W-pair production near threshold with unstable particle effective theory

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Introduction

W **mass measurement** at ILC from in GigaZ option:

threshold scan of $\sigma(e^+e^- \to 4f)$ between 160GeV $< \sqrt{s} < 170$GeV

(G.Wilson 2001)

Precision $m_W \lesssim 6$ MeV requires accuracy $\lesssim 0.6\%$ of theoretical prediction in threshold region $\sqrt{s} \sim 160 - 170$ GeV

Unstable particles near resonance: $k^2 - M^2 \sim M\Gamma \sim M^2\alpha$

systematic treatment of finite width effects important

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systematic treatment of finite width effects important

Effective theory approach to W-pair production at threshold:

Systematic expansion in α and Γ/M

(Beneke, Chapovsky, Signer, Zanderighi 03, Beneke, Kauer, Signer, Zanderighi 04)

NLO calculation of $e^+e^- \to \mu^-\bar{\nu}_\mu u\bar{d}$ (Beneke, Falgari,C.S., Signer, Zanderighi)

- comparison to NLO $e^+e^- \to 4f$ calculation (Denner et.al. 05)
- ambiguities in implementation of ISR

Methods for unstable particles

Resummation of self-energy insertions in propagator

$$\frac{i}{p^2 - M^2 + \Pi(p^2)}$$

Mixes different orders in perturbation theory

Pole scheme: (Stuart 91; Aeppli,v. Oldenbourgh, Wyler 93)

expand around complex pole of propagator $\bar{s} = M + iM\Gamma$

$$\mathcal{A}(s)|_{s\sim M^2} = \frac{\mathcal{R}(\bar{s})}{s - \bar{s}} + \mathcal{N}(s)$$

- Applied to W-pair production at NLO in the continuum

  (Berends et. al. 98; Denner et.al. 99)

- supposed to break down near threshold

Full NLO calculation $e^+e^- \to 4$ fermions

(Denner, Dittmaier, Roth ,Wieders 05)

- complex mass scheme for finite width effects: $M_W \to \bar{s}$
- need to treat six-point one-loop diagrams

Effective theory approach

Construct simultaneous expansion in

$$\alpha \sim \frac{\Gamma}{M} \sim \frac{k^2 - M^2}{M^2} \equiv \delta$$

(Chapovsky, Khoze, Signer,Stirling 01; Beneke, Chapovsky, Signer, Zanderighi 03/04)

(Similar approach to top pair production (Hoang, Reißer 04; Hoang, Mantry, Stewart 07))

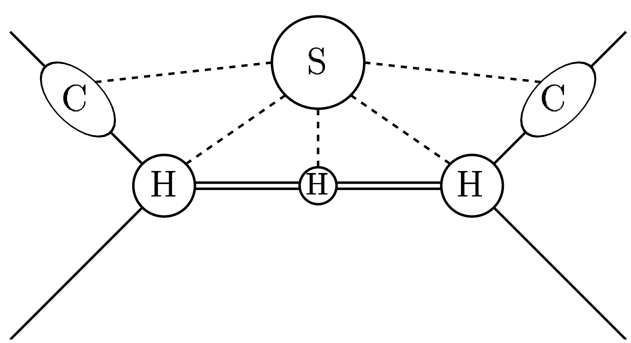
- Seperate scales δ, M:
  - hard fluctuations $k^\mu \sim M$, $k^2 \sim M^2$ are integrated out
  - $\Rightarrow$ Effective Lagrangian $\mathcal{L}_{\text{eff}}$ for modes with $k^2 - M^2 \sim M^2\delta$ ("soft", "potential")
- Equivalent view:
  Expand amplitude in δ by method of regions (Beneke, Smirnov 97)
- Treatment of self-energy
  - hard region: $k^2 \gg M\Gamma \Rightarrow$ treat Π insertions perturbatively
  - potential region: Π has to be resummed $\Rightarrow$ include in $\mathcal{L}_{\text{eff}}$

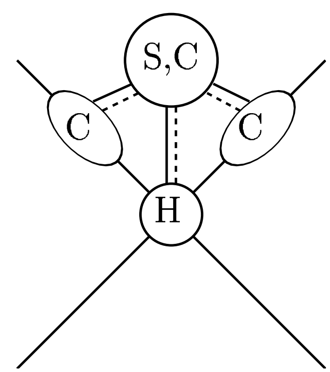
Effective theory approach

Example: $f_1\bar{f}_2 \to \Phi \to f_3\bar{f}_4$ in effective theory approach:

$$i\mathcal{A}(s)|_{s\sim M_\Phi^2} = \int d^4x \, \langle f_1\bar{f}_2|\,\mathrm{T}\left[i\mathcal{O}_{f_1\bar{f}_2\Phi}(0) i\mathcal{O}^\dagger_{f_3\bar{f}_4\Phi}(x)\right]|f_3\bar{f}_4\rangle + \langle f_1\bar{f}_2|i\mathcal{O}_{4f}(0)|f_3\bar{f}_4\rangle$$

