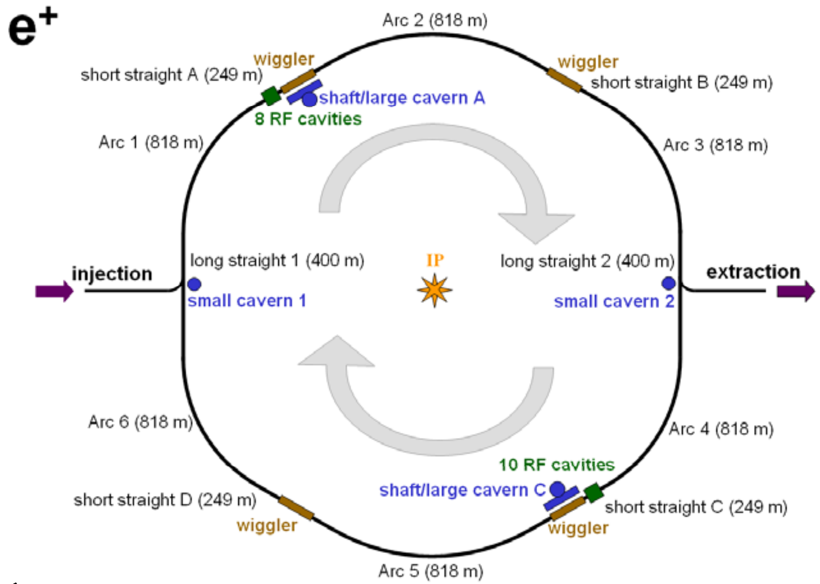
Dynamics with Transverse Coupled-Bunch Wake Fields in the ILC Damping Rings

Kai Hock and Andy Wolski



ILC Damping Ring

Wolski (2007)

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An International Centre for Research in Accelerator Science and Technology

Objective: 1. Effect of varying beta function in macroparticle model
2. Analytic solution for constant beta function.

Previous work – for constant beta function :

1. Equation of motion decouple into Fourier modes (Chao 1993).
2. Modes decay / grown exponentially, analytic formula available.
3. Analytic formula for bunch trajectories available but incomplete (Thompson and Ruth 1991).

Current work:

1. Varying beta function – modes remain coupled, decay modes grow.
2. Analytic formula for bunch trajectories completed with error bounds.

Example of an application

Proceedings of the Second Asian Particle Accelerator Conference, Beijing, China, 2001

OBSERVATION OF TRANSVERSE COUPLED BUNCH INSTABILITY AT KEKB

Su Su Win, Hitoshi Fukuma, Eiji Kikutani and Makoto Tobiyama
High Energy Accelerator Research Organization (KEK), Japan

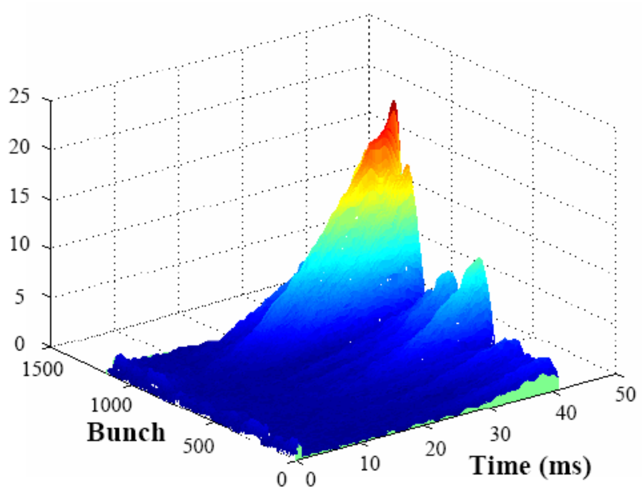


Fig. 3 The horizontal amplitude growth of bunch oscillation in HER at 400 mA

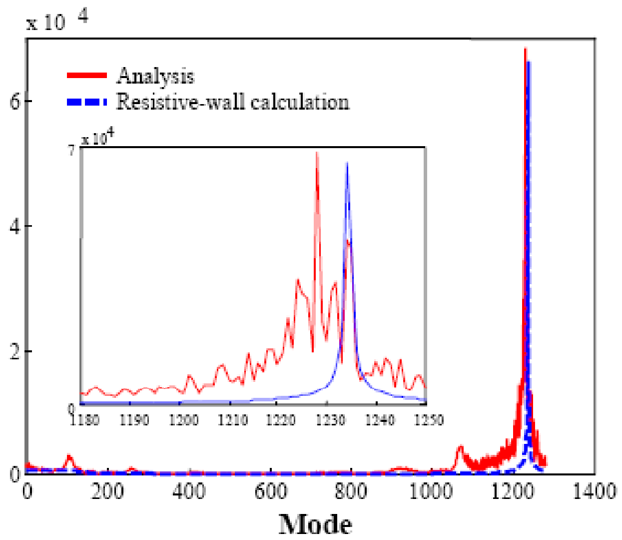
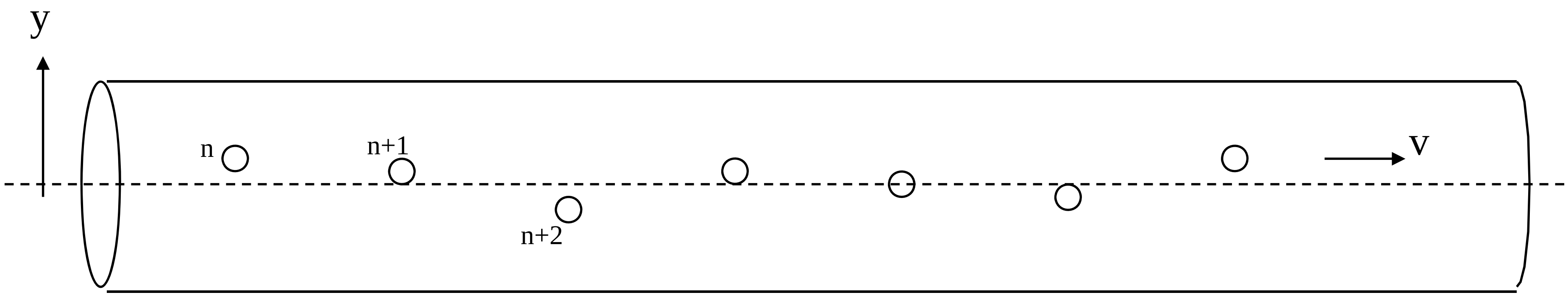


