$e^+e^- \to 3$ jets at NNLO

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Jet Observables

Observing ”free” quarks and gluons at colliders

QCD describes quarks and gluons; experiments observe hadrons

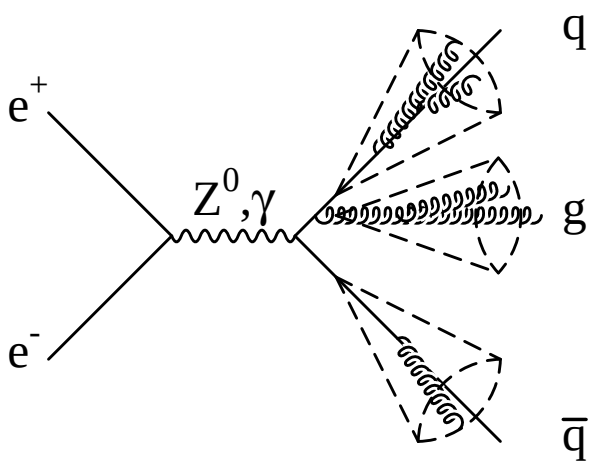
- describe parton $\longrightarrow$ hadron transition (fragmentation)
- define appropriate final states, independent of particle type in final state (jets)

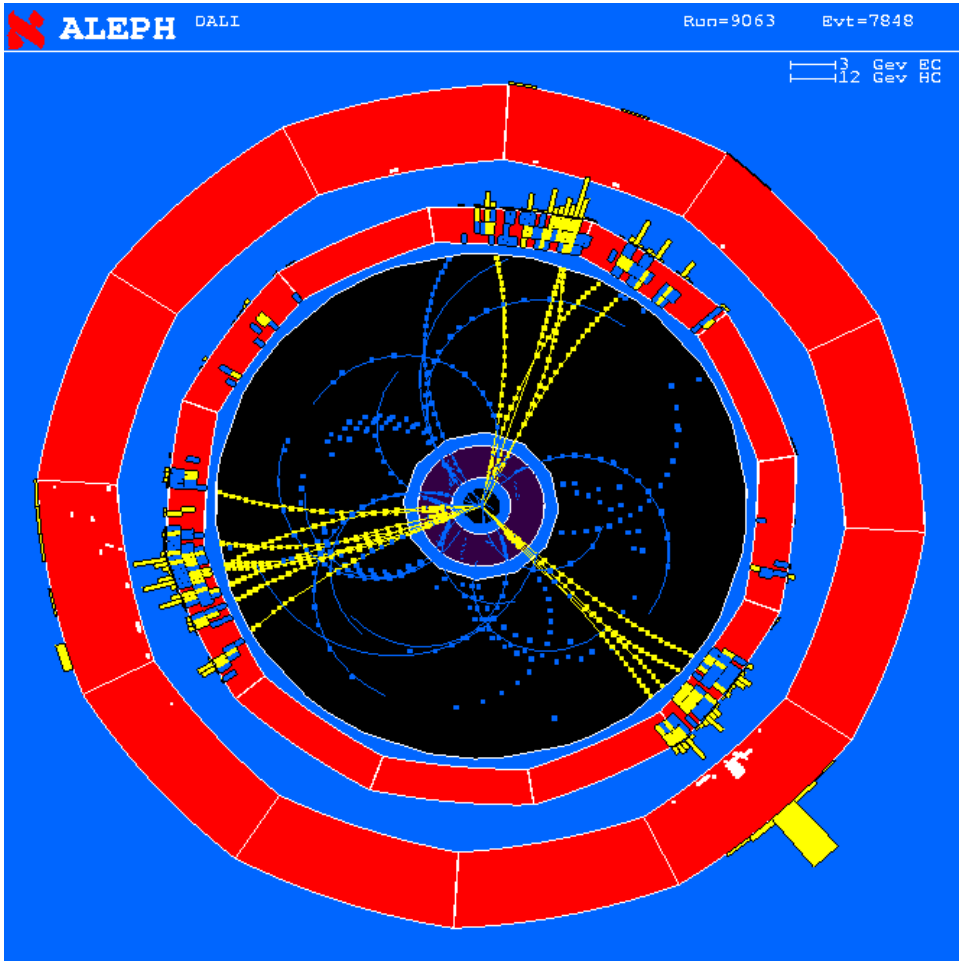
Jets

- experimentally: hadrons with common momentum direction
- theoretically: partons with common momentum direction

