τ polarisation and $\tilde{\chi}_1^0$ DM: mSUGRA, SUSY-GUTs.

- ◇ Introduction.
- ◇ τ polarisation probed using 1-prong hadronic decay.
- ◇ τ polarisation to be used to distinguish between mSUGRA and SUSY-GUTS in the $\tilde{\tau}$-$\tilde{\chi}_1^0$ coannihilation region.

Some of the refs:

M. Guchait, D.P. Roy and R.G., PLB **618**, 193, 2005. Use of the inclusive single π channel to measure P_τ at the ILC.

L. Cabibbi, Y. Mambrini and S. Vempati, hep-ph/0704.3518 SUSY-GUTs, SUSY-Seesaw and Neutralino Dark Matter.

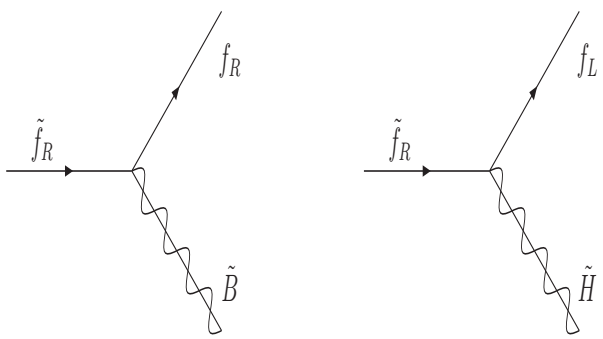
• τ has hadronic decay modes. The energy distribution of the π produced in the decay, $\tau \to \nu_\tau \pi$ as well as those in $\tau \to \rho \nu_\tau, \tau \to a_1 \nu_\tau$ depends on the handedness of the τ. Thus τ polarisation can be determined using decay π energy distribution.

K. Hagiwara, A.D. Martin and D. Zeppenfeld, PLB **235** 198 (1990), B.K.Bullock, K.Hagiwara and A.D.Martin, PRL **67**, 3055 (1991), NPB **395**, 499 (1993), D.P.Roy, PLB **277**, 183 (1992).

• Polarisation of the fermions produced in sfermion decays carry information on SUSY model parameters, sfermion or chargino/neutralino composition. $\tilde{\tau}$ is NLSP in $\tilde{\tau}$–$\tilde{\chi}_1^0$ coannihilation region. The polaristion of the τ produced in the decays of $\tilde{\tau}_1, \tilde{\tau}_2$ thus can be used to probe the SUSY models.

τ produced in stau decay.

M. Nojiri, PRD **51** (1995) 6281 [hep-ph/9412374]



- In MSSM mass eigenstates of $\tilde{\tau}$ $\tilde{\tau}_1, \tilde{\tau}_2$, are mixtures of $\tilde{\tau}_L$ and $\tilde{\tau}_R$.
- Mixing affects gauge couplings of $\tilde{\tau}_i, i = 1, 2$ and hence the production rates.
- The $\tilde{\chi}_j^0, j = 1, 4$ are mixtures of higgsinos and gauginos.

- Couplings of sfermions with higgsinos flip chirality whereas those with gauginos do not.

- Net helicity of produced τ in the decay $\tilde{\tau}_i \to \tilde{\chi}_j^0 f$ depends on the L–R mixing in the sfermion sector and on the gaugino-higgsino mixing.

Collinear approximation for the $\tilde{\tau}$ decay; i.e. $m_\tau \ll m_{\tilde{\tau}_1}$

$$
\begin{aligned}
P_\tau &= \frac{\left(a_{11}^R\right)^2 - \left(a_{11}^L\right)^2}{\left(a_{11}^R\right)^2 + \left(a_{11}^L\right)^2}, \\
a_{11}^R &= -\frac{2g}{\sqrt{2}} N_{11} \tan\theta_W \sin\theta_\tau - \frac{g m_\tau}{\sqrt{2} m_W \cos\beta} N_{13} \cos\theta_\tau, \\
a_{11}^L &= \frac{g}{\sqrt{2}} \left[N_{12} + N_{11} \tan\theta_W\right] \cos\theta_\tau - \frac{g m_\tau}{\sqrt{2} m_W \cos\beta} N_{13} \sin\theta_\tau, \qquad (1)
\end{aligned}
$$

where

$\tilde{\chi}_1 = N_{11}\tilde{B} + N_{12}\tilde{W} + N_{13}\tilde{H}_1 + N_{14}\tilde{H}_2,$

Essentially different SPS points, for $\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau$:

$\diamond$ mSUGRA: $\tilde{\chi}_1^0 \sim \tilde{B}$

Small $\tan\beta, \cos\theta_\tau$ small $\Rightarrow P_\tau \simeq +1$.