Fig. 1 The horizontal mode distribution of HER at 700 mA

A Macroparticle Model



No wake field

$$\ddot{y}_n(t) + \omega_\beta^2 y_n(t) = 0$$

Resistive wake force

$$b_1 y_{n+1}(t-\tau)$$

$$\ddot{y}_n(t) + \omega_\beta^2 y_n(t) = b_1 y_{n+1}(t-\tau) + b_2 y_{n+2}(t-2\tau) + \dots$$

Equation of motion

History

Sands (1969),
Thompson and Ruth (1991), …

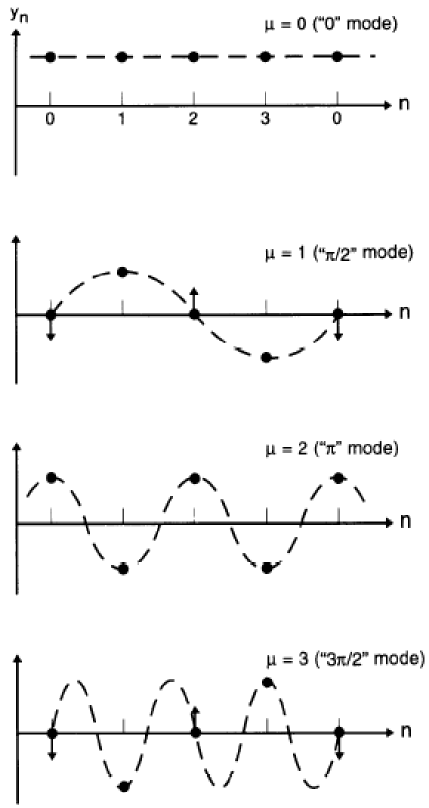
Normal Modes

$$\ddot{y}_m(t) + \omega_\beta{}^2 y_m(t) = \sum_{n=1}^{\infty} b_n y_{m+n}(t - n\tau)$$ coupled

DFT

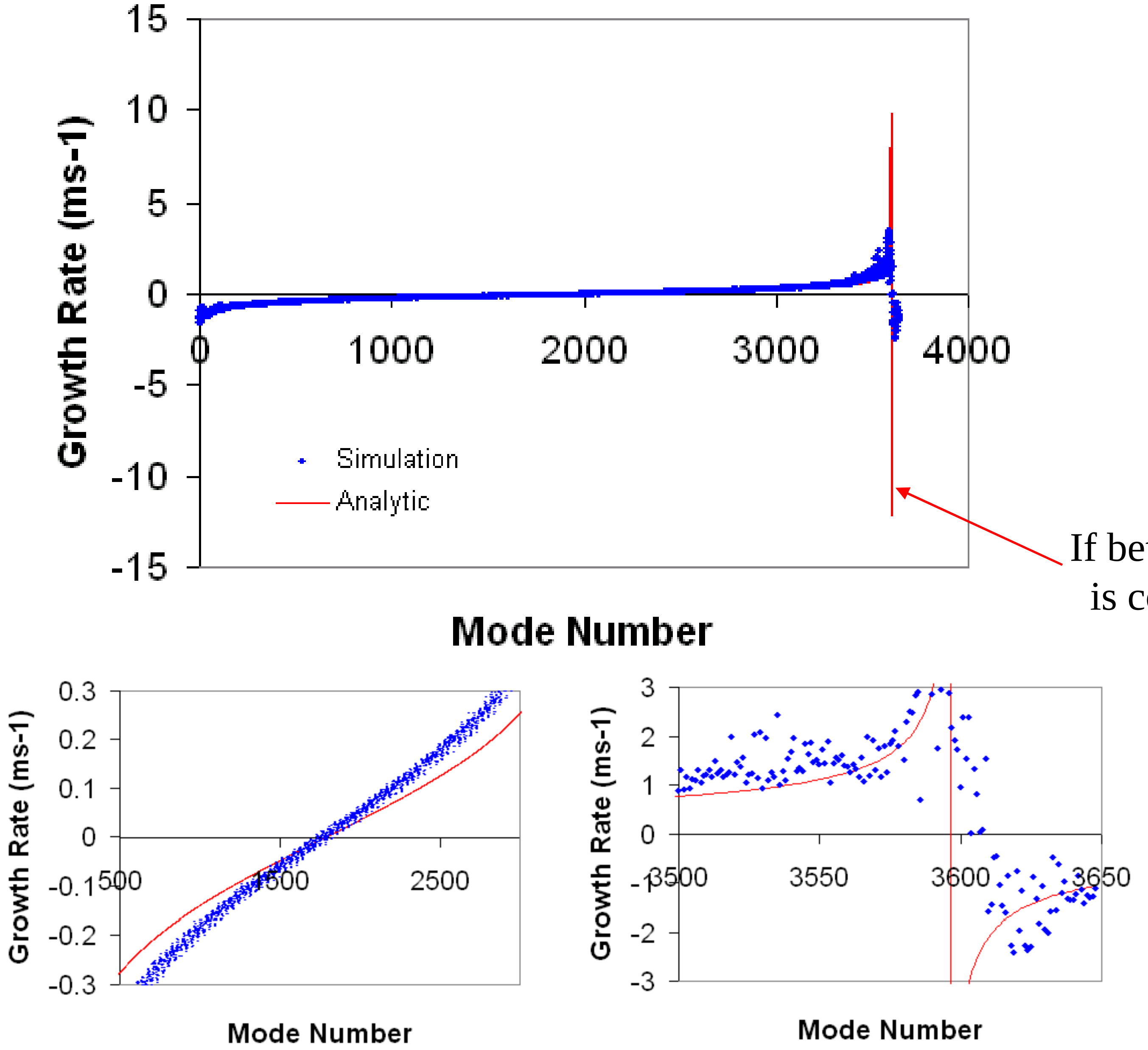
$$\frac{d^2}{dt^2}\tilde{y}_\mu(t) + \omega_\beta{}^2 \tilde{y}_\mu(t) = \sum_{n=1}^{\infty} b_n e^{i\frac{2\pi n\mu}{M}} \tilde{y}_\mu(t - n\tau)$$ decoupled

where $$\tilde{y}_\mu(t) = \sum_{m=0}^{M-1} y_m(t) e^{-i\frac{2\pi m\mu}{M}}$$ (Fourier modes)