- $\mathcal{O}_{f\bar{f}\Phi}$: Production/Decay operators matched at $\bar{s}$, contain hard fluctuations: "Factorizable corrections"
- Calculate matrix elements with effective lagrangian $\mathcal{L}_{\text{eff}}$ for soft, potential,... modes: "Non-Factorizable corrections"
- $\mathcal{O}_{4f}$: four-fermion operator: "Nonresonant contributions"





Effective theory for W-pair production

Goal: total cross section for

$$e^- e^+ \to \mu^- \bar{\nu}_\mu \, u \, \bar{d}$$

for $s \sim 4M_W^2$ to NLO in

$$\delta = \frac{s - 4M_W^2}{M_W^2} \sim v^2 \sim \frac{\Gamma_W}{M_W} \sim \alpha_{ew} \sim \alpha_s^2$$

Compute total cross section from cuts of $\mathcal{A}(e^- e^+ \to e^- e^+)$

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Modes of the effective theory: (Beneke, Kauer, Signer, Zanderighi 04)

- non-relativistic W: $k_0 \sim M_W \delta$, $|\vec{k}| \sim M_W \sqrt{\delta}$ — potential (p)
- External fermions: $k \sim M_W$, $k^2 \sim 0$ — collinear (c)
- Photons: $\begin{cases} k_0 \sim |\vec{k}| \sim M_W \delta & \text{soft (s)} \\ \text{potential} & (\Rightarrow \text{Coulomb correction}) \end{cases}$

Hard modes: non-resonant fluctuations of W, Z, H, t, ...

Effective theory for W-pair production

Momentum of non-relativistic Ws:

$$k^\mu = M_W\, v^\mu + r^\mu\,, \qquad r^0 \sim \delta,\, \vec{r} \sim \sqrt{\delta}\,, \qquad v^\mu \equiv (1, \vec{0})$$

Expansion of resummed propagator:

$$\frac{i}{k^2 - M_W^2 - \Pi_T^W(k^2)} \Rightarrow \frac{i R_W}{2M_W(r^0 - \frac{\vec{r}^{\,2}}{2M_W} + \frac{\Delta}{2})} + \ldots$$

Matching coefficient:

$$\Delta \equiv \frac{\bar{s} - M_W^2}{M_W} = -i\Gamma_W \qquad \text{(pole scheme)}$$

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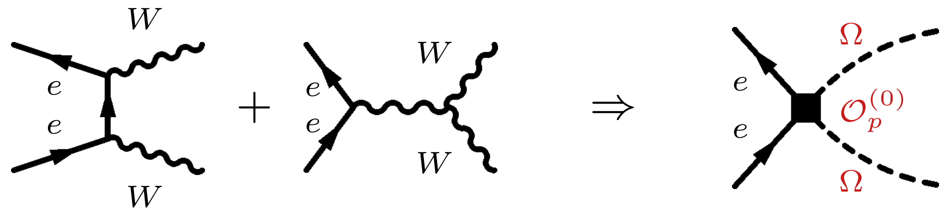
Effective Lagrangian: NRQED $\qquad \left(\Omega_\pm \equiv \sqrt{2M_W} W_\pm \right)$

$$\mathcal{L}_{\text{NRQED}} = \sum_\mp \left[\Omega_\mp^{\dagger i} \left(iD^0 + \frac{\vec{D}^2}{2M_W} - \frac{\Delta}{2} \right) \Omega_\mp^i + \Omega_\mp^{\dagger i} \frac{(\vec{D}^2 - M_W \Delta)^2}{8\hat{M}_W^3} \Omega_\mp^i \right]$$

(Only physical polarizations part of effective theory, modes with $M \sim \sqrt{\xi} M_W$ "hard")

Leading order amplitude

Matching of LO production operator $\mathcal{O}_p^0$:



$$\mathcal{O}_p^{(0)} = \frac{\pi\alpha_{ew}}{M_W^2}\left(\bar{e}_{c2,L}\left(\gamma^i n^j + \gamma^j n^i\right) e_{c1,L}\right)\left(\Omega_-^{\dagger i}\Omega_+^{\dagger j}\right)$$

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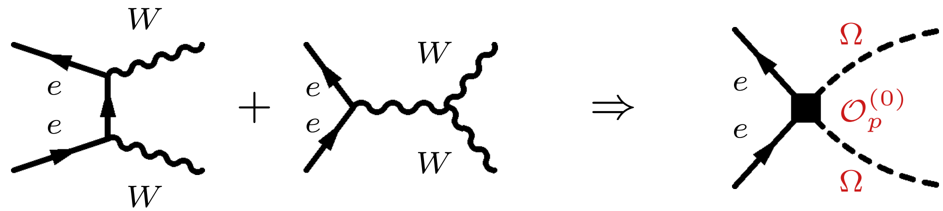