$\tan\beta \Rightarrow$ larger $(\cos\theta_\tau)$ P_τ still close to $+1 (> 0.90)$ over the allowed SUGRA parameter space due to cancellations.

$\diamond$ Nonuniversal SUGRA models. The gauge kinetic function determined by the non-singlet chiral superfield, at the GUT scale, representations 75,200. LSP dominated by the Higgsino component over most of the parameter space. $P_\tau \simeq \cos^2\theta_\tau - \sin^2\theta_\tau$,

$\diamond$ AMSB : $\tilde{\chi}_1^0$ is Wino like: expect $P_\tau = -1$.

$\diamond$ GMSB : the LSP is the gravitino $\tilde{G}$, while the $\tilde{\tau}_1$ is the NLSP over a large part of parameter space

$\tilde{\tau}_1 \to \tau\tilde{G} \Rightarrow P_\tau = \sin^2\theta_\tau - \cos^2\theta_\tau$.

If $\tilde{\tau}_1$ is heavier than the $\tilde{\chi}_1^0$, then GMSB is like mSUGRA.

$\diamond$ Expected polarisations: $1, -1/2, -1, +1/2$ if one uses $\cos\theta_\tau = 0.5$.

- How does one get information on τ polarisation using hadronic decay of τ.? K. Hagiwara, A.D. Martin and D. Zeppenfeld, PLB **235** 198 (1990).

- τ decays: $\tau \to \pi^\pm\nu, \rho^\pm\nu, a_1^\pm\nu$. The CM angular distribution of the decay meson [J=0, π] [J=1, $\rho, a_1 J = 1$] depends on τ polarisation:

$$\frac{1}{\Gamma_\pi}\frac{d\Gamma_\pi}{d\cos\theta} = \frac{1}{2}(1 + P_\tau \cos\theta)$$
$$\frac{1}{\Gamma_v}\frac{d\Gamma_{vL,T}}{d\cos\theta} = \frac{\frac{1}{2}m_\tau^2, m_v^2}{m_\tau^2 + 2m_v^2}(1 \pm P_\tau \cos\theta),$$

- L,T are longitudinal and transverse states of vector mesons v. These can be distinguished using the fact that transverse (longitudinal) vector mesons share the energy of parent meson evenly (unevenly) among the decay pions. Energy distribution of decay pions can be used then to measure the τ polarisation.

- A lot of nice analyses of τ polarisation and hence of the MSSM parameter determination at LC exist. They Use the $\tau \to \rho/a_1\nu_\tau$ (multiprong) mode. M.M.Nojiri, PRD **51** (1995) 6281 [hep-ph/9412374], M.M.Nojiri et al PRD **54**, 6756 (1996) [hep-ph/9606370], E.Boos et al, EPJC **30** (2003) 395 [hep-ph/0303110].

- Our addition to this: (M. Guchait, D.P. Roy and R.G.:[PLB 628, 131, 2005.])

- 1-prong π final state used previously to sharpen up $H^{\pm}$ signature S. Raychaudhuri, D. P. Roy, PRD52(1995)1556; D53(1996)4902; D.P. Roy, PLB459(1999)607.

Developed a variable for τ polarisation analysis.

- Look at $R = p_{\pi^{\pm}}/p_{\tau-jet}$. and study

$$f = \frac{\sigma(0.2 < R < 0.8)}{\sigma_{total}}$$

- f a good discriminator of τ polarisation.

- If region $R < 0.2$ is inacessible due to the difficulty in τ identification for a soft track, use the $\sigma(0.2 < R)$ for normalisation.

- We have studied its application to the ILC studies.

More on inclusive 1-prong τ decay τ identification best done through the hadronic decay.