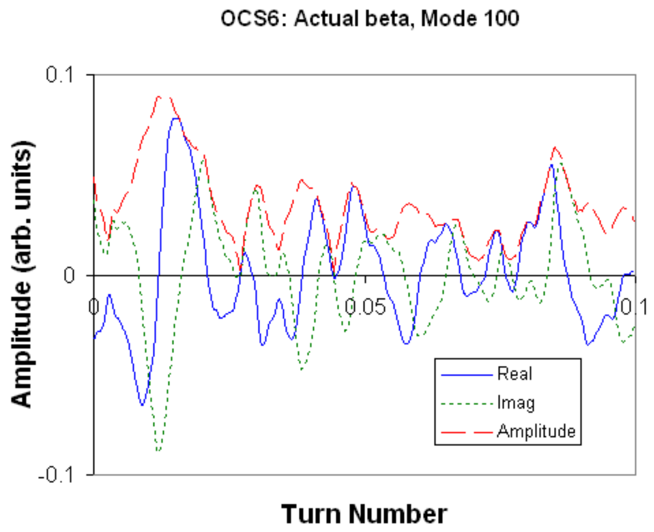
Chao (1993)

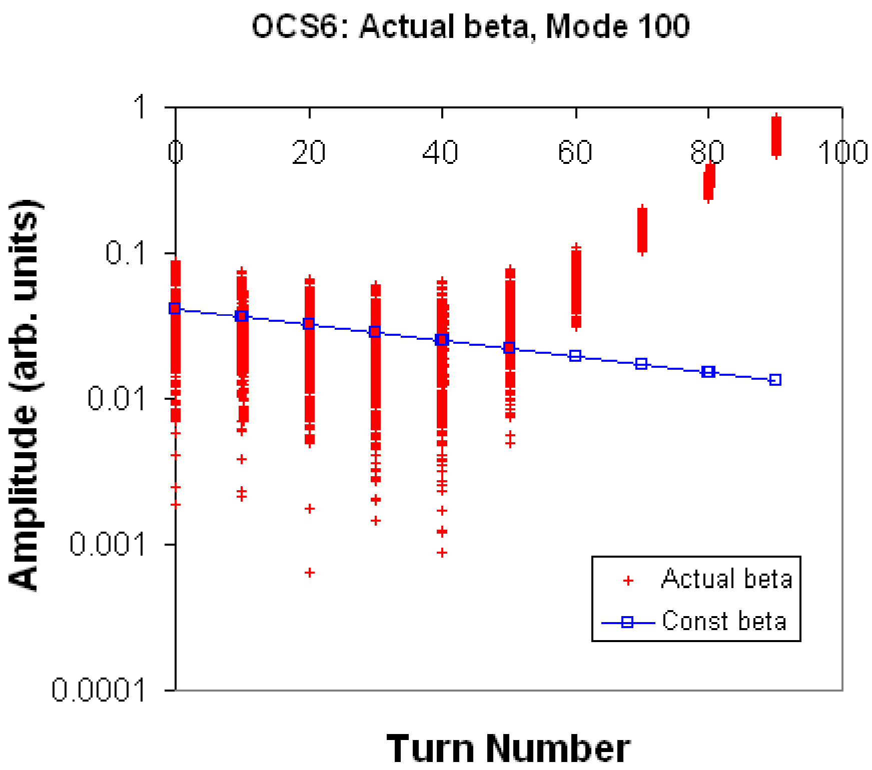
OCS6 Damping Ring: Uniform Fill

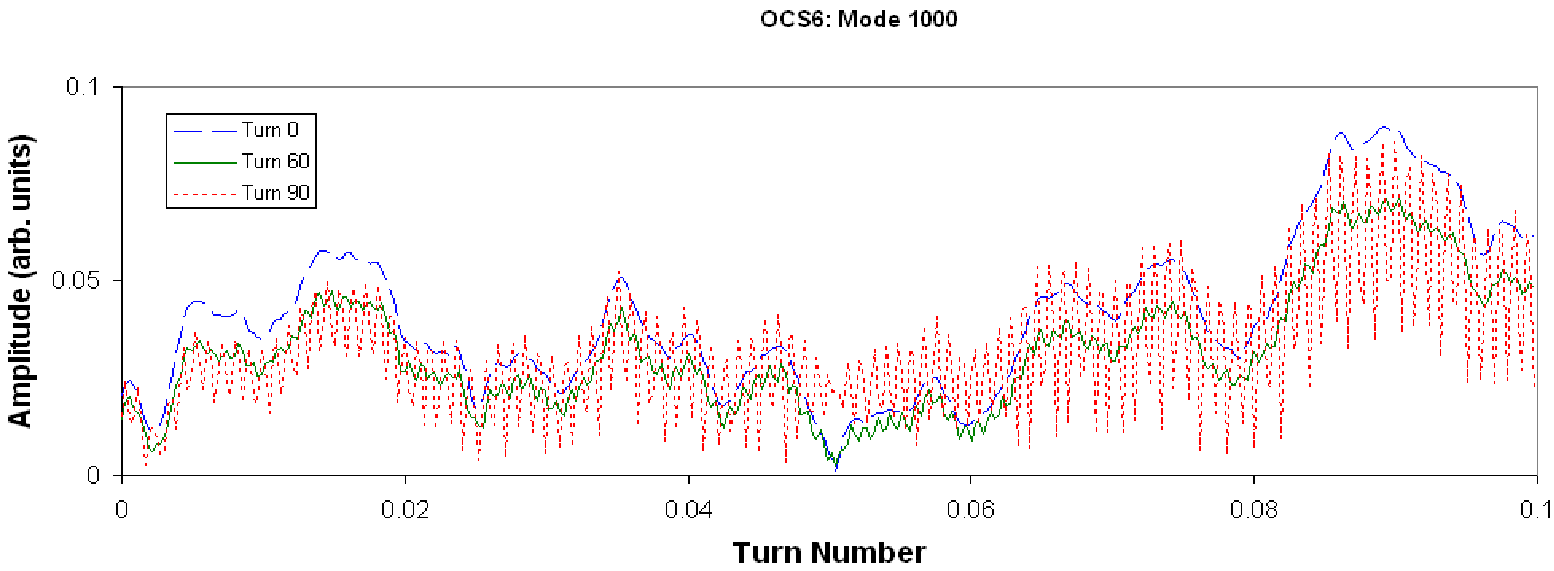


If beta function is constant

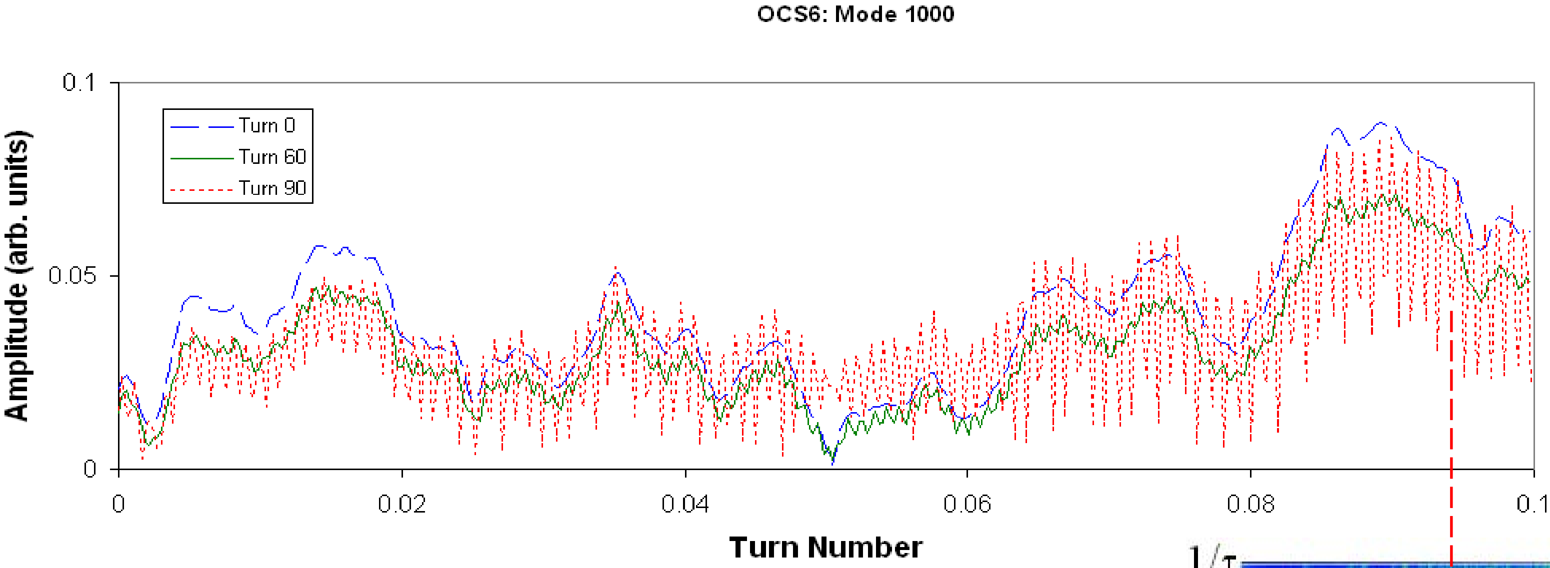
A few surprises …