Jet Observables

$e^+e^- \longrightarrow 3$ jets
event at LEP





Jet Observables

Formal requirements on jet observables

G. Sterman, S. Weinberg

Jet observable defined using n-particle final state: $O_n(p_1, \ldots, p_n)$

- collinear limit: $O_n(p_1, p_2, \ldots, p_n) \stackrel{p_1 \| p_2}{\longrightarrow} O_{n-1}(p_1 + p_2, \ldots, p_n)$
- soft limit: $O_n(p_1, p_2, \ldots, p_n) \stackrel{E_1 \to 0}{\longrightarrow} O_{n-1}(p_2, \ldots, p_n)$

Jet observables which fulfil these criteria are infrared-safe

Jet algorithms

measurement and recombination procedure to combine nearby particle momenta into jets, e.g. JADE-algorithm

recombine pair (ij) with lowest $s_{ij} = (p_i + p_j)^2 < s_{\rm cut}$

other jet algorithms: Durham (k_T), Cambridge, Cone

Jet Observables

Event shape variables

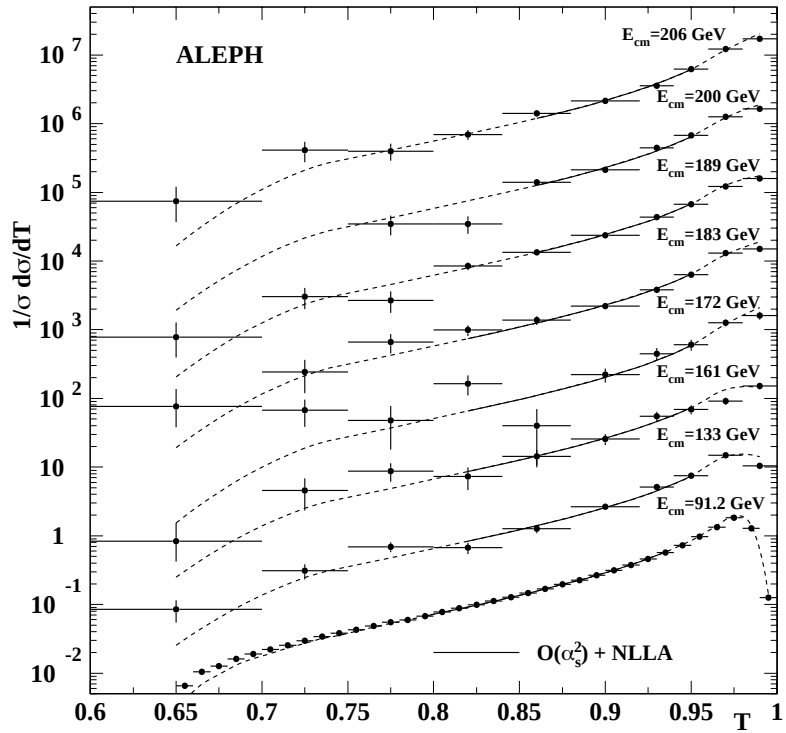
assign a number x to a set of final state momenta: $\{p\}_i \to x$

e.g. Thrust in e^+e^-

$$T = \max_{\vec{n}} \frac{\sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}|}{\sum_{i=1}^{n} |\vec{p_i}|}$$

limiting values:

- back-to-back (two-jet) limit: $T = 1$
- spherical limit: $T = 1/2$



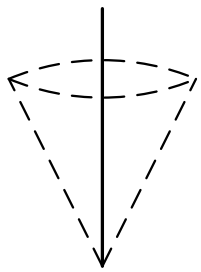
can be used as precision measurement of α_s: (based on NLO)

$$\alpha_s(M_Z) = 0.1202 \pm 0.0003(\text{stat}) \pm 0.0009(\text{sys}) \pm 0.0009(\text{had}) \pm 0.0047(\text{scale})$$