1-prong inclusive hadronic decay corresponds to 80% of hadronic decay and 50% of total width.

Main decay modes contributing to 1-prong decay (about 90% of total 1-prong decay) are: $\tau \to \nu_\tau \pi$, $\tau \to \rho \nu_\tau, \tau \to a_1 \nu_\tau$

Define x as the fraction of the τ lab momentum carried by its decay meson. In the collinear approximation x is given by:

$$x = \frac{1}{2}(1 + \cos\theta) + \frac{m^2_{\pi,v}}{2m^2_\tau}(1 - \cos\theta) = \frac{p_{\tau-jet}}{p_\tau}.$$

For τ decay the only measurable momentum is τ-jet momentum.

If $P_\tau = 1$: hard jets come from $\pi, \rho_L, a_{1L} \Rightarrow$ uneven sharing of momenta among the decay π coming from v. $\Rightarrow$ Distribution in R is peaked at $R < 0.2$ and $R > 0.8$.

If $P_\tau = -1$: hard jets come from $\rho_T, a_{1T} \Rightarrow$ even sharing of momenta among the decay π coming from v. $\Rightarrow$ R distribution peaked in the middle.

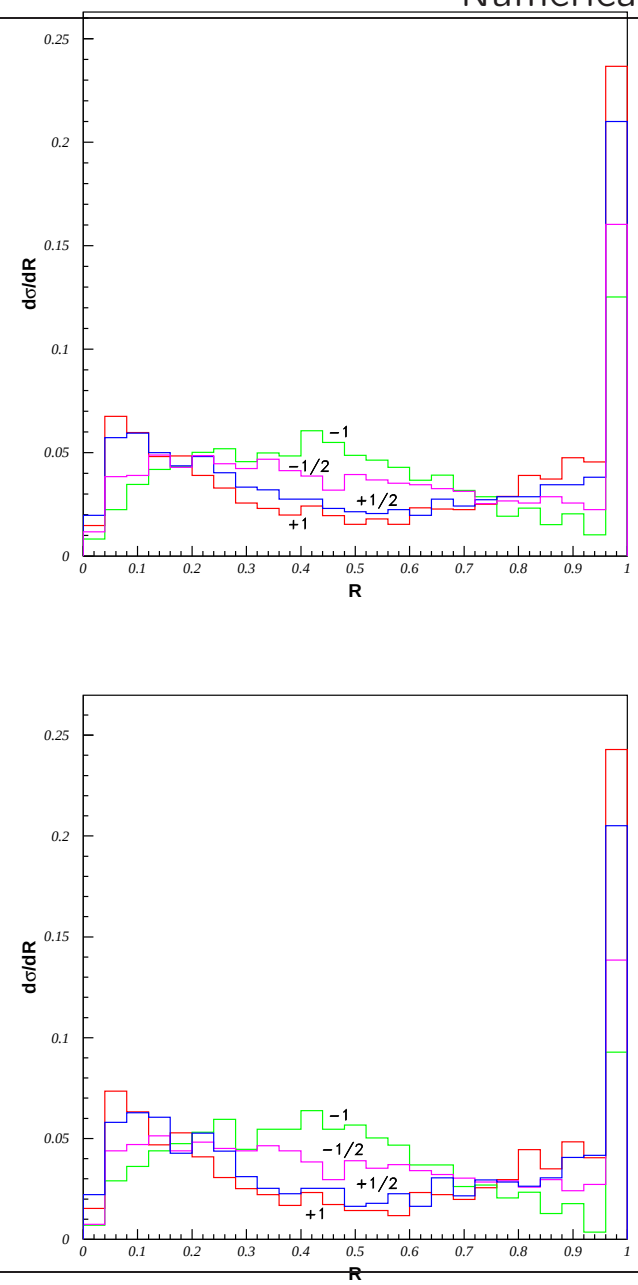
Results presented for

$\sqrt{s} = 350$ GeV, $m_{\tilde{\tau}_1} = 150$ GeV, $m_{\tilde{\chi}_1} = 100$ GeV

$p^T_{\tau-\text{jet}} > 25$ GeV, $\cos\theta_{\tau-\text{jet}} < 0.75$

Top :Distribution in R. Different curves for values of polarisation $P_\tau = -1, -.5, 0.5, 1.0$ indicated on the different curves.

