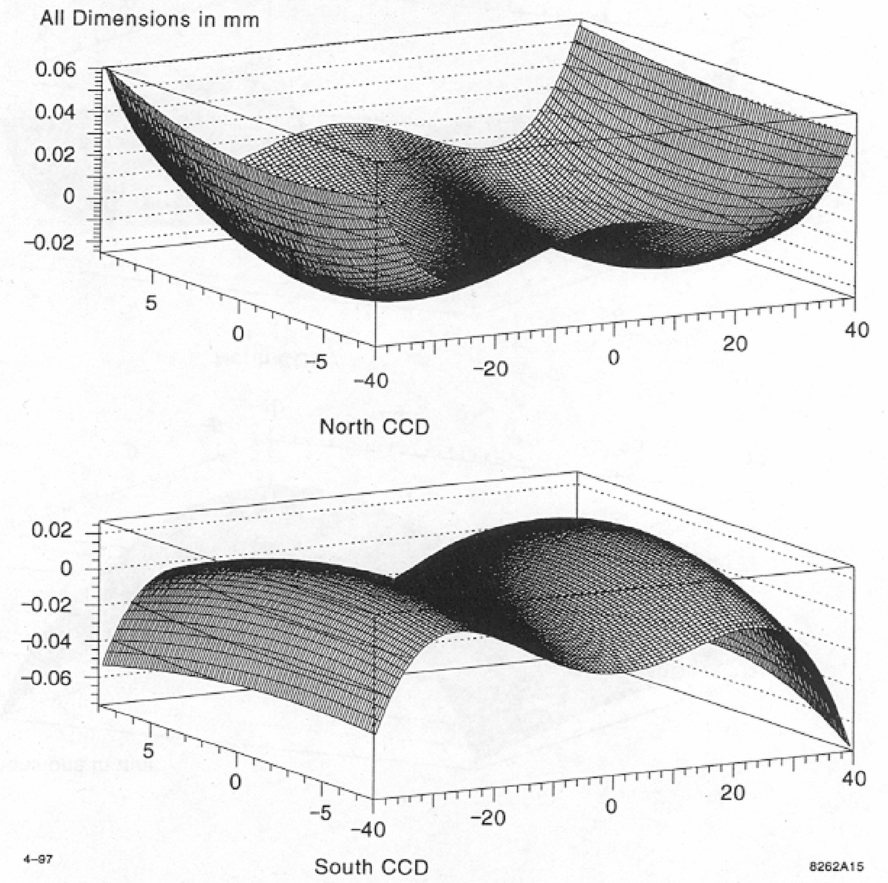
Then if $\mathbf{A}x = b$ (for vectors x,b)

The solution $x_0 = \mathbf{A}^+ b$ is such that: $|\mathbf{A}x_0 - b|$ has minimum length

That is, the SVD technique gives the closest 'least squares' solution for an over-constrained (and possibly singular) system

CCD shapes from optical survey

Fitted 14-parameter Chebychev polynomial shape, as well as CCD position, used as rigid body starting point for internal alignment



A large number of track residual distributions showed signs of the CCD shapes deviating from the optical survey data.

The biggest effects could be described my a 4th order polynomial as a function of the z axis

An arbitrary surface shape can be introduced by setting:

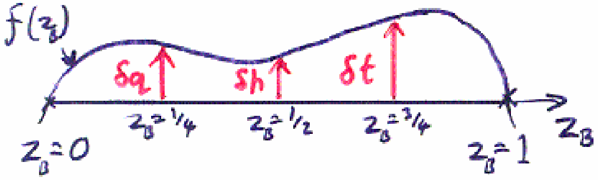
$$\delta r \rightarrow \delta r + f(z)$$

For convenience the base of the CCDs (each 8cm in length) was taken as:

$$z_B = (r \tan\lambda)/8$$

4TH ORDER POLYNOMIAL 'FIXED' AT EACH END

(RIGID BODY δr, $\delta\alpha$ CORRECTIONS ALLOW ENDS TO MOVE)



$$\delta q = f(1/4)$$
$$\delta h = f(1/2)$$
$$\delta t = f(3/4)$$

$$f(z_B) = c_1 z_B + c_2 z_B^2 + c_3 z_B^3 + c_4 z_B^4$$

.... A LITTLE ALGEBRA ....

$$c_1 = 16\,\delta q - 12\,\delta h + \frac{16}{3}\,\delta t$$
$$c_2 = -\frac{208}{3}\,\delta q + 76\,\delta h - \frac{112}{3}\,\delta t$$
$$c_3 = 96\,\delta q - 128\,\delta h + \frac{224}{3}\,\delta t$$
$$c_4 = -\frac{128}{3}\,\delta q + 64\,\delta h - \frac{128}{3}\,\delta t$$

With shape parameters included the same residual distributions were fitted to extended higher order functional forms:

		1	$\tan\lambda$	$\tan^2\lambda$	$\tan^3\lambda$	$\tan^4\lambda$	$\tan^5\lambda$	$L\phi$	$L^2\phi$	$L\phi\tan\lambda$	$L\phi\tan^2\lambda$	$L\phi\tan^3\lambda$	$L\phi\tan^4\lambda$	# FITS	# PARAM.
TRIPLETS	$\delta_z^t =$	t_1''	t_2''	t_3''	t_6'' ▲	t_7'' ▲	t_8'' ▲	t_5''		t_4''				80	640
	$\delta_{L\phi}^t =$	$t_1^\perp$	$t_5^\perp$	$t_9^\perp$ ▲	$t_{10}^\perp$ ▲	$t_{11}^\perp$ ▲		$t_2^\perp$	$t_3^\perp$	$t_4^\perp$	$t_6^\perp$ ▲	$t_7^\perp$ ▲	$t_8^\perp$ ▲	80	880
SHINGLES	$\delta_z^s =$	s_1''	s_2''	s_3''	s_4'' ▲	s_5'' ▲	s_6'' ▲							96	576
	$\delta_{L\phi}^s =$	$s_1^\perp$	$s_2^\perp$	$s_3^\perp$ ▲	$s_4^\perp$ ▲	$s_5^\perp$ ▲								96	480
DOUBLETS	$\delta_z^d =$	d_1''						d_2''						48	96
	$\delta_{L\phi}^d =$	$d_1^\perp$						$d_2^\perp$	$d_3^\perp$					48	144
PAIRS	$\delta_{rz}^p =$	p_1''	p_2''	p_3''	p_4'' ▲	p_5'' ▲	p_6'' ▲							28	168
	$\delta_{xy}^p =$	$p_1^\perp$	$p_2^\perp$	$p_3^\perp$ ▲	$p_4^\perp$ ▲	$p_5^\perp$ ▲								28	140
	$\delta_\phi^p =$	p_1^ϕ	p_2^ϕ	p_3^ϕ ▲	p_4^ϕ ▲	p_5^ϕ ▲								28	140
CDC	$\delta_\lambda^c =$	c_1^λ	c_2^λ	c_3^λ	c_4^λ ▲	c_5^λ ▲	c_6^λ ▲							56	336
	$\delta_\phi^c =$	c_1^ϕ	c_2^ϕ	c_3^ϕ ▲	c_4^ϕ ▲	c_5^ϕ ▲								56	280
IP	$\delta_\perp^i =$	$i_1^\perp$	$i_2^\perp$	$i_3^\perp$ ▲	$i_4^\perp$ ▲	$i_5^\perp$ ▲								56	280
														700	4,160

The required new fit coefficients ▲ roughly doubling the total number to 4,160

Six examples of the 28 Pair δ*rz* residual fits

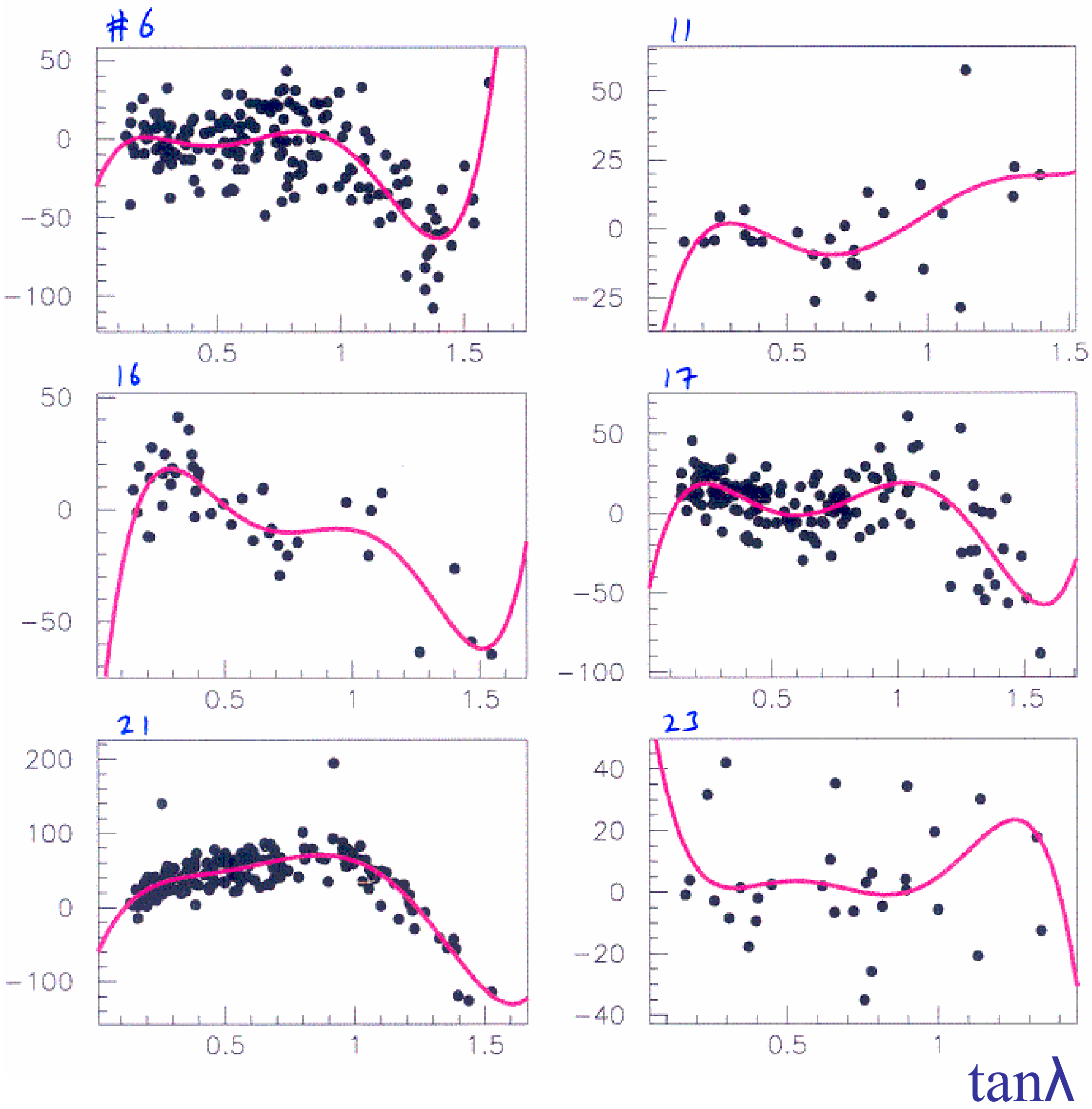
(would take quadratic form without shape corrections)

