Tools for NNLO QCD calculations

Thomas Gehrmann

Universität Zürich

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QCD

Precision physics with QCD

- precise determination of
  - strong coupling constant
  - quark masses
  - electroweak parameters
- precise predictions for
  - new physics effects
  - and their backgrounds

Precision QCD Observables at ILC

Standard model parameters from

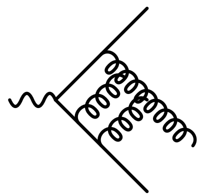
- three-jet production and event shapes: α_s
- forward-backward asymmetry of heavy quarks: $\sin^2\Theta_W$
- top quark pair pair production in the continuum: top quark properties

All precision QCD observables contain detailed final state information (jet clustering, top quark reconstruction) $\longrightarrow$ exclusive observables (jet cross sections)

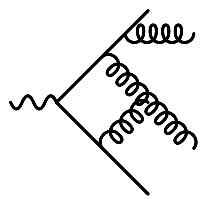
Jets in Perturbation Theory

Ingredients to NNLO m-jet:

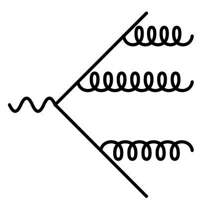
- Two-loop matrix elements

$|\mathcal{M}|^2_{\text{2-loop},m \text{ partons}}$  explicit infrared poles from loop integrals

- One-loop matrix elements

$|\mathcal{M}|^2_{\text{1-loop},m+1 \text{ partons}}$  explicit infrared poles from loop integral and implicit infrared poles due to single unresolved radiation

- Tree level matrix elements

$|\mathcal{M}|^2_{\text{tree},m+2 \text{ partons}}$  implicit infrared poles due to double unresolved radiation

Infrared Poles cancel in the sum

NNLO calculations

Infrared poles

- infrared poles appear in all contributions
- can not add contributions before integration
- must compute each individual divergent contribution
  (typically in dimensional regularisation)
- must separate poles and finite terms, Laurent expansion in regulator $\epsilon = (4-d)/2$

Possible approaches: loop integrals

- Analytical computation
- Numerical computation of all Laurent coefficients
  (sector decomposition, contour deformation, Mellin-Barnes)

Possible approaches: phase space integrals

- Numerical computation of all Laurent coefficients (sector decomposition)
- Analytical extraction of infrared poles (subtraction method),
  numerical computation of finite remainder

NNLO techniques

Sector decomposition

K. Hepp; M. Roth, A. Denner; T. Binoth, G. Heinrich;
C. Anastasiou, K. Melnikov, F. Petriello

- start from parameter representation
- disentangle overlapping singularities by partial fractioning
- expand regulators in distributions
- decompose integration regions into sectors containing only single type of singularity
- compute Laurent coefficients of sector integrals numerically
- Applications
  - virtual two-loop and three-loop four-point functions
    T. Binoth, G. Heinrich
  - NNLO corrections to $pp \to (H/V) + X$, μ-decay
    C. Anastasiou, K. Melnikov, F. Petriello
  - sparticle mass effects in SUSY Higgs production
    C. Anastasiou, S. Beerli, A. Daleo

NNLO techniques

Reduction to Master Integrals

- analytically reduce large number of different loop integrals to few master integrals
- Integration-by-parts identities (IBP)
  K. Chetyrkin, F. Tkachov

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{\partial}{\partial a^\mu} \left[b^\mu f(k,l,p_i) \right] = 0$$

with: $a^\mu = k^\mu, l^\mu$ and $b^\mu = k^\mu, l^\mu, p_i^\mu$

- Lorentz invariance identites (LI)
  E. Remiddi, TG

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \delta\varepsilon^\mu_\nu \left(\sum_i p_i^\nu \frac{\partial}{\partial p_i^\mu} \right) f(k,l,p_i) = 0$$

- automated solution (S. Laporta)
- also possible for phase space integrals (C. Anastasiou, K. Melnikov)

NNLO techniques

Mellin-Barnes integration

V. Smirnov, J.B. Tausk

- disentangle loop propagators using Mellin-Barnes representation
- perform analytical continuation in all integration variables to allow $\epsilon \longrightarrow 0$
  MB-package (M. Czakon)
- perform Mellin-Barnes integrals analytically or numerically
  M. Czakon; C. Anastasiou, A. Daleo
- Applications
  - massless two-loop four-point functions: $q\bar{q} \longrightarrow q\bar{q}$
    V. Smirnov, J.B. Tausk
  - expansion of massive two-loop four-point functions: $e^+e^- \longrightarrow e^+e^-$
    S. Actis, M. Czakon, J. Gluza, T. Riemann

