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Two-loop electroweak NLL corrections: from massless to massive fermions

Bernd Jantzen

Paul Scherrer Institut (PSI), CH-Villigen

In collaboration with **A. Denner** and **S. Pozzorini**

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Overview

I Electroweak corrections at high energies

II 2-loop next-to-leading logarithmic (NLL) corrections

- extraction of mass-singular logs at 1 and 2 loops
- factorizable and non-factorizable contributions
- treatment of UV singularities

III Results for massless fermionic processes

- calculation of loop integrals
- factorization & exponentiation
- comparison to existing results & applications

IV From massless to massive fermions

- fermion mass effects in 1-loop results
- new complications from fermion masses at 2 loops
- structure of the corrections
- preliminary results: the Abelian form factor

V Summary

I Electroweak (EW) corrections at high energies

EW collider experiments

- today (LEP, Tevatron): energy scales $\lesssim M_{\mathrm{W,Z}}$
- upcoming colliders (LHC, ILC) $\longrightarrow$ explore TeV regime
  $\hookrightarrow$ new energy domain $\sqrt{s} \gg M_{\mathrm{W}}$ becomes accessible!

EW radiative corrections at high energies $\sqrt{s} \gg M_{\mathrm{W}}$

$\Rightarrow$ enhanced by large Sudakov logarithms

$$\ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \sim 25 \quad \text{at } \sqrt{s} \sim 1\,\text{TeV}$$

Logs present in exclusive observables with only virtual W and Z bosons (this project), but also in inclusive observables due to Bloch–Nordsieck violations

Ciafaloni, Ciafaloni, Comelli '00,'01; Ciafaloni, Comelli '05

General form of EW corrections for $s \gg M_\mathrm{W}^2$

$$\left[L = \ln\left(\frac{s}{M_\mathrm{W}^2}\right)\right]$$

1 loop: $\alpha\left[C_1^{\mathsf{LL}}\,L^2 + C_1^{\mathsf{NLL}}\,L + C_1^{\mathsf{N^2LL}}\right] + \mathcal{O}\left(\frac{M_\mathrm{W}^2}{s}\right)$

$$\downarrow \quad -17\,\% \qquad \downarrow \quad +12\,\% \qquad \downarrow \quad -3\,\%$$

2 loops: $\alpha^2\left[C_2^{\mathsf{LL}}\,L^4 + C_2^{\mathsf{NLL}}\,L^3 + C_2^{\mathsf{N^2LL}}\,L^2 + C_2^{\mathsf{N^3LL}}\,L + C_2^{\mathsf{N^4LL}}\right] + \mathcal{O}\left(\frac{M_\mathrm{W}^2}{s}\right)$

$$\downarrow \quad +1.7\,\% \qquad \downarrow \quad -1.8\,\% \qquad \downarrow \quad +1.2\,\% \qquad \downarrow \quad -0.3\,\%$$

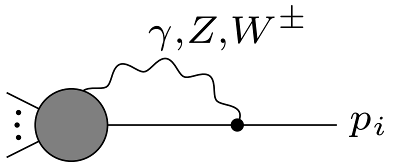
[percentages for $\sigma(u\bar{u} \to d\bar{d})$ at $\sqrt{s} = 1\,\mathrm{TeV}$
B.J., Kühn, Penin, Smirnov '05]

Theoretical prediction with accuracy $\sim 1\,\%$ required

$\Rightarrow$ 2-loop corrections important

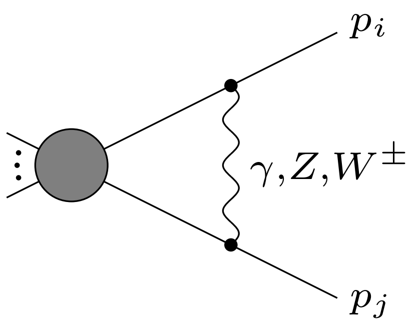
$\Rightarrow$ 2-loop LL approximation not sufficient

Origin of logarithms $\ln(s/M_\mathrm{W}^2)$ in virtual corrections



mass singularities from virtual gauge bosons $(\gamma, Z, W^\pm)$
coupling to on-shell external leg

$\longrightarrow$ single logs from collinear region



special case:
gauge bosons exchanged between **2** on-shell external legs
$\longrightarrow$ double logs from soft–collinear region

For massless photons: $\log \rightsquigarrow \dfrac{1}{\epsilon}$ in $D = 4 - 2\epsilon$ dimensions

- count $1/\epsilon$ poles like logs for logarithmic approximations (LL, NLL, ...)
- fermion masses regularize collinear singularities $\rightarrow \ln(m_\text{top}^2)$

EW LLs & NLLs are universal, at least at 1 loop for arbitrary — Denner, Pozzorini '00, '01
and at 2 loops for massless fermionic processes — Denner, B.J., Pozzorini '06
$\hookrightarrow$ depend only on quantum numbers of external particles

Approaches for virtual 2-loop EW corrections at high energies

Resummation of 1-loop result:

- LL for arbitrary processes — Fadin, Lipatov, Martin, Melles '99
- NLL for arbitrary processes ($M_Z = M_W$) — Melles '00, '01
- N^{2}LL for massless $f\bar{f} \to f'\bar{f}'$ ($M_Z = M_W$) — Kühn, Penin, Smirnov '99, '00; Kühn, Moch, Penin, Smirnov '01