Pairs, using

$Z^0 \rightarrow \mu^+\mu^-$

$Z^0 \rightarrow e^+e^-$

events, were the most limited in statistics.

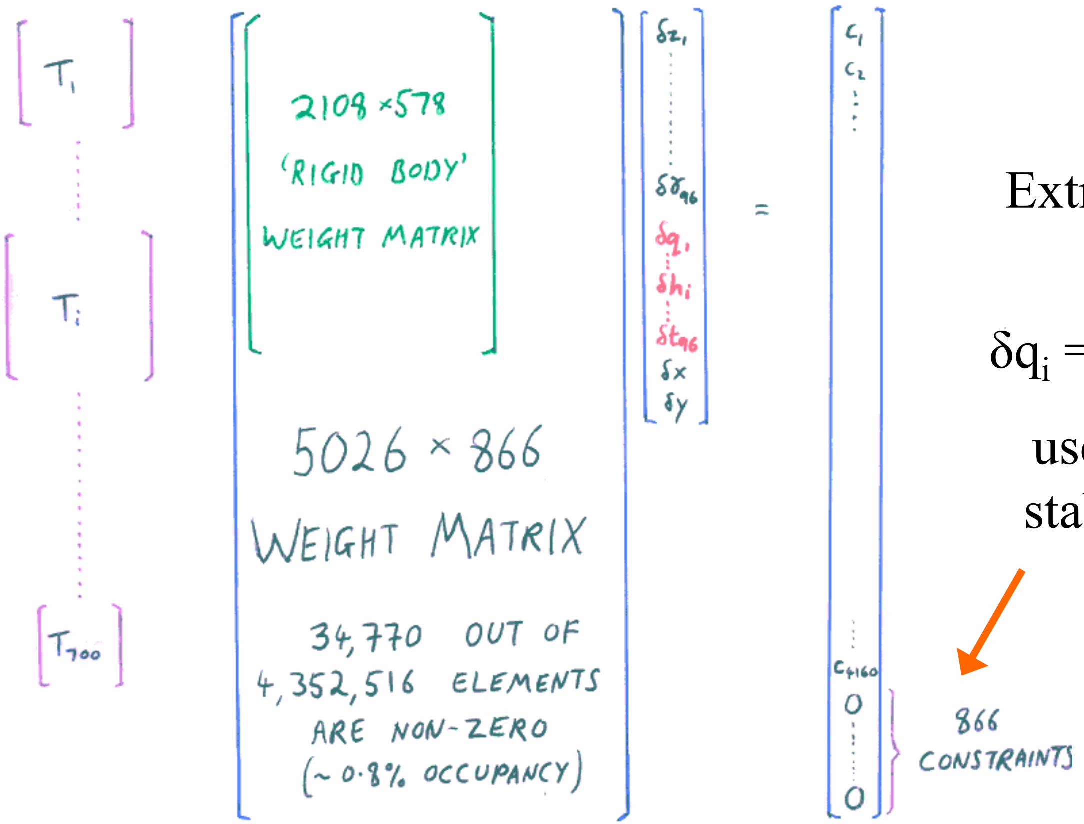


Important to correctly take into account correlations in each fit.

Internal Alignment Matrix Equation II

Each of 700 residual fit error matrices used to determine linearly independent basis in each case.

The SVD technique is improved from a ‘least squares’ to an optimal χ^2 fit.



Extra constraints such as

$\delta q_i = 0.0 \pm 5.0\ \mu m$

used to ensure stable solution.

