Factorization Approach to Top Mass Reconstruction in the Continuum: What mass is measured.

André H. Hoang

Max-Planck-Institute for Physics
Munich

Based on work with:

Sean Fleming, Sonny Mantry and Iain Stewart (hep-ph/0703207)

… more work in progress

Outline

- Why do we want a precision m_t ? What kind of precision.
- Previous ILC studies & experimental issues.
- Factorization theorem for t and $\bar{t}$ invariant mass distribution in electron-positron annihilation ($Q \gg m_t \gg \Gamma_t$)

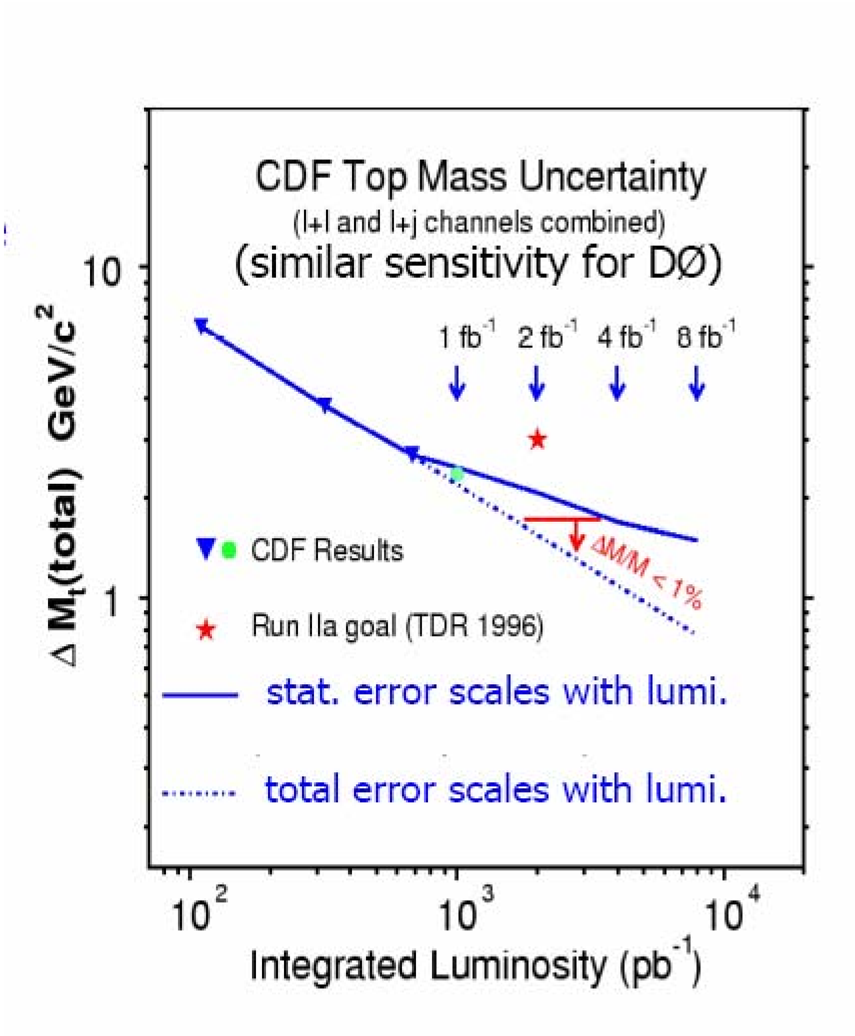
$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m},\Gamma,\mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\Big)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

  - Factorization discussion at LO in $m/Q,\ \Gamma_t/m$
  - Top mass determination to better than $\Lambda_{\rm QCD}$ (at least in principle)
  - Phenomenology
  - How general is our result
- Summary

This talk concentrates on concepts and applications !

See talk in the Loopverein for details on SCET & HQET and radiative corrections

Top Quark is Special !



) SSB?)

ecting many observables

dronization” ($\Gamma_t \approx 1.5$ GeV)

lts

FERMILAB-TM-2380-E
TEVEWWG/top 2007/01
CDF Note 8735
DØ Note 5378
13th March 2007

$$M_t = 170.9 \pm 1.8 \text{ GeV}/c^2$$

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Taking
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ponds to
ecision of

1% precision !

How shall we theorists judge the error ?

What is the theoretical error ?

What mass is it ?

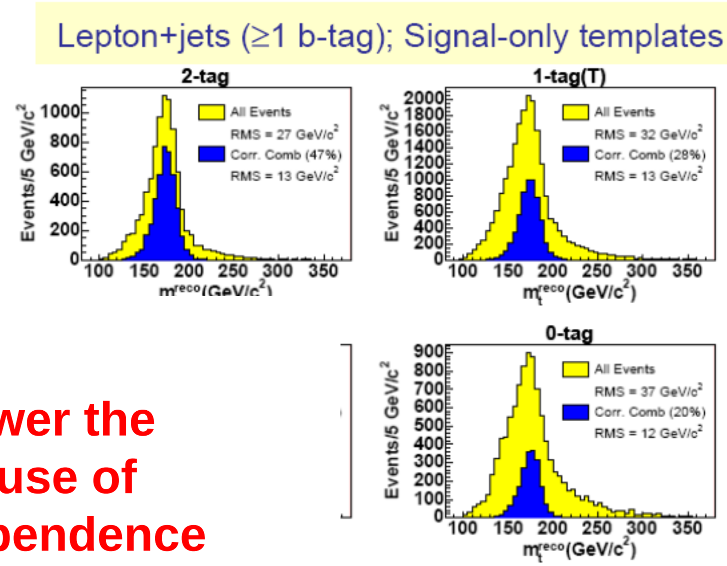
Methods at Tevatron

Template Method

- Principle: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit}-p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=x,y}\frac{(p_j^{UE,fit}-p_j^{UE,meas})^2}{\sigma_j^2} + \frac{(M_{\ell\nu}-M_W)^2}{\Gamma_W^2}+\frac{(M_{jj}-M_W)^2}{\Gamma_W^2}$$

Usually pick solution with



Not easy to answer the questions because of complicated dependence on the top quark mass

from Aurelio Juste

Dynamics Metho

- Principle: compute eve
a function of m_t making
objects in the events (i
Maximize sensitivity by:

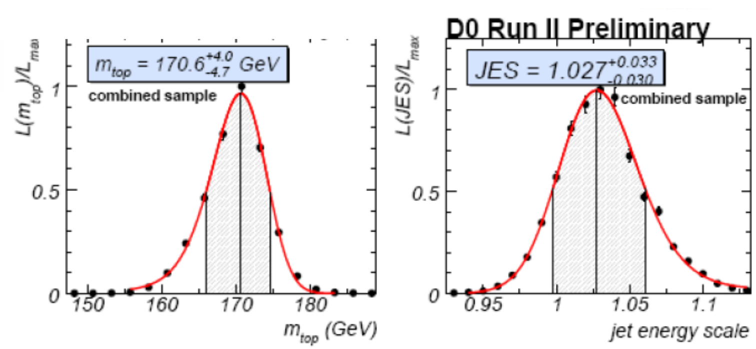
b^{-1})

$$P(x;m_t)=\frac{1}{\sigma}\int d^n\sigma(y;m_t)\,dq_1dq_2\,f(q_1)f(q_2)\,W(x|y)$$

parton distribution functions

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)

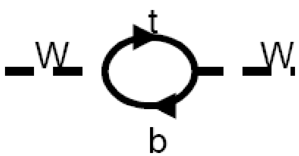


Need for a precise Top mass

even more

Small error in m_t is only meaningful if the mass definition is exactly known.

Electroweak precision observables

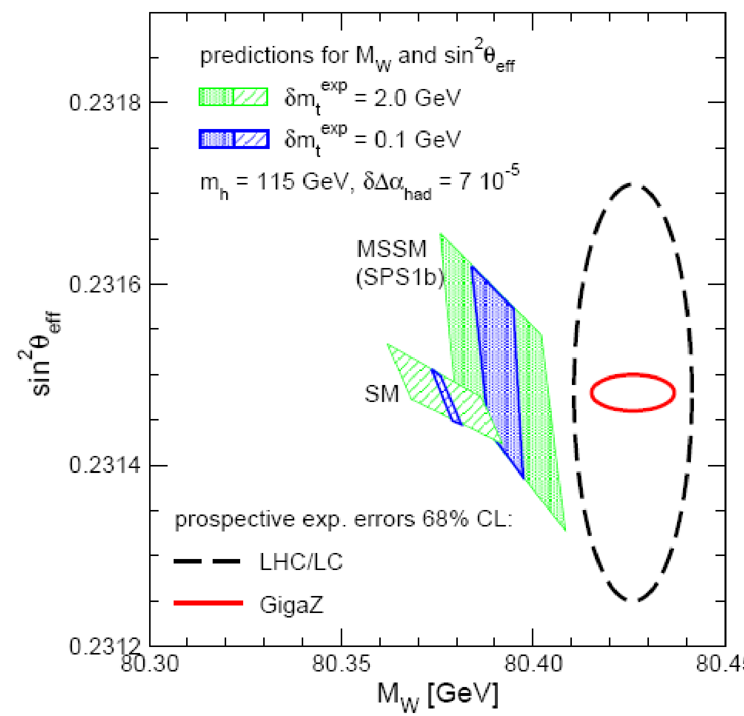


$\delta M_W \propto m_t^2$

$\delta M_W \propto \ln(M_H)$

$$\sin\theta_W \times \left(1 + \delta(m_t, m_H, \ldots)\right)$$

$$= 1 - \frac{M_W^2}{M_Z^2}$$



Need for a precise Top mass

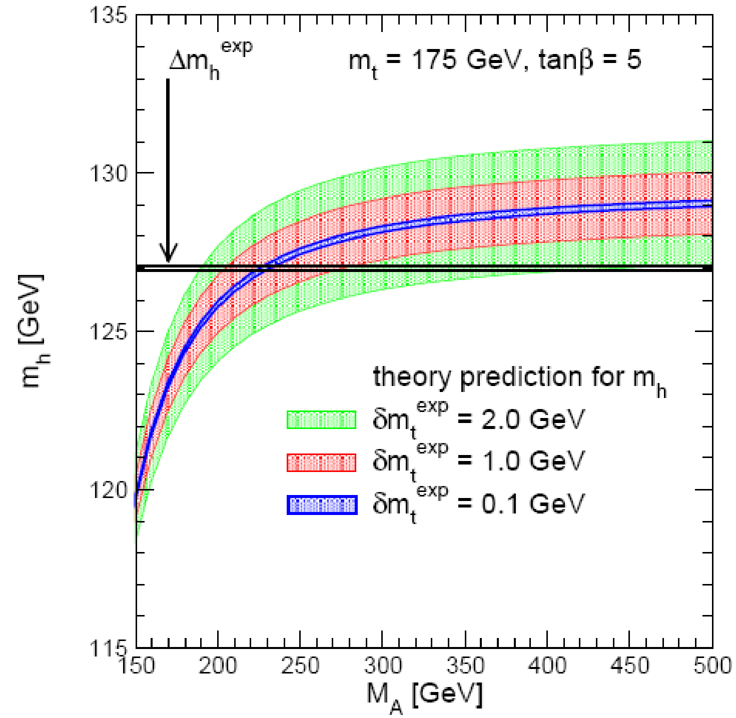
even more

Small error in m_t is only meaningful if the mass definition is exactly known.

Mass of Lightest MSSM Higgs Boson

$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

	LHC	LC
δm_h	1 GeV	50 MeV
needed δm_t	4 GeV	0.2 GeV
expected δm_t	1-2 GeV	~ 0.1 GeV



Top Mass at the ILC

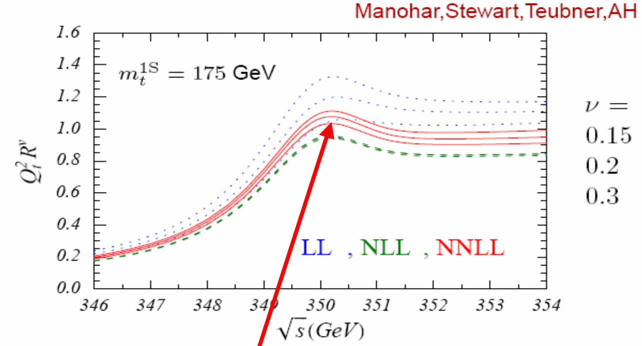
Threshold Scan

$Q \approx 2m_t$

ILC

- count number of $t\bar{t}$ events
- color singlet state
- background is non-resonant
- physics well understood
  (renormalons, summations)

Manohar,Stewart,Teubner,AH



$\rightarrow \delta m_t^{\mathrm{exp}} \simeq$ 50 MeV

$\rightarrow \delta m_t^{\mathrm{th}} \simeq$ 100 MeV

What mass?