$\rightarrow$ apply evolution equations to spontaneously broken SU(2)×U(1) EW model

$\hookrightarrow$ rely on splitting of EW theory into symmetric SU(2)×U(1) and QED regime

Diagrammatic 2-loop calculations to check & extend resummation predictions:

- LL for fermionic form factor — Melles '00; Hori, Kawamura, Kodaira '00
- LL for arbitrary processes — Beenakker, Werthenbach '00, '01
- angular-dependent NLLs for arbitrary processes — Denner, Melles, Pozzorini '03
- complete NLL for massless fermionic form factor ($M_Z \neq M_W$) — Pozzorini '04
- N^{3}LL for massless fermionic form factor ($M_Z = M_W$)

  $\hookrightarrow$ N^{3}LL for massless $f\bar{f} \to f'\bar{f}'$ ($M_Z \approx M_W$) via evolution equations

  B.J., Kühn, Moch '03; B.J., Kühn, Penin, Smirnov '04, '05

II 2-loop next-to-leading logarithmic corrections

Goal: virtual 2-loop EW corrections for arbitrary processes in NLL accuracy

$\hookrightarrow$ further check of evolution equation predictions

$\hookrightarrow$ provide better accuracy for arbitrary $2 \to n$ processes

Implement

- different large kinematical invariants $|(p_i + p_j)^2| \sim Q^2 \gg M_\mathrm{W}^2$
- different heavy particle masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\text{top}^2 \sim M_\text{Higgs}^2$
- massive (top quark) and massless fermions

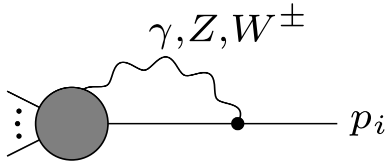
$\Rightarrow$ Logs $L = \ln\left(\dfrac{Q^2}{M_\mathrm{W}^2}\right)$ and $\dfrac{1}{\epsilon}$ poles (from virtual photons)

1 loop: LL $\to \epsilon^{-2},\ L\epsilon^{-1},\ L^2,\ L^3\epsilon,\ L^4\epsilon^2;$ NLL $\to \epsilon^{-1},\ L,\ L^2\epsilon,\ L^3\epsilon^2$

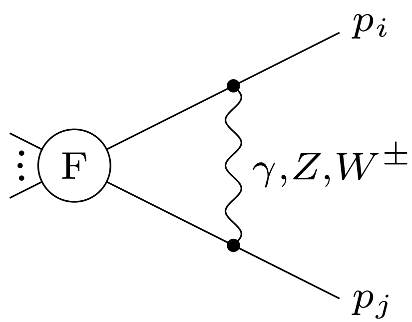
2 loops: LL $\to \epsilon^{-4},\ L\epsilon^{-3},\ L^2\epsilon^{-2},\ L^3\epsilon^{-1},\ L^4;$ NLL $\to \epsilon^{-3},\ L\epsilon^{-2},\ L^2\epsilon^{-1},\ L^3$

$\Rightarrow$ NLL coefficients involve small logs $\ln\left(\dfrac{|(p_i+p_j)^2|}{Q^2}\right)$ and $\ln\left(\dfrac{M_\mathrm{Z}^2, m_\text{top}^2}{M_\mathrm{W}^2}\right)$

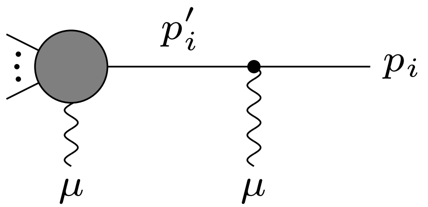
Extraction of NLL mass singularities at 1 loop

Contributions originate from 

in the collinear region

Isolate factorizable contributions: 

- gauge boson momentum set to zero in tree subdiagram F
- soft–collinear approximation for loop vertices combined with propagators:



$i\not{p}_i' \cdot i\gamma^{\mu} \to -2p_i'^{\mu}$ (for massless external fermions),
extension of *eikonal approximation*,
valid for soft and/or collinear gauge boson momenta

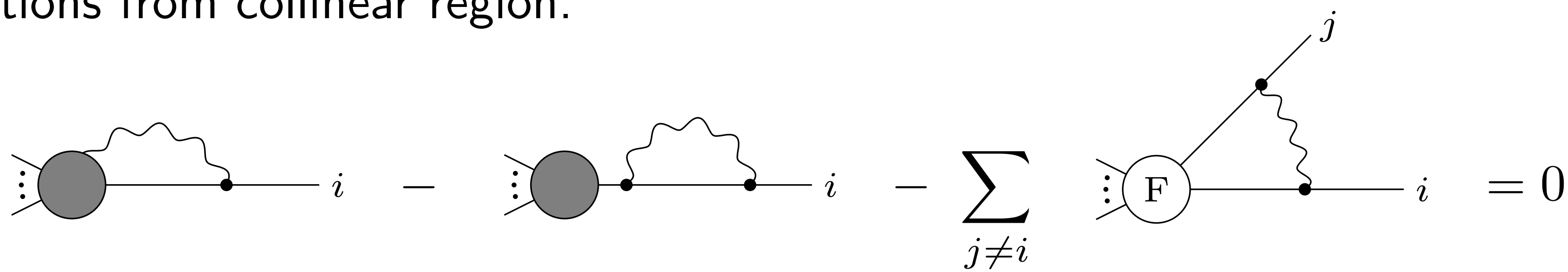
$\hookrightarrow$ eliminates Dirac structure of loop corrections

$\Rightarrow$ loop integrals independent of structure of Born matrix element

The factorizable contributions contain all soft and/or collinear NLL mass singularities.

