The Noncommutative Standard Model Phenomenology at the ILC

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Quantum mechanics: position and momentum measurements complementary

$$[\hat{x}_i, \hat{p}_j] = \hat{x}_i \hat{p}_j - \hat{p}_j \hat{x}_i = i\hbar\delta_{ij} \quad \Rightarrow \quad \Delta x_i \cdot \Delta p_j \geqslant \frac{\hbar}{2}\delta_{ij}$$

Analog: postulation of noncommutative space-time

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu} = i\frac{C_{\mu\nu}}{{\Lambda_{NC}}^2} \quad \Rightarrow \quad \Delta\hat{x}_\mu \cdot \Delta\hat{x}_\nu \geqslant \frac{\theta_{\mu\nu}}{2}$$

- no experimental evidence yet
- possible, as long as the characteristic length scale $l_{NC} = \frac{1}{\Lambda_{NC}}$ small enough compared to the characteristic scales of present experiments
- introduces a minimal area / maximal energy:

$$A_{NC} = \frac{1}{\Lambda_{NC}^2}$$

- Motivation and Bounds
- QFT on noncommutative (NC) space-time
- NCSM at $\mathcal{O}(\theta)$
  - The model
  - Partonic cross sections
  - NCSM @ LHC $\Rightarrow$ sensitivity bounds
  - NCSM @ ILC $\Rightarrow$ sensitivity bounds
- Conclusions

Theoretically, hypothesis/appearance of length/area scale:

- historically regularisation of infinites in QFT [Heisenberg/Pauli/Snyder]
- NCQFT as low energy limit of string theories [Seiberg/Witten]
- incorporation of quantum gravity: quantization of space-time
- NCQFT as intermediate regime between QFT and string theory

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Current model dependent bounds on the noncommutative scale Λ_{NC}:

- $\Lambda_{NC} > 141$ GeV (OPAL: $e^+e^- \to \gamma\gamma$ in NCQED [Abbiendi et al.])
- $\Lambda_{NC} \sim 80$ GeV … 10^8 TeV (supernova cooling, …, γ/p above GZK cutoff)
- $\Lambda_{NC} \gtrsim 10$ TeV … $5 \cdot 10^{14}$ GeV (interferometry, …, Lorentz-tests)

Canonical noncommutativity: $\theta^{\mu\nu}$ constant 4×4-matrix:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} = i\frac{1}{\Lambda_{NC}{}^2}C^{\mu\nu} = i\frac{1}{\Lambda_{NC}{}^2}\begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{pmatrix}$$

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Effective lagrangians

$$\mathcal{L}_{\text{eff.}} = \cdots + g\bar{\psi}(\hat{x})\gamma_\mu(1-\gamma_5)\psi(\hat{x})W^\mu(\hat{x}) + \cdots$$

with product of functions of noncommuting variables

$$(fg)(\hat{x}) = f(\hat{x})g(\hat{x})$$

realised by Moyal-Weyl $\star$-products of functions of commuting variables:

$$(f\star g)(x) = f(x)e^{\frac{i}{2}\overleftarrow{\partial^\mu}\theta_{\mu\nu}\overrightarrow{\partial^\nu}}g(x) = f(x)g(x) + \frac{i}{2}\theta_{\mu\nu}\frac{\partial f(x)}{\partial x_\mu}\frac{\partial g(x)}{\partial x_\nu} + \mathcal{O}(\theta^2)$$

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Note: $[x_\mu \overset{\star}{,} x_\nu](x) = (x_\mu \star x_\nu)(x) - (x_\nu \star x_\mu)(x) = i\theta_{\mu\nu} = [\hat{x}_\mu, \hat{x}_\nu]$

NC generalization of the SM Lagrangian:

Replace products in $\mathcal{L}$ with Moyal-Weyl $\star$-products:

$$\mathcal{L}_{\mathsf{SM}} = \mathrm{i}\bar{\psi}\not{A}\psi + \cdots \longrightarrow \quad \mathcal{L}_{\mathsf{NC}} = \mathrm{i}\bar{\psi}\star\not{A}\star\psi + \cdots$$