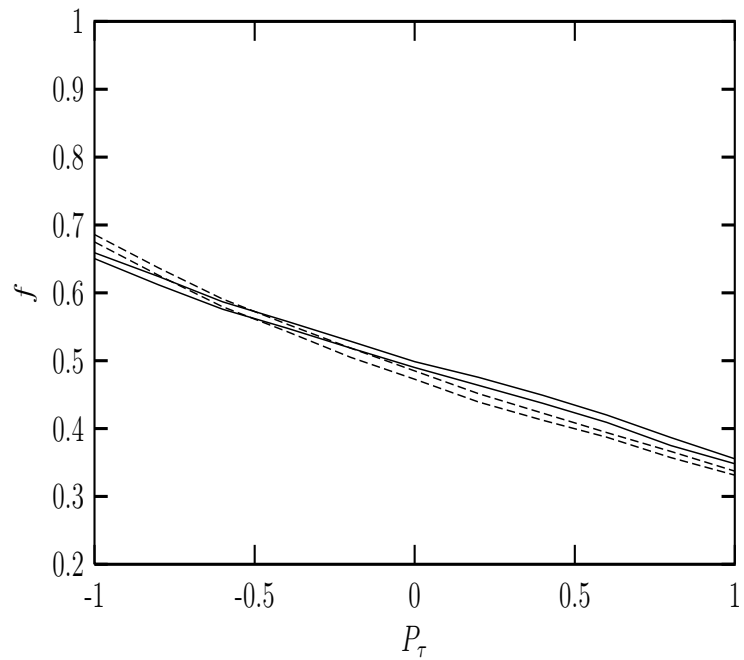
Bottom: The same thing for a cut on $P_\tau >$ 50 GeV.



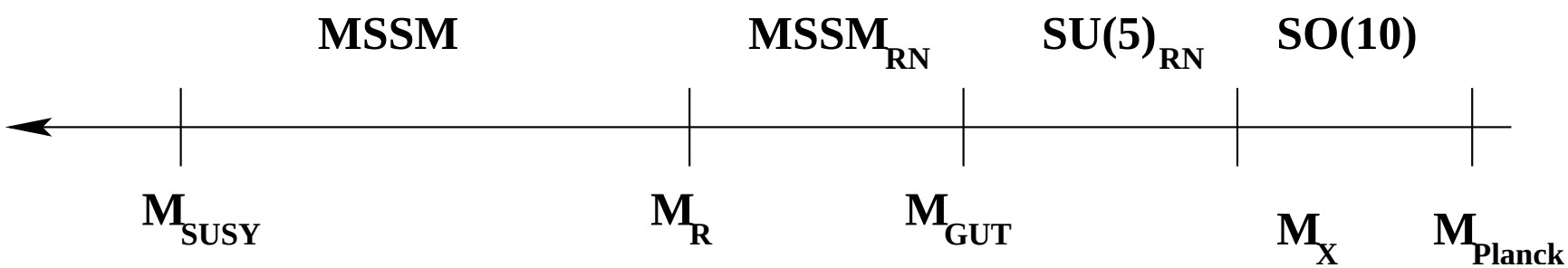
f as a function of P_τ. Uncertainty due to the difft. parametrisations of the a_1 and non-resonant contributions to the π. Estimated using Tauola. $\Delta P_\tau = \pm 0.03(\pm 0.05)$ for $P_\tau = -1(+1)$ Additional error might come from measurement of f. Still measurement upto 10%($\Delta P_\tau = 0.1$) possible.

- $p^T_{\tau-jet}$ >25 GeV cut (solid lines): f changes from 0.65 to 0.35 for $P_\tau = -1$ to 0.35 at $P_\tau = +1$. For $p^T_{\tau-jet} > 50$ GeV (dashed line) decrease is steeper.

- Use of inclusive 1-prong channel, robust method of determining τ polarisation. If the aim is only polarisation determination this has the advantage of higher statistics and smaller systematic errors, compared to the exclusive channel.



