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# Recent advances at the top threshold: summation of logs and finite lifetime effects

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**Based on work with:**

**C. Farrell, A. Manohar, Ch. Reisser, P. Ruiz-Femenia, M. Stahlhofen,  
I. Stewart, T. Teubner,**



# Outline

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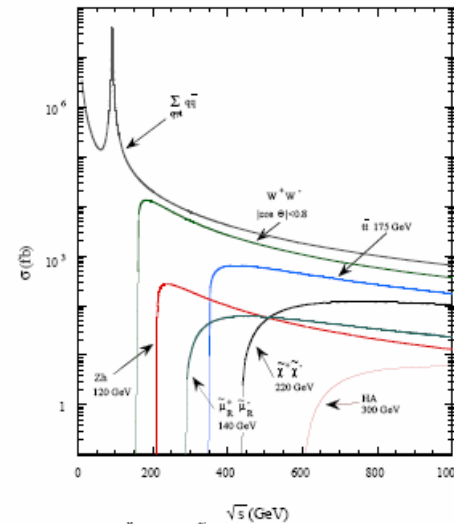
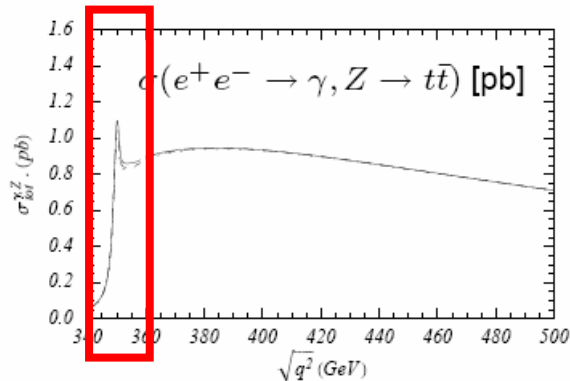
- Physics at the Top Pair Threshold
  - Measurements, experimental issues
  - Theory issues
- Effective Theory (A): stable top quarks
- Effective Theory (B): unstable particle EFT
  - Finite lifetime effects
  - Interference effects
  - Phase space UV-divergences, summation of logs
  - Phase space matching
- Applications:
  - $e^+e^- \rightarrow t\bar{t}$  at NNLL
  - $e^+e^- \rightarrow t\bar{t}H$  at NLL
  - squark pair production  $\longrightarrow$  **A. Sopczak's talk**



# Top Physics and the ILC

- $e^+e^-$  collider:  $E_{\text{cm}} = 350 \text{ GeV} - 1 \text{ TeV}$
- Luminosity:  $10^{34} - 10^{35} \text{ cm}^{-2} \text{ s}^{-1} \rightarrow 100 - 1000 \text{ fb}^{-1} / \text{year}$ 
  - LC  $\sim 10^5 t\bar{t}$  pairs       $[\sigma_{\text{tot}} < 1 \text{ pb}] (e^+e^- \rightarrow t\bar{t})$
  - LHC  $\sim 10^8 t\bar{t}$  pairs       $[\sigma_{\text{tot}} \approx 850 \text{ pb}] (gg \rightarrow t\bar{t})$
- Initial state tunable and very well known  $\rightarrow$  threshold & continuum

- Centre of mass energy variable



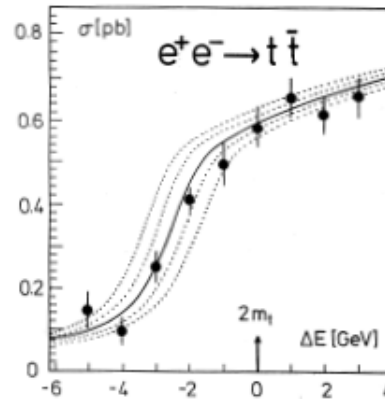
- $e^\pm$  beam polarization:  $P_{e^-} \sim 80\%$ ,  $P_{e^+} \sim 60\%$
- $\gamma\gamma, \gamma e$  options:  $e^+e^- \rightarrow t\bar{t} (^3S_1)$ ,       $\gamma\gamma \rightarrow t\bar{t} (^1S_0)$



# Threshold Measurements

Threshold Scan:  $\sqrt{s} \simeq 350 \text{ GeV}$  (Phase I)

- ▷ count number of  $t\bar{t}$  events
- ▷ color singlet state
- ▷ background is non-resonant
- ▷ physics well understood  
(renormalons, summations)