Jets in Perturbation Theory

Theoretically

- Partons are combined into jets using the same jet algorithm as in experiment



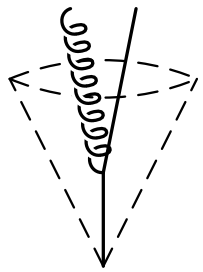
LO

each

parton

forms 1 jet

on its own



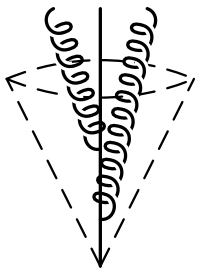
NLO

2 partons in

1 jet, 1 parton

experimentally

unresolved



NNLO

3 partons in

1 jet, 2 partons

experimentally

unresolved

Current state-of-the-art: NLO

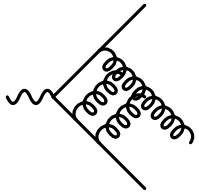
Need for higher orders:

- reduce error on α_s
- better matching of parton level and hadron level jet algorithm

Ingredients to NNLO $e^+e^- \to$ 3-jet

- Two-loop matrix elements

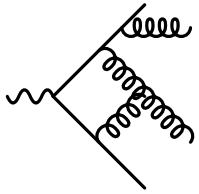
  $|\mathcal{M}|^2_{\text{2-loop},3}$ partons

  explicit infrared poles from loop integrals
  L. Garland, N. Glover, A. Koukoutsakis, E. Remiddi, TG;
  S. Moch, P. Uwer, S. Weinzierl

- One-loop matrix elements

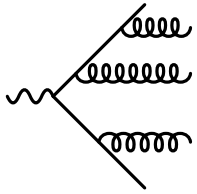
  $|\mathcal{M}|^2_{\text{1-loop},4}$ partons

  explicit infrared poles from loop integral and
  implicit infrared poles due to single unresolved radiation
  Z. Bern, L. Dixon, D. Kosower, S. Weinzierl;
  J. Campbell, D.J. Miller, E.W.N. Glover