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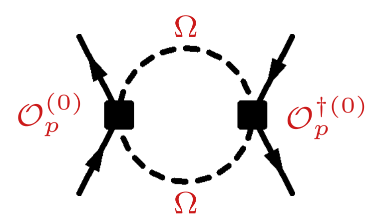
Leading order forward scattering amplitude $e^-e^+ \to e^-e^+$:

$$i\mathcal{A}_p^{(0)} = \int d^4x \,\langle e^+e^-|\,\mathrm{T}[i\mathcal{O}_p^{(0)\dagger}(0) i\mathcal{O}_p^{(0)}(x)]|e^+e^-\rangle$$

$=$  $= -i\pi\alpha_{ew}^2\sqrt{-\frac{(\sqrt{s}-2M_W+i\Gamma_W)}{M_W}}$

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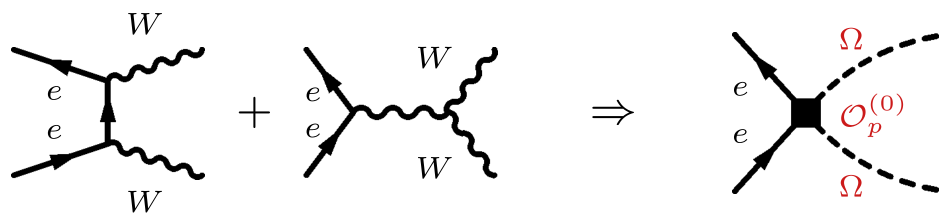

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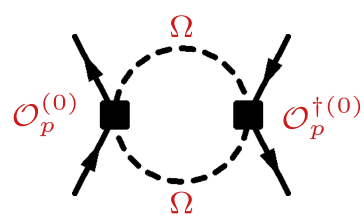
$=$  $= -i\pi\alpha_{ew}^2 \sqrt{-\frac{(\sqrt{s}-2M_W+i\Gamma_W)}{M_W}}$

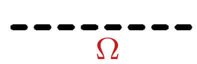
Leading order total cross section:

$$\sigma^{(0)}(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d}) = \frac{\Gamma^{(0)}_{W^- \to \mu^- \bar{\nu}} \Gamma^{(0)}_{W^+ \to u\bar{d}}}{\Gamma_W^{(0)\,2}} \frac{1}{s} \mathrm{Im}\mathcal{A}_p^0$$

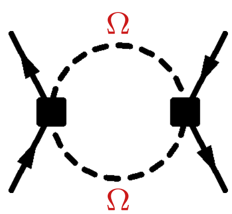
Estimate of contributions up to NLO

Systematic power counting in δ using momentum scaling:

potential: $k_0 \sim M_W\delta,\ |\vec{k}| \sim M_W\sqrt{\delta}$ soft: $k_0 \sim |\vec{k}| \sim M_W\delta$

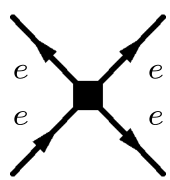


$$\sim \left(r^0 - \frac{\vec{r}^2}{2M_W} + \frac{\Delta}{2}\right)^{-1} \qquad \sim\delta^{-1}$$



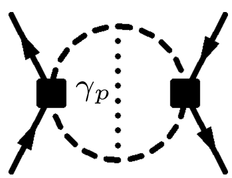
$$\sim \int \underbrace{d^4k}_{\sim\ \delta^{5/2}}\ \delta^{-2} \qquad \sim\delta^{1/2}$$

LO



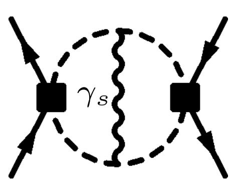
$$\sim\alpha \sim \delta$$

"N$^{1/2}$LO"



$$\sim \alpha \int \underbrace{d^4k d^4q}_{\delta^5}\ \underbrace{\frac{1}{|\vec{q}|^2}}_{\delta^{-1}}\ \delta^{-4} \qquad \sim\alpha \sim \delta$$

N$^{1/2}$LO



$$\sim \alpha \int \underbrace{d^4k}_{\delta^{5/2}}\ \underbrace{d^4q}_{\delta^4}\ \underbrace{\frac{1}{q^2}}_{\delta^{-2}}\ \delta^{-4} \qquad \sim\alpha\delta^{1/2} \sim \delta^{3/2}$$

NLO

NLO expansion of the born cross section

Matching calculations:

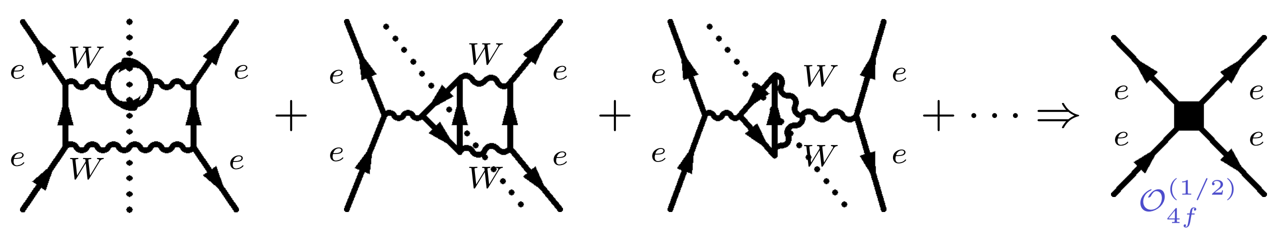
Production operators $\mathcal{O}_p^{(1/2)}$ and $\mathcal{O}_p^{(1)}$: (Beneke, Kauer, Signer, Zanderighi 04)