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Problem: only $U(N)$-gauge theory on NC space-time possible

$$[A_\mu^a T^a \stackrel{\star}{,} A_\nu^b T^b] = \frac{1}{2}\{A_\mu^a \stackrel{\star}{,} A_\nu^b\}[T^a, T^b] + \frac{1}{2}[A_\mu^a \stackrel{\star}{,} A_\nu^b]\{T^a, T^b\}$$

is not in Lie Algebra, but in Enveloping Algera

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Solution: express noncommutative fields by commutative fields

$$A_\mu,\ \Psi,\ \lambda \to \widehat{A}_\mu[A, \theta],\ \widehat{\Psi}[\Psi, A, \theta],\ \widehat{\lambda}[\lambda, A, \theta]$$

and NC gauge transformations through ordinary gauge transformations:

$$\begin{aligned} \widehat{A}_\rho(A, \theta) + \delta_\lambda \widehat{A}_\rho(A, \theta) &\stackrel{!}{=} \widehat{A}_\rho(A + \delta_\lambda A, \theta) \\ \widehat{\Psi}(\Psi, A, \theta) + \delta_\lambda \widehat{\Psi}(\Psi, A, \theta) &\stackrel{!}{=} \widehat{\Psi}(A + \delta_\lambda A, \Psi + \delta_\lambda \Psi, \theta) \end{aligned}$$

"Gauge Equivalence Conditions"

⇒ Seiberg-Witten-maps(SWM), as power series in θ

$$\hat{A}_\mu(x) = A_\mu(x) + \frac{1}{4}\theta^{\rho\sigma}\{A_\sigma(x), \partial_\rho A_\mu(x) + F_{\rho\mu}(x)\} \qquad + \mathcal{O}((\theta^{\mu\nu})^2)$$

$$\hat{\Psi}(x) = \Psi(x) + \frac{1}{2}\theta^{\rho\sigma}A_\sigma(x)\partial_\rho\Psi(x) + \frac{i}{8}\theta^{\rho\sigma}[A_\rho(x), A_\sigma(x)]\Psi(x) \qquad + \mathcal{O}((\theta^{\mu\nu})^2)$$

$$\hat{\lambda}(x) = \lambda(x) + \frac{1}{4}\theta^{\rho\sigma}\{A_\sigma(x), \partial_\rho\lambda(x)\} \qquad + \mathcal{O}((\theta^{\mu\nu})^2)$$

- Leave Lie Algebra and enter Universal Enveloping Algebra (UEA)
- Not unique: homogenous solution to gauge equivalence condition
  - at $\mathcal{O}(\theta)$: field redefinitions ⇒ no influence on observables

Noncommutative $SU(3)_C \times SU(2)_L \times U(1)_Y$ effective th. as expansion in $\mathcal{O}(\theta)$:

- replace usual "$\cdot$" products by "$\star$" products
- replace fields by their Seiberg-Witten maps

$$\text{e.g.} \quad S_{\text{fermions}} = \int d^4x \left(\sum_f \overline{\widehat{\Psi}}_{fL} \star i\widehat{\not{D}}\widehat{\Psi}_{fL} + \sum_f \overline{\widehat{\Psi}}_{fR} \star i\widehat{\not{D}}\widehat{\Psi}_{fR} \right)$$

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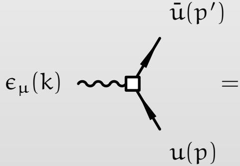
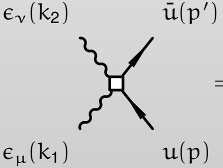
$\Rightarrow$ NC-corrections to SM-interactions:



$$= \quad -\frac{g}{2}\left[k\theta^\mu \not{p} - p\theta^\mu \not{k} - (k\theta p)\gamma_\mu\right]$$