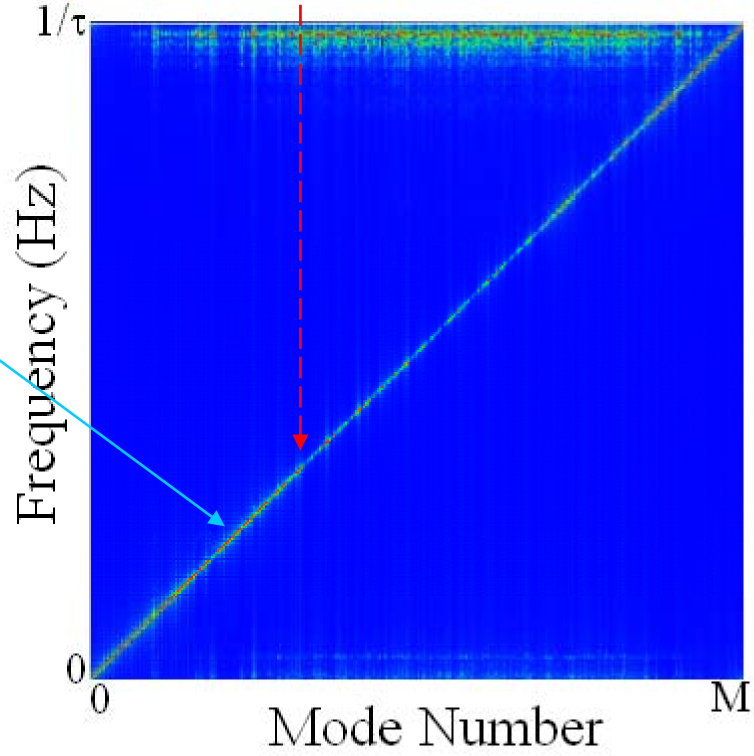


High frequency oscillation comes from …



$$\ddot{\tilde{y}}_{\mu}(t)+\overline{K}\tilde{y}_{\mu}(t)=\sum_{n=1} b_n e^{i\frac{2\pi n\mu}{M}}\tilde{y}_{\mu}(t-n\tau)-\sum_{m=0}^{M-1} e^{-i\frac{2\pi m\mu}{M}}k(t+m\tau)y_m(t)$$