Corrections $\sim v$ and v^2 e.g.

$$\mathcal{O}_p^{(1/2,a)} \sim \left(\bar{e}_L \gamma^j e_L\right)\left(\Omega_-^{*i}(-i)D^j\Omega_+^{*i}\right) + 3 \text{ more}$$

Corrections to residue of W-propagator: $\mathcal{O}_p^{(0)} R_W$

Four-electron operators at N$^{1/2}$LO: ($e^-e^+ \to W^- u\bar{d}$ and $e^-e^+ \to W^+\mu^-\bar{\nu}_\mu$)



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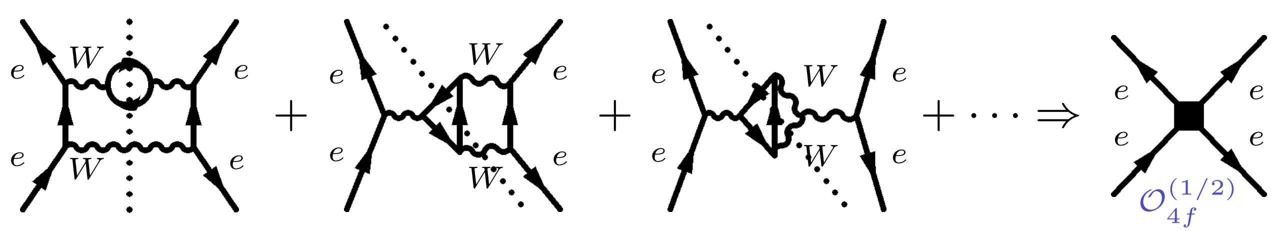
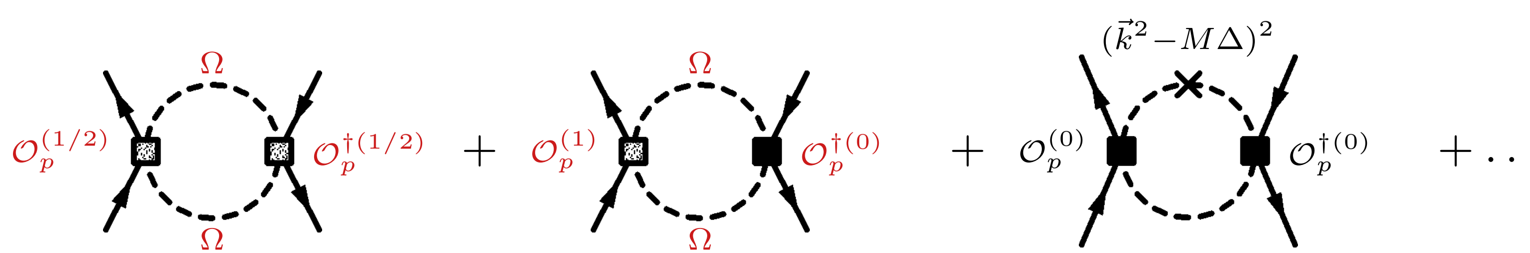


Calculation in the EFT:

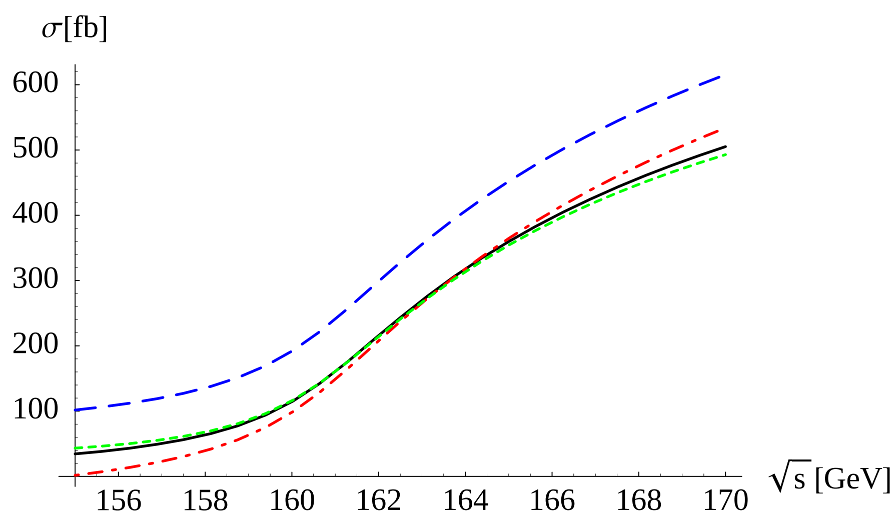


Equivalently: Expand $\mathcal{A}(e^- e^+ \to W^+ W^- \to e^- e^+)$ in potential region.

