High-precision tests of the MSSM with GigaZ

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In collaboration with *S. Heinemeyer, W. Hollik, A. Weber*

- Introduction
- New results for electroweak precision observables in the MSSM with $\mathcal{CP}$-violating phases
- Numerical analysis
- Conclusions

Introduction

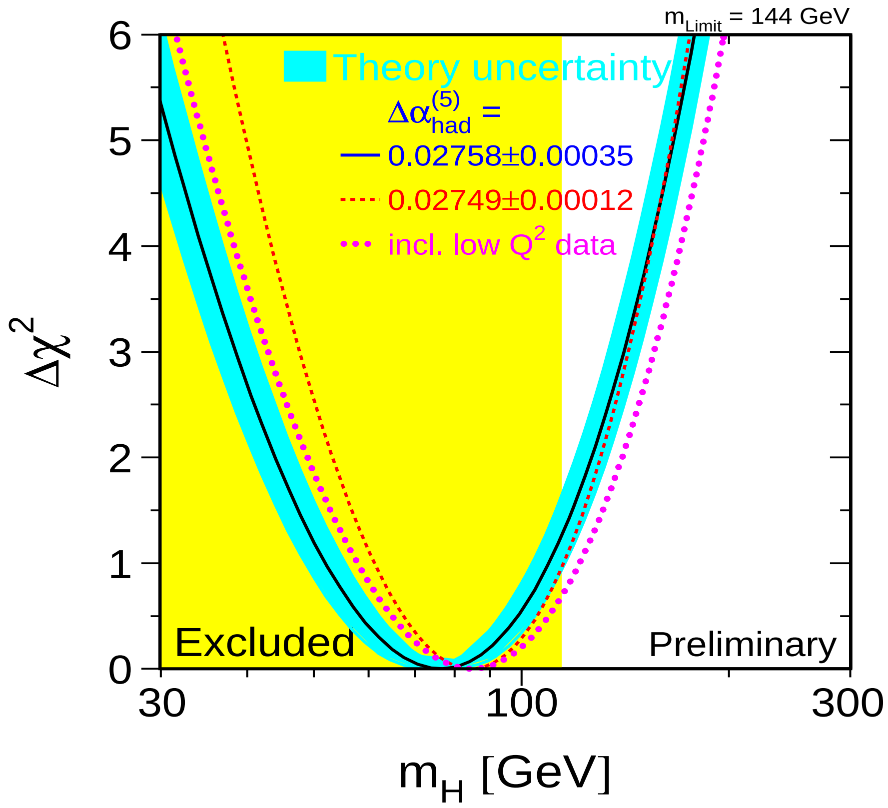
Electroweak precision physics $\Leftrightarrow$ sensitivity to loop effects

Introduction

Electroweak precision physics ⇔ sensitivity to loop effects

Example: indirect constraints on M_{H} in the SM

[*LEPEWWG '07*]