$\Rightarrow$ New interactions:



$$= \begin{cases} -\frac{g^2}{2}\left[k_2\theta^\mu\gamma^\nu - k_1\theta^\mu\gamma^\nu - \theta^{\mu\nu}\not{k}_1 \right. \\ \left. \qquad + (\mu \leftrightarrow \nu, k_1 \leftrightarrow k_2)\right] \end{cases}$$

In the enveloping algebra, the trace

$$S_{\text{gauge}} = -\frac{1}{2}\int d^4x \ \text{Tr}\left(\widehat{F}_{\mu\nu}\star\widehat{F}^{\mu\nu}\right)$$

depends on the representation:

- Minimal NCSM $\rightarrow$ no triple neutral gauge boson interactions
- Nonminimal NCSM $\rightarrow$ new interactions: $\gamma\gamma\gamma$, $Z\gamma\gamma$, $ZZ\gamma$, ...

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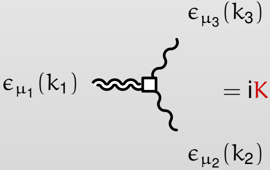
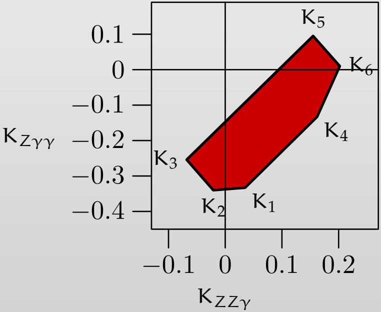


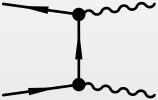
$= iK_{Z\gamma\gamma}\cdot\ldots$

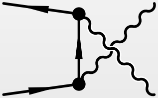


- coupling constants not unique, yet constrained from matching the SM at $\theta \to 0$

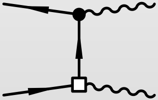
$$f\bar{f} \to \gamma\gamma,\ Z\gamma,\ ZZ \ @\ \mathcal{O}(\theta)$$

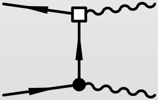
Standard Model:

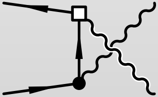
$A_t^{SM} =$ 

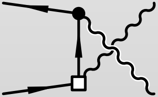
$A_u^{SM} =$ 

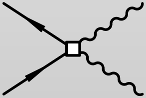
NCSM:

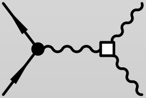
$A_{t,1}^{NC} =$ 

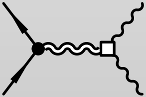
$A_{t,2}^{NC} =$ 

$A_{u,1}^{NC} =$ 

$A_{u,2}^{NC} =$ 

$A_c^{NC} =$ 

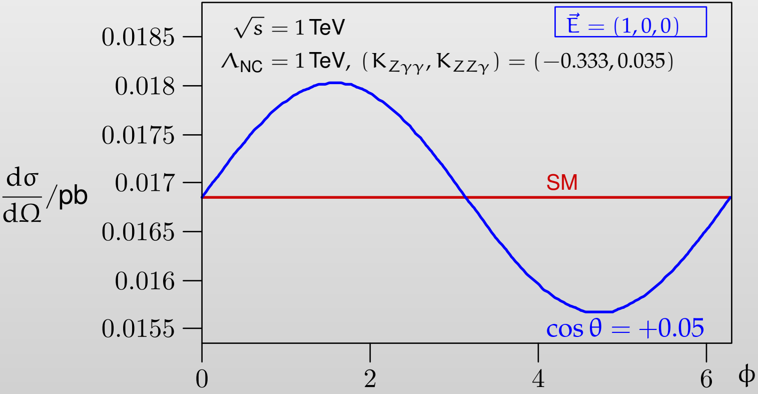
$A_{s,\gamma}^{NC} =$ 

$A_{s,Z}^{NC} =$ 

$f\bar{f} \to Z\gamma$

Discriminative effect: azimuthal dependence of cross sections

- antisymmetric in $\cos\theta$ for $\vec{E} \neq 0$ (symmetric for $\vec{B} \neq 0$!)
- dependency on $\vec{E}$ stronger than on $\vec{B}$