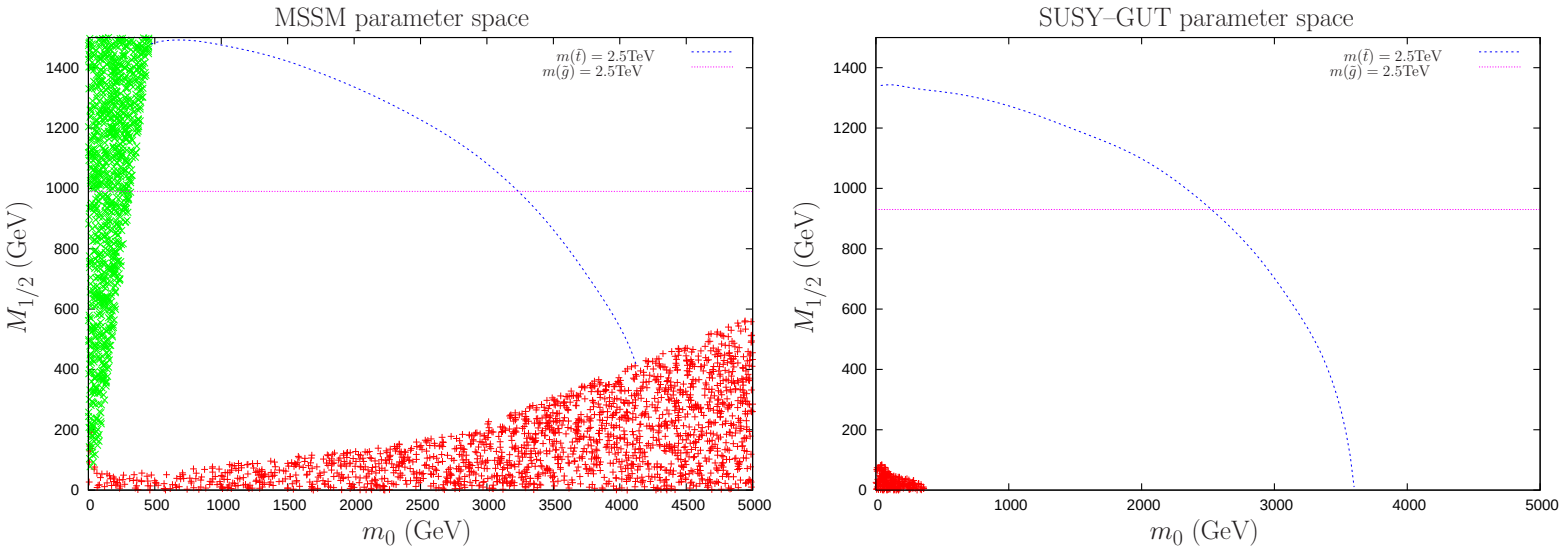
Recent analysis of CDM in a SUSY-GUTs: $SU(5)$ with a Right handed (s)neutrino. The RN: right handed neutrino, has Yukawa couplings close to top Yuakawa coupling.



$$\mathcal{W}_{RN} = \frac{1}{2} h^u_{ij} 10_i 10_j 5_u + h^\nu_3 \bar{5}_3 1 5_u + h^d_{ij} 10_i \bar{5}_j \bar{5}_d$$

H_2 is contained in 5_u and H_1 in 5_d, i, j : generation indices.