$\sqrt{s}_{\mathrm{rise}} \sim 2m_t^{\mathrm{thr}} +$ pert.series

(short distance mass: 1S $\leftrightarrow \overline{\mathrm{MS}}$)

"threshold masses"

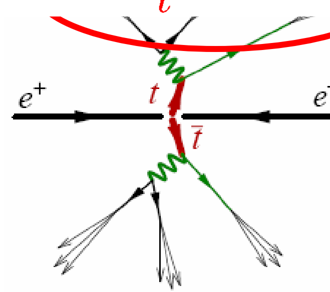
Invariant Mass Reconstruction

$Q \geq 2m_t$

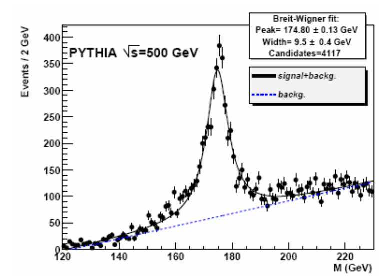
LHC + ILC

$$M_{\mathrm{peak}} \simeq 2M_{1S} = 2m_t^{\mathrm{pole}} - 0.22\,\alpha_s^2 m_t^{\mathrm{pole}}$$

- useful as a cross check
- measures different top mass
- uncertainties (much) more involved
- addresses some issues relevant for LHC/Tevatron



M_t^2



Reconstruction Simulations

Chekanov, Morgunov
hep-ex/0301014

All-hadronic channel $e^+e^- \to t\bar{t} \to 6$ jets

ILC special

Event Selection

- k_T (Durham) jet algorithm $y_6^{cut} > \Delta_y$
  all particles are assigned to exactly 6 jets
- $\left|\frac{E_{vis}}{\sqrt{s}} - 1\right| < \Delta_E, \quad \frac{|\sum \vec{p}_{\|i}|}{\sum |\vec{p}_i|} < \Delta_{PL}, \quad \frac{|\sum \vec{p}_{Ti}|}{\sum |\vec{p}_i|} < \Delta_{PT}$

→ every particle is assigned to top or antitop (fully inclusive !!)

→ reduces events for massless quark pair production

→ selects hadronic $t\bar{t}$ events

Top Reconstruction

- 6 jets with momenta $p_i, (i = 1, \ldots, 6)$
- group in pairs of 3: $M_I(1)$ and $M_I(2)$
- $|M_I(1) - M_I(2)| < \Delta_M$ → both invariant masses close to m_t
  $|\vec{P}_I(1) + \vec{P}_I(2)| < \Delta_P$ → selects back-to-back events
- b-tagging
- massless jet pair mass: $|M_{jj} - M_W| < \Delta_W$

→ reduces combinatorics

Analysis:

- 500 GeV with 16 fb^{-1}
- 800 GeV with 33 fb^{-1}

→ $\delta m_t^{\text{stat}} \sim 400$ MeV

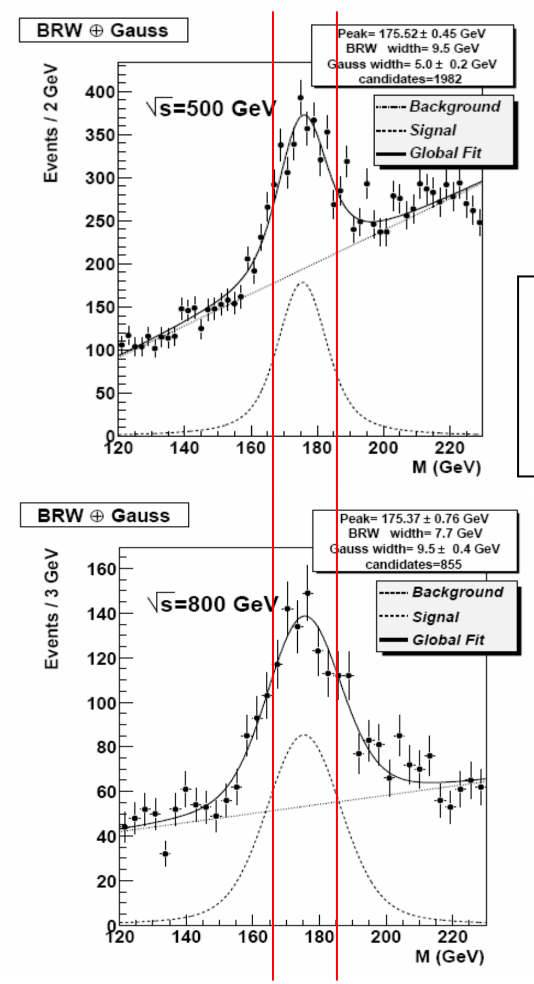
- 800 GeV with 300 fb^{-1} → $\delta m_t^{\text{stat}} \sim 100$ MeV

Reconstruction Simulations

Reconstruction for all-hadronic events at the LC

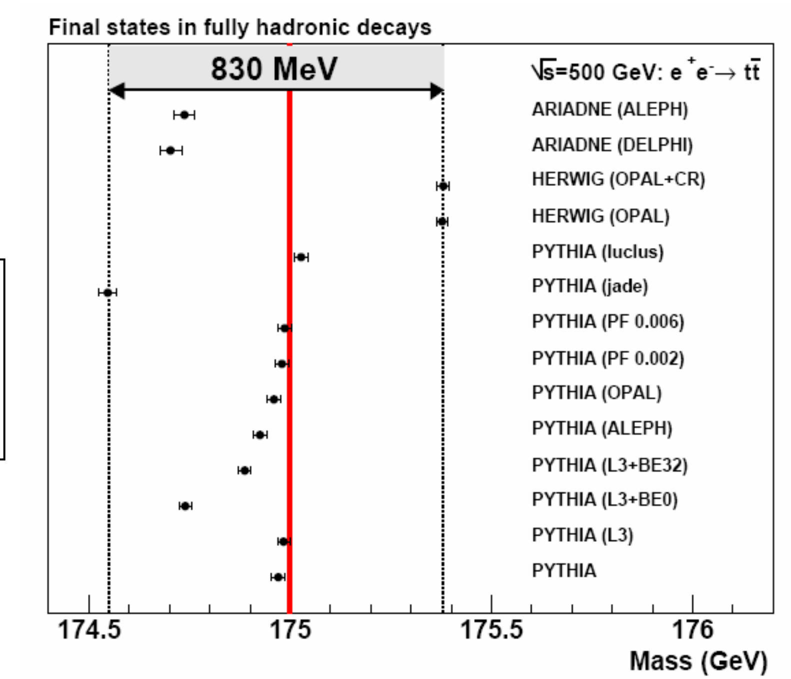
Chekanov; Morgunov

hep-ex/0301014



Q dependence of observed width

Dependence on parton shower, hadroniz. models, jet algorithms



- What mass is measured? (Cannot be pole mass !)
- Depedendence on jet algorithms, event selection?
  OR Totally free of theory input? (→ error estimate)
- How can presciption dependence implemented in theoretical predictions?