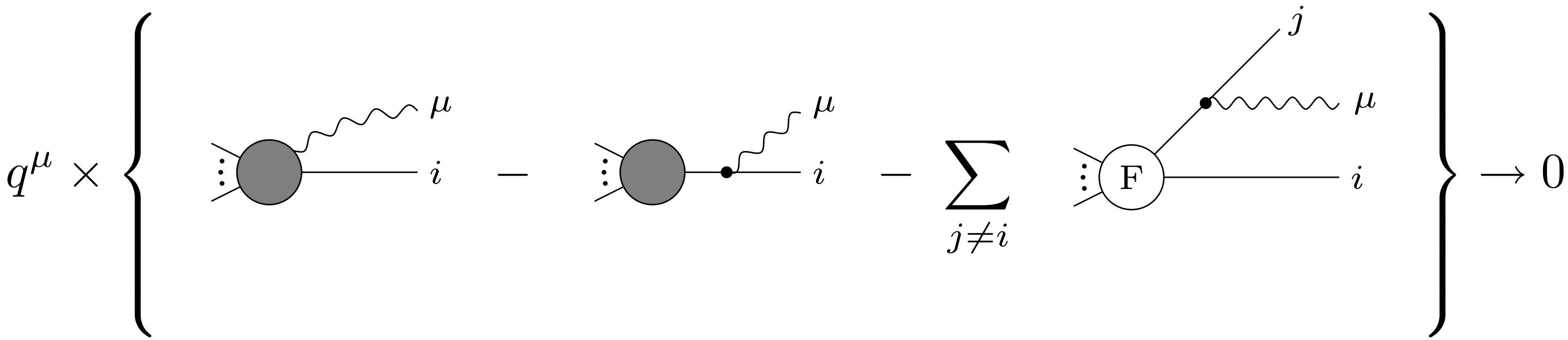
Remaining non-factorizable contributions

Contributions from collinear region:



Cancellation mechanism:

- collinear vertex $\propto$ gauge boson momentum q^μ
- collinear Ward identities for EW theory: Denner, Pozzorini '00, '01

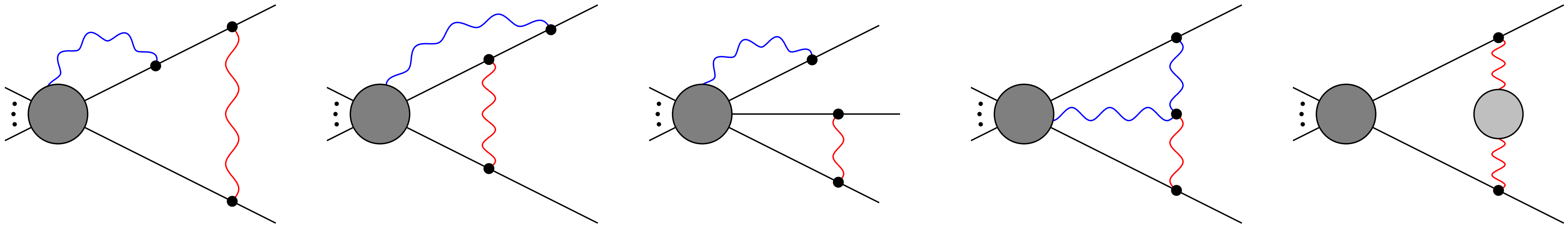


in the collinear limit

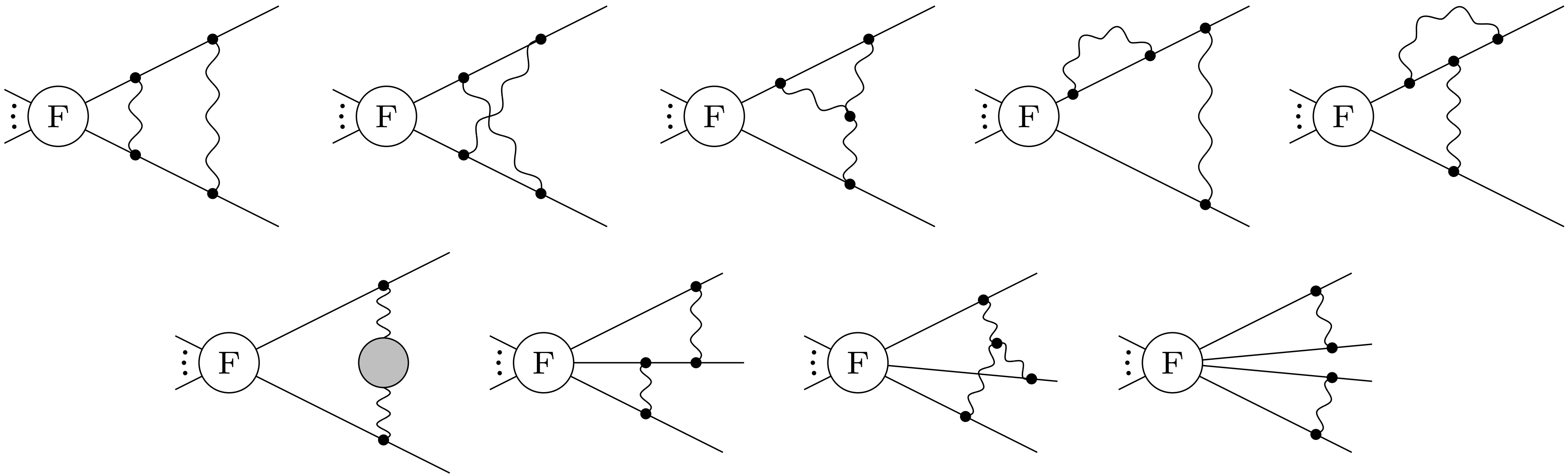
$\Rightarrow$ only calculation of factorizable contributions needed