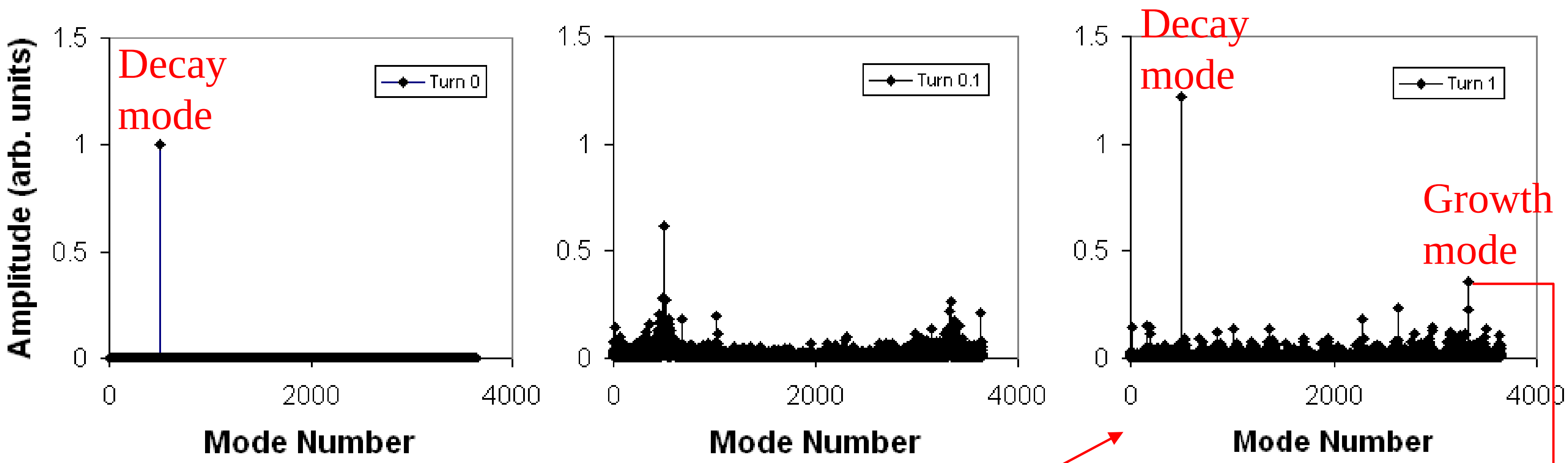
Parametric force



Mode Frequency Spectra

Turn 90

Decay modes grow because …



$$\ddot{\tilde{y}}_{\mu}(t)+\overline{K}\tilde{y}_{\mu}(t)=\sum_{n=1}^{\infty} b_n e^{i\frac{2\pi n\mu}{M}}\tilde{y}_{\mu}(t-n\tau)-\frac{1}{M}\sum_{\mu'=0}^{M-1}\tilde{k}_{\mu-\mu'}(t)\tilde{y}_{\mu'}(t)$$

Mode Coupling

?

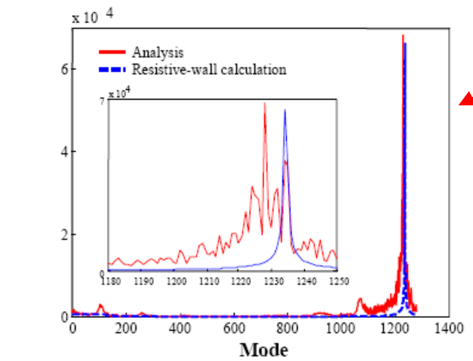
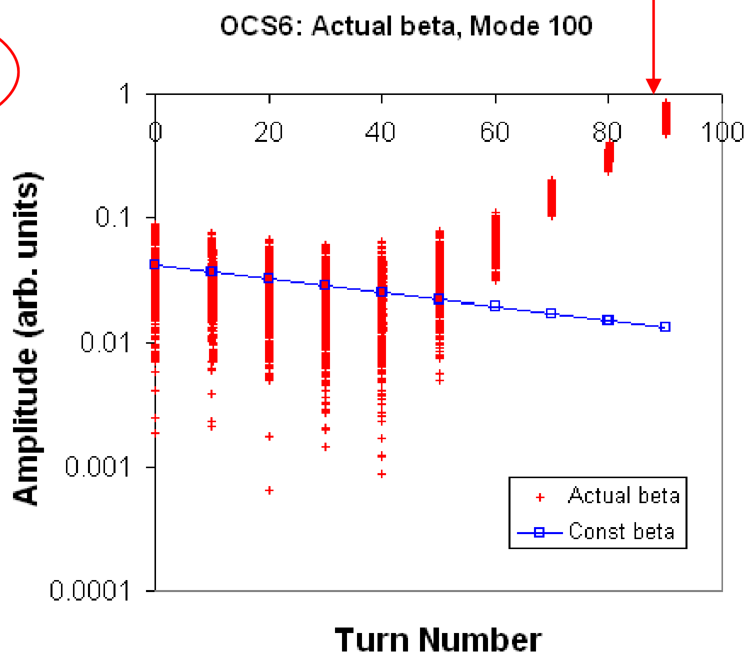


Fig. 1 The horizontal mode distribution of HER at 700 mA



Bunch Trajectories

Equation of motion

$$\ddot{x}_i(t)+2\zeta\dot{x}_i(t)+\omega_\beta^2 x_i(t)=\frac{Ne^2c}{E_0T_0}\sum_{j=1}^{n}\sum_{q=0}^{\infty}W_\perp(L_{ij}+qT_0c)x_j(t-qT_0-L_{ij}/c)$$