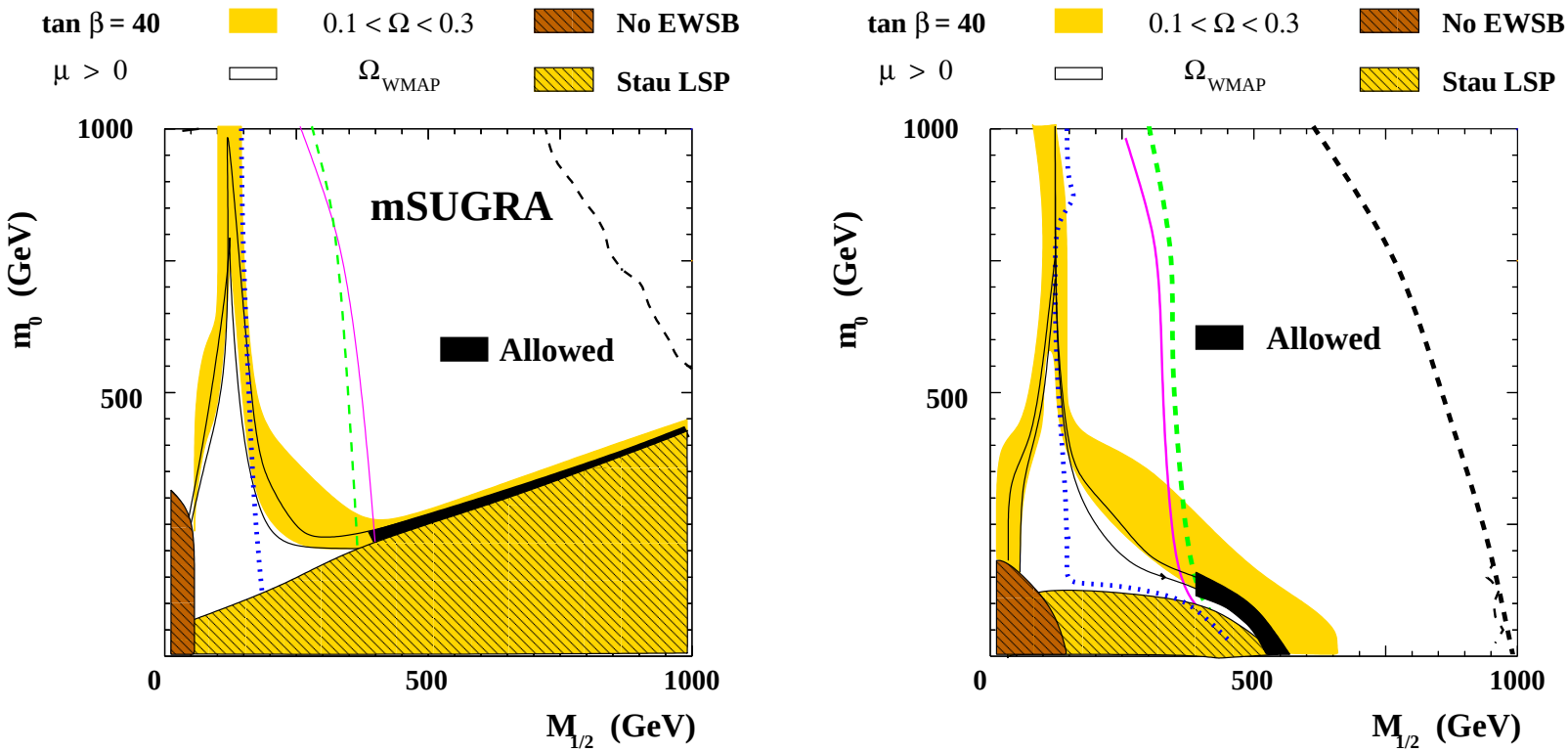
M_X : scale of breaking of some higher GUTs chosen around 10^{17} GeV.



$SU(5)+$ RHN running between M_X and M_{GUT}: nontrivial effects of the large h_3^ν on the RGE running of H_2 mass **and** nontrivial contributions to masses of the right sfermions.

After M_R the regular mSUGRA RGE takes over

Universal masses at M_X. Due to the large additional contribution from h_3^ν the EWSB is possible almost always.



Effects of running between M_X and M_{GUT} as well as the effect of RN neutrino.

$\tilde{\tau}_L$ and $\tilde{\tau}_R$ maximally mixed.