- Tree level matrix elements

  $|\mathcal{M}|^2_{\text{tree},5}$ partons

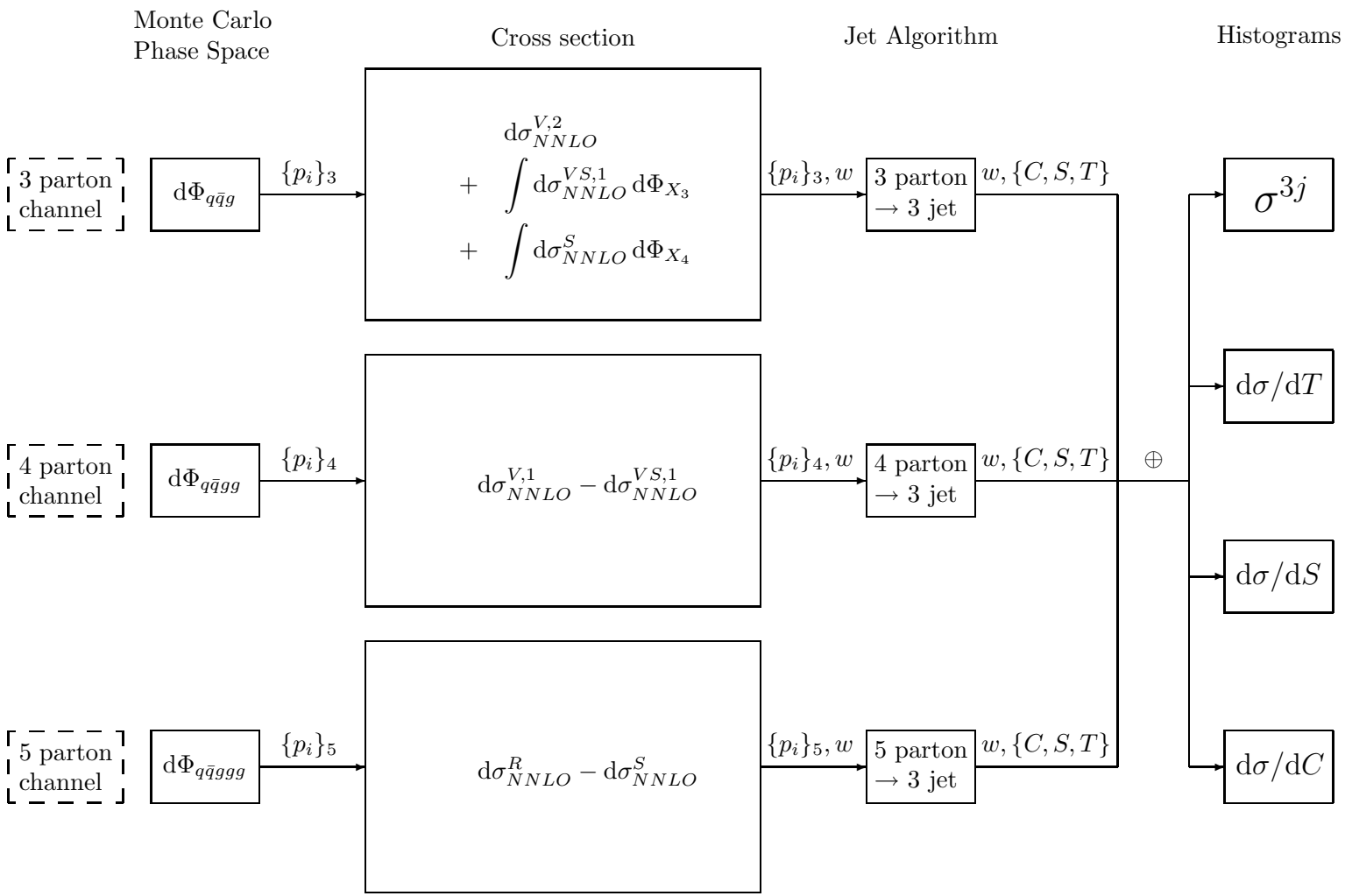
  

  implicit infrared poles due to double unresolved radiation
  K. Hagiwara, D. Zeppenfeld;
  F.A. Berends, W.T. Giele, H. Kuijf;
  N. Falck, D. Graudenz, G. Kramer

Infrared Poles cancel in the sum

Numerical Implementation

Structure of $e^+e^- \longrightarrow 3$ jets program:



Numerical Implementation

Antenna subtraction

NLO: M. Cullen, J. Campbell, E.W.N. Glover; D. Kosower; A. Daleo, D. Maitre, TG
NNLO: A. Gehrmann-De Ridder, E.W.N. Glover, TG

- construct subtraction terms from physical $1 \to 3$ and $1 \to 4$ matrix elements
- each antenna function interpolates between all limits associated to one or two unresolved partons
- integrated subtraction terms cancel infrared pole structure of two-loop matrix element
  S. Catani; G. Sterman, M.E. Yeomans-Tejeda

Checks

- cancellation of infrared poles in 3-parton and 4-parton channel
- convergence of subtraction terms towards matrix elements along phase space trajectories
- distributions in raw phase space variables
- independence on phase space cut y_0