Massive from massless amplitudes

- exploit universal infrared structure to construct high energy limit of massive amplitudes, up to m^2/s; Application: $q\bar{q} \rightarrow Q\bar{Q}$
  M. Czakon, A. Mitov, S. Moch

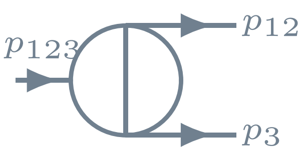
NNLO techniques

Differential equations

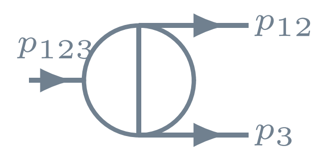
A. Kotikov; E. Remiddi, TG

- Master integrals fulfil inhomogeneous differential equations in external invariants
- For example:

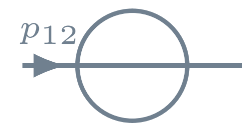
$$s_{123} \frac{\partial}{\partial s_{123}}$$



$$= + \frac{d-4}{2} \frac{2 s_{123} - s_{12}}{s_{123} - s_{12}}$$



$$- \frac{3d-8}{2} \frac{1}{s_{123} - s_{12}}$$



- boundary conditions are simpler integrals
- Applications:
  - master integrals for $\gamma^* \to q\bar{q}g$
    E. Remiddi, TG
  - master integrals for $\gamma^* \to Q\bar{Q}$
    R. Bonciani, P. Mastrolia, E. Remiddi
  - some master integrals for $e^+e^- \to e^+e^-$
    R. Bonciani, A. Ferroglia, P. Mastrolia, E. Remiddi, J. van der Bij
    M. Czakon, J. Gluza, T. Riemann

Virtual Corrections at NNLO

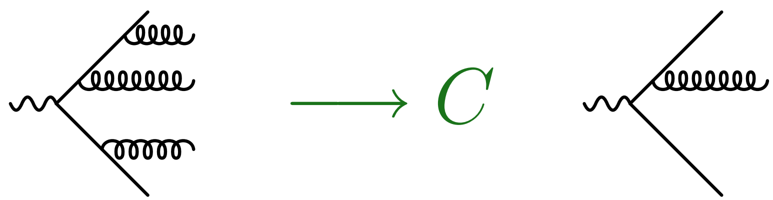
Virtual two-loop matrix elements are available for:

- Bhabha-Scattering: $e^+e^- \to e^+e^-$
  Z. Bern, L. Dixon, A. Ghinculov
  R. Bonciani, A. Ferroglia, P. Mastrolia, E. Remiddi, J. van der Bij
  S. Actis, M. Czakon, J. Gluza, T. Riemann
- Hadron-Hadron 2-Jet production: $qq' \to qq'$, $q\bar{q} \to q\bar{q}$, $q\bar{q} \to gg$, $gg \to gg$
  C. Anastasiou, N. Glover, C. Oleari, M. Yeomans-Tejeda
  Z. Bern, A. De Freitas, L. Dixon
- Photon pair production at LHC: $gg \to \gamma\gamma$, $q\bar{q} \to \gamma\gamma$
  Z. Bern, A. De Freitas, L. Dixon
  C. Anastasiou, N. Glover, M. Yeomans-Tejeda
- Three-jet production: $e^+e^- \to \gamma^* \to q\bar{q}g$
  L. Garland, N. Glover, A.Koukoutsakis, E. Remiddi, TG
  S. Moch, P. Uwer, S. Weinzierl
- DIS (2+1) jet production: $\gamma^* g \to q\bar{q}$, Hadronic (V+1) jet production: $qg \to Vq$
  E. Remiddi, TG
- Matrix elements with internal masses: $\gamma^* \to Q\bar{Q}$
  W.Bernreuther, R.Bonciani, R.Heinesch, T.Leineweber, P.Mastrolia, E.Remiddi, TG

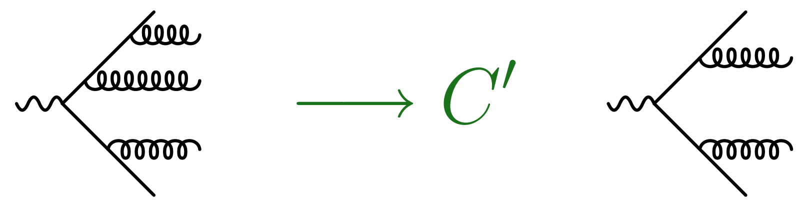
Real Corrections at NNLO

Infrared subtraction terms

$m+2$ partons $\to m$ jets:



$m+2 \to m+1$ pseudopartons $\to m$ jets:



- Double unresolved configurations:
  - triple collinear
  - double single collinear
  - soft/collinear
  - double soft
- Single unresolved configurations:
  - collinear
  - soft

J. Campbell, E.W.N. Glover; S. Catani, M. Grazzini

Issue: find subtraction functions which

- approximate full $m+2$ matrix element in all singular limits
- are sufficiently simple to be integrated analytically

NLO Subtraction

Structure of NLO m-jet cross section (subtraction formalism):

Z. Kunszt, D. Soper

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO}\right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^S_{NLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^V_{NLO}\right]$$

- $\mathrm{d}\sigma^S_{NLO}$: local counter term for $\mathrm{d}\sigma^R_{NLO}$
- $\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO}$: free of divergences, can be integrated numerically

General methods at NLO

- Dipole subtraction
  S. Catani, M. Seymour; NNLO: S. Weinzierl
- $\mathcal{E}$-prescription
  S. Frixione, Z. Kunszt, A. Signer;
  NNLO: S. Frixione, M. Grazzini; G. Somogyi, Z. Trocsanyi, V. Del Duca
- Antenna subtraction
  D. Kosower; J. Campbell, M. Cullen, N. Glover; A. Daleo, D. Maitre, TG
  NNLO: A. Gehrmann-De Ridder, E.W.N. Glover, TG

NNLO Infrared Subtraction

Structure of NNLO m-jet cross section:

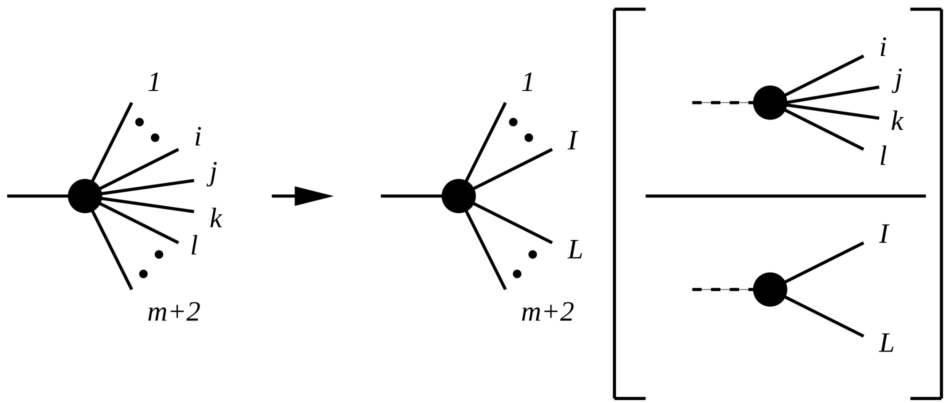
$$\begin{aligned}
\mathrm{d}\sigma_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} \,,
\end{aligned}$$

- $\mathrm{d}\sigma^{S}_{NNLO}$: real radiation subtraction term for $\mathrm{d}\sigma^{R}_{NNLO}$
- $\mathrm{d}\sigma^{VS,1}_{NNLO}$: one-loop virtual subtraction term for $\mathrm{d}\sigma^{V,1}_{NNLO}$
- $\mathrm{d}\sigma^{V,2}_{NNLO}$: two-loop virtual corrections

Each line above is finite numerically and free of infrared ϵ-poles $\longrightarrow$ numerical programme

Antenna Subtraction: Double Real

Two colour-connected unresolved partons



$$X^0_{ijkl} = S_{ijkl,IL} \frac{|M^0_{ijkl}|^2}{|M^0_{IL}|^2}$$

$$\mathrm{d}\Phi_{X_{ijkl}} = \frac{\mathrm{d}\Phi_4}{P_2}$$

Phase space factorisation

$$\mathrm{d}\Phi_{m+2}(p_1,\ldots,p_{m+2};q) = \mathrm{d}\Phi_m(p_1,\ldots,\tilde{p}_I,\tilde{p}_L,\ldots,p_{m+2};q)\cdot\mathrm{d}\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l;\tilde{p}_I+\tilde{p}_L)$$

Integrated subtraction term (analytically)

$$|\mathcal{M}_m|^2\, J_m^{(m)}\, \mathrm{d}\Phi_m \int \mathrm{d}\Phi_{X_{ijkl}}\, X^0_{ijkl} \sim |\mathcal{M}_m|^2\, J_m^{(m)}\, \mathrm{d}\Phi_m \int \mathrm{d}\Phi_4 |M^0_{ijkl}|^2$$