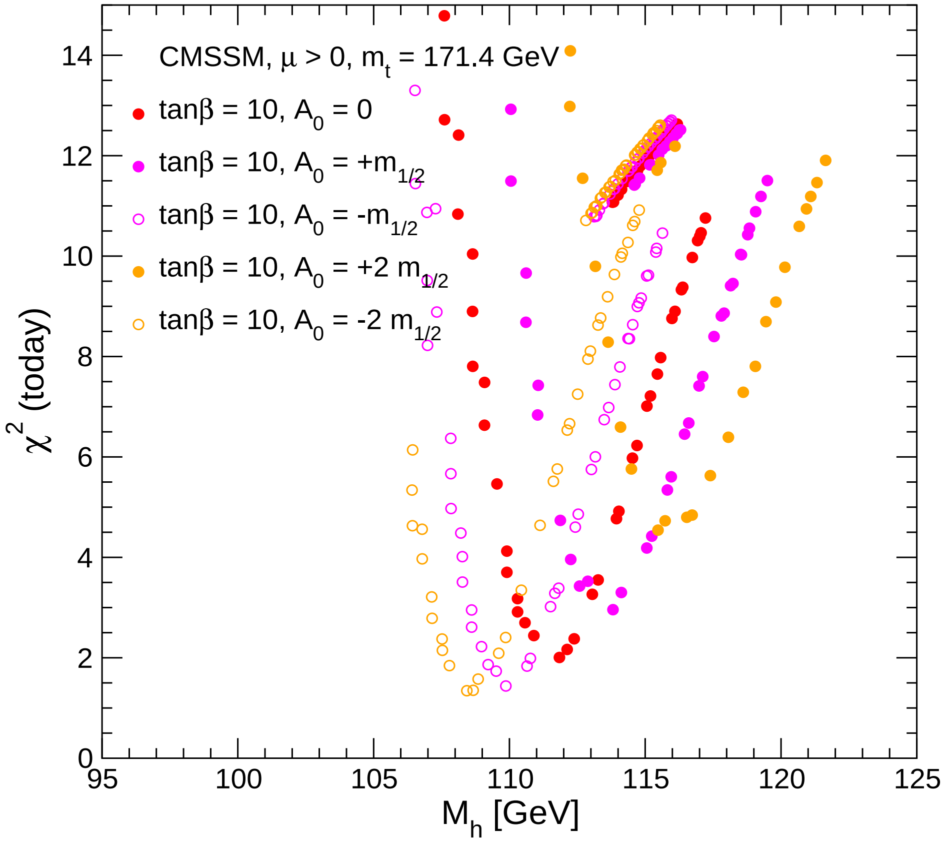


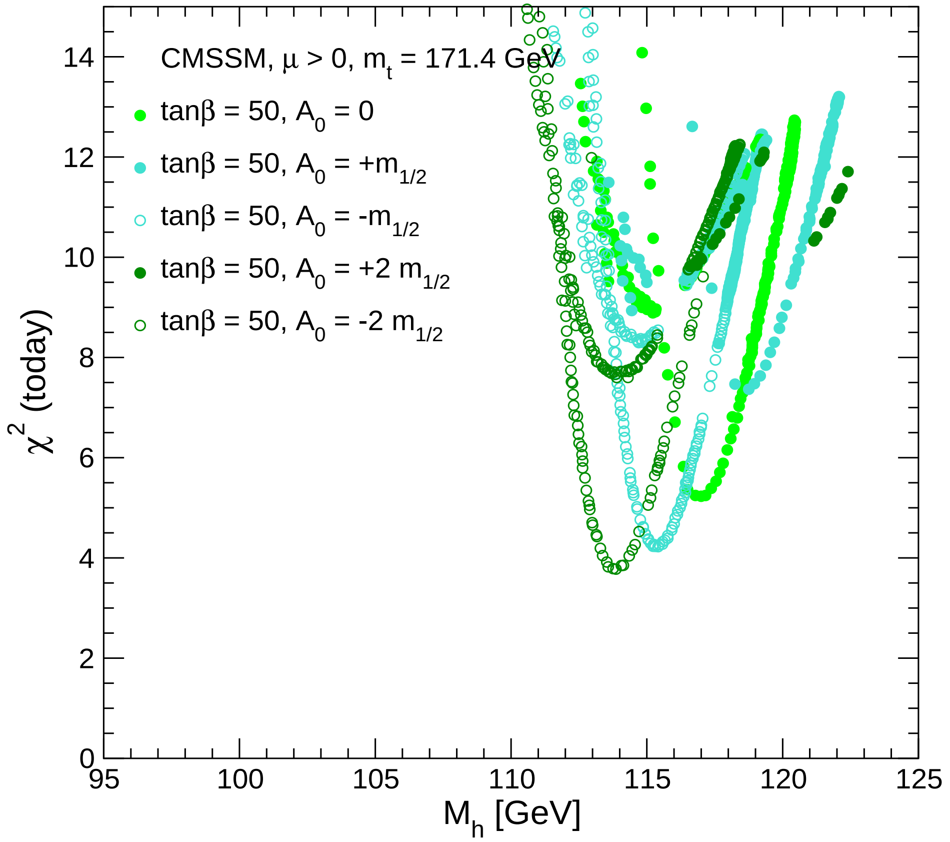
⇒ Increasing tension between indirect bounds on M_{H} in the SM and direct search limit

Bounds on the light Higgs mass in the CMSSM with dark matter constraints

χ^2 fit for M_h from electroweak precision observables and b-physics observables, without imposing direct search limit:

[*J. Ellis, S. Heinemeyer, K. Olive, A. Weber, G. W. '07*]





⇒ Much less tension than in SM, best fit value $\gtrsim 110$ GeV

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles:

$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_L$ Spin $\frac{1}{2}$

$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_L$ Spin 0

$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0}$ Spin 1 / Spin 0

$\tilde{g} \qquad \tilde{\chi}_{1,2}^{\pm} \qquad \tilde{\chi}_{1,2,3,4}^{0}$ Spin $\frac{1}{2}$

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Enlarged Higgs sector: two Higgs doublets, physical states: $h^0, H^0, A^0, H^\pm$

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Enlarged Higgs sector: two Higgs doublets, physical states: $h^0, H^0, A^0, H^{\pm}$

General parametrisation of possible SUSY-breaking terms

$\Rightarrow$ free parameters, no prediction for SUSY mass scale

Constrained MSSM (CMSSM) with restrictions from dark matter relic density

CMSSM characterised by five parameters:

$m_{1/2}$, m_0, A_0 (GUT scale), $\tan\beta$, sgn(μ) (weak scale)

$\Rightarrow$ Low-energy spectrum from renormalisation group running

lightest SUSY particle: $\tilde{\chi}_1^0$

Cold dark matter (CDM) density (WMAP, . . .):

$$0.094 < \Omega_{\rm CDM}\, h^2 < 0.129$$

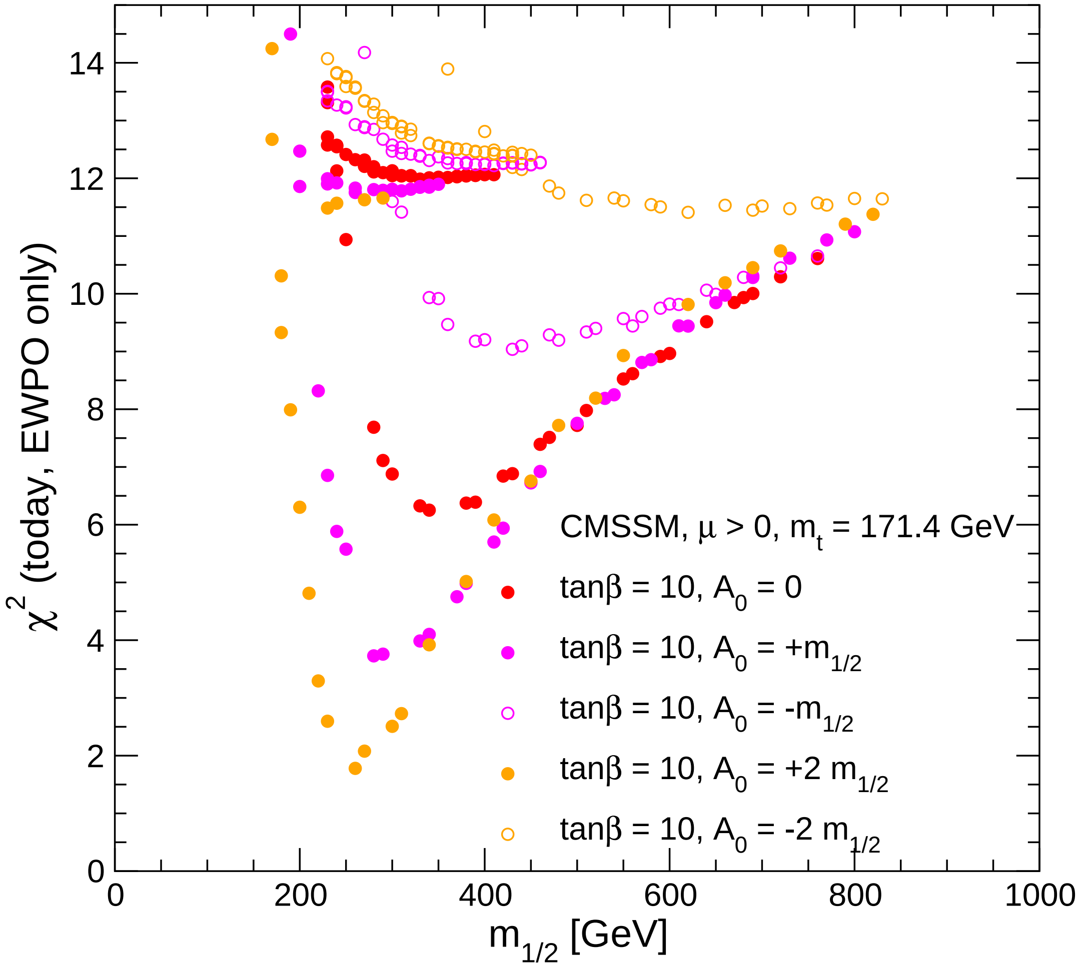
$\Rightarrow$ Constraints on SUSY parameter space

χ^2 *fit in CMSSM with dark matter constraints:*

M_W, $\sin^2\theta_{\mathrm{eff}}$, $(g-2)_\mu$, $\mathrm{BR}(b \to s\gamma)$

[*J. Ellis, S. Heinemeyer, K. Olive, A. Weber, G. W. '07*]