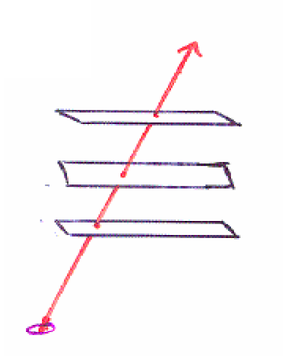
700 RESIDUAL FITS ⇒

4,160 PARAMETERS c_i

+16,332 CORRELATION TERMS IN T_i

866 (9 x 96 + 2) alignment corrections to be determined

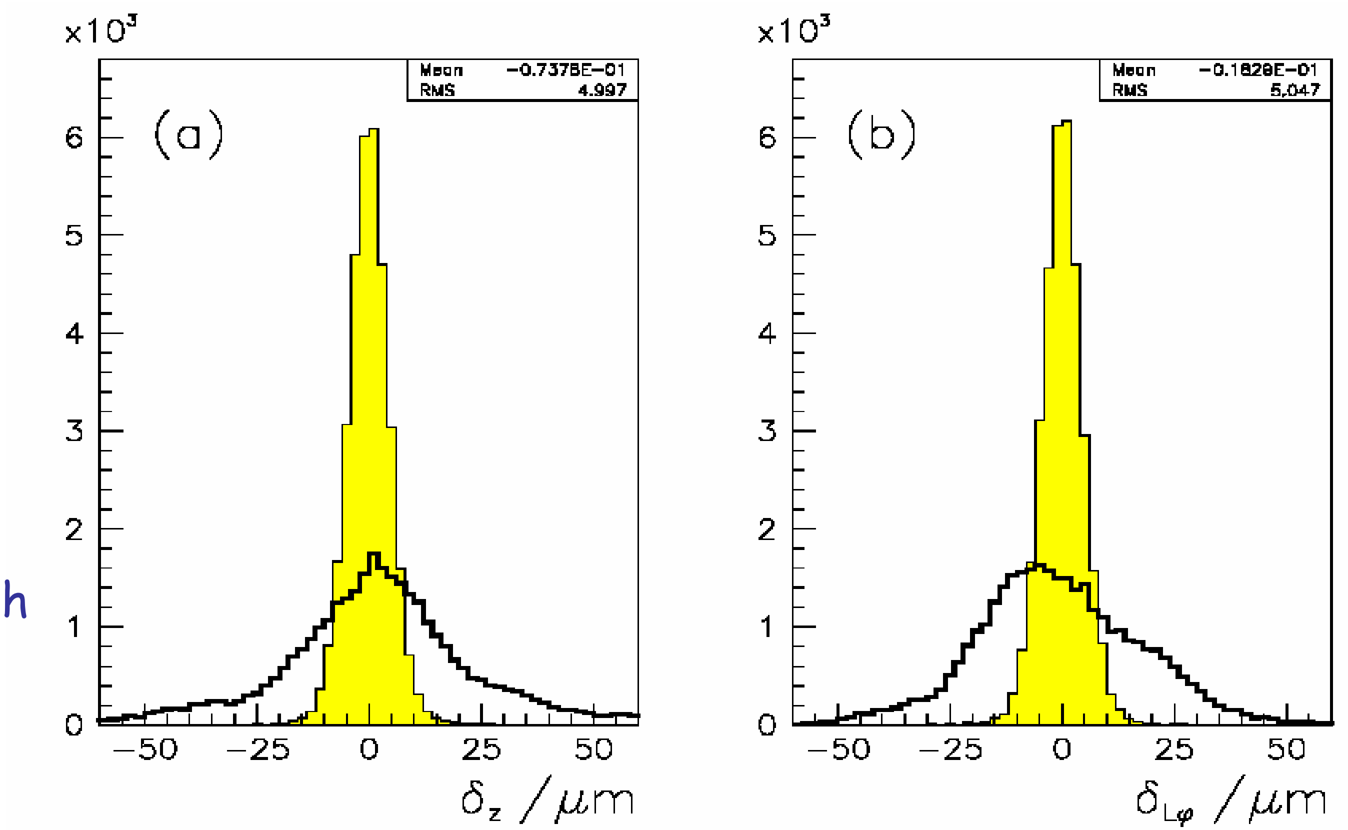
'Before' and 'After' Triplet Residuals



Using optical survey geometry

After track-based alignment

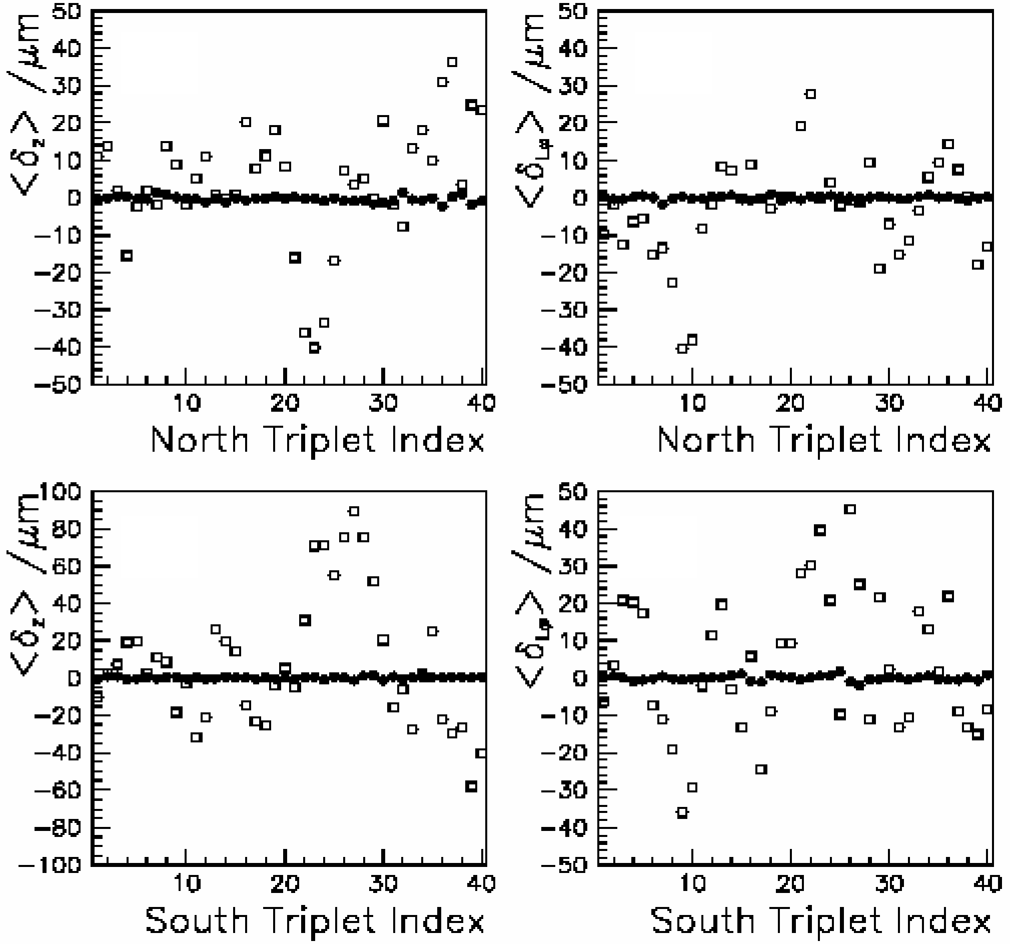
Tracks with
P > 5 GeV



Post-alignment single hit resolution ~ 3.6 μm

Triplet residual mean as function of φ-dependent index

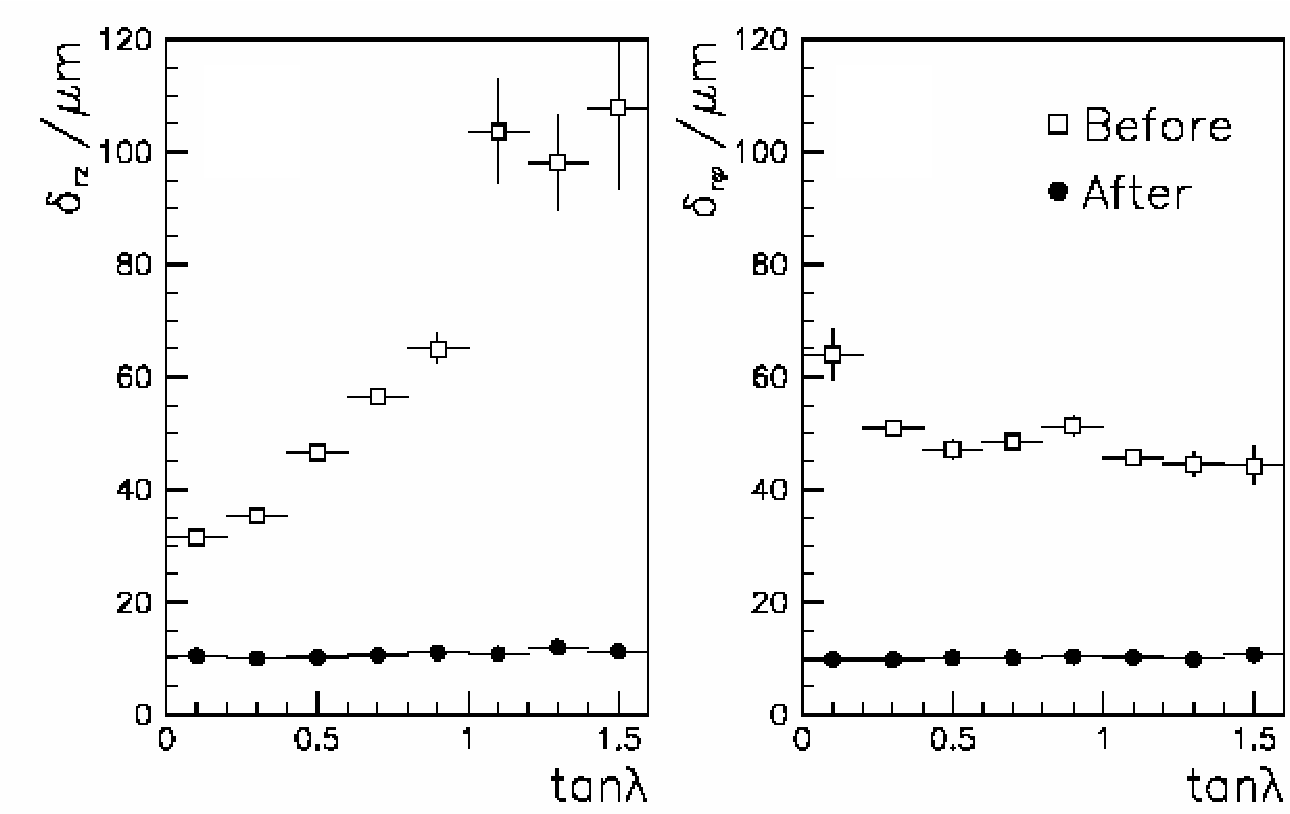
- □ Before Alignment
- ● After Alignment



Systematic effects $\lesssim$ 1μm level

Pair Residuals rms at Interaction Point

(divided by √2 to give single track contribution)



Impact Parameter resolution (for full track fit):

$$\sigma_{rz} = 9.7 \oplus \frac{33}{p \sin^{3/2}\theta} \mu\mathrm{m} \qquad \sigma_{r\phi} = 7.8 \oplus \frac{33}{p \sin^{3/2}\theta} \mu\mathrm{m}$$

…design performance achieved

Comments for SiD tracker I

- **Singular Value Decomposition** – this alignment technique allowed a robust unbiased solution for SLD; but the method is somewhat secondary in that any technique will have similar statistical dependence on the data and geometry.