Conceptual Goals

- relate top jet observables with a given Lagrangian mass (define suitable short-distance mass with good convergence properties → What mass is measured?)
- proof of factorization of dynamics at different length scales (→ What has to be computed by theorists ?)
- combined treatement of top production & decay
- separate perturbative from non-perturbative effects
- hopefully better understand & reduce theoretical & experimental uncertainties

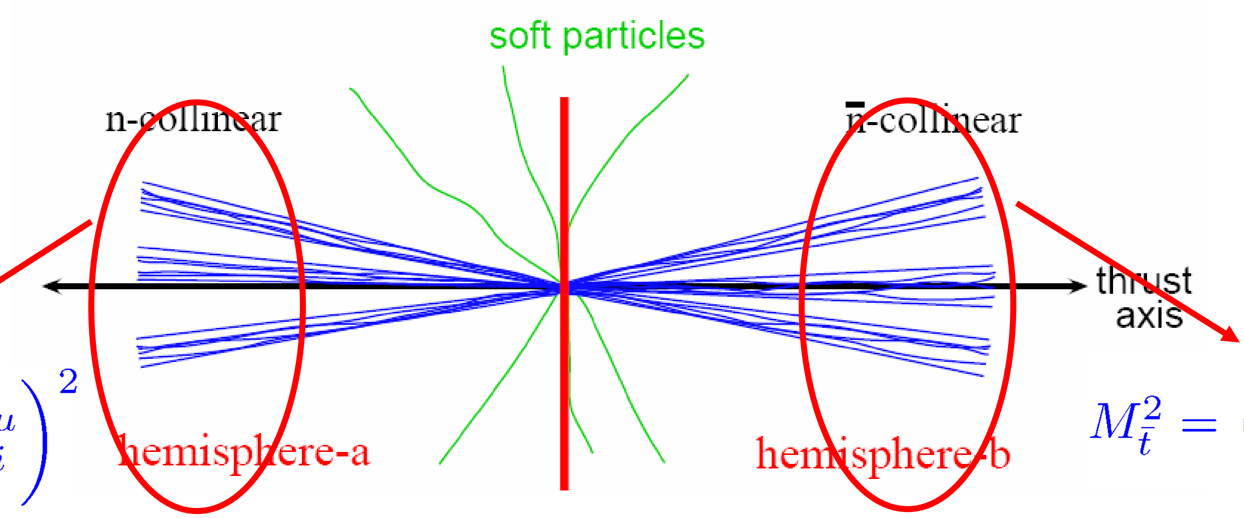
➡ **Tool: Sequence of Effective Field Theories**

Basic Idea

Hemisphere invariant mass distribution

$$Q \gg m_t \gg \Gamma_t > \Lambda_{\rm QCD}$$

$$\frac{d^2\sigma}{dM_t\, dM_{\bar t}}$$



$$M_t^2 = \Big(\sum_{i\,\epsilon\, a} p_i^\mu\Big)^2$$

$$M_{\bar t}^2 = \Big(\sum_{i\,\epsilon\, b} p_i^\mu\Big)^2$$

Resonance region: Lagrangian mass

$$M_{\bar t} - m_t \sim \Gamma_t \qquad M_t - m_t \sim \Gamma_t$$

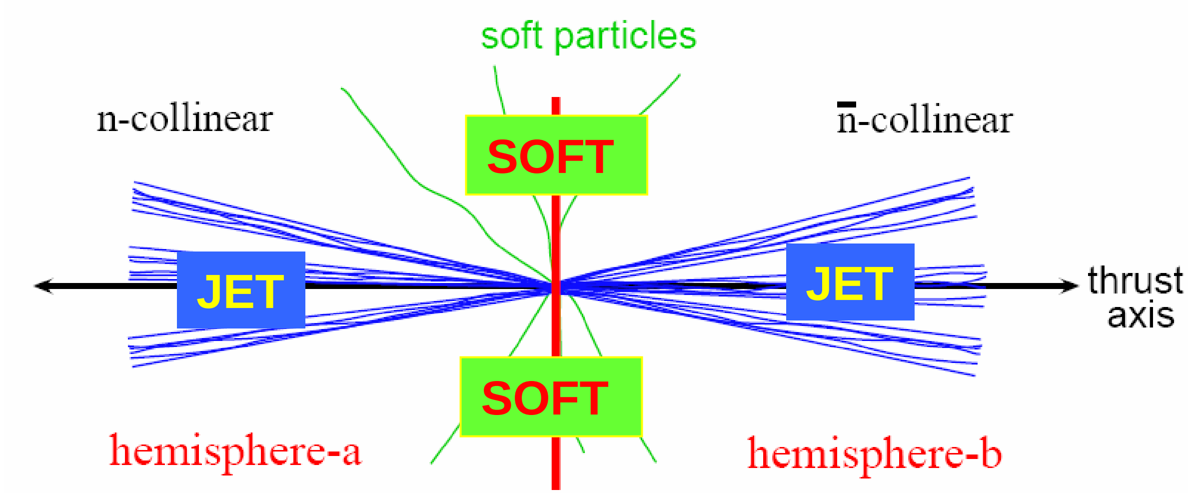
- realistic ILC jet observable
- closely related to thrust and heavy jet mass event shapes
- set up similar to Chekanov/Morgunov analyis except for use of hemispheres instead of k_T