Extraction of NLL mass singularities at 2 loops

↪ soft×soft and soft×collinear contributions (massless fermions):



Factorizable contributions:



- calculated with soft–collinear approximation and projection techniques
- remaining non-factorizable contributions vanish due to collinear Ward identities

Denner, B.J., Pozzorini '06

Treatment of ultraviolet (UV) singularities & renormalization

UV $1/\epsilon$ poles in subdiagrams with scale μ^2_{loop} and renormalization at scale μ^2_{R} $\Big\}$ $\Rightarrow$ logs $\ln\left(\frac{\mu^2_{\text{R}}}{\mu^2_{\text{loop}}}\right)$ $\Rightarrow$ possibly NLL

UV subtraction: remove $1/\epsilon$ poles from UV-singular subdiagrams and counterterms

Advantages:

- UV NLL contributions from hard subdiagrams completely shifted to counterterms
  $\hookrightarrow$ no need to calculate UV-singular loops in internal parts of tree subdiagrams
- can use soft–collinear approximation (not valid in UV regime!) also for hard subdiagrams which produce UV NLL contributions

Renormalization:

- use couplings in Born matrix elements renormalized at $\mu^2_{\text{R}} = Q^2$
  $\hookrightarrow$ no counterterm contributions from Born amplitude
  $\hookrightarrow$ loop corrections universal & independent of inner structure of Born amplitude
  $\hookrightarrow$ renormalization scale can be changed for specific matrix elements
- renormalization of couplings in soft–collinear loops at arbitrary scale μ^2_{R}

III Results for massless fermionic processes

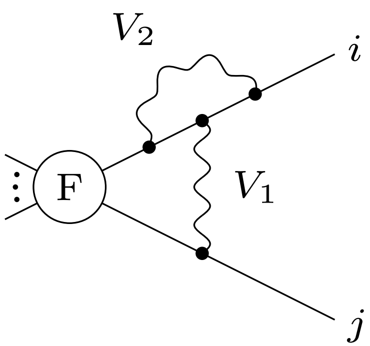
Evaluation of factorizable contributions

Calculate & check loop integrals with 2 independent methods:

- automatized algorithm based on sector decomposition — Denner, Pozzorini '04
- combination of expansion by regions & Mellin–Barnes representations — Smirnov, B.J. '06 & refs. therein

$\hookrightarrow$ extended for different hard scales $(p_i + p_j)^2 \sim Q^2$ and soft scales $M_i^2 \sim M_\mathrm{W}^2$

Example:



$$= \underbrace{\mathcal{M}_0}_{\text{Born amplitude factorized}} \underbrace{\sum_{V_1,V_2=\gamma,Z,W^\pm}}_{\text{sum over various gauge bosons}} \overbrace{I_i^{V_2} I_i^{V_1} I_i^{\bar{V}_2} I_j^{\bar{V}_1}}^{\text{isospin matrices @ external legs}} \underbrace{D(M_{V_1}, M_{V_2}; p_i \cdot p_j)}_{\text{scalar 2-loop integral}}$$

$+$ sum over external legs i, j

NLL result for massless fermionic processes $f_1 f_2 \longrightarrow f_3 \cdots f_n$ up to 2 loops

$$\boxed{\mathcal{M} = \mathcal{M}_0 \, F^{\mathsf{sew}} \, F^{\mathrm{Z}} \, F^{\mathsf{em}}}$$

symmetric-electroweak factor: $$F^{\mathsf{sew}} = \exp\left[\frac{\alpha}{4\pi} F_1^{\mathsf{sew}} + \left(\frac{\alpha}{4\pi}\right)^2 G_2^{\mathsf{sew}}\right]$$

electromagnetic factor: $$F^{\mathsf{em}} = \exp\left[\frac{\alpha}{4\pi} \Delta F_1^{\mathsf{em}} + \left(\frac{\alpha}{4\pi}\right)^2 \Delta G_2^{\mathsf{em}}\right]$$

terms from $M_{\mathrm{Z}} \neq M_{\mathrm{W}}$: $$F^{\mathrm{Z}} = 1 + \frac{\alpha}{4\pi} \Delta F_1^{\mathrm{Z}}$$