Alignment is aided by:

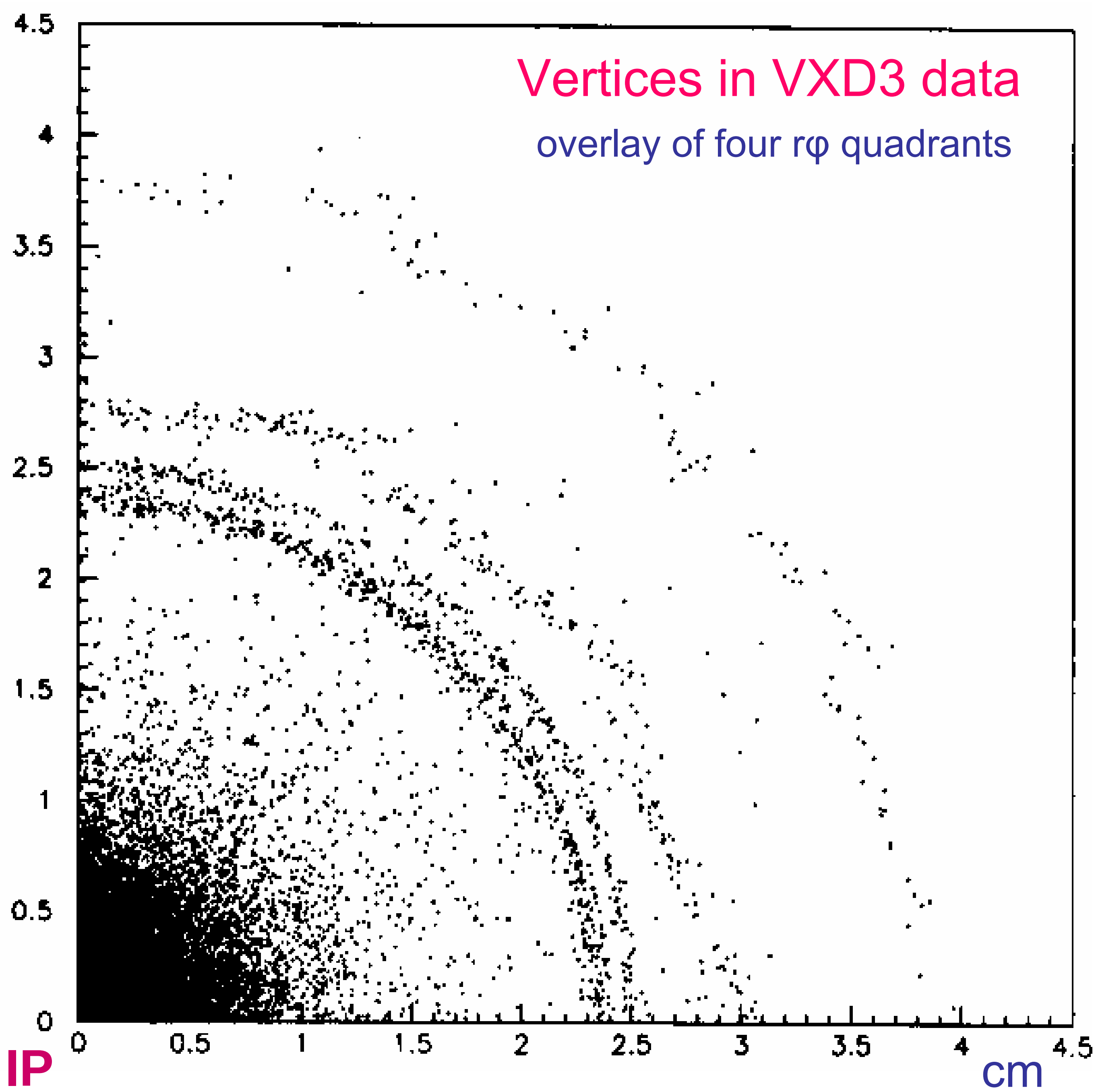
- **Symmetry of the detector** – greatly assists book-keeping and allows comparison of different parts of the detector.
- **Overlap regions** – allows devices to be stitched together with favourable lever arm (data α area of overlap).
- **Large devices** – obviously better to have a single element than two with an overlap.