$\tan\beta = 10$:



Higgs bound from LEP:

full likelihood information and theory uncertainty included in the fit

⇒ very good description of the data

preference for relatively small mass values

⇒ good prospects for the LHC and the ILC

Electroweak precision observables (EWPO): present status vs. GigaZ / MegaW precision

obs.	exp. cent. value	σ^{today}	σ^{LHC}	σ^{ILC}
M_{W} [GeV]	80.398	0.025	0.015	0.007
$\sin^2\theta_{\mathrm{eff}}$	0.23153	0.00016	$20\text{–}14 \times 10^{-5}$	1.3×10^{-5}
Γ_Z [GeV]	2.4952	0.0023	—	0.001
R_l	20.767	0.025	—	0.01
R_b	0.21629	0.00066	—	0.00014
σ^0_{had}	41.540	0.037	—	0.025

⇒ Large improvement at the ILC

Theoretical predictions for EWPO

Sources of theoretical uncertainties:

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Theoretical predictions for EWPO

Sources of theoretical uncertainties:

- Unknown higher-order corrections
- Parametric uncertainty induced by the experimental errors of the input parameters

  Dominant effect: experimental error of m_{t}

  $\Rightarrow$ ILC will yield improvement by an order of magnitude

  exp. error on m_{t}: $\approx 1\ \mathrm{GeV} \xrightarrow{\text{ILC + GigaZ}} 0.1\ \mathrm{GeV}$

New results for electroweak precision observables in the MSSM $\mathcal{CP}$-violating phases

New results for M_W and Z observables $\sin^2\theta_\mathrm{eff}$, Γ_Z, R_l, R_b, σ^0_had:

Complete one-loop results with complex parameters $+$ inclusion of all available higher-order corrections

[*S. Heinemeyer, W. Hollik, D. Stöckinger, A.M. Weber, G. W. ’06*]

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Theoretical evaluation in the SM is more advanced than in the MSSM $\Rightarrow$ incorporation of state–of–the–art SM results using

$$O^\mathrm{MSSM} = \underbrace{O^\mathrm{SM}}_{(a)} + \underbrace{O^\mathrm{MSSM-SM}}_{(b)}$$

(a): full SM result

(b): difference between SM and MSSM, evaluated at the level of precision of the known MSSM corrections

New result for $\Gamma(Z \to \tilde{\chi}^0_1 \tilde{\chi}^0_1)$

Complete one-loop results with complex parameters $+$ higher-order corrections
[*S. Heinemeyer, W. Hollik, A. Weber, G. W. '07*]

If $Z \to \tilde{\chi}^0_1 \tilde{\chi}^0_1$ is kinematically allowed

$\Rightarrow$ yields contribution to invisible width of the Z boson

$\mathcal{CP}$*-violating loop effects*

Higher-order corrections to M_{W}, $\sin^2\theta_{\mathrm{eff}}$, $M_{\mathrm{h}}, \ldots$ are affected by $\mathcal{CP}$-violating effects from complex phases

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MSSM Higgs sector is $\mathcal{CP}$-conserving at tree level

Complex parameters enter via (often large) loop corrections:

– μ: Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings

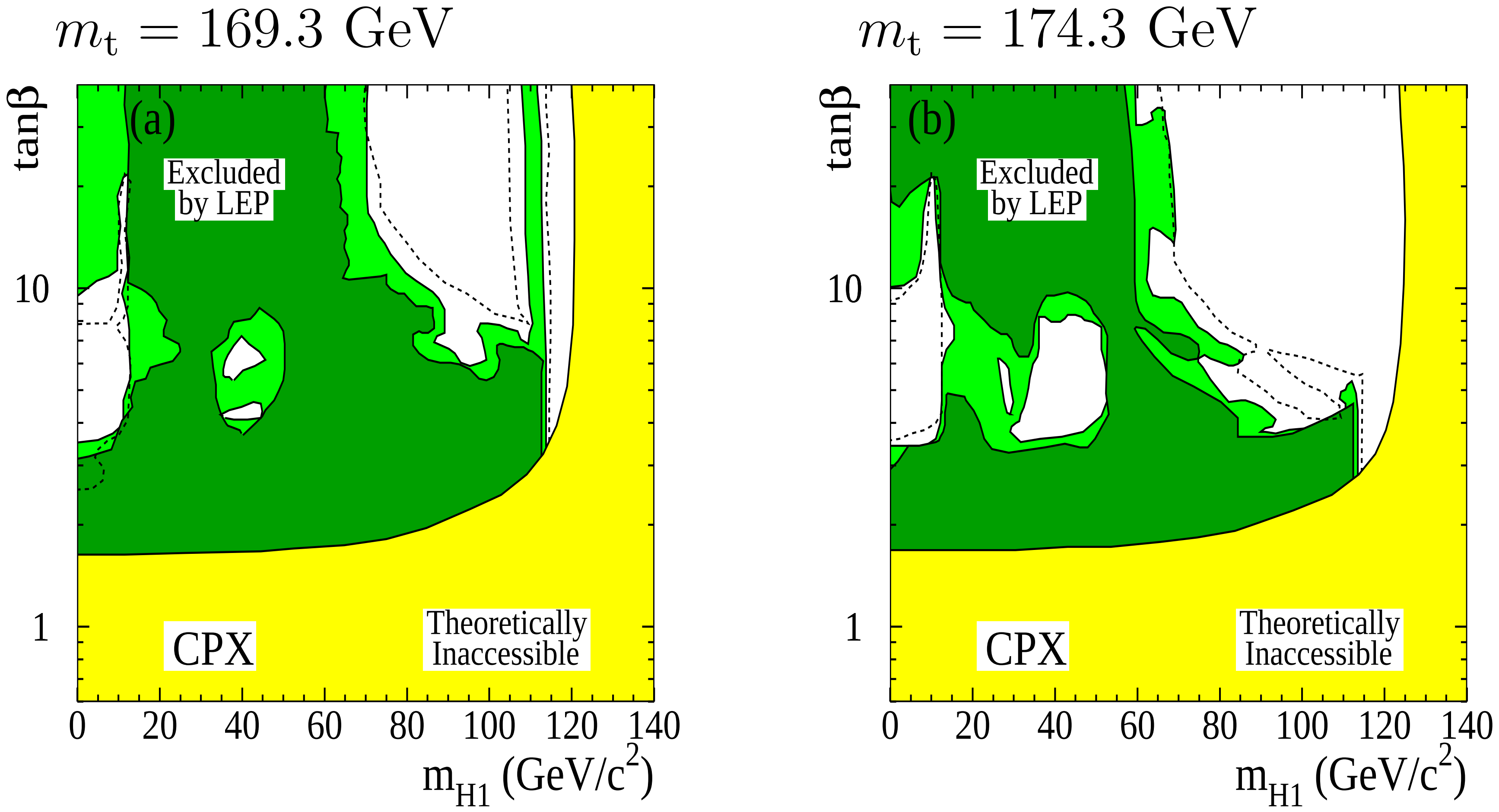
– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– $m_{\tilde{\mathrm{g}}}$: gluino mass