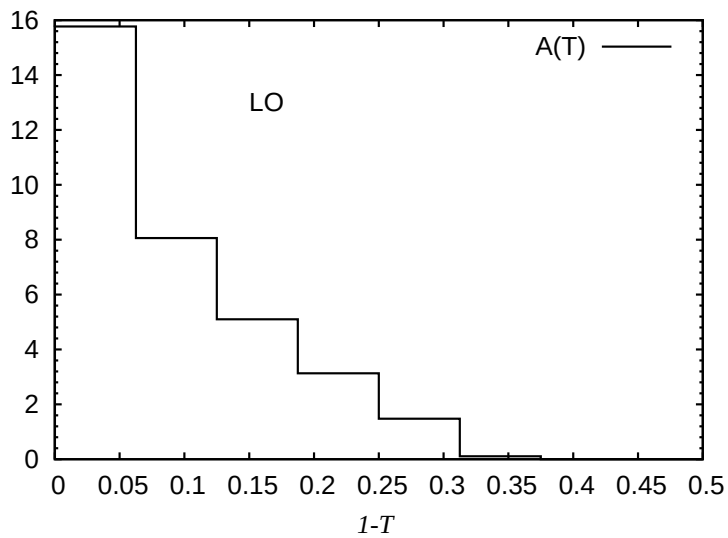
Event shapes at NNLO

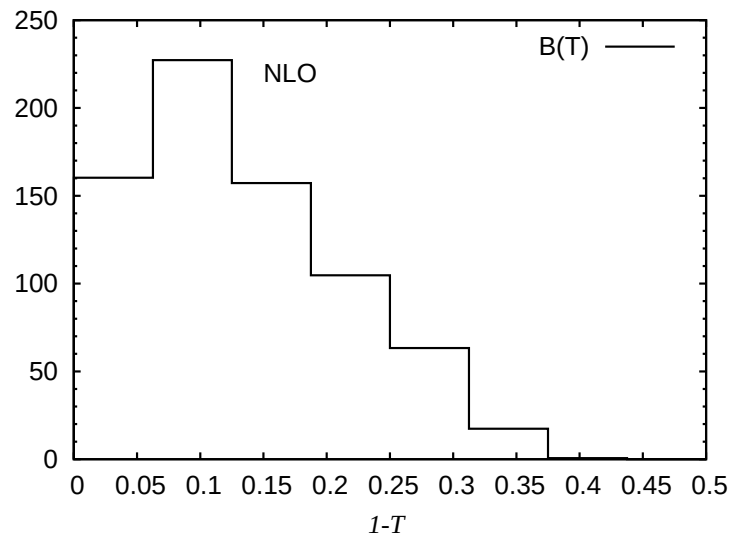
NNLO expression for Thrust

$$(1-T)\,\frac{1}{\sigma_{\rm had}}\,\frac{{\rm d}\sigma}{{\rm d}T} = \left(\frac{\alpha_s}{2\pi}\right) A(T) + \left(\frac{\alpha_s}{2\pi}\right)^2 (B(T) - 2A(T)) + \left(\frac{\alpha_s}{2\pi}\right)^3 (C(T) - 2\,B(T) - 1.64\,A(T))$$