Basic Idea

Hemisphere invariant mass distribution

$$Q \gg m_t \gg \Gamma_t > \Lambda_{\rm QCD}$$

$$\frac{d^2\sigma}{dM_t\, dM_{\bar t}}$$



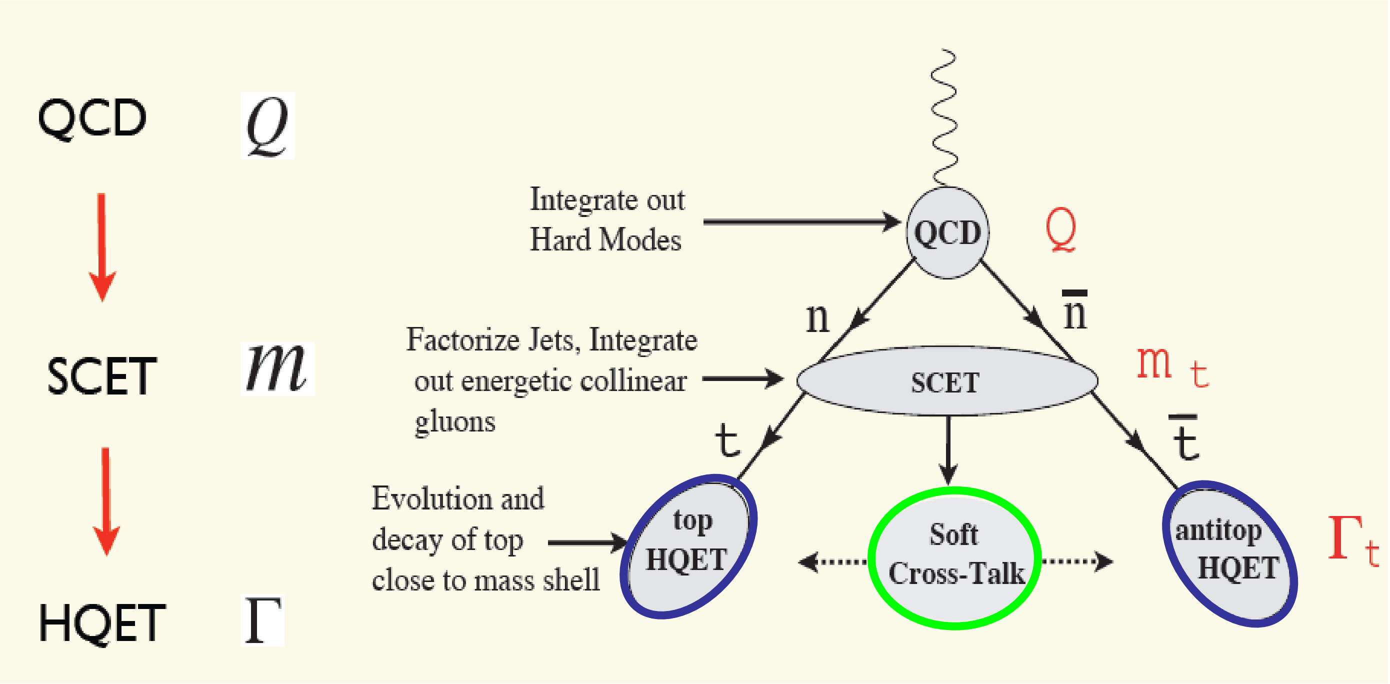
LO factorization theorem

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \qquad \hat s = \frac{M_t^2 - m_J^2}{m_J}$$

$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big) B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

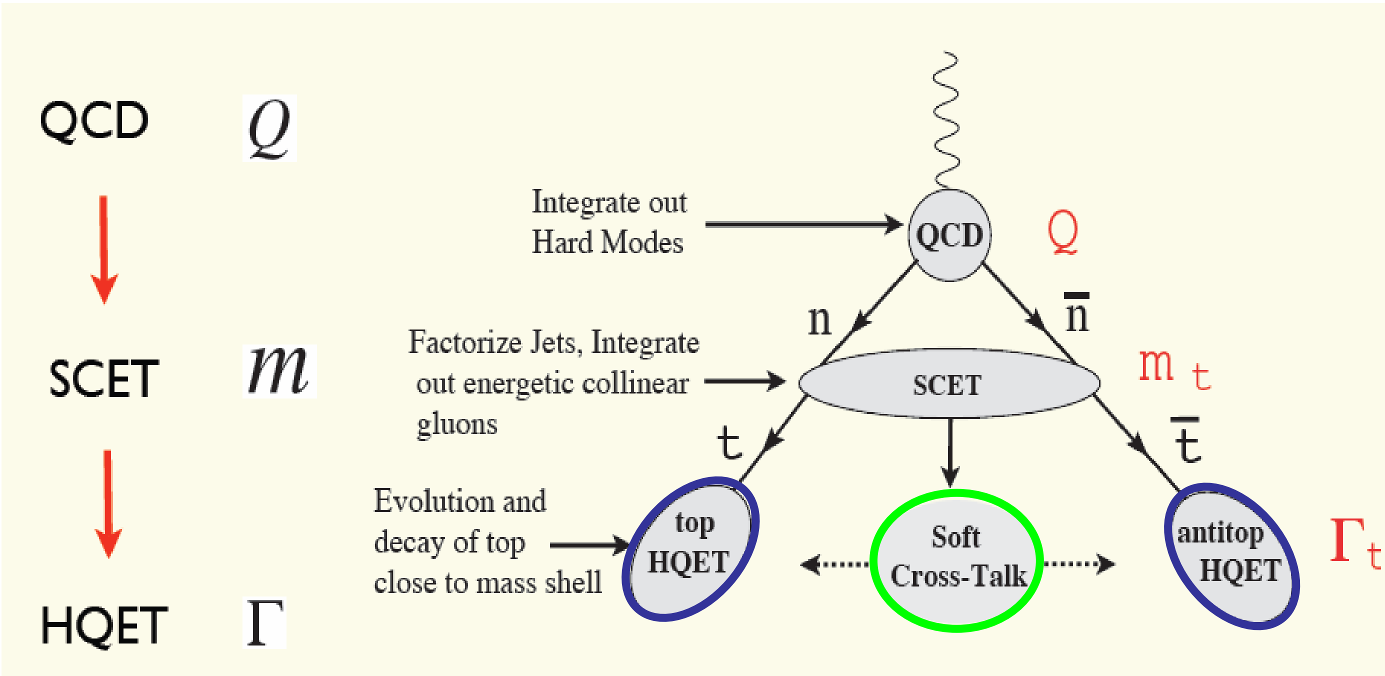
JET JET SOFT

Scheme of EFT's



$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Scheme of EFT's



Program technically analogous to combination of

threshold resummation ($M_t - m_t \to 0$) Korshemsky, Sterman, etal.