Convergence of δ-expansion?

⇒ compare EFT tree cross section to `Whizard` (Kilian 01)

⇒ Good convergence near threshold



	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d})$(fb)				
$\sqrt{s}$[GeV]	LO	$\sqrt{\text{NLO}}$	NLO	$N^{3/2}$LO	Whizard
155	101.61	1.62	43.28	31.30	34.43(1)
161	240.85	148.44	160.45	160.89	160.62(6)
170	615.5	533.9	492.9	505.4	505.1(2)

- NLO not sufficient for $\sqrt{s} = 155 - 170$ GeV

⇒ Partial inclusion of $N^{3/2}$LO corrections (matrix element corrections)

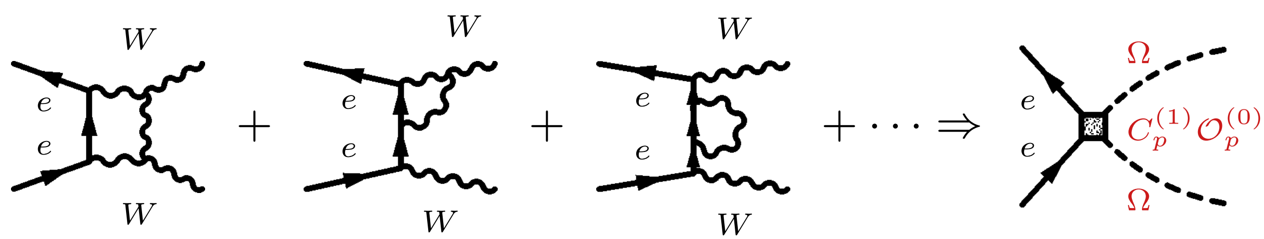
⇒ Convolution with ISR: match to full theory below $\sqrt{s} = 155$ GeV or use full theory every-where

Matching calculations

Corrections to $\Delta = -i\Gamma_W$ to order $\alpha_{ew}\alpha_s$ ($N^{1/2}$LO) α_{ew}^2 and $\alpha_{ew}\alpha_s^2$ (NLO).

$$\Gamma_W^{(1/2)} = \frac{2\alpha_s}{3\pi}\Gamma_W^{(0)}, \; \Gamma_W^{(1)} = \left[\Gamma_W^{(1,ew)} + 1.409\,\frac{2\alpha_s^2}{3\pi^2}\Gamma_W^{(0)}\right]$$

Renormalization of production operator:



$$C_p^{(1)} = \frac{\alpha}{2\pi}\left[\left(-\frac{1}{\varepsilon^2} - \frac{3}{2\varepsilon}\right)\left(-\frac{4M_W^2}{\mu^2}\right)^{-\varepsilon} + C_p^{(1\ \mathrm{fin})}\right]$$

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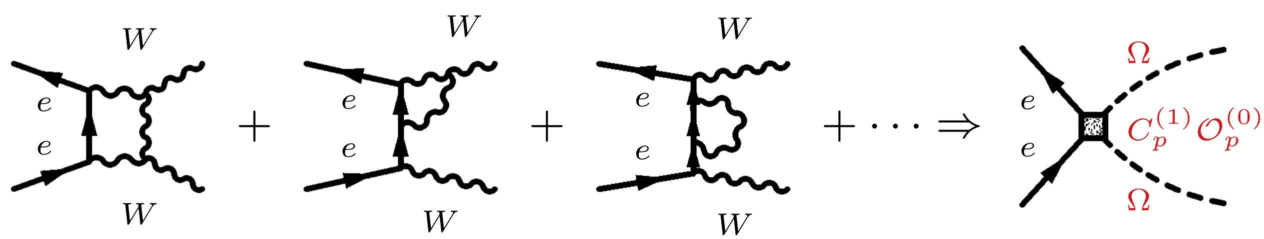

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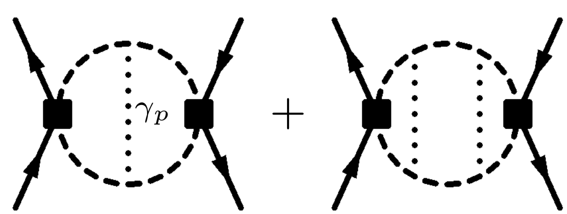
Loops corrections in the EFT