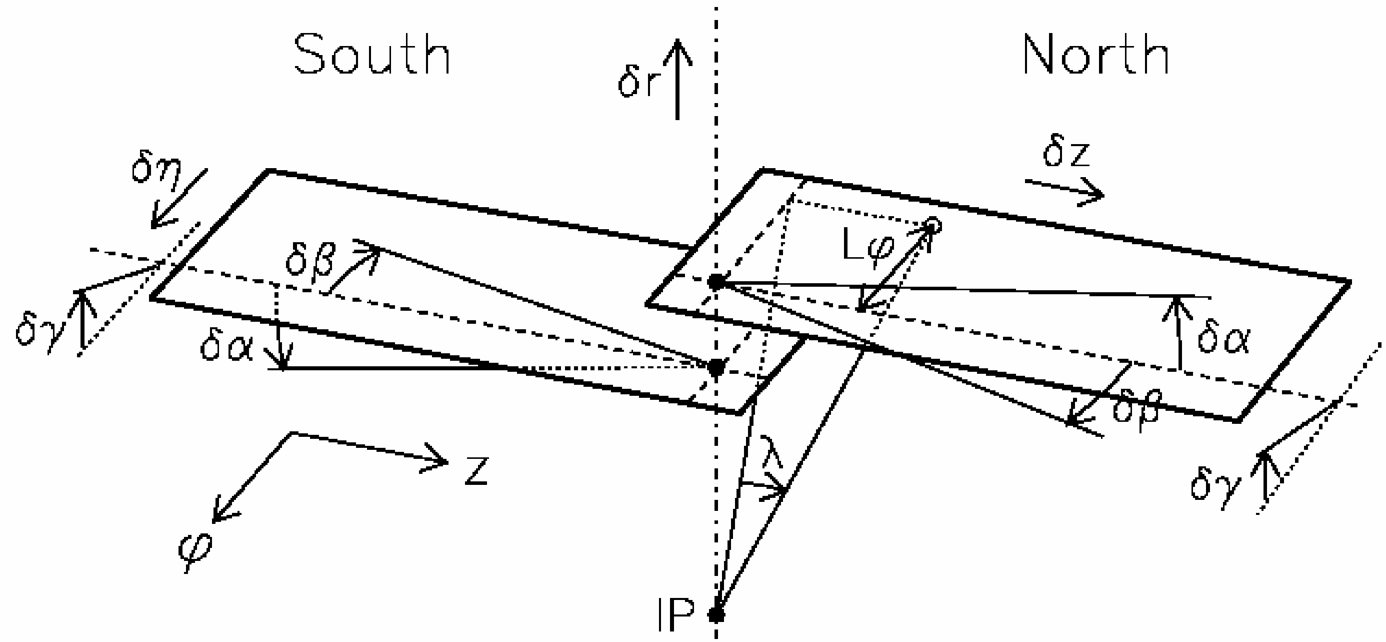
Comments for SiD tracker II

- **Stability** - the geometry (devices and support structure) should be stable with respect to time. Changes due to temperature fluctuations, cycling of magnetic field, ageing under gravity/elastic forces, should be 'small'; at least over a period of time long enough to collect sufficient track data for alignment.

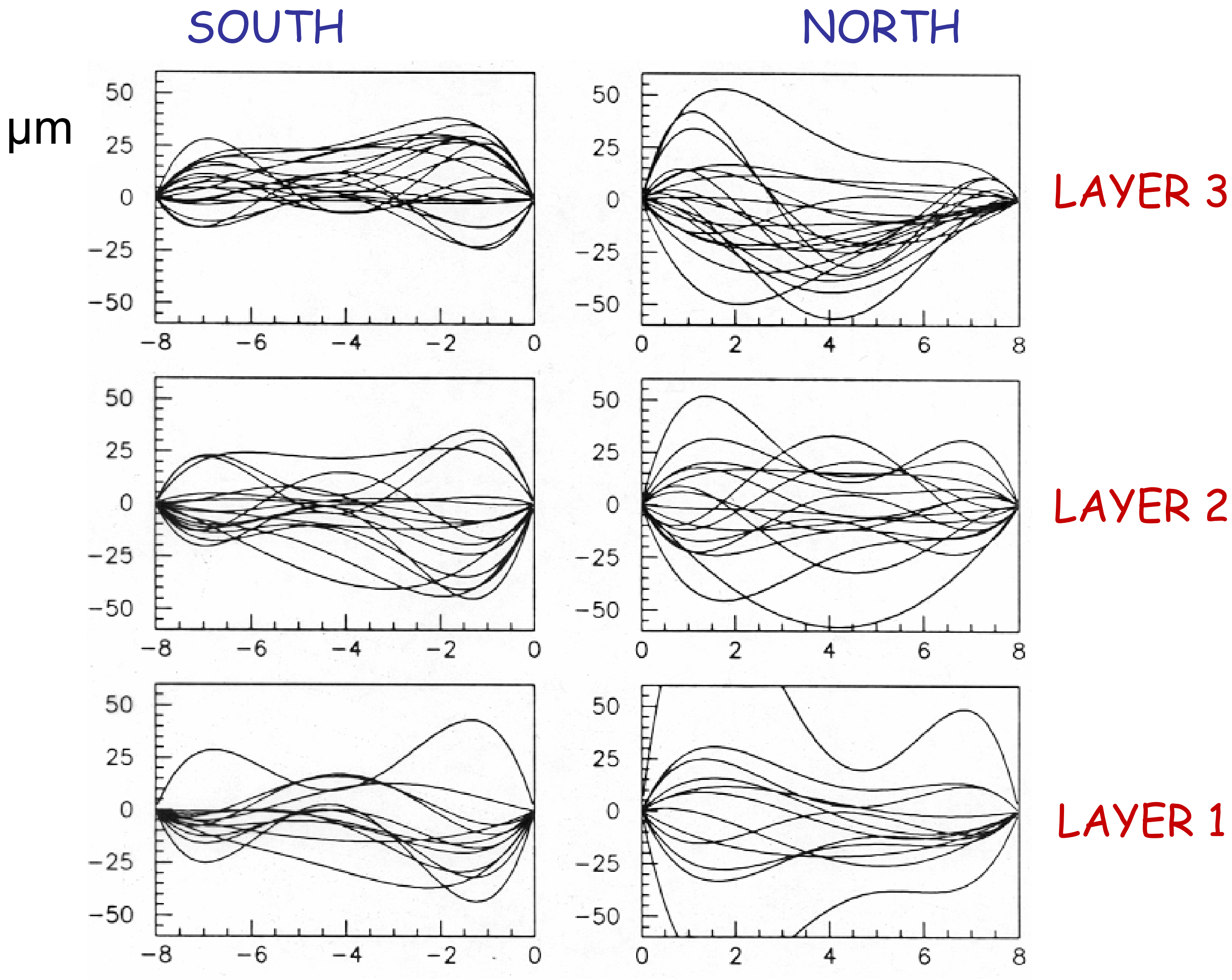
- **Shape** - within reason the shape of the device is irrelevant; only the uncertainty in the shape is important and the ability to describe the shape correction with as few parameters as possible. Making the devices 'flat' is somewhat arbitrary; introducing a deliberate bow of around 1% could greatly increase mechanical stability and decrease shape uncertainty without effecting tracking performance.