&

method of unstable particle EFT

Fadin, Khoze

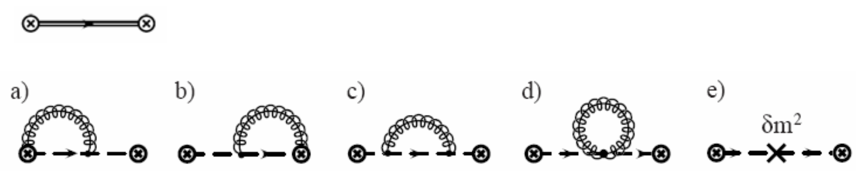
Beenacker etal., Beneke, etal.

Reisser, AH

Factorization Theorem

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Jet functions: $$B_+(2v_+\cdot k) = \frac{-1}{8\pi N_c m}\int d^4x\, e^{ik\cdot x}\, {\rm Disc}\, \langle 0|{\rm T}\{\bar h_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x)\}|0\rangle$$



- perturbative, any mass scheme
- depends on m_t, Γ_t
- Breit-Wigner at tree level

$$B_\pm(\hat s, \Gamma_t) = \frac{1}{\pi m_t}\frac{\Gamma_t}{\hat s^2 + \Gamma_t^2} \qquad \hat s = \frac{M^2 - m_t^2}{m_t}$$

Soft function: $$S_{\rm hemi}(\ell^+, \ell^-, \mu) = \frac{1}{N_c}\sum_{X_s} \delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$$

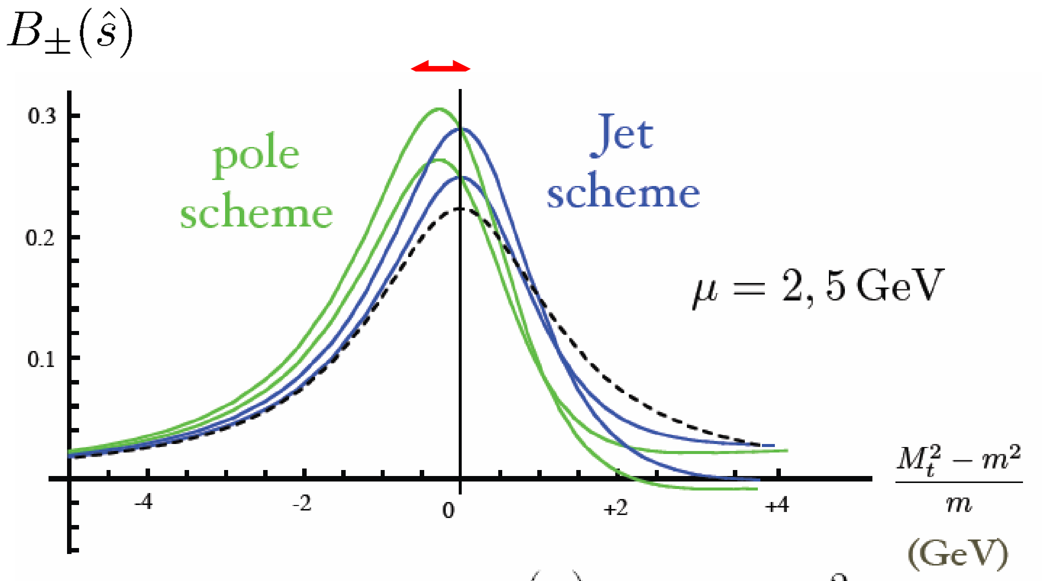
- non-perturbative
- renormalized due to UV divergences
- also governs **massless dijet thrust and jet mass event distributions**

Korshemsky, Sterman, etal.

Bauer, Manohar, Wise, Lee

➡ **Short distance top mass can (in principle) be determined to better than $\Lambda_{\rm QCD}$.**

Short-distance Top Jet Mass



$$m_J(\mu) = m_{\mathrm{pole}} - \Gamma \frac{\alpha_s(\mu)}{3}\left[\ln\left(\frac{\mu}{\Gamma}\right) + \frac{3}{2}\right]$$

- **One-loop: shift in the pole scheme 300 MeV**
- **shift in the pole scheme contains $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ renormalon**
- **jet mass scheme: defined such that peak located at the mass to all orders**

Top Jet mass is the scheme where we expect that a LO analysis contains the least theoretical uncertainties.

What mass is measured?

Answer: the one that gives the best convergence in the theoretical expansion.

LO Numerical Analysis

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

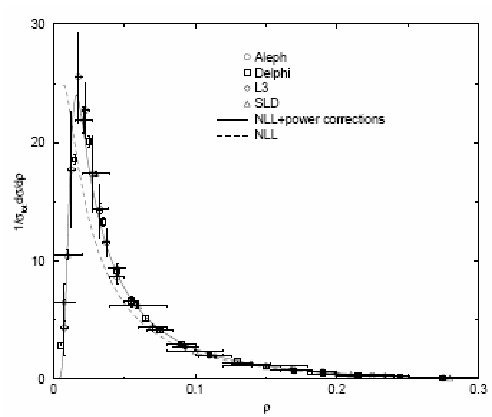
Jet functions:

$$B_\pm(\hat s, \Gamma_t) = \frac{1}{\pi m_t}\frac{\Gamma_t}{\hat s^2 + \Gamma_t^2}$$