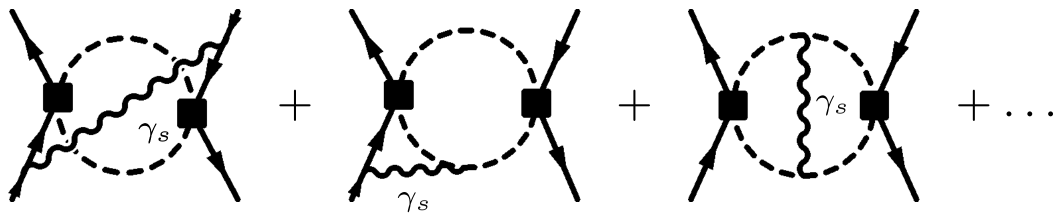
First and second Coulomb correction



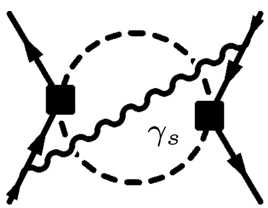
$$= \frac{\pi\alpha^2}{s_w^4 s}\,\mathrm{Im}\left[-\frac{\alpha}{27}\log\sqrt{-\frac{E+i\Gamma_W^{(0)}}{M_W}} + \frac{\pi^2\alpha^2}{324}\sqrt{-\frac{M_W}{E+i\Gamma_W^{(0)}}}\right]$$

NLO EFT calculation of $e^+e^- \to \mu^-\bar{\nu}_\mu u\bar{d}$

Soft photon corrections



For σ_{tot} only ISR nonzero: (Melnikov, Yakovlev; Fadin, Martin, Khoze; 93)



$$= \frac{4\pi^2\alpha_{ew}^2}{M_W^2}\frac{\alpha}{\pi}\int\frac{d^dr}{(2\pi)^d}\frac{1}{\eta_-\eta_+}\left[\left(\frac{1}{\varepsilon^2}+\frac{5}{12}\pi^2\right)\left(-\frac{4\eta_-^2}{\mu^2}\right)^{-\varepsilon}\right]$$

($\eta_- = r_0 - \frac{|\vec{r}|^2}{2M_W} + i\frac{\Gamma^{(0)}}{2}$ and $\eta_+ = E - r_0 - \frac{|\vec{r}|^2}{2M_W} + i\frac{\Gamma^{(0)}}{2}$)

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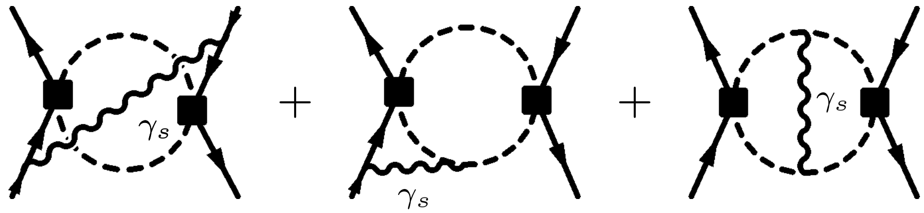

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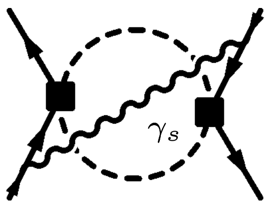

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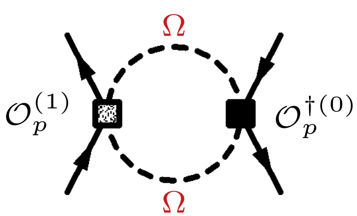
Insertion of one loop production vertex:



$$= \frac{4\pi^2\alpha_{ew}^2}{M_W^2}\frac{\alpha}{\pi}\int\frac{d^dr}{(2\pi)^d}\frac{1}{\eta_-\eta_+}\left[\left(-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}\right)\left(-\frac{4M_W^2}{\mu^2}\right)^{-\varepsilon}+C_p^{(1\ \text{fin})}\right]$$

ϵ^{-2} poles cancel; collinear initial state singularity $-\frac{1}{\epsilon}\left(2\log\left(\frac{\eta_-}{M_W}\right)+\frac{3}{2}\right)$

Results

Treatment of collinear initial state singularity

Factorize into electron PDF? (DimReg for IR singularities, $\overline{MS}$)

not standard in EW calculations

$\Rightarrow$ Introduce m_e: additional momentum regions:

hard-collinear: $k^\mu \sim M_W\,,\; k^2 \sim m_e^2$ soft-collinear: $k^\mu \sim \Gamma_W\,,\; k^2 \sim m_e^2 \frac{\Gamma_W}{M_W}$

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Fixed order NLO result for cross section:

$$\sigma_{\text{NLO}} = \left(1 + \delta_{W\to\mu\bar\nu_\mu} + \delta_{W\to u\bar d}\right)\sigma_{\text{tree}} + \Delta\sigma_{\text{Coulomb}} + \frac{1}{27s}\,\text{Im}\left\{\mathcal{A}^{(0)}\delta_{\text{NLO}}\right\}$$

$$\delta_{\text{NLO}} = \frac{\alpha}{\pi}\left[\left(4\log\left(-\frac{4(\sqrt{s}-2M_W+i\Gamma_W)}{M_W}\right) - 5\right)\log\left(\frac{2M_W}{m_e}\right) + C_p^{(1\ \text{fin})} - \frac{\pi^2}{4} + 3\right]$$