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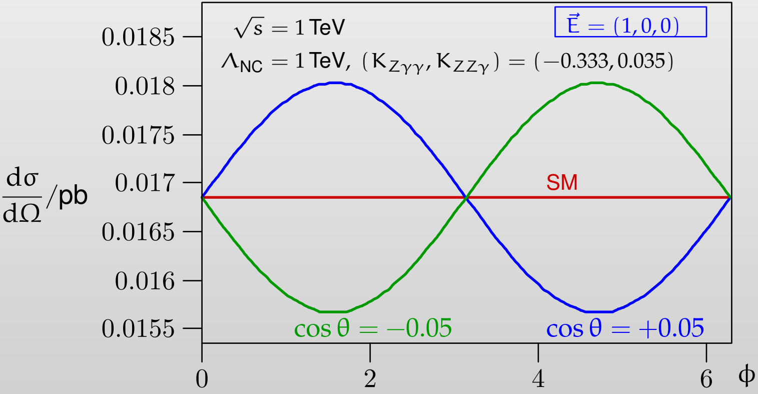

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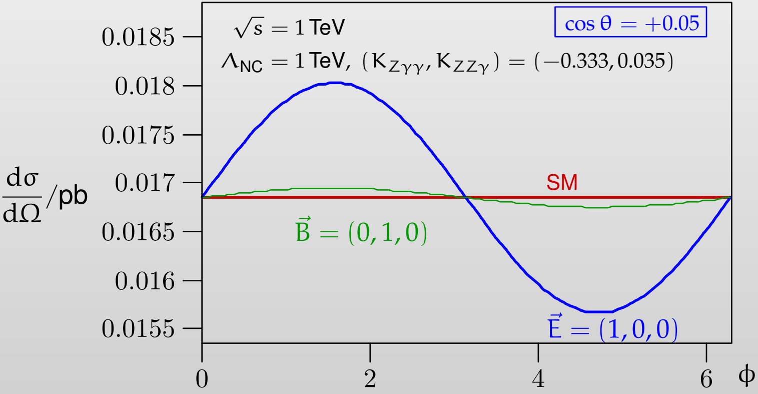

$$f\bar{f} \to Z\gamma$$

Nonminimal NCSM ($K_i \neq (0,0)$):

- (differential) cross sections depend on $sign(Q_f)$:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{SM} + \frac{d\sigma'}{d\Omega}_{NC}(\theta, Q_f^4) + \frac{d\sigma''}{d\Omega}_{NC}(\theta, |Q_f|^3, sign(Q_f), K_{Z\gamma\gamma}, K_{ZZ\gamma})$$

- contribution of s-channel diagrams ($K_i \neq (0,0)$): cancellation or enhancement of noncommutative effects, depending on the scattered particles

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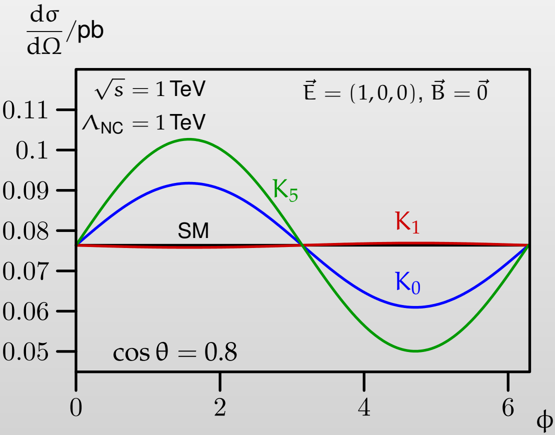
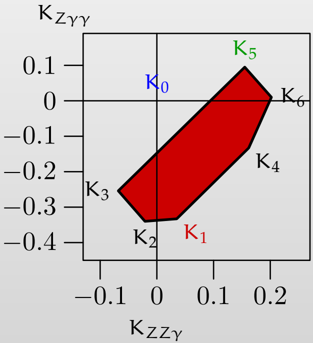
- dependency on the coupling constants K_i:

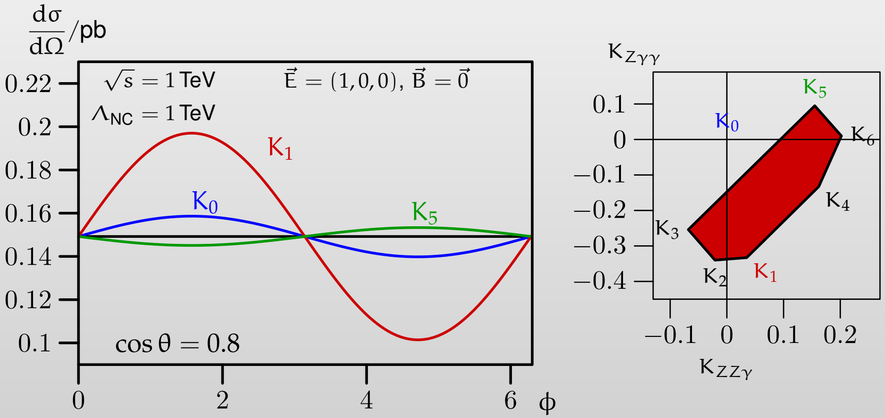




- p(uud)p(uud) scattering (LHC) partially sensitive on noncommutative effects

$e^+e^- \to Z\gamma$

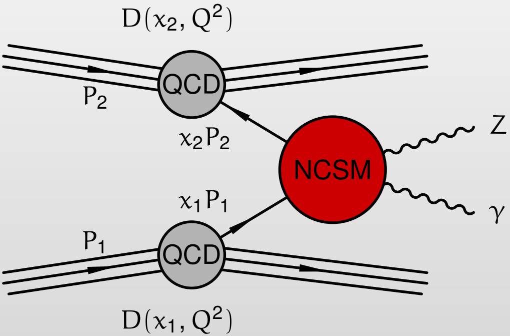
- dependency on the coupling constants K_i:



- sensitivity of e^+e^- scattering (ILC) complementary to LHC

Experiment in the near future: LHC

Convolution with Parton Distribution Functions:



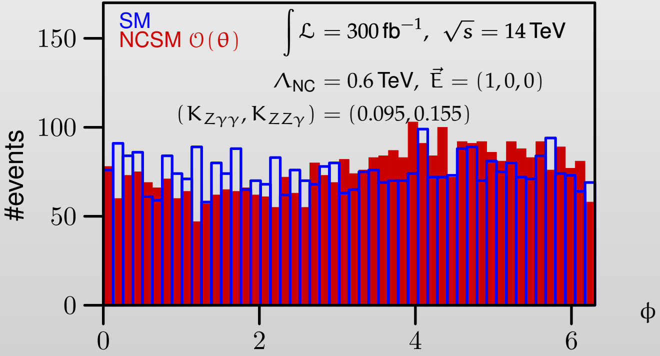
CMS(quarks) $\neq$ CMS(protons) = laboratory frame

- Significative boost along beam axis
- Complicated kinematics

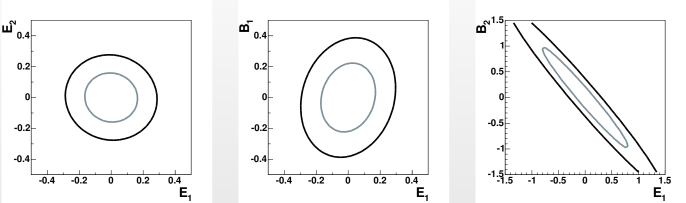
$$pp \to Z\gamma \to e^+e^-\gamma \text{ @ } \mathcal{O}(\theta)$$