$\Rightarrow$ $\mathcal{CP}$-violating mixing between neutral Higgs bosons h_1, h_2, h_3

$\mathcal{CP}$-violating case (CPX scenario): LEP exclusion bounds

[*LEP Higgs Working Group ’06*]



⇒ no lower limit on M_{h_1}: light SUSY Higgs not ruled out!

sensitive dependence on m_t

Incorporation of higher-order corrections from the Higgs sector

Higgs sector enters EWPO only via loop corrections

$\Rightarrow$ For one-loop corrections to EWPO it would in principle be sufficient to treat the Higgs sector in leading order, i.e. at the tree level

However:

Tree-level mass of light MSSM Higgs boson is below the SM exclusion bound on $M_{\rm H}$

$\Rightarrow$ Treating the MSSM Higgs sector at tree level leads to artificially large contributions to EWPO from the light MSSM Higgs boson

Incorporation of higher-order corrections from the Higgs sector

Large higher-order corrections in the MSSM Higgs sector:

$\Rightarrow$ Correction to upper bound on $m_{\rm h}$ of about 50%

large corrections to Higgs couplings

$\mathcal{CP}$-violating mixing

$\Rightarrow$ Important to consistently incorporate leading higher-order corrections in the MSSM Higgs sector into the predictions for the EWPO

Higher-order corrections in the MSSM Higgs sector with $\mathcal{CP}$-violating phases

Mixing between h, H, A

$\Rightarrow$ loop-corrected masses obtained from propagator matrix

$$\Delta_{hHA}(p^2) = -\left(\hat{\Gamma}_{hHA}(p^2)\right)^{-1}, \quad \hat{\Gamma}_{hHA}(p^2) = i\left[p^2 \mathbb{1} - \mathrm{M_n}(p^2)\right]$$

where

$$\mathrm{M_n}(p^2) = \begin{pmatrix} m_{\mathrm{h}}^2 - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hH}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ -\hat{\Sigma}_{hH}(p^2) & m_{\mathrm{H}}^2 - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ -\hat{\Sigma}_{hA}(p^2) & -\hat{\Sigma}_{HA}(p^2) & m_{\mathrm{A}}^2 - \hat{\Sigma}_{AA}(p^2) \end{pmatrix}$$

$\Rightarrow$ Higgs propagators: $\Delta_{ii}(p^2) = \dfrac{i}{p^2 - m_i^2 + \hat{\Sigma}_{ii}^{\mathrm{eff}}(p^2)}$

Higher-order corrections in the MSSM Higgs sector with $\mathcal{CP}$-violating phases

$$\hat{\Sigma}^{\text{eff}}_{ii}(p^2) = \hat{\Sigma}_{ii}(p^2) - i\frac{2\hat{\Gamma}_{ij}(p^2)\hat{\Gamma}_{jk}(p^2)\hat{\Gamma}_{ki}(p^2) - \hat{\Gamma}^2_{ki}(p^2)\hat{\Gamma}_{jj}(p^2) - \hat{\Gamma}^2_{ij}(p^2)\hat{\Gamma}_{kk}(p^2)}{\hat{\Gamma}_{jj}(p^2)\hat{\Gamma}_{kk}(p^2) - \hat{\Gamma}^2_{jk}(p^2)}$$

Complex pole $\mathcal{M}^2$ of each propagator is determined from

$$\mathcal{M}_i^2 - m_i^2 + \hat{\Sigma}^{\text{eff}}_{ii}(\mathcal{M}_i^2) = 0,$$

where

$$\mathcal{M}^2 = M^2 - iM\Gamma,$$

Expansion up to first order in Γ around M^2:

$$M_i^2 - m_i^2 + \text{Re}\,\hat{\Sigma}^{\text{eff}}_{ii}(M_i^2) + \frac{\text{Im}\,\hat{\Sigma}^{\text{eff}}_{ii}(M_i^2)\,\left(\text{Im}\,\hat{\Sigma}^{\text{eff}}_{ii}\right)'(M_i^2)}{1 + \left(\text{Re}\,\hat{\Sigma}^{\text{eff}}_{ii}\right)'(M_i^2)} = 0$$

Effective couplings

Effective mixing matrix $\mathrm{U}_{\mathrm{eff}}$ obtained from propagator matrix in approximation where all Higgs self-energies are evaluated at $p^2 = 0$:

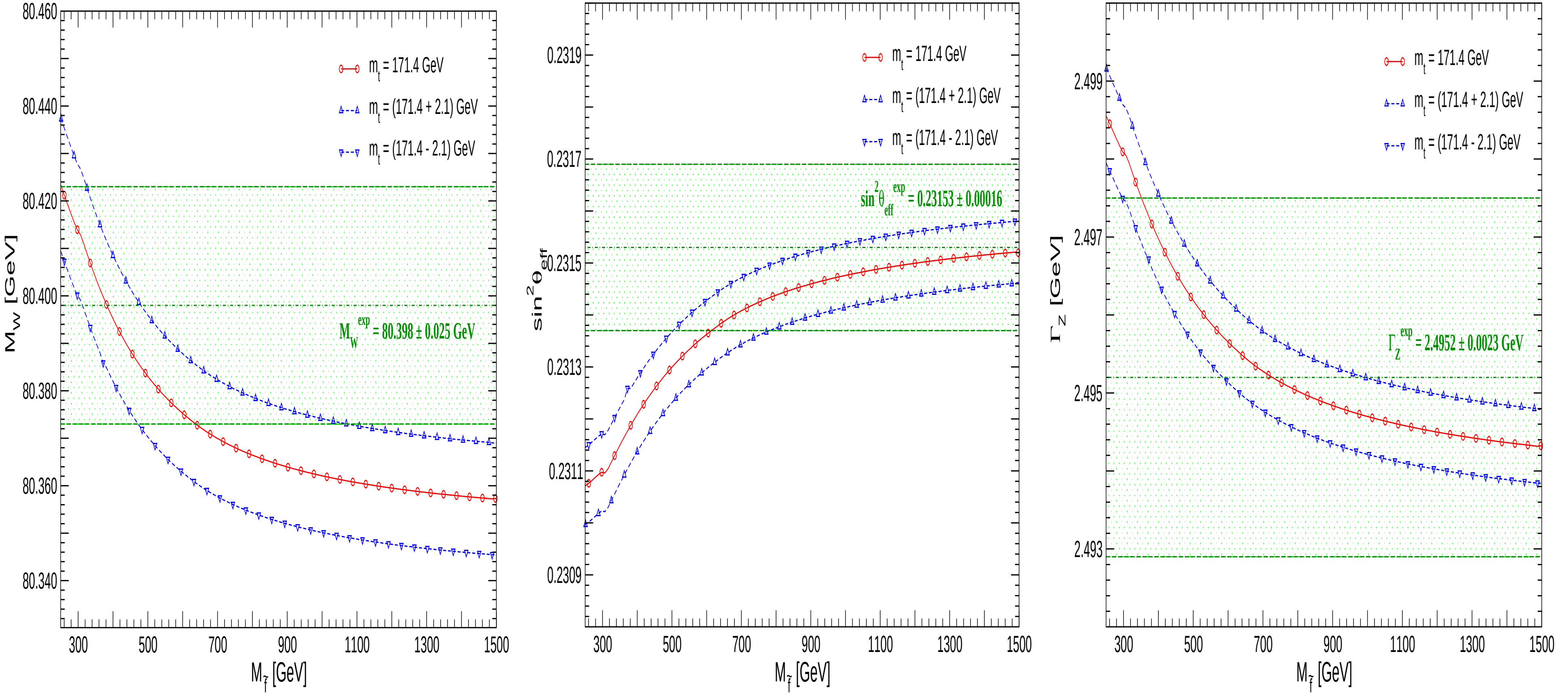
$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}_{p^2=0} = \mathbf{U}_{\mathrm{eff}} \begin{pmatrix} h \\ H \\ A \end{pmatrix}$$