Treatment of collinear initial state singularity

Factorize into electron PDF? (DimReg for IR singularities, $\overline{MS}$)

not standard in EW calculations

$\Rightarrow$ Introduce m_e: additional momentum regions:

hard-collinear: $k^\mu \sim M_W\,,\; k^2 \sim m_e^2$ soft-collinear: $k^\mu \sim \Gamma_W\,,\; k^2 \sim m_e^2 \frac{\Gamma_W}{M_W}$

Fixed order NLO result for cross section:

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Comparison to $e^+e^- \to 4f$ calculation (Denner, Dittmaier, Roth ,Wieders 05)

$\sqrt{s}$ [GeV]	LO(Whizard)	EFT(α)	ee4f [DDRW]	DPA [DDRW]
161	150.05(6)fb	104.97(6)	105.71(7)	103.15(7)
170	481.2(2)fb	373.24(2)	377.1(2)	376.9(2)

($M_W = 80.425$ GeV, $\alpha(0)$ in rad.corr.; EFT(α): Whizard tree, 2nd Coulomb + QCD switched off)

Results

Resumm $\log\left(\frac{2M_W}{m_e}\right)$ in electron structure function

$$\sigma(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{ee}(x_1)\Gamma_{ee}(x_2)\sigma_{\mathsf{tree}}(x_1 x_2 s) + \Delta\widehat{\sigma}_{\mathsf{NLO}}(s)$$

To avoid double-counting: $\left(\beta_e = \frac{2\alpha}{\pi}\left(2\log\left(\frac{2M_W}{m_e}\right) - 1\right)\right)$

$$\Delta\widehat{\sigma}_{\mathsf{NLO}}(s) = \Delta\sigma_{\mathsf{NLO}}(s) - 2\frac{\beta_e}{4}\int_0^1 dx P(x)\sigma^{(0)}(xs)$$

$$\Rightarrow \widehat{\delta}_{\mathsf{NLO}} = \frac{\alpha}{\pi}\left[2\log\left(-\frac{4(\sqrt{s}-2M_W+i\Gamma_W)}{M_W}\right) + C_p^{(1\ \mathrm{fin})} - \frac{\pi^2}{4} + \frac{1}{2}\right]$$

Resumm $\log\left(\frac{2M_W}{m_e}\right)$ **in electron structure function**

$$\sigma(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{ee}(x_1)\Gamma_{ee}(x_2)\sigma_{\text{tree}}(x_1 x_2 s) + \Delta\widehat{\sigma}_{\text{NLO}}(s)$$

To avoid double-counting: $\left(\beta_e = \frac{2\alpha}{\pi}\left(2\log\left(\frac{2M_W}{m_e}\right) - 1\right)\right)$

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$$\Rightarrow \widehat{\delta}_{\text{NLO}} = \frac{\alpha}{\pi}\left[2\log\left(-\frac{4(\sqrt{s}-2M_W+i\Gamma_W)}{M_W}\right) + C_p^{(1\ \text{fin})} - \frac{\pi^2}{4} + \frac{1}{2}\right]$$

Comparison to $e^+e^- \to 4f$ calculation (Denner, Dittmaier, Roth ,Wieders 05)

$\sqrt{s}$ [GeV]	LO+ISR(Whizard)	EFT(α, α_s)	EFT(C2)	ee4f [DDRW]	DPA [DDRW]
161	107.10(5)fb	117.41(5)	117.79(5)	118.12(8)	115.48(7)
170	381.0(2)fb	399.9(2)	400.1(2)	401.8(2)	402.1(2)

($M_W = 80.425$ GeV, $\alpha(0)$ in rad.corr.;EFT: tree+ISR from Whizard, C2: 2nd Coulomb)

Ambiguities in higher order ISR

ISR1: convolute tree with structure function

ISR2: convolute σ_{NLO}:

$$\sigma_{\text{ISR2}}(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{ee}(x_1)\Gamma_{ee}(x_2)\hat{\sigma}_{\text{NLO}}(x_1 x_2 s)$$

$\sqrt{s}$ [GeV]	LO(Whizard)	EFT(ISR1)	EFT(ISR2)	$\delta_{\text{ISR1-ISR2}}$
161	154.19(6)fb	119.93(5)	117.73(5)	1.9%
164	303.3(1)fb	237.0(1)	235.1(1)	0.8%
170	481.9(2)fb	398.3(2)	397.9(2)	0.1%

($m_W = 80.377$ GeV, α_{G_F} scheme, α_s^2 included)