- F^{sew} equals result from symmetric SU(2)×U(1) theory with $M_\gamma = M_{\mathrm{Z}} = M_{\mathrm{W}}$
- exponentiation of 1-loop terms F_1^{sew} and ΔF_1^{em} (found from fixed-order calculation!)
- electromagnetic terms in F^{em} factorize and exponentiate separately
  $\hookrightarrow$ separation of photonic singularities possible
- loop correction factors F^{sew}, F^{em} and F^{Z} are universal,
  $\hookrightarrow$ depend only on quantum numbers of external particles

Exponentiated 1-loop terms: LLs & NLLs

$$F_1^{\text{sew}} = -\frac{1}{2}\left(L^2 + \frac{2}{3}L^3\epsilon + \frac{1}{4}L^4\epsilon^2 - 3L - \frac{3}{2}L^2\epsilon - \frac{1}{2}L^3\epsilon^2\right)\sum_{i=1}^{n}\left(\frac{Y_i^2}{4c_{\text{w}}^2} + \frac{C_i}{s_{\text{w}}^2}\right)$$
$$+\left(L + \frac{1}{2}L^2\epsilon + \frac{1}{6}L^3\epsilon^2\right)\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\ln\left(\frac{-(p_i+p_j)^2}{Q^2}\right)\sum_{V=\gamma,Z,W^\pm} I_i^{\bar{V}} I_j^{V}$$

$$\Delta F_1^{\text{em}} = -\frac{1}{2}\left(2\epsilon^{-2} - L^2 - \frac{2}{3}L^3\epsilon - \frac{1}{4}L^4\epsilon^2 + 3\epsilon^{-1} + 3L + \frac{3}{2}L^2\epsilon + \frac{1}{2}L^3\epsilon^2\right)\sum_{i=1}^{n} Q_i^2$$
$$-\left(\epsilon^{-1} + L + \frac{1}{2}L^2\epsilon + \frac{1}{6}L^3\epsilon^2\right)\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\ln\left(\frac{-(p_i+p_j)^2}{Q^2}\right) Q_i Q_j$$

$$\Delta F_1^{\text{Z}} = \left(L + L^2\epsilon + \frac{1}{2}L^3\epsilon^2\right)\ln\left(\frac{M_{\text{Z}}^2}{M_{\text{W}}^2}\right)\sum_{i=1}^{n}\left(\frac{c_{\text{w}}}{s_{\text{w}}}T_i^3 - \frac{s_{\text{w}}}{c_{\text{w}}}\frac{Y_i}{2}\right)^2$$

Additional 2-loop terms with 1-loop β-function coefficients: only NLLs

$$G_2^{\text{sew}} = \frac{1}{6}L^3\sum_{i=1}^{n}\left(b_1^{(1)}\frac{Y_i^2}{4c_{\text{w}}^2} + b_2^{(1)}\frac{C_i}{s_{\text{w}}^2}\right)$$

$$\Delta G_2^{\text{em}} = \left(\frac{3}{4}\epsilon^{-3} + L\epsilon^{-2} + \frac{1}{2}L^2\epsilon^{-1}\right) b_{\text{QED}}^{(1)}\sum_{i=1}^{n} Q_i^2$$

$[\mu_{\text{R}}^2 = M_{\text{W}}^2]$

Comparison to existing results

- previous results for form factor and angular-dependent NLLs reproduced and extended — Denner, Melles, Pozzorini '03; Pozzorini '04
- structure of symmetric-electroweak NLL corrections in complete analogy with Catani's formula for massless QCD — Catani '98
- agreement with general resummation predictions based on evolution equations — Melles '00, '01

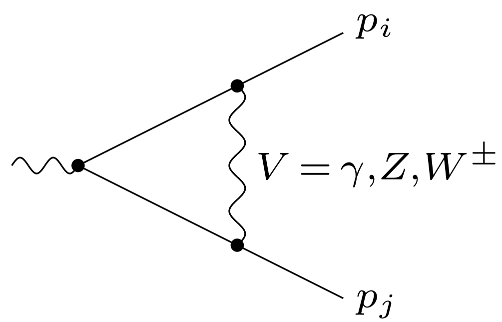
Application to massless 4-fermion scattering

- neutral current $f\bar{f} \to f'\bar{f}'$: agreement (NLL), — B.J., Kühn, Penin, Smirnov '05
  additional contributions $\propto s_\mathrm{w}^2 \ln(M_\mathrm{Z}^2/M_\mathrm{W}^2)$
- charged current $f_1\bar{f}_2 \to f_3\bar{f}_4$: new NLL result

IV From massless to massive fermions

Fermion mass effects in 1-loop results

Form-factor contribution from vertex diagram
and wave-function renormalization in NLL accuracy:



Massive $V = Z, W^\pm$:

$$F_1^M = -L_V^2 - \frac{2}{3}L_V^3\epsilon - \frac{1}{4}L_V^4\epsilon^2 + 3L + \frac{3}{2}L^2\epsilon + \frac{1}{2}L^3\epsilon^2 \qquad \left[L_V = \ln\left(\frac{Q^2}{M_V^2}\right)\right]$$

$\Rightarrow$ independent of fermion masses p_i^2, p_j^2 (checked by explicit calculation)!