$\Rightarrow$ unitary matrix

Elements of $\mathrm{U}_{\mathrm{eff}}$ can be interpreted as effective couplings of the Higgs bosons, incorporate leading higher-order corrections from Higgs-boson self-energies

Numerical analysis

Dependence on the sfermion mass scale

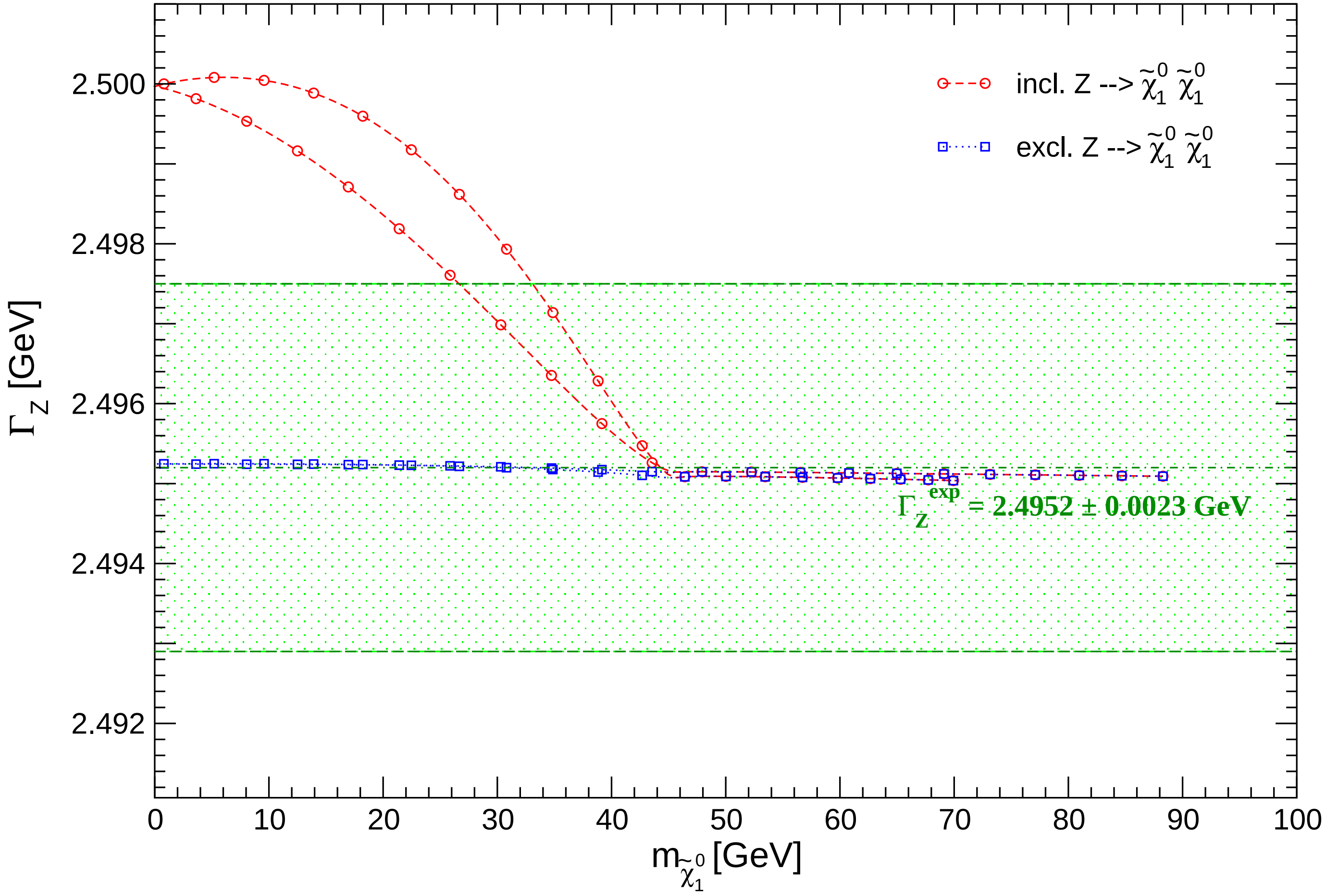


$\Rightarrow$ Sizable dependence on the sfermion mass scale

Drastic improvement with ILC prec. on M_{W}, $\sin^2\theta_{\mathrm{eff}}$, Γ_{Z}, m_{t}