Soft function:

$$S^{\rm M1}_{\rm hemi}(\ell^+, \ell^-) = \theta(\ell^+)\theta(\ell^-)\frac{\mathcal{N}(a,b)}{\Lambda^2}\Big(\frac{\ell^+\ell^-}{\Lambda^2}\Big)^{a-1} \exp\Big(\frac{-(\ell^+)^2 - (\ell^-)^2 - 2b\ell^+\ell^-}{\Lambda^2}\Big)$$

$$a = 2\,, \quad b = -0.4$$
$$\Lambda = 0.55 \text{ GeV}$$



Fit to heavy jet mass distribution

Korchemsky, Tafat
hep-ph/0007005

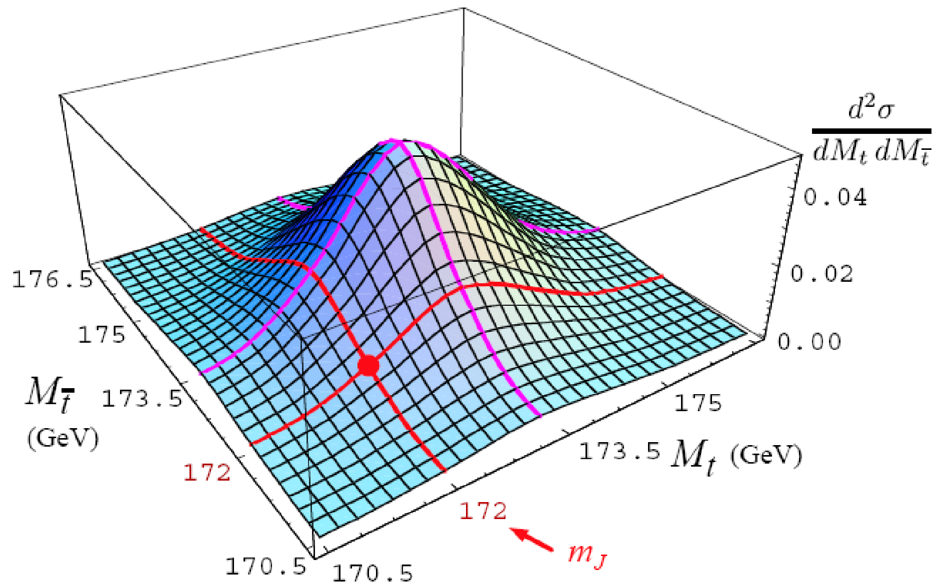
LO Numerical Analysis

Double differential invariant mass distribution:

$$Q = 745 \text{ GeV}$$

$$\Gamma = 1.43 \text{ GeV}$$

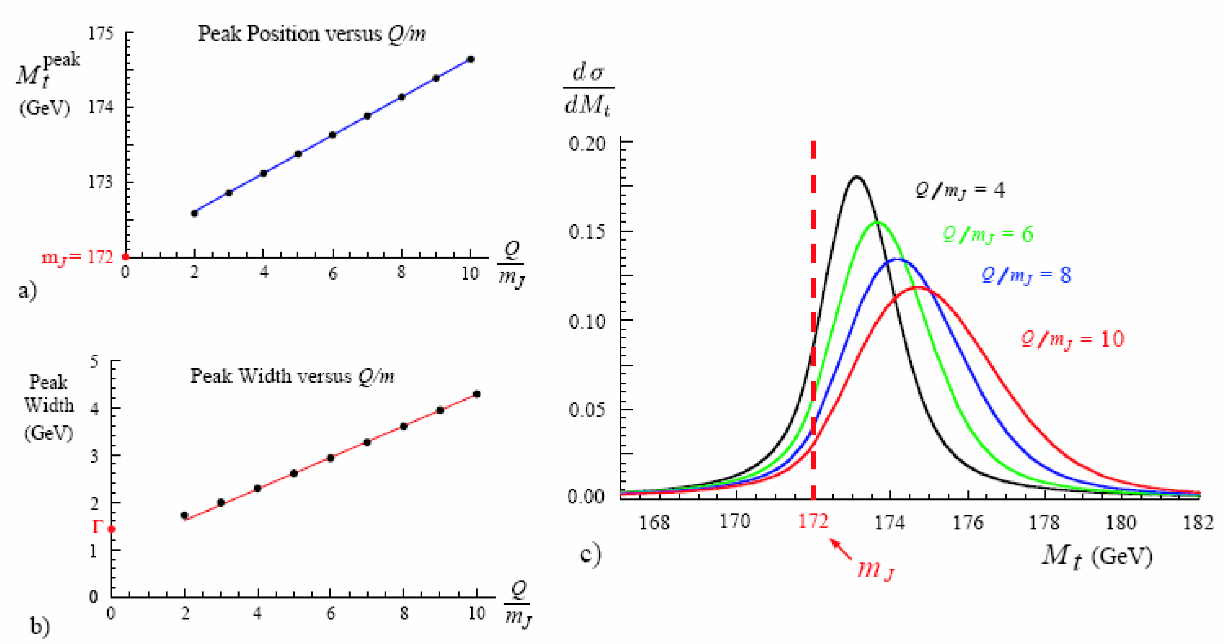
$$m_J = 172 \text{ GeV}$$



Non-perturbative effects shift the peak to higher energies and broaden the distribution.

LO Numerical Analysis

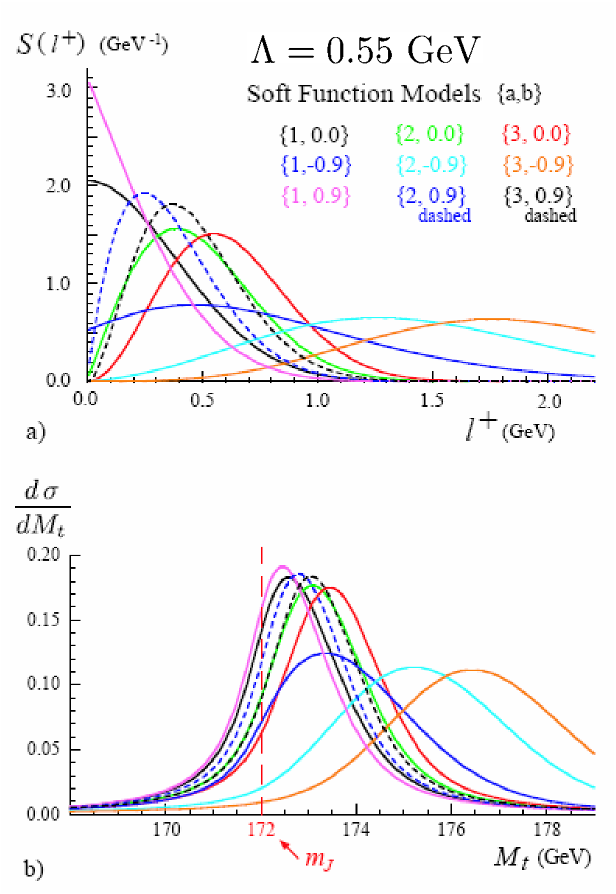
Single differential distribution:

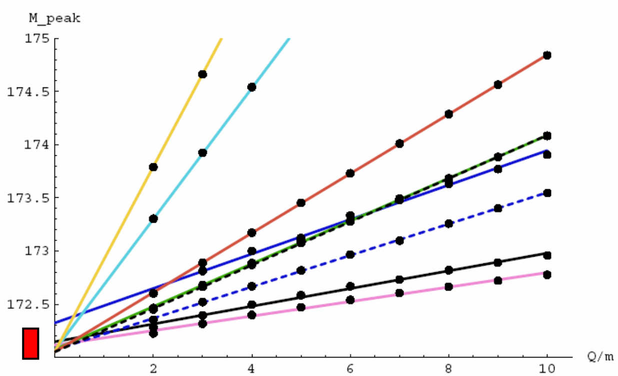


Non-perturbative effects shift the peak to higher energies and broaden the distribution.