Massless $V = \gamma$:

$$F_1^0(p_i^2, p_j^2) = F_1^0(0,0) + \Delta F_1^0(p_i^2) + \Delta F_1^0(p_j^2)\,, \qquad \left[L_\mathrm{t} = \ln\left(\frac{Q^2}{m_\mathrm{top}^2}\right)\right]$$

$$F_1^0(0,0) = -2\epsilon^{-2} - 3\epsilon^{-1}\,, \qquad \Delta F_1^0(0) = 0\,,$$

$$\Delta F_1^0(m_\mathrm{top}^2) = \epsilon^{-2} + L_\mathrm{t}\epsilon^{-1} + \frac{1}{2}L_\mathrm{t}^2 + \frac{1}{6}L_\mathrm{t}^3\epsilon + \frac{1}{24}L_\mathrm{t}^4\epsilon^2 + \frac{1}{2}\epsilon^{-1} + \frac{1}{2}L + \frac{1}{4}L^2\epsilon + \frac{1}{12}L^3\epsilon^2$$

$\Rightarrow$ dependence on p_i^2 and p_j^2 separated!

Expansion by regions with massive external fermions

Asymptotic expansion for loop integrals with the following recipe:

1. divide the integration domain into regions for the loop momenta
2. in every region, expand the integrand appropriately
3. integrate the expanded terms over the whole integration domain

Expand before integration $\rightsquigarrow$ need to eliminate small invariants $p_i^2, p_j^2 \ll 2p_i \cdot p_j \sim Q^2$

$\Rightarrow$ shift to lightlike momenta: $p_{i,j} = \tilde{p}_{i,j} + \dfrac{p_{i,j}^2}{2\tilde{p}_i \cdot \tilde{p}_j}\,\tilde{p}_{j,i}$ with $\tilde{p}_i^2 = \tilde{p}_j^2 = 0$

Relevant regions for each loop momentum k: $[M \sim M_{\mathrm{W,Z}} \sim m_{\text{top}}]$

	$k_{\parallel \tilde{p}_i}$	$k_{\parallel \tilde{p}_j}$	$k_{\perp(\tilde{p}_i,\tilde{p}_j)}$
hard	Q	Q	Q
soft	M	M	M
ultrasoft	M^2/Q	M^2/Q	M^2/Q

	$k_{\parallel \tilde{p}_i}$	$k_{\parallel \tilde{p}_j}$	$k_{\perp(\tilde{p}_i,\tilde{p}_j)}$	
i-collinear	Q	M^2/Q	M	
j-collinear	M^2/Q	Q	M	
i-ultracollinear	M^2/Q	M^4/Q^3	M^3/Q^2	new!
j-ultracollinear	M^4/Q^3	M^2/Q	M^3/Q^2	new!

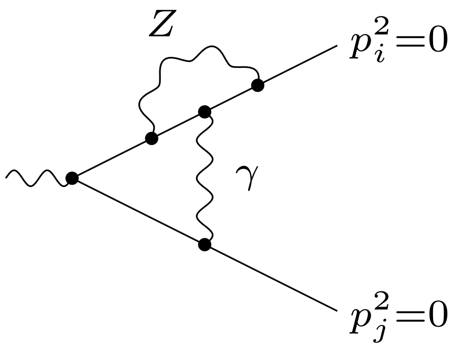
Power singularities Q^2/M^2

Asymptotic expansion for $Q^2 \gg M^2_{\mathrm{W,Z}},\ m^2_{\mathrm{top}}$

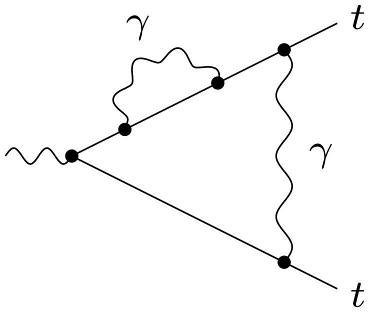
$\hookrightarrow$ logarithmic singularities $\ln(Q^2/M^2_{\mathrm{W,Z}}),\ \ln(Q^2/m^2_{\mathrm{top}})$

$\hookrightarrow$ power singularities $Q^2/M^2_{\mathrm{W,Z}},\ Q^2/m^2_{\mathrm{top}}$

E.g. in scalar diagrams (master integrals)



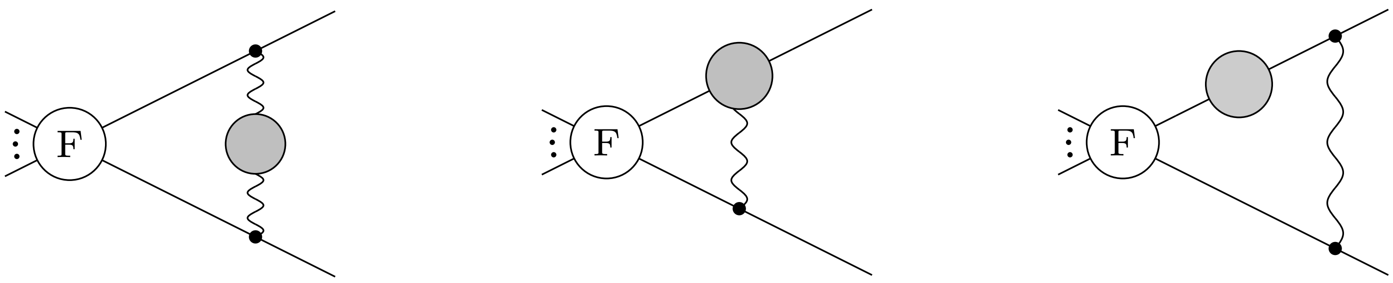
$$= -\frac{1}{Q^4}\,\frac{Q^2}{M_{\mathrm{Z}}^2}\left(\frac{3}{4}\epsilon^{-3} + \frac{3}{2}L\epsilon^{-2} + \frac{3}{2}L^2\epsilon^{-1} + L^3\right)$$