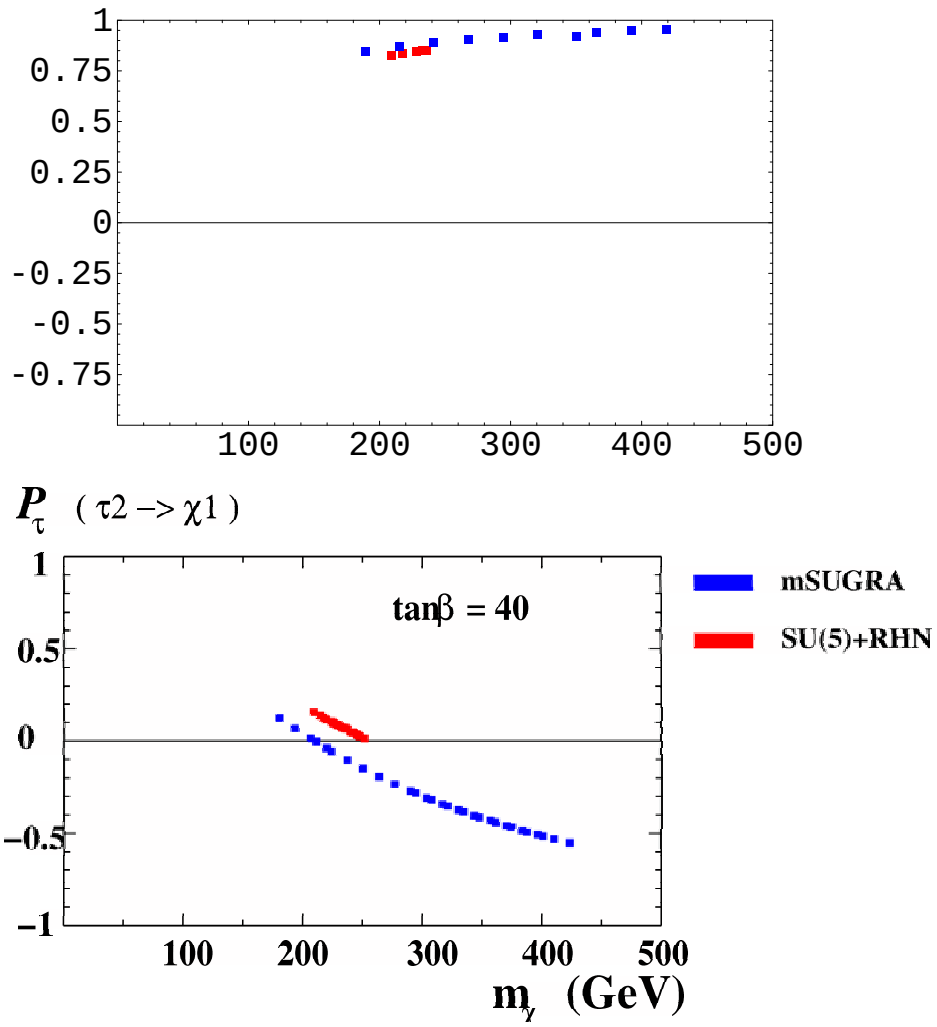
Can τ polarisation in $\tilde{\tau}$ decay be used to differentiate between the two models?

What decides the mixing?

$$m^2_{\tilde{\tau}} = \begin{pmatrix} m^2_{\mathsf{LL}} & m^2_{\mathsf{LR}} \\ m^2_{\mathsf{LR}} & m^2_{\mathsf{RR}} \end{pmatrix} = \mathbf{R}^T \begin{pmatrix} m^2_{\tilde{\tau}_1} & 0 \\ 0 & m^2_{\tilde{\tau}_2} \end{pmatrix} \mathbf{R} \tag{2}$$

where

$$\begin{aligned} m^2_{\mathsf{LL}} &\approx (1-\rho_L)m_0^2 + c_L M^2_{1/2} \\ m^2_{\mathsf{RR}} &\approx (1-\rho_R)m_0^2 + c_R M^2_{1/2} \\ m^2_{\mathsf{LR}} &= m_\tau(A_\tau - \mu \tan\beta) \approx -m_\tau \mu \tan\beta \end{aligned} \tag{3}$$



- For $\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0$ the P_τ is the same for mSUGRA and $SU(5)+$ RHN.

- For $\tilde{\tau}_2 \to \tau\tilde{\chi}_1^0$ is completely different.

- Can be a clear way to distinguish. Do we understand it?

In the coannihilation region, $\tilde{\chi}_1^0 \sim \tilde{B}$: The polarisation of τ in $\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0$ is:

$$P_\tau = \frac{4\sin^2\theta_\tau - \cos^2\theta_\tau}{4\sin^2\theta_\tau + \cos^2\theta_\tau}$$

and for $\tilde{\tau}_2 \to \tau\tilde{\chi}_1^0$

$$P_\tau = \frac{4\cos^2\theta_\tau - \sin^2\theta_\tau}{4\cos^2\theta_\tau + \sin^2\theta_\tau}$$

Recall the mixing angle determined by entries in the $\tilde{\tau}$ mass matrix.