Four-particle inclusive phase space integrals are known

A. Gehrmann-De Ridder, G. Heinrich, TG

Antenna Subtraction: Real/Virtual

Single unresolved limit of one-loop amplitudes

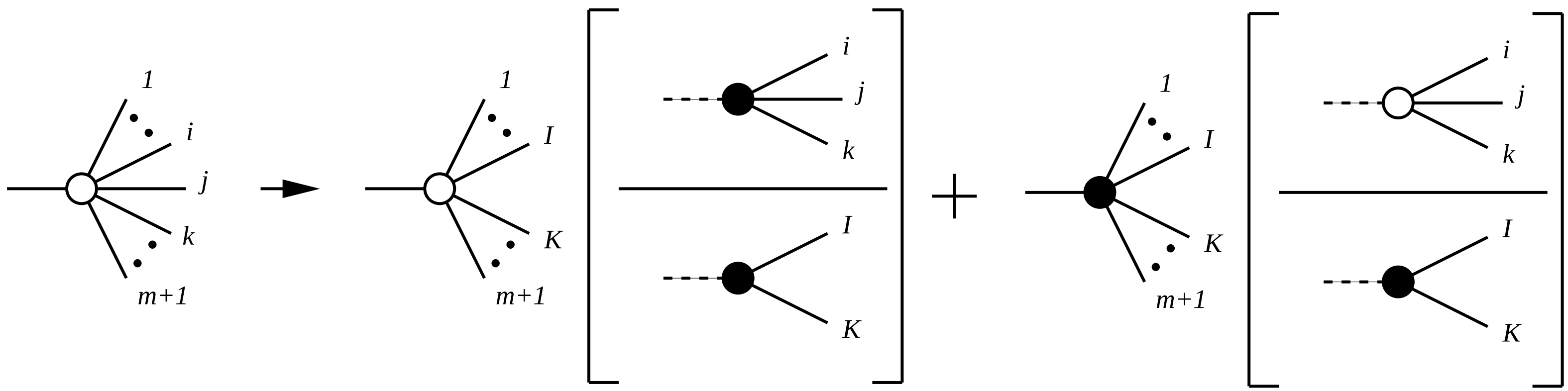
$$Loop_{m+1} \stackrel{j\ unresolved}{\longrightarrow} Split_{tree} \times Loop_m + Split_{loop} \times Tree_m$$

Z. Bern, L.D. Dixon, D. Dunbar, D. Kosower; S. Catani, M. Grazzini; D. Kosower, P. Uwer

Z. Bern, V. Del Duca, W.B. Kilgore, C.R. Schmidt

Z. Bern, L.D. Dixon, D. Kosower; S. Badger, E.W.N. Glover

Accordingly: $Split_{tree} \to X^0_{ijk}$, $Split_{loop} \to X^1_{ijk}$



$$X^1_{ijk} = S_{ijk,IK} \frac{|\mathcal{M}^1_{ijk}|^2}{|\mathcal{M}^0_{IK}|^2} - X^0_{ijk} \frac{|\mathcal{M}^1_{IK}|^2}{|\mathcal{M}^0_{IK}|^2}$$