$$= -\frac{1}{Q^4}\,\frac{Q^2}{m_{\mathrm{top}}^2}\left(\frac{1}{4}L\epsilon^{-2} + \frac{3}{4}L^2\epsilon^{-1} + \frac{7}{6}L^3\right)$$

Power singularities Q^2/M^2 (2)

- not present at 1 loop
- appear at 2 loops in (scalar) diagrams with loop insertions at soft–collinear lines:



- expansion by regions predicts where power singularities can appear: simply combine the factors of M from propagators and integration measures for each region

Complete Feynman diagrams:

- Q^2/M^2 singularities always compensated by factors of M^2/Q^2 from Feynman rules or reductions [e.g. $k^2/(k^2 - M^2) \to 1 + M^2/(k^2 - M^2)$]
  $\hookrightarrow$ results for Feynman diagrams are free from power singularities
- **massless fermions:** power singularities do <u>not</u> affect fermion lines
- **massive fermions:** mass terms in numerator of fermion lines important!
  $\hookrightarrow$ soft–collinear approximation not possible for the subdiagrams from above
  $\Rightarrow$ treat these subdiagrams with projection techniques (like already in massless case)

Power singularities: complications from fermion masses

Mass terms in numerator of fermion lines important:

- fermion propagators: $\dfrac{\not{k} + m_{\text{top}}}{k^2 - m_{\text{top}}^2}$
- spinors: $(\not{p} - m_{\text{top}})\, u(p) = 0, \;\; (\not{p} + m_{\text{top}})\, v(p) = 0$

$\Rightarrow$ need more complicated projection on Dirac structure than in massless case:

$$\mathcal{M}^{f_i} = G_0^{f_i}\, \Gamma(p_i, p_j)\, u(p_i) = \Gamma_1 \underbrace{G_0^{f_i}\, u(p_i)}_{\mathcal{M}_0^{f_i}} \quad + \quad \underbrace{\Gamma_2\, G_0^{f_i}\, \not{p}_j\, u(p_i)}_{\text{suppressed for } Q^2 \gg m_{\text{top}}^2}$$

$\Rightarrow$ fermion mass terms mix chiralities, $\not{p}\, u(p, \mathsf{L}) = m_{\text{top}}\, u(p, \mathsf{R})$:

$$\mathcal{M}^{f_i,\mathsf{L}} = G_0^{f_i}\, \Gamma(p_i, p_j)\, u(p_i, \mathsf{L}) \simeq \Gamma_{1,\mathsf{L}} \underbrace{G_0^{f_i}\, u(p_i, \mathsf{L})}_{\mathcal{M}_0^{f_i,\mathsf{L}}} + \Gamma_{1,\mathsf{R}} \underbrace{G_0^{f_i}\, u(p_i, \mathsf{R})}_{\mathcal{M}_0^{f_i,\mathsf{R}}}$$

From preliminary results, valid for Abelian vertex corrections:

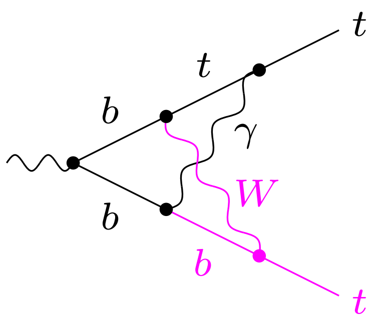
In NLL accuracy, power singularities seem to be relevant only for QED corrections (without W, Z in soft–collinear loops), where gauge couplings are not chiral.

Structure of the corrections: new logarithms

General structure of soft–collinear LLs with some mass scale $\Delta \sim M_\mathrm{W} \ll Q^2$:

$$\ln^n\left(\frac{-(p_i+p_j)^2}{\Delta}\right) \overset{\text{NLL}}{=} \ln^n\left(\frac{Q^2}{M_\mathrm{W}^2}\right) + n\left[\ln\left(\frac{-(p_i+p_j)^2}{Q^2}\right) - \ln\left(\frac{\Delta}{M_\mathrm{W}^2}\right)\right]\ln^{n-1}\left(\frac{Q^2}{M_\mathrm{W}^2}\right)$$

- massless fermions: only $\Delta = M_\mathrm{Z}^2$
- massive fermions: also $\Delta = m_\text{top}^2$ and $\Delta = M_\mathrm{W}^2 - m_\text{top}^2 - i0$ (at W–t–b vertices):



scalar diagram $\propto \frac{5}{6}L^4 - \frac{4}{3}\left[\ln\left(\frac{m_\text{top}^2}{M_\mathrm{W}^2}\right) + \ln\left(\frac{M_\mathrm{W}^2 - m_\text{top}^2}{M_\mathrm{W}^2}\right)\right]L^3$