with LO contribution $A(T)$, NLO contribution $B(T)$

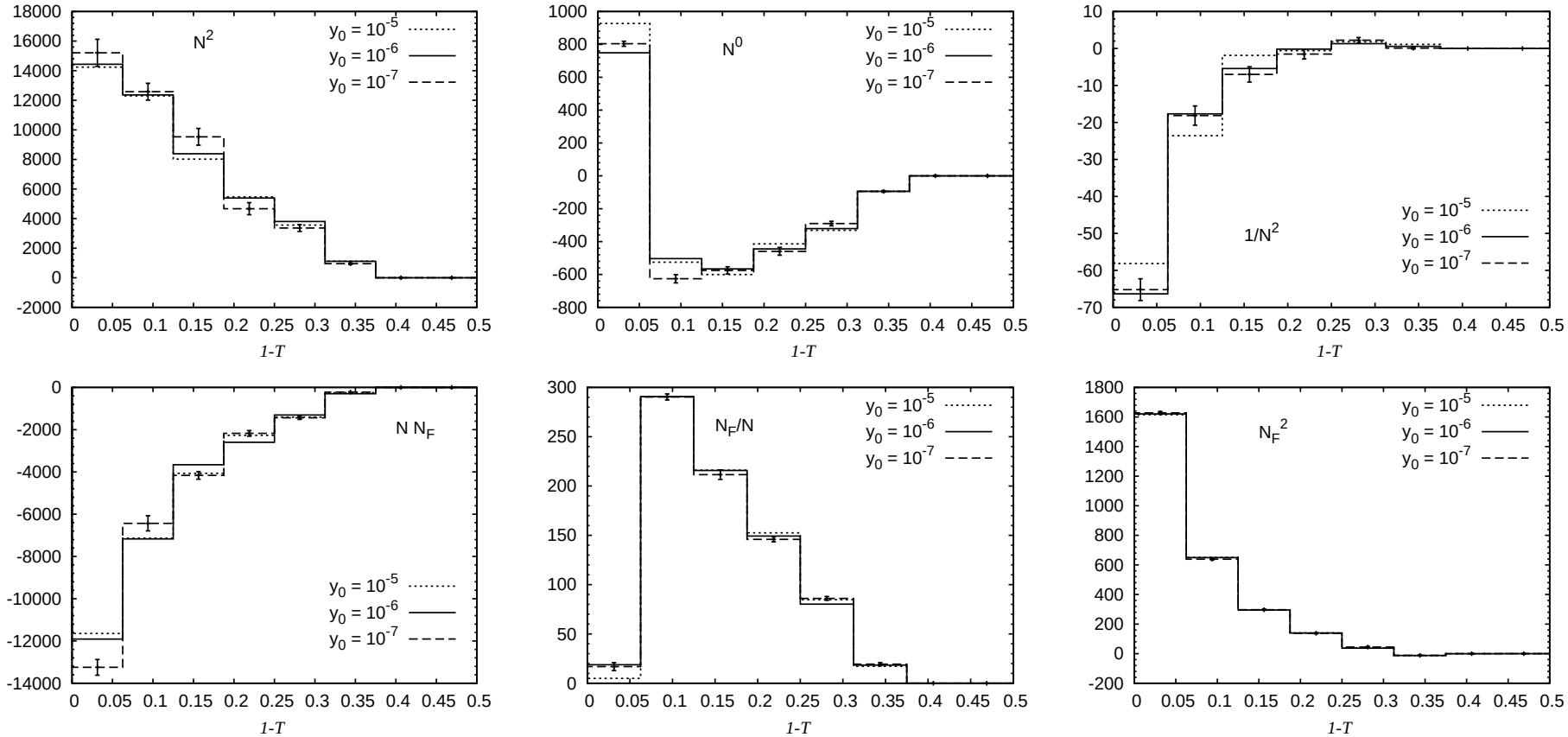
R.K. Ellis, D.A. Ross, A. Terrano





Results

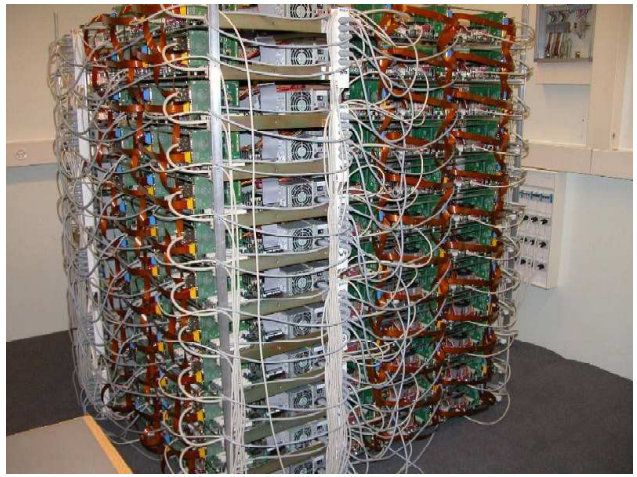
NNLO coefficient $C(T)$ of thrust



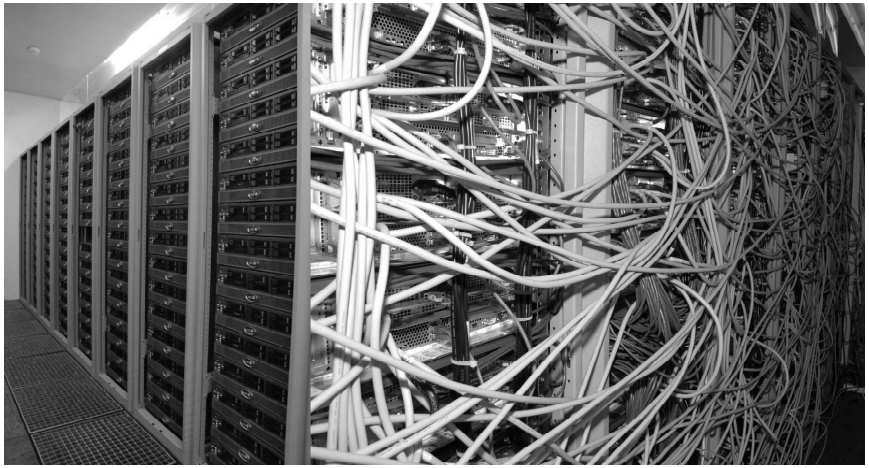
independent on y_0

Numerical computation

zBox1 and zBox2 supercomputers