For MSSM:

$$\rho_L = \frac{y_\tau^2}{8\pi^2}(3 + a_0^2) \ln \frac{M_{\mathrm{GUT}}}{M_{\mathrm{susy}}}$$

$$\rho_R = \frac{y_\tau^2}{8\pi^2}(6 + 2a_0^2) \ln \frac{M_{\mathrm{GUT}}}{M_{\mathrm{susy}}}$$

For the GUTS:

$$\rho_L = \frac{3 + a_0^2}{8\pi^2}(y_\tau^2(4 \ln \frac{M_{\mathrm{X}}}{M_{\mathrm{GUT}}} + \ln \frac{M_{\mathrm{GUT}}}{M_{\mathrm{susy}}}) + y_\nu^2 \ln \frac{M_{\mathrm{X}}}{M_{\mathrm{R}}})$$

$$\rho_R = \frac{3 + a_0^2}{8\pi^2}(2y_\tau^2 \ln \frac{M_{\mathrm{X}}}{M_{\mathrm{susy}}} + 3y_t^2 \ln \frac{M_{\mathrm{X}}}{M_{\mathrm{GUT}}})$$

Using these an approximate expression for the condition $P_\tau > 0$ can be obtained.

For $\tilde{\tau}_2 \to \tau\tilde{\chi}_1^0$ this means

$$4\cos^2\theta_\tau > \sin^2\theta_\tau \Rightarrow 4(m_{\mathrm{RR}}^2 - m_{\tilde{\tau}_1}^2)^2 > m_{\mathrm{LR}}^4$$

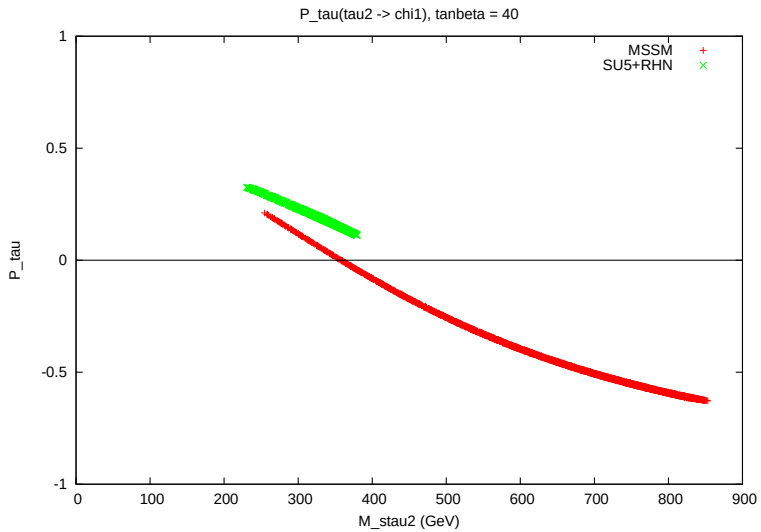
Due to the RHN effect and the additional running, this can be satisfied for the $SU(5)+$ RHN always, in contrast to the mSUGRA case.

For $\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0$

$$(m_{\mathrm{RR}}^2 - m_{\tilde{B}}^2)^2 < 4m_{\mathrm{LR}}^4$$

Which is sastisfied always for both mSUGRA and $Su(5)+$ RHN.

Using an approximate value of μ one can calculate p_τ again:



We need to see what are the production c.sections for the $\tilde{\tau}_2$, but certainly mass values are such that 500 GeV ILC will be able to distinguish between the two cases using polarisation.

- τ polarisation can be measured to a good accuracy at ILC, using the inclusive pion energy spectrum.

- The parameter space where $\tilde{\tau}$ coannihilation gives the right amount of DM is different in $SU(5)+$ RHN from that in mSUGRA, so is the mixing in $\tilde{\tau}$ sector.

- The polarisation of τ in $\tilde{\tau}_2 \to \tilde{\chi}_1^0 \tau$ is different in the two cases and the difference is a pure GUT effect. Effect of the running and the large Yukawa of the RHN.

- Possible to distinguish the two cases at a 500 GeV NLC.