- Discriminative effect: azimuthal dependence of cross sections
- Effect $\propto \sqrt{s^*}/\Lambda_{NC} \Rightarrow m_{e^+e^-\gamma} > 220$ GeV
- Validity of $\mathcal{O}(\theta)$ approximation $\Rightarrow m_{e^+e^-\gamma} < 1$ TeV
- Effect antisymmetric in $\cos\theta^*_\gamma \Rightarrow 0 < \cos\theta^*_\gamma < 0.9$
- Separation of $q\bar{q}$ from $\bar{q}q$ initial state: $\langle x_{\text{valence}}\rangle \gg \langle x_{\text{sea}}\rangle$
  $\Rightarrow Z$ and γ in the same hemisphere: $\cos\theta_{Z,\gamma} > 0$

event generation using WHIZARD



Likelihood-Fits: e^+e^- channel, $\sqrt{s} = 14\,\text{TeV}$, $100\,\text{fb}^{-1}$, $\Lambda_{NC} = 500\,\text{GeV}$



- just kinematical correlations of (E_1, B_2) and (E_2, B_1) due to Lorentz-boost along beam axis x_3

$$\theta^{\mu\nu} = \frac{C^{\mu\nu}}{\Lambda_{NC}^2} : \quad C^{\mu\nu} \rightarrow \Lambda^{\mu}{}_{\rho} \Lambda^{\nu}{}_{\sigma} C^{\rho\sigma}$$

$$\begin{aligned} E_1 &\rightarrow \gamma(E_1 - \beta B_2) & \quad B_1 &\rightarrow \gamma(B_1 + \beta E_2) \\ E_2 &\rightarrow \gamma(E_2 + \beta B_1) & \quad B_2 &\rightarrow \gamma(B_2 - \beta E_1) \\ E_3 &\rightarrow E_3 & \quad B_3 &\rightarrow B_3 \end{aligned}$$

with $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$

- measurements of (E_1, B_2) and of (E_2, B_1) correlated by $\langle\beta\rangle$

- Parton CMS: dependency on $\vec{E}$ much stronger than on $\vec{B}$, (except for $\cos\theta_\gamma^* = 0$)
- Proton CMS: $\vec{E}$ and $\vec{B}$ mixed by Lorentz-boost

$$\Downarrow$$

$\vec{E} \neq 0$ and $\vec{B} = 0$: boost-induced $\vec{B}$ component negligible
$\vec{B} \neq 0$ and $\vec{E} = 0$: boost-induced $\vec{E}$ component major contribution

$$\Downarrow$$

$$(0 < \cos\theta_\gamma^* < 0.9 \wedge -0.9 < \cos\theta_\gamma^* < 0)$$

effective measurement only in $\vec{E}$ direction

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effective measurement only in $\vec{E}$ direction

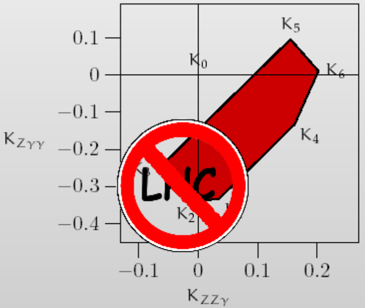
- NC effect antisymmetric in $\cos\theta^*_\gamma$ for $\vec{E} \neq 0$
- NC effect symmetric in $\cos\theta^*_\gamma$ for $\vec{B} \neq 0$

$\Downarrow$

$(-0.9 < \cos\theta^*_\gamma < 0.9)$
cancellation of $\vec{E}$-type noncommutativity,
pure measurement of $\vec{B}$ type noncommutativity

LHC sensitivity on Λ_{NC}:

$(K_{Z\gamma\gamma}, K_{ZZ\gamma})$	$\lvert\vec{E}\rvert^2 = 1, \vec{B} = 0$	$\vec{E} = 0, \lvert\vec{B}\rvert^2 = 1$
$K_0 \equiv (0,0)$ (mNCSM)	$\Lambda_{NC} \gtrsim 1$ TeV	-
$K_1 \equiv (-0.333, 0.035)$(nmNCSM)	-	-
$K_5 \equiv (0.095, 0.155)$(nmNCSM)	$\Lambda_{NC} \gtrsim 1.2$ TeV	-



- LHC not sensitive on $\vec{B}$ directions
- LHC not sensitive for certain values of the triple gauge boson couplings