zBox1



zBox2

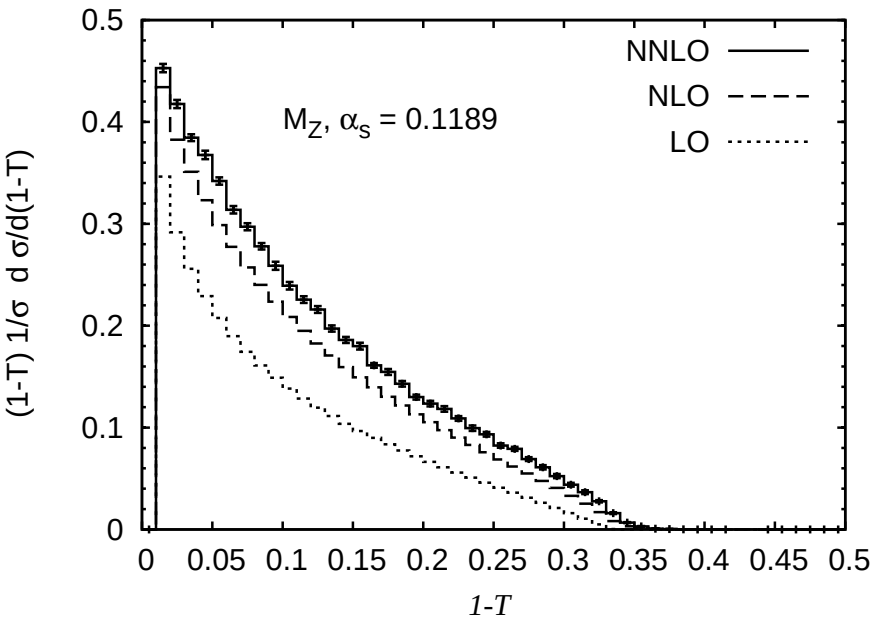
- 288 processors,
  2.2 GHz AMD Athlon
- 0.57 TFlops
- built in-house from
  off-the-shelf components
  J. Stadel, B. Moore

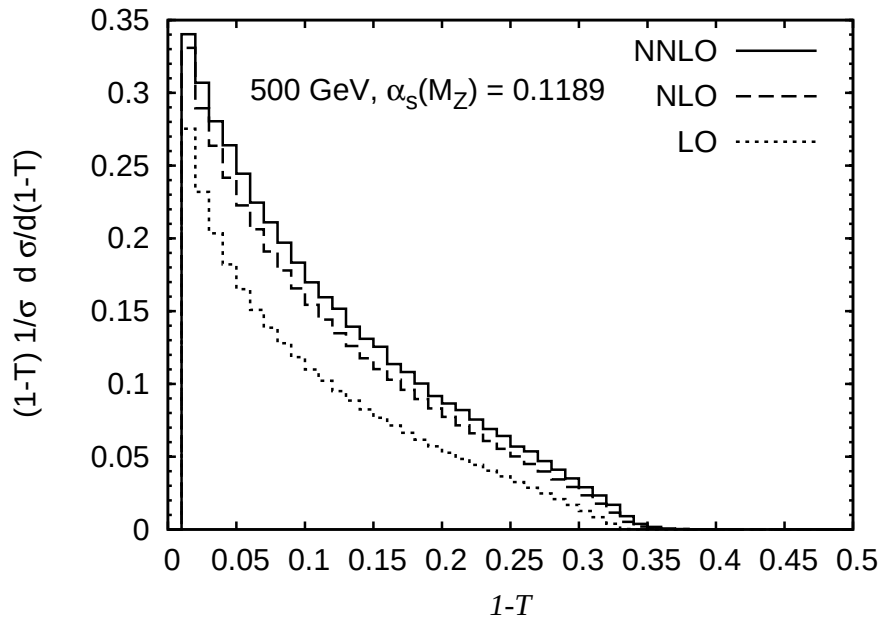
- 500 processors,
  2.6 GHz Opteron 852
- 5.2 TFlops
- built by Sun
  microsystems