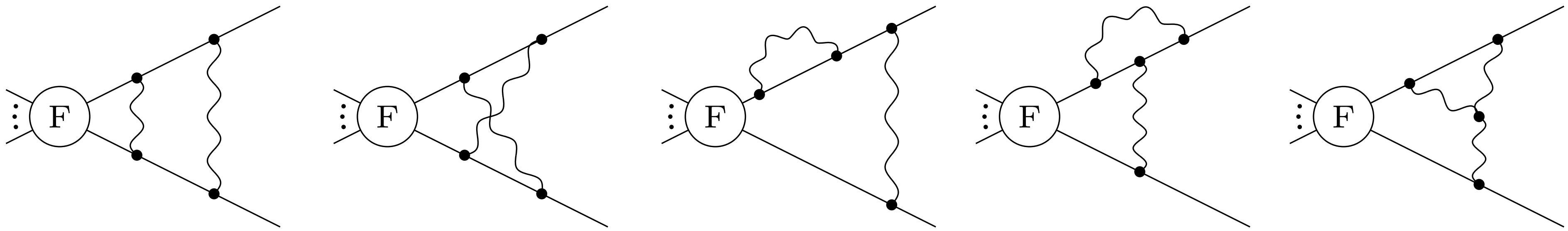
Feynman diagram $\propto \frac{2}{3}L^4 - \left[4 + \frac{2}{3}\ln\left(\frac{m_\text{top}^2}{M_\mathrm{W}^2}\right)\right]L^3$

Complete Feynman diagrams:

- diagrams **without photons:** independent of fermion masses, only $\Delta = M_\mathrm{Z}^2$
- diagrams **with photons:** $\Delta = M_\mathrm{Z}^2$ and $\Delta = m_\text{top}^2$, but not $\Delta = M_\mathrm{W}^2 - m_\text{top}^2$; additionally $1/\epsilon$ poles

Preliminary results

Factorizable contributions already calculated for massless & massive fermions:



First 4 $\longrightarrow$ determine 2-loop Abelian form factor of $\mathrm{U}(1)^{\alpha}_{\text{massive}} \times \mathrm{U}(1)^{\alpha'}_{\text{massless}}$ model:

1-loop result: $F_1(p_i^2, p_j^2) = \alpha\, F_1^M + \alpha' \left[F_1^0(0,0) + \Delta F_1^0(p_i^2) + \Delta F_1^0(p_j^2)\right]$

$$F_1^M = -L_V^2 - \frac{2}{3} L_V^3 \epsilon - \frac{1}{4} L_V^4 \epsilon^2 + 3L + \frac{3}{2} L^2 \epsilon + \frac{1}{2} L^3 \epsilon^2 \,, \qquad \left[L_V = \ln\left(\frac{Q^2}{M_V^2}\right)\right]$$

$$F_1^0(0,0) = -2\epsilon^{-2} - 3\epsilon^{-1} \,, \qquad \Delta F_1^0(0) = 0 \,, \qquad \left[L_{\mathrm{t}} = \ln\left(\frac{Q^2}{m_{\text{top}}^2}\right)\right]$$

$$\Delta F_1^0(m_{\text{top}}^2) = \epsilon^{-2} + L_{\mathrm{t}} \epsilon^{-1} + \frac{1}{2} L_{\mathrm{t}}^2 + \frac{1}{6} L_{\mathrm{t}}^3 \epsilon + \frac{1}{24} L_{\mathrm{t}}^4 \epsilon^2 + \frac{1}{2} \epsilon^{-1} + \frac{1}{2} L + \frac{1}{4} L^2 \epsilon + \frac{1}{12} L^3 \epsilon^2$$

2-loop result: $F_2(p_i^2, p_j^2) = \frac{1}{2} \left[F_1(p_i^2, p_j^2)\right]^2$

$\hookrightarrow$ exponentiation: $\exp\left[F_1(p_i^2, p_j^2)\right]$

V Summary

Massless fermionic processes $f_1 f_2 \longrightarrow f_3 \cdots f_n$

with different $|(p_i + p_j)^2| \gg M_\mathrm{W}^2$ and different masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathrm{top}^2 \sim M_\mathrm{Higgs}^2$:

- complete EW NLL corrections in $D = 4 - 2\epsilon$ dimensions
  $\hookrightarrow$ Denner, B.J., Pozzorini, Nucl. Phys. B 761 (2007) 1
- factorizable contributions calculated with 2 independent methods:
  1.) sector decomposition, 2.) expansion by regions & Mellin–Barnes
- non-factorizable contributions shown to vanish due to collinear Ward identities
- result expressed by exponentiated 1-loop terms and β-function coefficients
- universal correction factors, in agreement with existing results

From massless to massive fermions

- method also works for massive fermions
- treat fermion mass terms carefully (power singularities $1/m_\mathrm{top}^2$, mixing of chiralities)
- contributions to Abelian 2-loop form factor completed, exponentiates 1-loop result
- remaining diagrams will soon be finished . . .

$\rightsquigarrow$ **Goal: electroweak NLL corrections for arbitrary processes**