Analytic solution (Thompson, Ruth 1991)

$$x_i(t)=\sum_{l=1}^{n}\frac{\sum_{j=1}^{n}c_{ji}(-i\Omega_l)[(-i\Omega_l+2\zeta)x_j(0)+\dot{x}_j(0)]}{(-2i\omega_\beta)\prod_{\substack{k=1\\k\neq l}}^{n}(-\Omega_l+\Omega_k)}e^{-i\Omega_l t}$$

Incomplete – Zero wake field limit is SHM, need $e^{+i\Omega_l t}$

– Else cannot have arbitrary $x(0),\dot{x}(0)$

If beta function is constant:

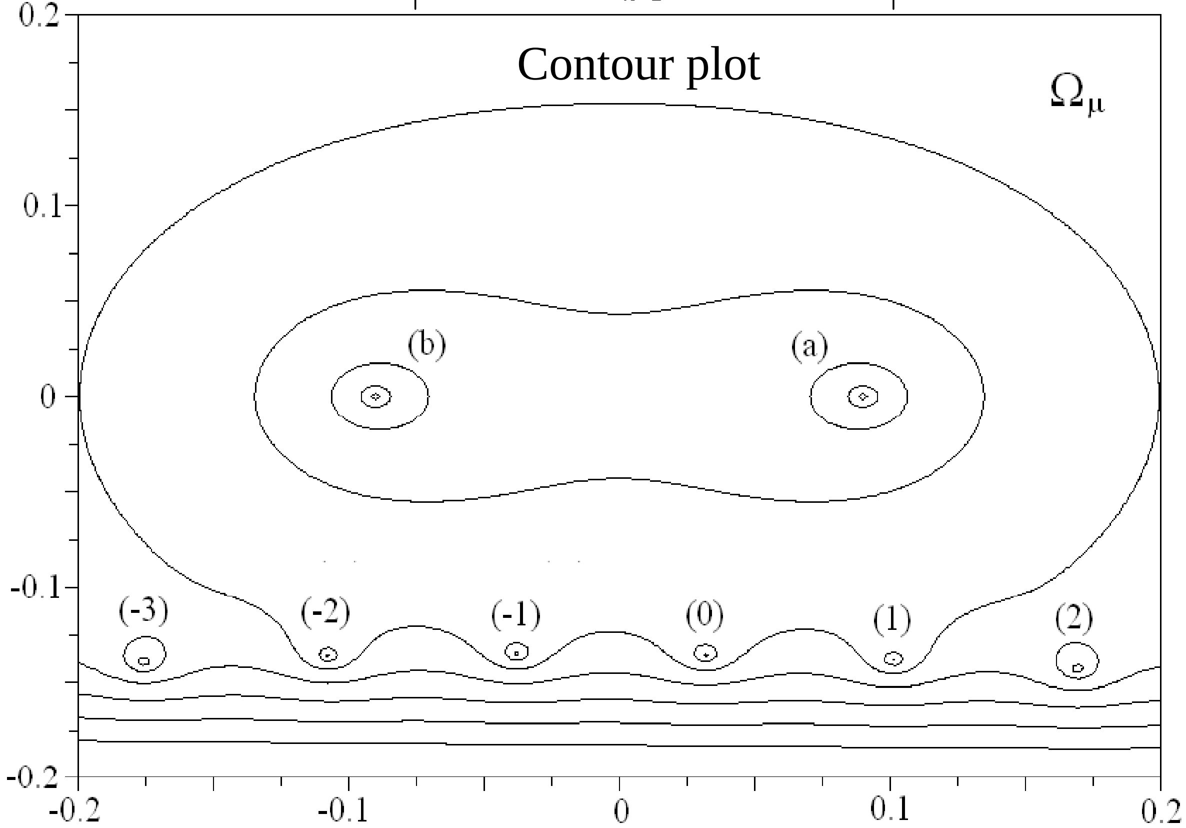
Elementary solution

$$\tilde{y}_\mu(t) = \tilde{y}_\mu(0) e^{-i\Omega_\mu t}$$

Characteristic Equation

$$-\Omega_\mu{}^2 + \omega_\beta^2 = \sum_{n=1}^{N} b_n e^{i\frac{2\pi n\mu}{M}} e^{i\Omega_\mu \tau}$$

$$\left| \Omega_\mu{}^2 - \omega_\beta^2 + \sum_{n=1}^{N} b_n e^{i\frac{2\pi n\mu}{M}} e^{i\Omega_\mu \tau} \right|^2$$