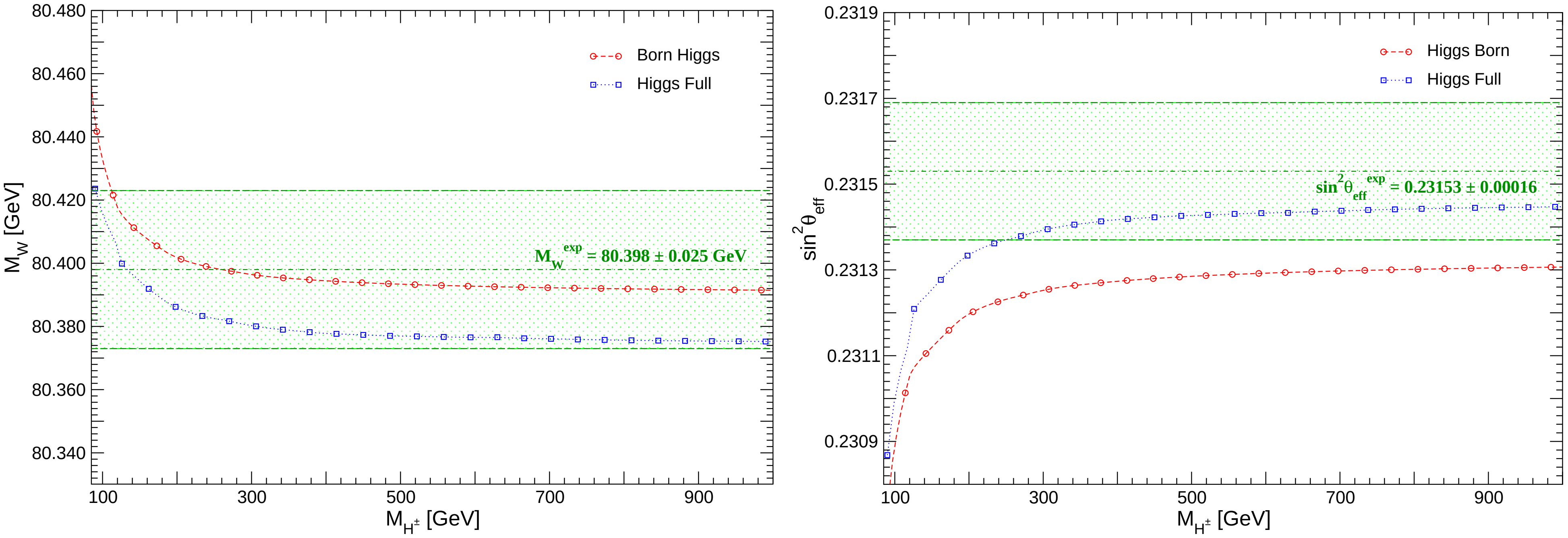
Impact of $\Gamma(Z \to \tilde{\chi}_1^0 \tilde{\chi}_1^0)$ ***on the total Z width***

$\mu \approx M_1$, $M_1 \lesssim \frac{1}{2} M_2$



⇒ Large effects possible

Higgs sector at higher orders:
impact on M_W and $\sin^2\theta_\mathrm{eff}$



⇒ Sizable effects

ILC can probe loop-induced effects from the Higgs sector

Impact of the complex phases ϕ_{A_t}, ϕ_{A_b} in the sfermion sector

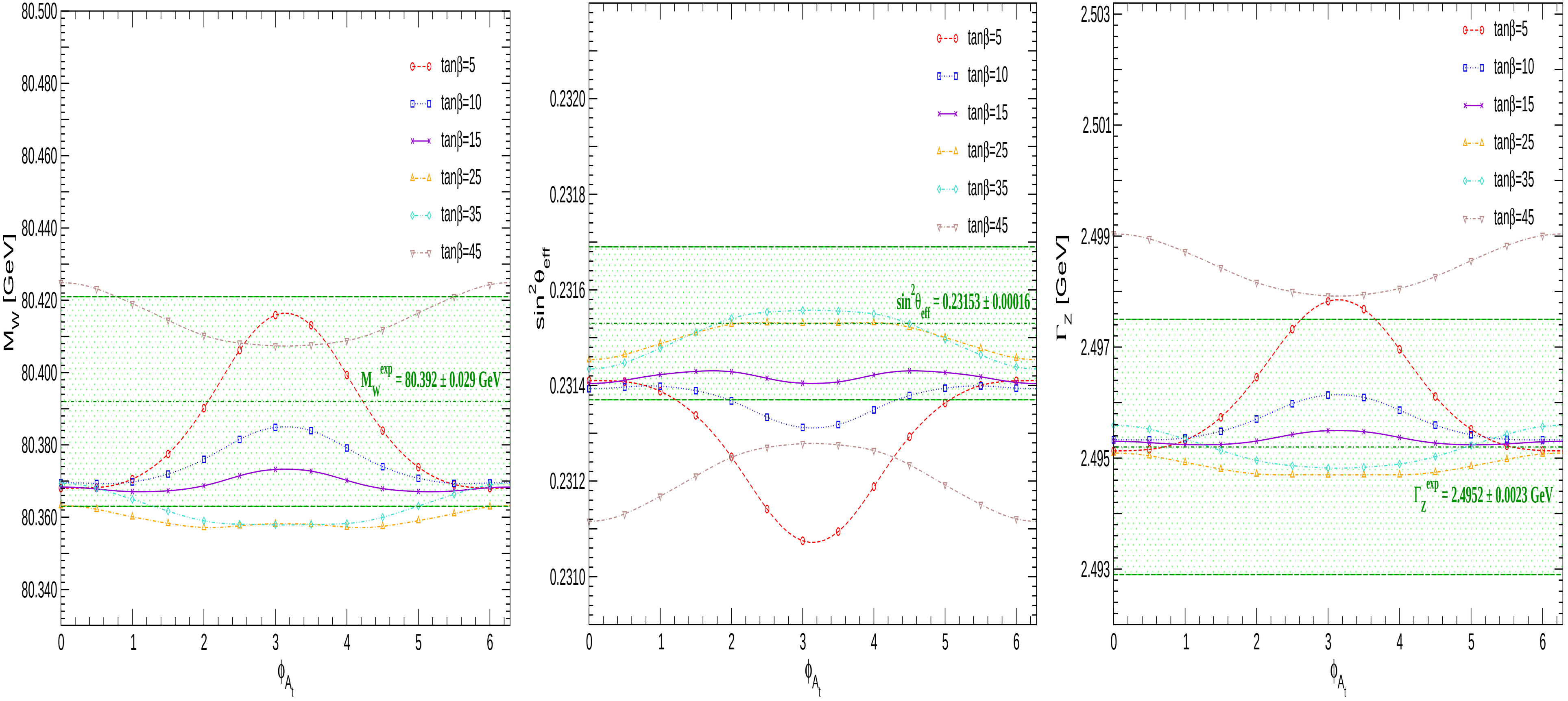
Enter only via

$$\begin{aligned} |X_\mathrm{t}|^2 &= |A_\mathrm{t}|^2 + |\mu \cot\beta\,|^2 - 2|A_\mathrm{t}| \cdot |\mu| \cot\beta\ \cos(\phi_{A_\mathrm{t}} + \phi_\mu) \\ |X_\mathrm{b}|^2 &= |A_\mathrm{b}|^2 + |\mu \tan\beta|^2 - 2|A_b| \cdot |\mu| \tan\beta\ \cos(\phi_{A_\mathrm{b}} + \phi_\mu) \end{aligned}$$

where $X_\mathrm{t} = A_\mathrm{t} - \mu^*/\tan\beta$, $X_\mathrm{b} = A_\mathrm{b} - \mu^* \tan\beta$, $\tan\beta \equiv v_2/v_1$