Ambiguities in higher order ISR

ISR1: convolute tree with structure function

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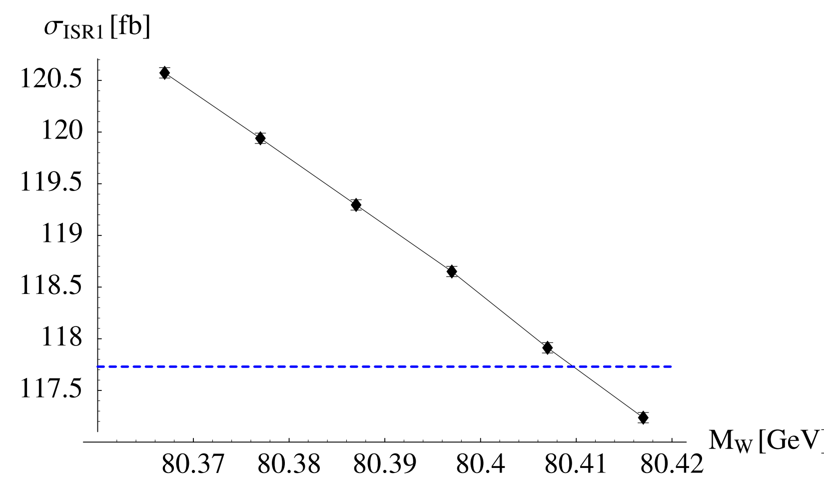
($m_W = 80.377$ GeV, α_{G_F} scheme, α_s^2 included)

Different treatments:

$\sim 2\%$ difference at threshold.

First estimate:

~ 30 MeV uncertainty in M_W



Summary

W **mass measurement at ILC:** with error $\lesssim 6$ MeV needs
$\sigma(e^+e^- \to 4f)$ at W pair threshold to accuracy $\lesssim 0.6\%$

Effective theory approach to unstable particle production:

- Systematic expansion in α and Γ/M
- Simpler than full NLO $e^+e^- \to 4f$ calculation

NLO results for W-pair production near threshold

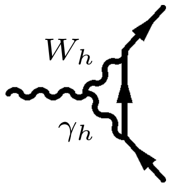
- Calculation of total cross section $e^+e^- \to \mu^-\bar{\nu}_\mu u\bar{d}$ completed
- Agreement $\sim 0.6\%$ at threshold with full $e^+e^- \to 4f$ calculation in complex mass scheme (Denner et.al. 05)
- Large $\sim 2\%$ uncerainties from ambiguities in ISR

Outlook

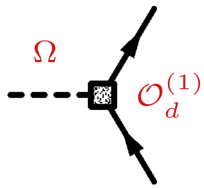
- Resummation of $\log\left(\frac{\Gamma_W}{M_W}\right)$ terms
- Differential cross sections

Example for split into regions

Hard region: $q^2 \sim M_W^2$: don't resum Π, no further approximations:



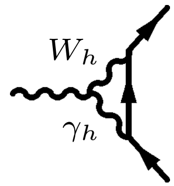
$$\sim \int \frac{d^D q}{(2\pi)^D} \frac{1}{((k+q)^2 - M_W^2)(q-p_f)^2 q^2}$$

$\Rightarrow$ 

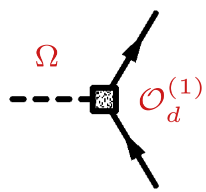
$$\sim \left[-\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \log\left(\frac{m_W^2}{\mu^2}\right) - \frac{1}{2} \log^2\left(\frac{m_W^2}{\mu^2}\right) - 1 - \frac{\pi^2}{12} \right]$$

Example for split into regions

Hard region: $q^2 \sim M_W^2$: don't resum Π, no further approximations:

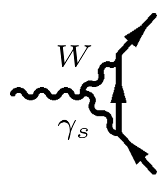


$$\sim \int \frac{d^D q}{(2\pi)^D} \frac{1}{((k+q)^2 - M_W^2)(q-p_f)^2 q^2}$$

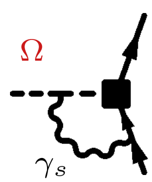
$\Rightarrow$ 

$$\sim \left[-\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\log\left(\frac{m_W^2}{\mu^2}\right) - \frac{1}{2}\log^2\left(\frac{m_W^2}{\mu^2}\right) - 1 - \frac{\pi^2}{12} \right]$$

Soft region: $q \sim M_W \delta$: resum Π, approximate: $\left(\eta_W = r^0 - \frac{\vec{r}^2}{2M_W} + \mathrm{i}\frac{\Delta}{2} \right)$



$$\sim \int \frac{d^D q}{(2\pi)^D} \frac{1}{((k+q)^2 - M_W^2 + \mathrm{i}\Pi_W(p^2))(p_f - q)^2 q^2}$$

$\Rightarrow$ 

$$\sim \int \frac{d^D q}{(2\pi)^D} \frac{1}{2M_W(r^0 - \frac{\vec{r}^2}{2M_W} + \mathrm{i}\frac{\Delta}{2} + q^0)(-2q\cdot p_f)q^2}$$

$$\sim \left[\frac{1}{\epsilon^2} - \frac{2}{\epsilon}\log\left(-\frac{2\eta_W}{\mu}\right) + 2\log^2\left(-\frac{2\eta_W}{\mu}\right) + \frac{5\pi^2}{12} \right]$$