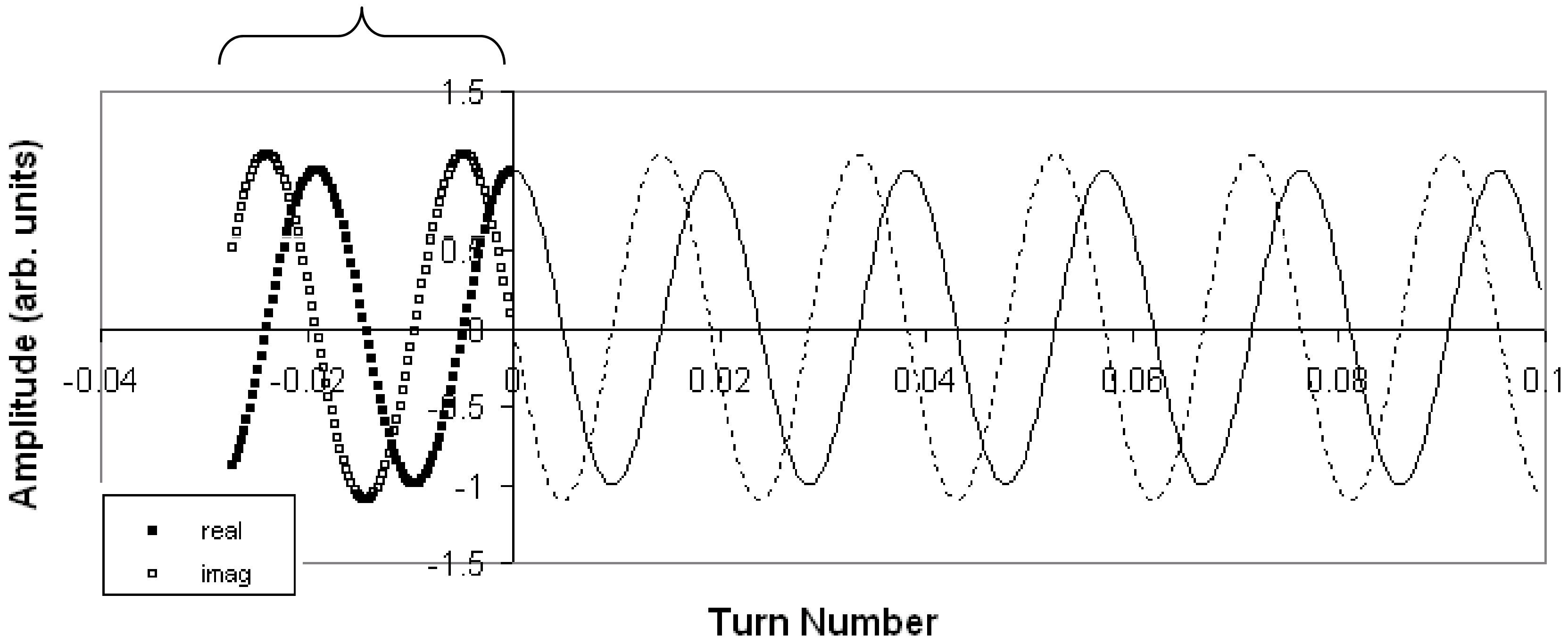
General solution

$$\tilde{y}_\mu = A e^{-i\Omega_\mu^{(a)} t} + B e^{-i\Omega_\mu^{(b)} t} + \sum_{n=-\infty}^{\infty} C_n e^{-i\Omega_\mu^{(n)} t}$$

Integration of

$$\ddot{y}_m(t) + \omega_\beta{}^2 y_m(t) = \sum_{n=1}^{\infty} b_n y_{m+n}(t - n\tau)$$

Depends on initial history



If wake field very small

$$\ddot{y}_m(t) + \omega_\beta{}^2 y_m(t) \approx 0$$

Conjecture

$$\tilde{y}_\mu = Ae^{-i\Omega_\mu^{(a)}t} + Be^{-i\Omega_\mu^{(b)}t} + \sum_{n=-\infty}^{\infty} C_n e^{-i\Omega_\mu^{(n)}t}$$

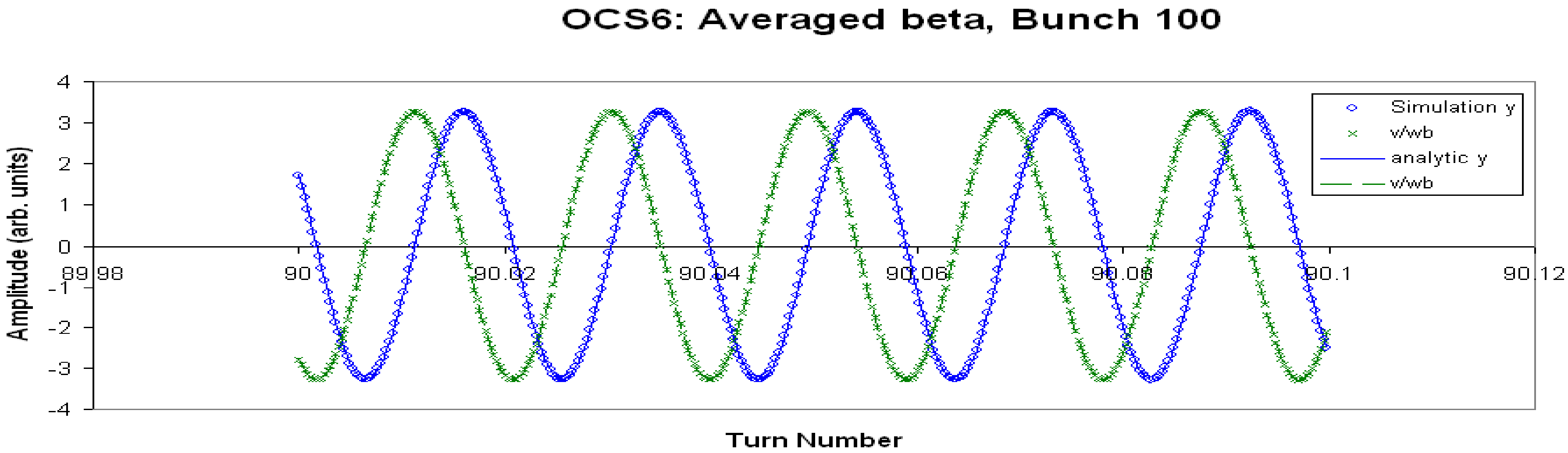
Depends on initial condition at t = 0

Depends on initial history

$$\tilde{y}_\mu \approx Ae^{-i\Omega_\mu^{(a)}t} + Be^{-i\Omega_\mu^{(b)}t}$$

Just like SHM

An analytic solution

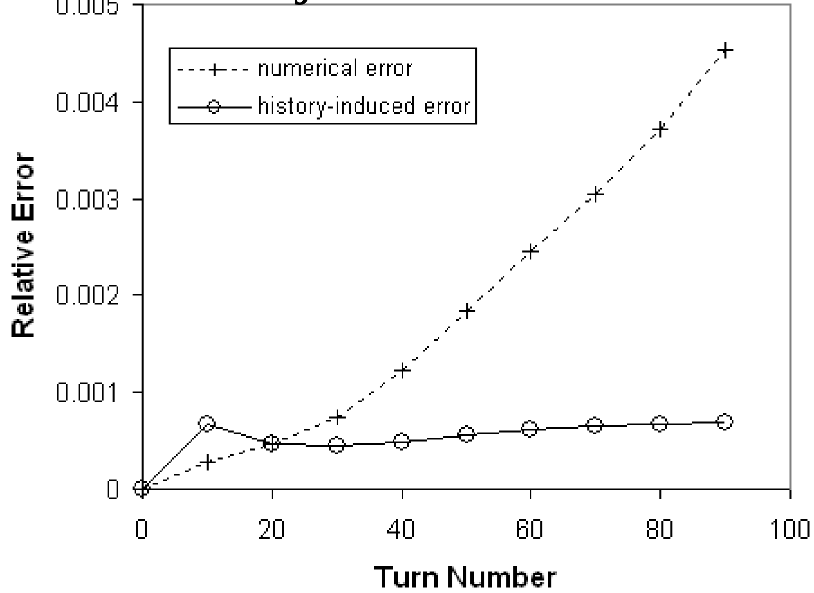


$$y_m(t) = \frac{1}{M}\sum_{\mu=0}^{M-1}\left[A_\mu e^{-i\Omega_\mu^{(a)}t} + B_\mu e^{-i\Omega_\mu^{(b)}t}\right]e^{i\frac{2\pi m\mu}{M}}$$