$\Rightarrow$ phase dependence only enters via the squark masses and mixing angles

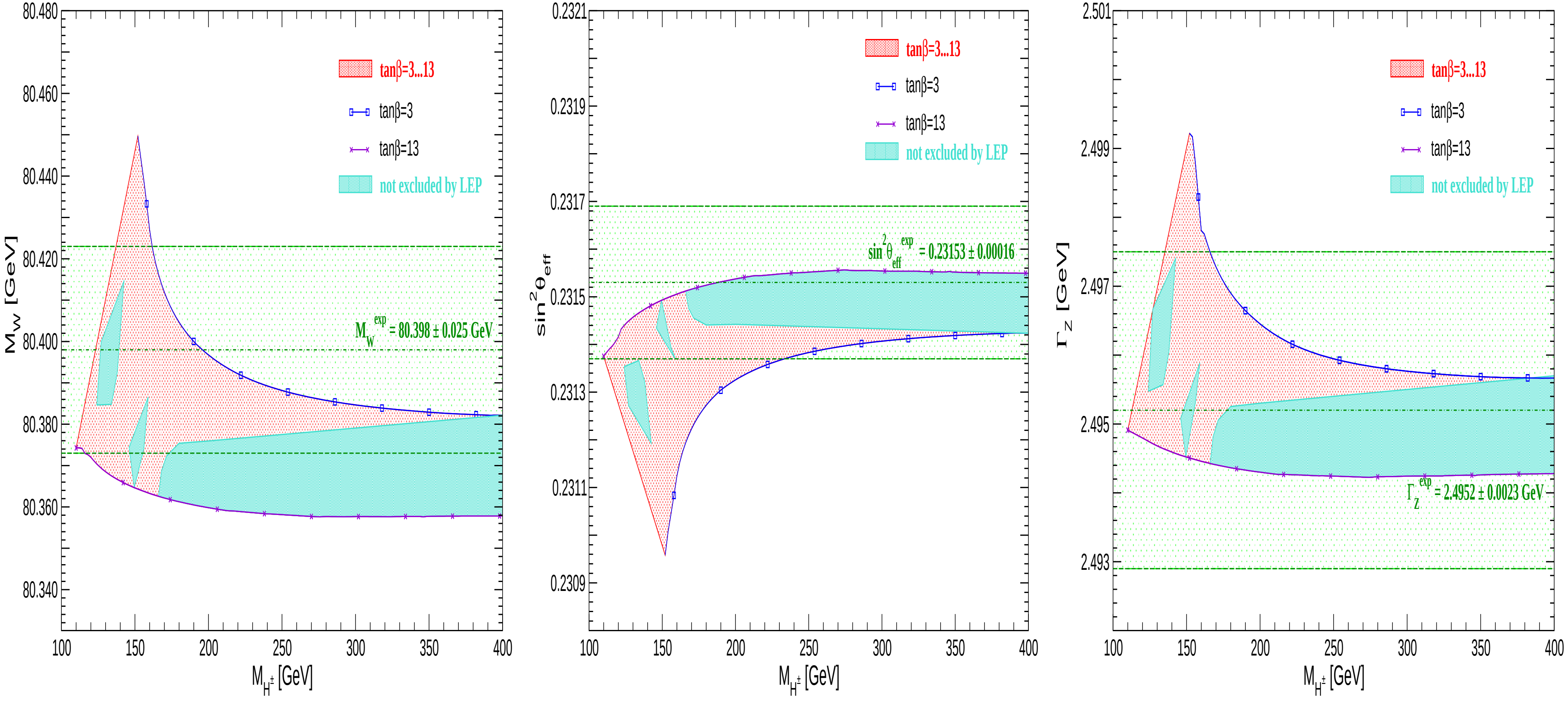
Effects of varying the complex phase ϕ_{A_t} *on* M_W, $\sin^2\theta_{\rm eff}$, Γ_Z



⇒ Shift in M_W, $\sin^2\theta_{\rm eff}$, Γ_Z predictions by 1–2 σ for small $\tan\beta$

Largely improved sensitivity at the ILC

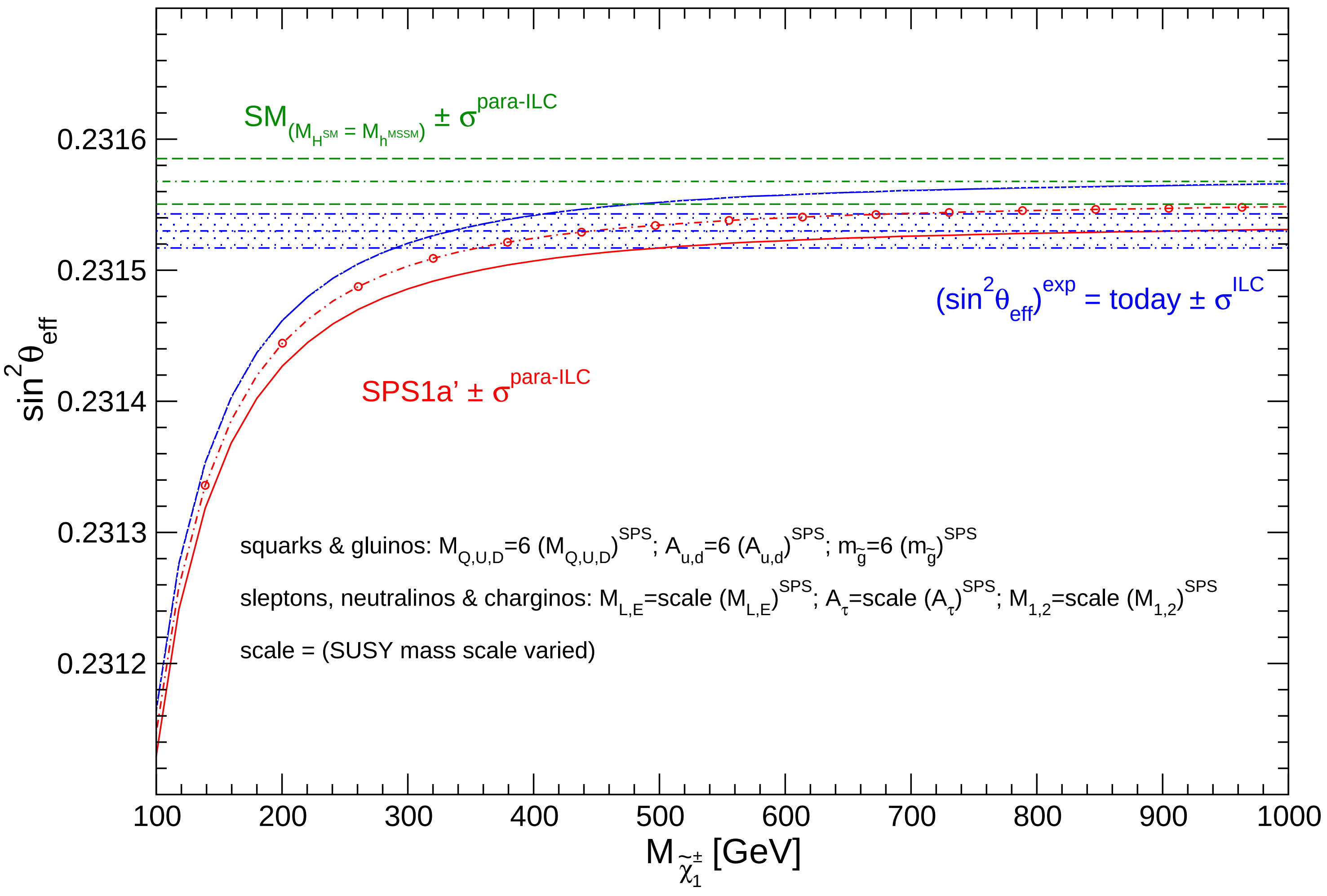
Are the “CPX holes” in agreement with electroweak precision data?