LO Numerical Analysis

Different invariant mass prescriptions/soft functions:





$$M_t^{\text{peak}} \approx m_J + \frac{Q}{m_J}\,\text{const}$$

Fairly precise determination of jet mass from determination of Q-dependence of the peak position and extrapolation Q to zero

How general is the approach?

Why Q>>m is crucial

- top and antitop boosted in opposite directions,

➡ top and antitop jet axes $\vec{n}$ and $\vec{\bar{n}}$ can be defined

➡ allows factorization $(\text{jet}_n \times \text{jet}_{\bar{n}}) \otimes \text{soft}$

- combinatorial background, wrong assignment suppressed by $\left(\frac{m}{Q}\right)^2$

ILC: ok for $Q \sim 0.5 - 1\ \text{TeV}$

LHC: probably ok for tops with $p_T > 200\ \text{GeV}$

Tev: ?

Generality of the approach

We don't have to assume a hemisphere mass definition:

- any jet algorithm that combines soft particles with the hard jets from the top decay
- wide cone definition $R > m/Q$ that contains the top/antitop jet axes and top decay products (collinear radiation off top)

➡ **Different soft function, same factorization formula**

[The soft functions for most cases at unknown at this time and migh need to be fitted together with the top mass OR determined from MC's OR by other means.]

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Theory Issues for $pp \rightarrow t\bar{t} + X$

- ★ definition of jet observables
- ★ initial state radiation
- ★ final state radiation
- • underlying events
- ★ color reconnection & soft gluon interactions
- ★ beam remnant
- ★ parton distributions
- ★ summing large logs $Q \gg m_t \gg \Gamma_t$
- ★ relation to Lagrangian short distance mass

★ **Can be addressed in the framework of the ILC.**

★ **Requires extensions of ILC concepts and other known concepts**

Summary & Outlook

- established factorization theorem for invariant mass distributions: separation of perturbative and non-perturbative effects
- applicable for many other systems and setups: (any colored unstable particle, W mass reconstruction, etc..)
- exact and systematic relation of peak to a Lagrangian mass: What mass is measured ? "Jet-mass"
- resummation of large logarithms $Q \gg m_t \gg \Gamma_t$
- soft gluon color reconnection power suppressed
- Here: $e^+e^- \to t\bar{t} + X_{\rm soft}$
- Planned: $pp \to t\bar{t} + X_{\rm soft}$ ... $Q \approx 2m_t$

  $pp \to t\bar{t} + {\rm jet} + X_{\rm soft}$

  different mass definitions (cone, k_T)

Backup Slides

Event Shapes

e.g. Thrust distribution on the peak region:

$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|}{Q}$$

$$\frac{d\sigma}{dT} = \sigma_0^H(\mu) \int_{-\infty}^{\infty} ds_t\, ds_{\bar t}\ \tilde{B}_+\Big(\frac{s_t}{m_J}, \Gamma, \mu\Big)\, \tilde{B}_-\Big(\frac{s_{\bar t}}{m_J}, \Gamma, \mu\Big)\, S_{\rm thrust}\Big(1 - T - \frac{(2m_J^2 + s_t + s_{\bar t})}{Q^2}, \mu\Big)$$

$$S_{\rm thrust}(\tau, \mu) = \int_0^{\infty} d\ell^+\, d\ell^- \delta\Big(\tau - \frac{(\ell^+ + \ell^-)}{Q}\Big) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

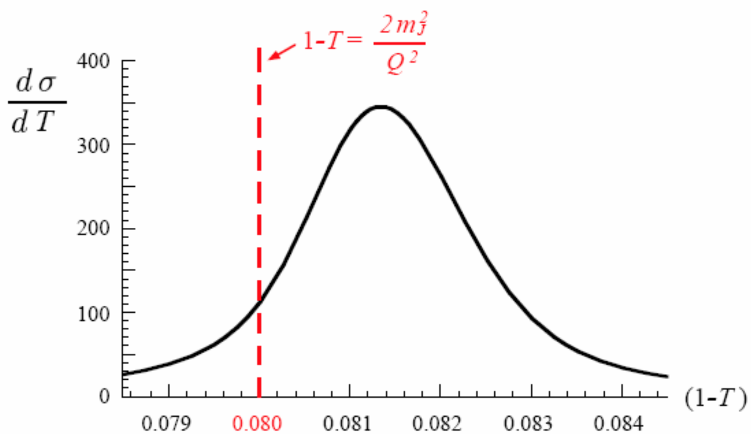
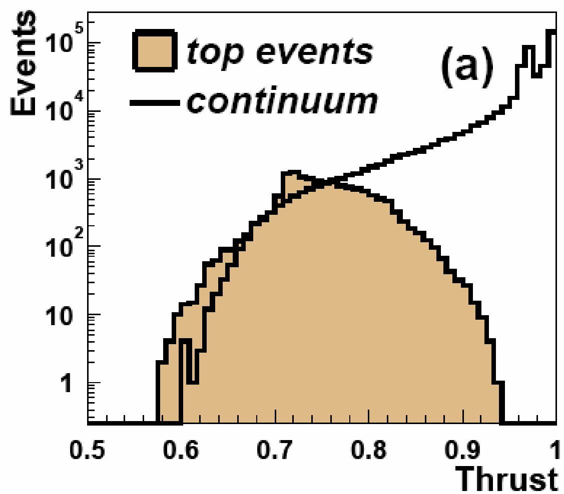


FIG. 8: Plot of the thrust distribution, $d\sigma/dT$ in units of σ_0^H, for top-initiated events in the peak region. We use $Q/m_J = 5$, $m_J = 172\,\text{GeV}$ and the soft function parameters in Eq. (115).



Chekanov; Morgunov

hep-ex/0301014