Future experiment: ILC

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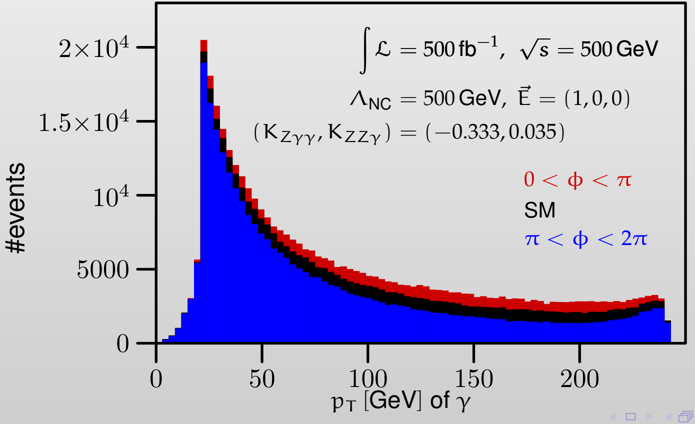
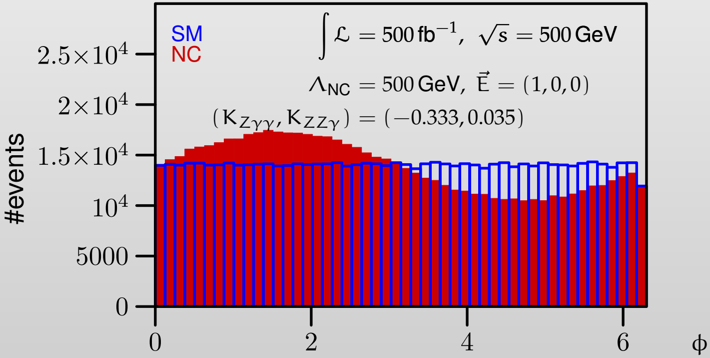
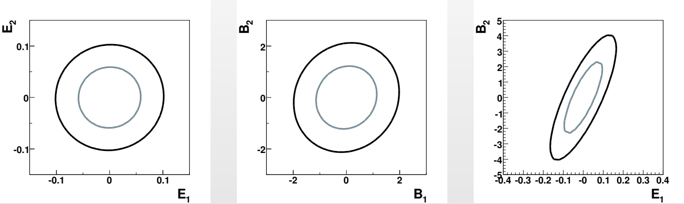


$$e^+e^- \to Z\gamma \ @ \ \mathcal{O}(\theta)$$

- Discriminative effect: azimuthal dependence of cross sections
- Simple initial state $\Rightarrow$ simpler cuts
- Only beam-strahlung $\Rightarrow$ negligible boost
- $\vec{E} \neq 0$: antisymmetry in $\cos\theta_\gamma \Rightarrow 0 < \cos\theta_\gamma < 0.9$
- $\vec{B} \neq 0$: symmetry in $\cos\theta_\gamma \Rightarrow -0.9 < \cos\theta_\gamma < 0.9$



Likelihood-Fits for $e^+e^- \to Z\gamma$: $\sqrt{s} = 500\,\text{GeV}$, $500\,\text{fb}^{-1}$, $\Lambda_{NC} = 500\,\text{GeV}$



ILC sensitivity on Λ_{NC}:

$(K_{Z\gamma\gamma}, K_{ZZ\gamma})$	$\lvert\vec{E}\rvert^2 = 1, \vec{B} = 0$	$\vec{E} = 0, \lvert\vec{B}\rvert^2 = 1$
$K_0 \equiv (0, 0)$ (mNCSM)	$\Lambda_{NC} \gtrsim 2$ TeV	$\Lambda_{NC} \gtrsim 0.4$ TeV
$K_1 \equiv (-0.333, 0.035)$ (nmNCSM)	$\Lambda_{NC} \gtrsim 5.9$ TeV	$\Lambda_{NC} \gtrsim 0.9$ TeV
$K_5 \equiv (0.095, 0.155)$ (nmNCSM)	$\Lambda_{NC} \gtrsim 2.6$ TeV	$\Lambda_{NC} \gtrsim 0.25$ TeV
$K_3 \equiv (-0.254, -0.048)$(nmNCSM)	$\Lambda_{NC} \gtrsim 5.4$ TeV	$\Lambda_{NC} \gtrsim 0.9$ TeV