used mostly by our computational astrophysics group

Results

NNLO thrust distribution

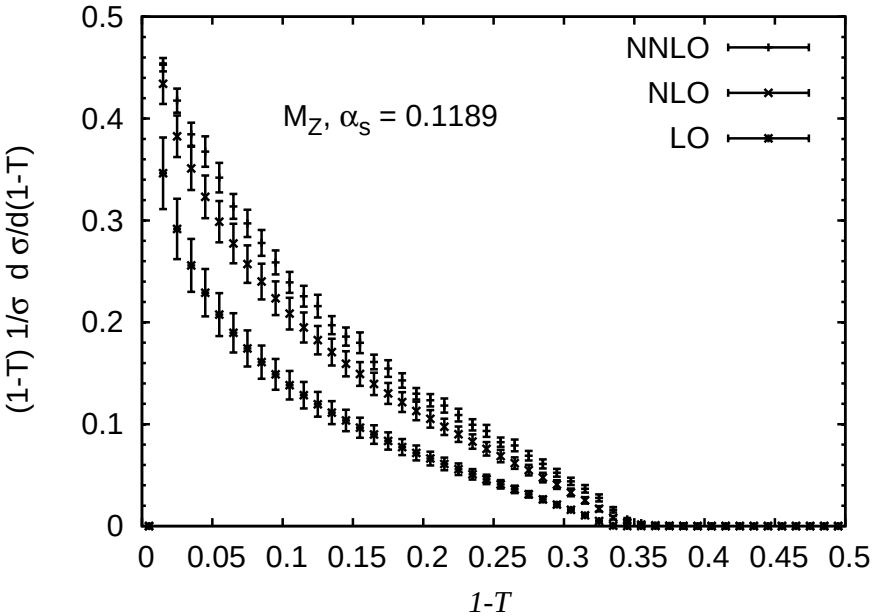


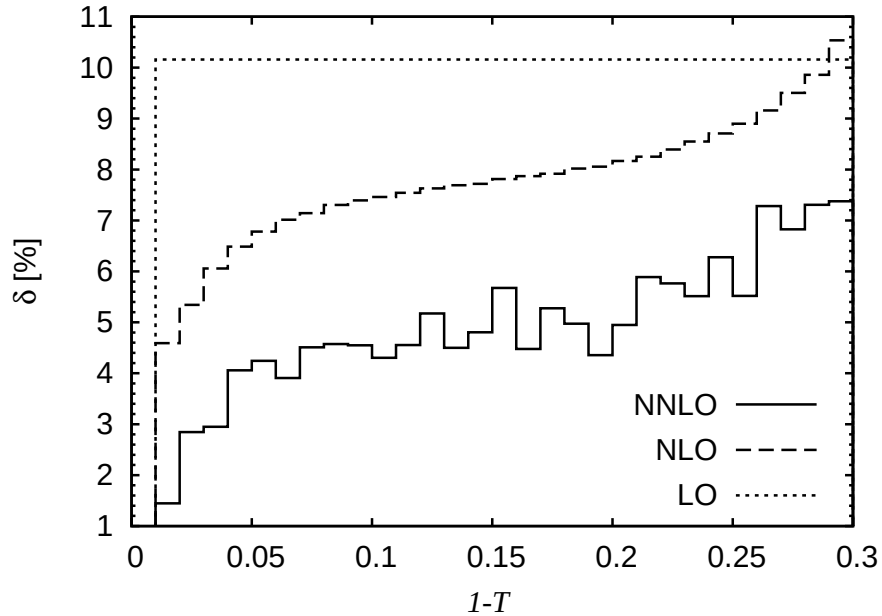


- numerical integration errors after 1 week on zBox1/zBox2
- NNLO corrections sizable, even at ILC energies
- may have to revisit hadronisation corrections
- small $1 - T$: two-jet region, need resummation

Results

NNLO thrust distribution





- varied $\mu = [M_Z/2; 2\,M_Z]$
- NNLO on the edge of NLO theory uncertainty
- renormalisation scale dependence decreases considerably
- started comparison with LEP data $\longrightarrow \alpha_s$

Summary and Conclusions

- completed calculation of NNLO corrections to thrust distribution in e^+e^- annihilation
- constructed parton-level event generator, based on antenna subtraction method
- corrections sizable, possible impact on α_s
- comparison with data just started
- next steps: other event shapes, three-jet rate