$\Rightarrow$ EWPO yield constraints on parameter space of CPX scenario

ILC precision can have large impact

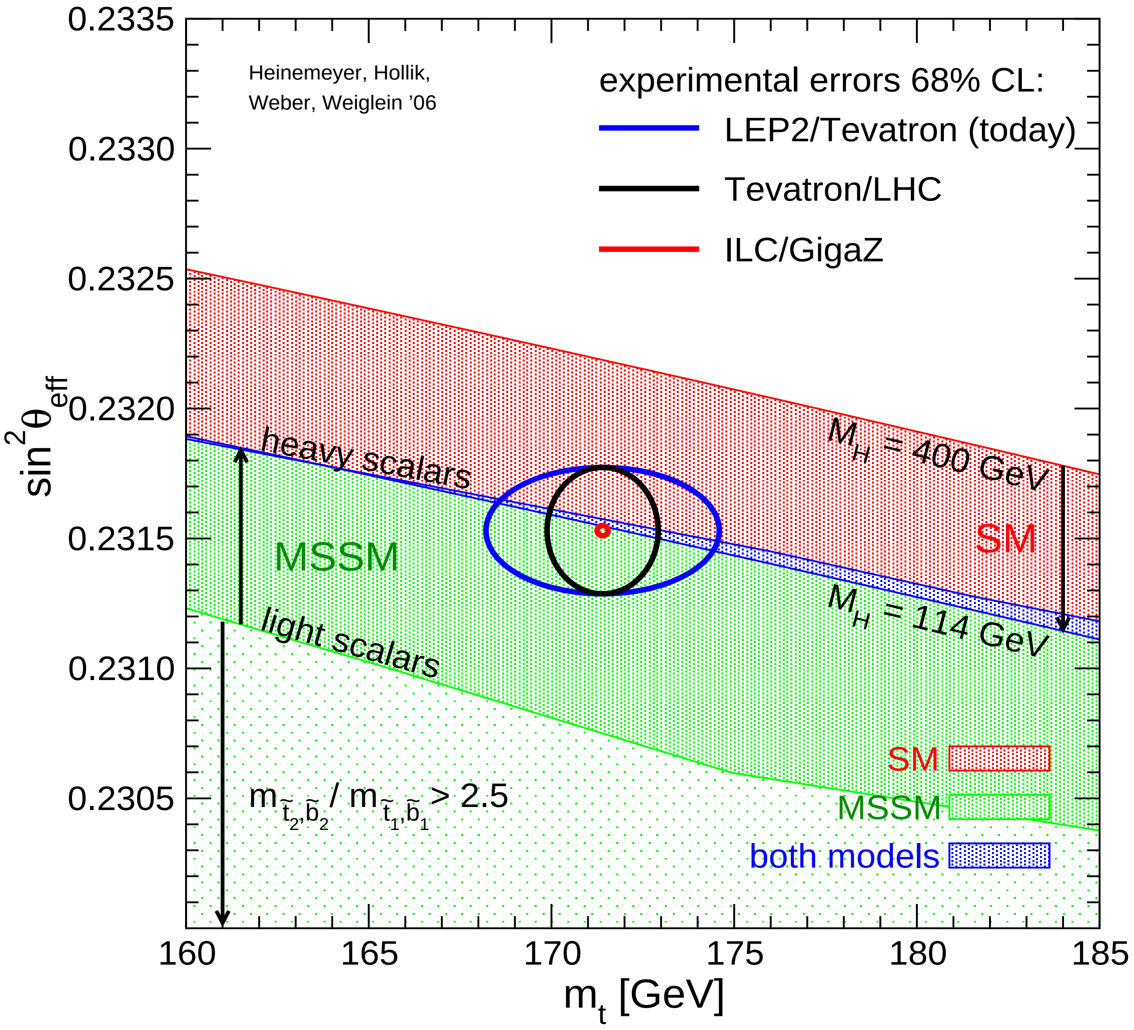
Sensitivity to the scale of SUSY in a scenario where no SUSY particles are observed at the LHC



⇒ GigaZ measurement provides sensitivity to SUSY scale, extends the direct search reach of ILC(500)

Prediction for $\sin^2\theta_{\rm eff}$ (parameter scan): SM vs. MSSM

Prediction for $\sin^2\theta_{\rm eff}$ in the SM and the MSSM:



[S. Heinemeyer, W. Hollik, A.M. Weber, G. W. '07]

MSSM: SUSY parameters varied

SM: $M_{\rm H}$ varied

⇒ ILC precision on $\sin^2\theta_{\rm eff}$ and $m_{\rm t}$ yields drastic improvement

Conclusions

- Results for M_W and Z observables $\sin^2\theta_\mathrm{eff}$, Γ_Z, R_l, R_b, σ^0_had:

  complete one-loop results with complex parameters $+$ inclusion of all available higher-order corrections

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⇒ GigaZ is a highly powerful tool for probing the structure of new physics