VXD3 alignment: D.J.Jackson, D.Su, F.J.Wickens; NIM **A510**, 233 (2003)

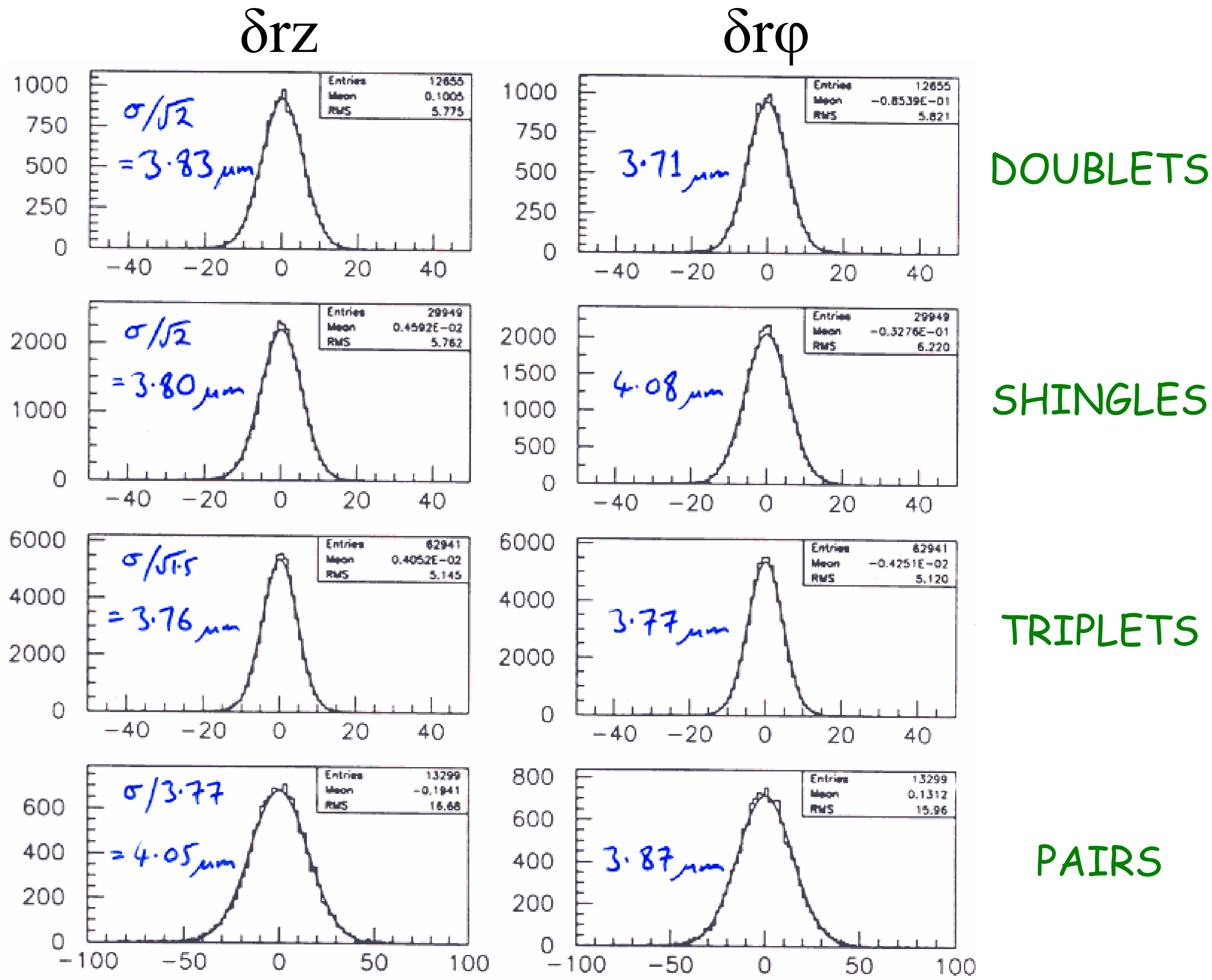

Vertices in VXD3 data
overlay of four rφ quadrants
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
IP
cm


South
North
δr
δz
δη
δβ
Lφ
δγ
δα
δα
δβ
δγ
z
φ
λ
IP

Alignment Shape Corrections



Single hit resolution



hit resolution consistently ~ 3.8 μm