$$A_\mu = \frac{i\Omega_\mu^{(b)}\tilde{y}_\mu(0) + \dot{\tilde{y}}_\mu(0)}{i\Omega_\mu^{(b)} - i\Omega_\mu^{(a)}}$$

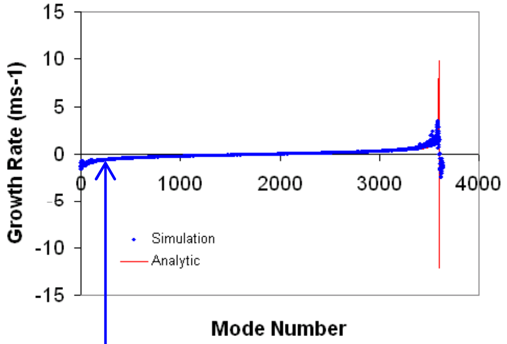
$$B_\mu = \frac{i\Omega_\mu^{(a)}\tilde{y}_\mu(0) + \dot{\tilde{y}}_\mu(0)}{i\Omega_\mu^{(a)} - i\Omega_\mu^{(b)}}$$

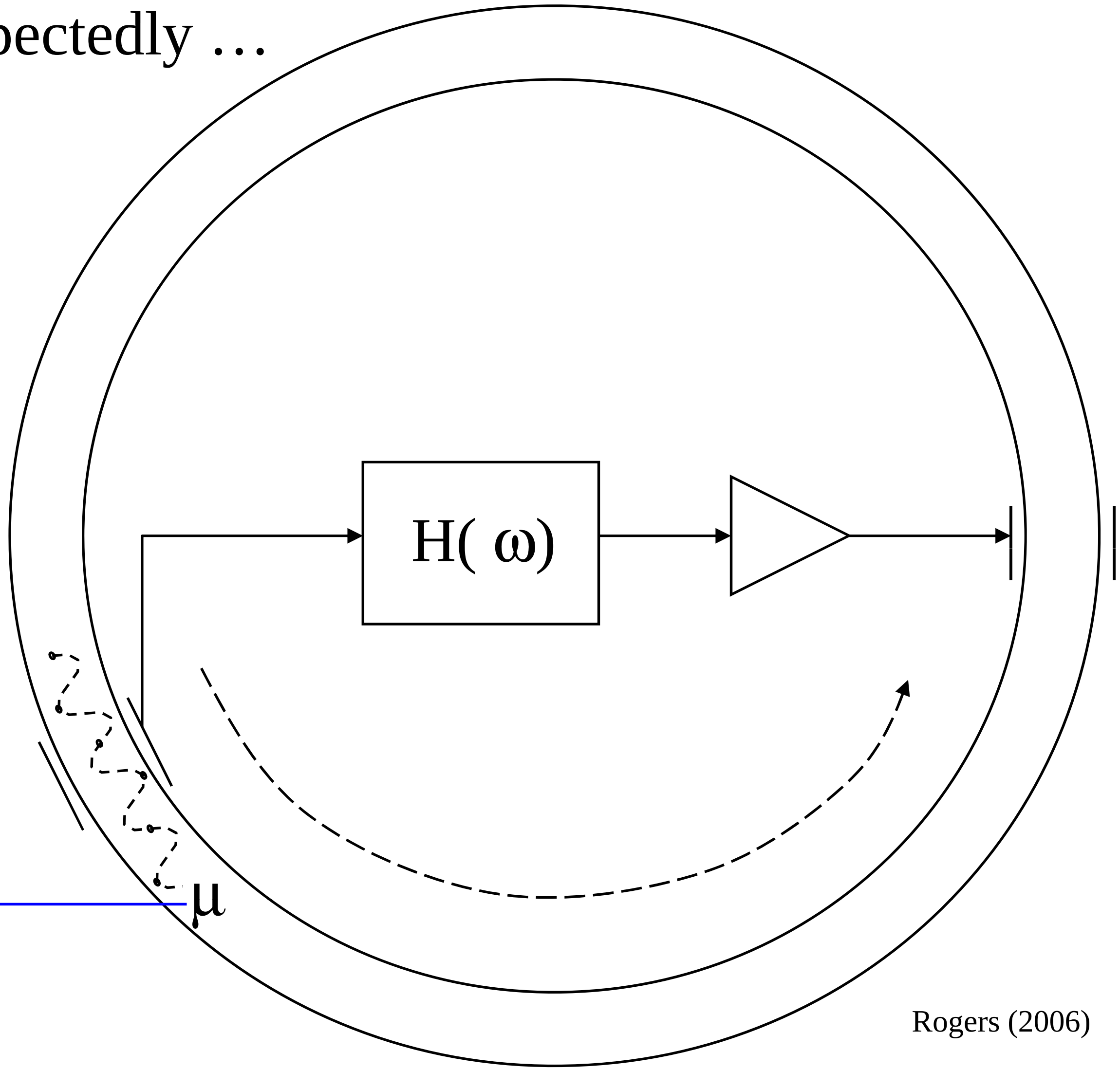
Conjecture validated



Feedback system bandwidth

Frequency $\propto$ mode number
Bandwidth must cover growth modes
If decay modes grow unexpectedly …





Rogers (2006)

Thank You