Antenna Subtraction

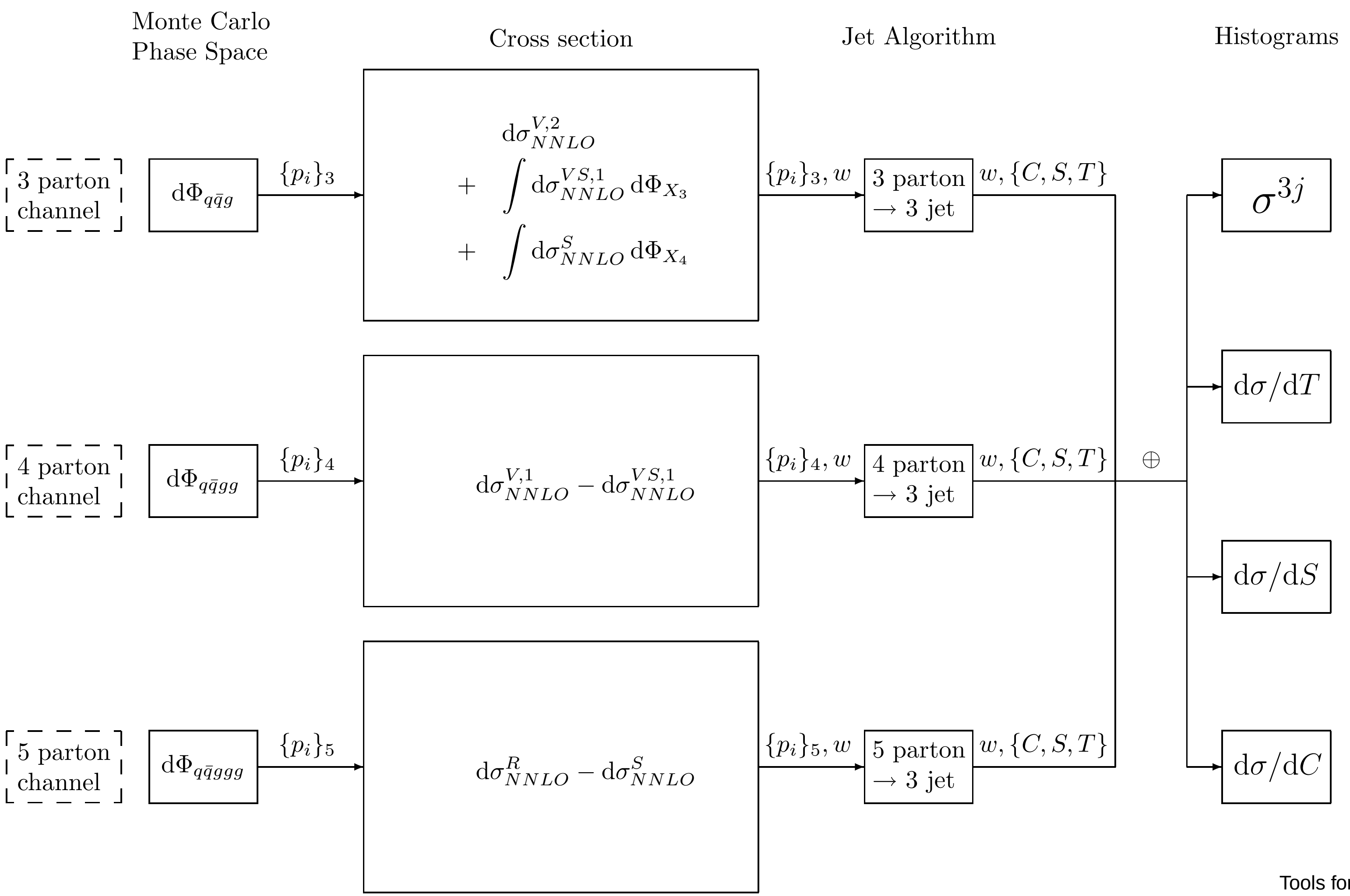
Antenna Functions

- colour-ordered pair of hard partons (radiators) with radiation in between
  - hard quark-antiquark pair
  - hard quark-gluon pair
  - hard gluon-gluon pair
- three-parton antenna $\longrightarrow$ one unresolved parton
- four-parton antenna $\longrightarrow$ two unresolved partons
- can be at tree level or at one loop
- all three-parton and four-parton antenna functions can be derived from physical matrix elements, normalised to two-parton matrix elements
  - $q\bar{q}$ from $\gamma^* \to q\bar{q} + X$
  - qg from $\tilde{\chi} \to \tilde{g}g + X$
  - gg from $H \to gg + X$
- recent results: $e^+e^- \to 3j$, $e^+e^- \to Q\bar{Q}$ (ongoing)

Numerical Implementation

Structure of $e^+e^- \to 3$ jets program:

A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG



Numerical Implementation

Parton-level event generator

Starting point $e^+e^- \to 4$ jets at NLO (EERAD2: J. Campbell, M. Cullen, N. Glover)

- contains already 4-parton and 5-parton matrix elements
- is based on NLO antenna subtraction

modified phase space generation: matrix element

- decompose phase space into wedges, according to relative size of invariants
- each wedge contributes only to some unresolved regions
- angular correlations cancel out (at least to large part) by combining several wedges

modified phase space generation: antenna subtraction terms

- uniform mapping of antenna phase space (D. Kosower)
- requires ordering of unresolved emissions

checks

- independence on phase space cut y_0
- local cancellations along phase space trajectories
- distributions in raw phase space variables

Summary and Conclusions

- Interpretation of precision data often requires NNLO corrections
- wide spectrum of new techniques
- analytical approaches to loop and phase space integrals
  - reduction to master integrals
  - Mellin-Barnes integration
  - differential equations
  - mass expansions, massive/massless relations
- numerical approaches to loop and phase space integrals
  - sector decomposition
  - Mellin-Barnes integration
- implementation into parton-level event generator
  - allows computation of exclusive observables
  - requires subtraction method, e.g. antenna subtraction
- NNLO exclusive results:
  $pp \to H + X$, $pp \to V + X$, $e^+e^- \to 3j$, more in progress