Another advantage of the clean initial state at ILC:

- fixed initial scattering energy $\Rightarrow$ control over $s/\Lambda_{\mathrm{NC}}^2$
- at LHC: regions in phase space with large $\sqrt{\hat{s}}/\Lambda_{\mathrm{NC}}$ $\Rightarrow$ negative cross section
- $\Rightarrow$second order in θ needed

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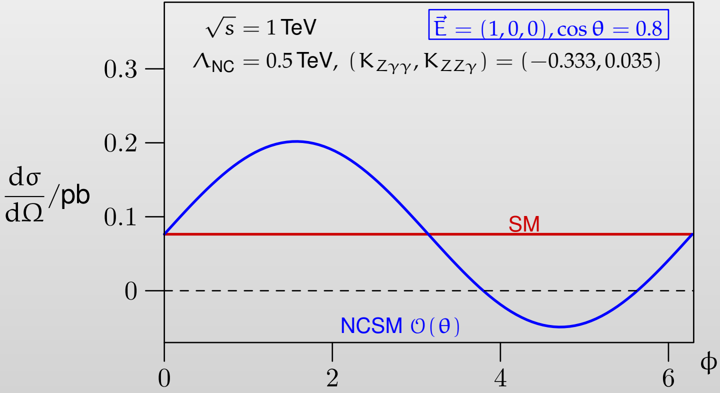

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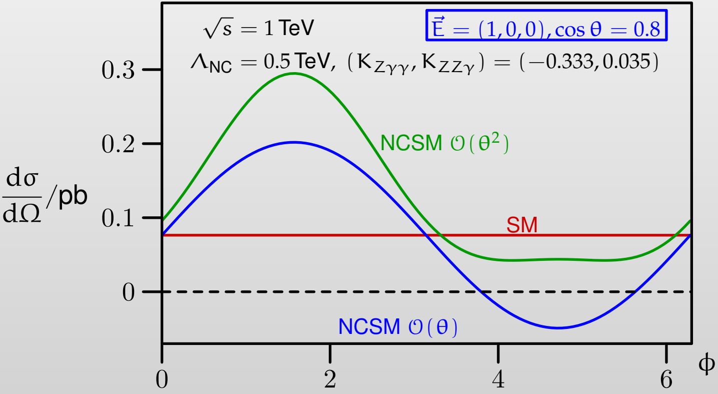

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QFT on NC space-time realised by means of Moyal-Weyl $\star$ product and Seiberg-Witten maps
$\Rightarrow$ NCSM as an effective theory, here up to $\mathcal{O}(\theta)$

- LHC: sensitive only on time-like noncommutativity and **certain** values of $(K_{Z\gamma\gamma}, K_{ZZ\gamma})$
- LHC: $\Lambda_{NC} \gtrsim 1.2\,\text{TeV}$

- ILC: complementary to LHC
- ILC: sensitivity on time- and space-like noncommutativity and **all** values of $(K_{Z\gamma\gamma}, K_{ZZ\gamma})$
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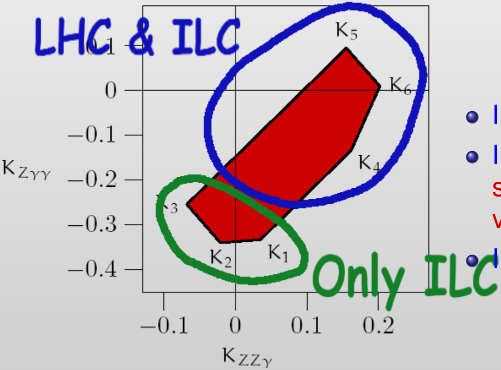

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