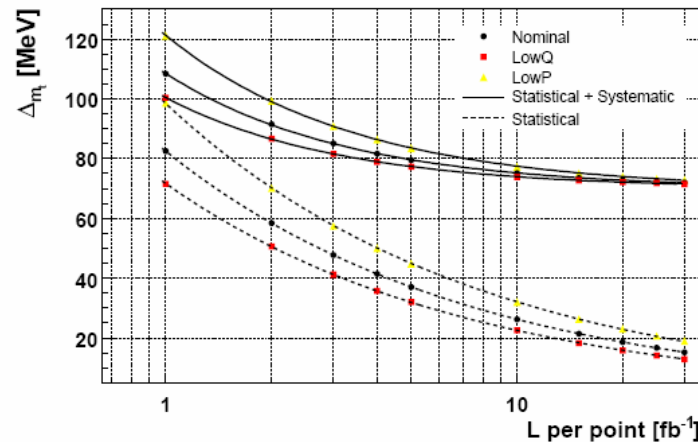
$$\rightarrow \delta m_t^{\text{exp}} \simeq 50 \text{ MeV}$$

$$\rightarrow \delta m_t^{\text{th}} \simeq 100 \text{ MeV}$$

What mass?

$$\sqrt{s}_{\text{rise}} \sim 2m_t^{\text{thr}} + \text{pert. series}$$

(short distance mass:  $1S \leftrightarrow \overline{MS}$ )



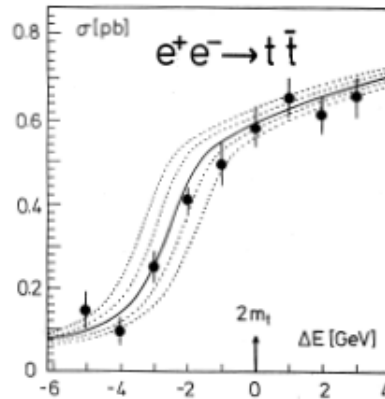
Boogert, Gounaris 2007  
9+1 scan points



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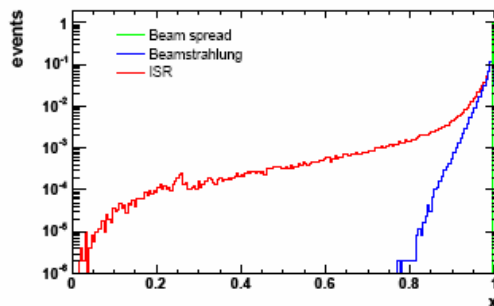
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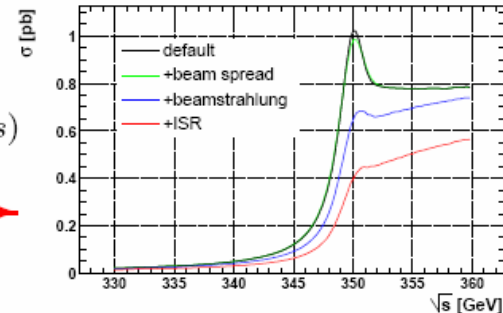
Simulations

Influence of Luminosity spectrum (LO QED effect!)

Boogert, Gounaris



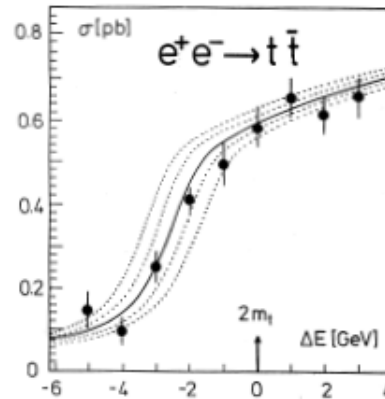
$$\sigma(s) = \int_0^1 dx L(x) \sigma^0(x^2 s)$$



# Threshold Measurements

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What mass?

$$\sqrt{s}_{\text{rise}} \sim 2m_t^{\text{thr}} + \text{pert. series}$$

(short distance mass:  $1S \leftrightarrow \overline{MS}$ )

Simulations  $300 \text{ fb}^{-1}$ , 9+1 scan points  $\rightarrow$  goal:

$$(\delta\sigma/\sigma)^{\text{theo}} \leq 3\%$$

$$(\delta m_t)^{\text{stat}} \sim 20 \text{ MeV}$$

$$(\delta m_t)^{\text{syst}} = 50 \text{ MeV}$$

$$(\delta m_t)^{\text{theo}} \simeq 100 \text{ MeV}$$

$$(\delta\lambda_t/\lambda_t)^{\text{stat}} = 15 - 50\%$$

$$(\delta\lambda_t/\lambda_t)^{\text{syst}} = ?$$

$$(\delta\lambda_t/\lambda_t)^{\text{theo}} \sim ?$$

$$(\delta\alpha_s(M_Z))^{\text{stat}} = 0.001$$

$$(\delta\alpha_s(M_Z))^{\text{syst}} = 0.002$$

$$(\delta\alpha_s(M_Z))^{\text{theo}} \sim ?$$

$$(\delta\Gamma_t)^{\text{stat}} = 50 \text{ MeV}$$

$$(\delta\Gamma_t)^{\text{syst}} = 15 \text{ MeV}$$

$$(\delta\Gamma_t)^{\text{theo}} \sim ?$$

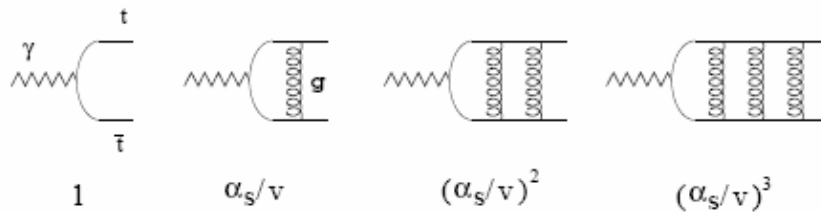


# Theory Issues

$$m_t \text{ (hard)} \gg p \sim m_t v \text{ (soft)} \gg E \sim m_t v^2 \text{ (ultrasoft)}$$

- perturbation theory in  $\alpha_s$  breaks down

$$(\alpha_s/v)^n$$



“Coulomb singularities”

→ Schrödinger Equation

- perturbation theory in  $\alpha_s$  breaks down → large logs  $(\alpha_s \ln v)^n$

$$m_t = 175 \text{ GeV}, \quad p \sim 25 \text{ GeV}, \quad E \sim 4 \text{ GeV} \quad \Rightarrow \quad \ln \left( \frac{m_t^2}{E^2} \right) = 8 \quad \rightarrow \text{RGE's}$$

“multi-scale problem”



# Theory Issues

- $\Gamma_t \approx 1.5 \text{ GeV} \gg \Lambda_{\text{QCD}} \Rightarrow v = \sqrt{\frac{E}{m}} \rightarrow v_{\text{eff}} = \sqrt{\frac{E+i\Gamma_t}{m}}$   
(Fadin,Khoze)

$\Rightarrow m_t \gg p = mv_{\text{eff}} \gg E = mv_{\text{eff}}^2 \gtrsim \Lambda_{\text{QCD}}$  always true !

$\Rightarrow$  top threshold entirely perturbative !  $\rightarrow$  “Schrödinger theory”

- $E \sim \Gamma_t$ : top quarks are always produced off-shell !

$\rightarrow$  methods for on-shell production do not apply for

- theoretical computations
- experimental analysis  $\rightarrow$  dependence on exp. cuts

“theory for unstable particles”





# EFT – Shopping List

$$\sigma_{t\bar{t}} \sim v \sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^m \left[ 1, \{v, \alpha_s\}, \{v^2, \alpha_s v, \alpha_s^2\}, \dots \right]$$

LL      NLL      NNLL

- Action from **Lagrangian** in terms of field operators
  - ▷ Manifest power counting in  $v$  at all times  
consistently obeye  $E \ll p \ll m$  and  $\Gamma_t \sim E \sim p^2/m$
  - ▷ Symmetries: gauge, spin, ...
  - ▷ Regulator independence → matching in different schemes
- **Renormalization** of the theory → anomalous dimensions, summation of  $(\alpha_s \ln v)^m$
- **Dimensional regularization**

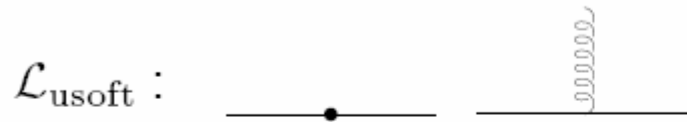


# vNRQCD (stable quarks)

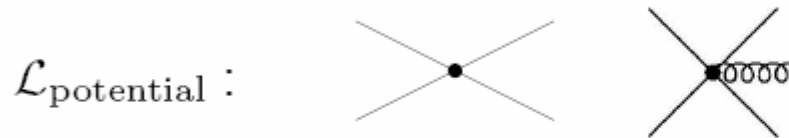
$$\mathcal{L} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{potential}} + \mathcal{L}_{\text{soft}}$$

Luke, Manohar, Rothstein, Stewart, A.H.

$$D^\mu = \partial^\mu + i\mu_U^\epsilon g_s(m\nu^2)A^\mu$$



$$\psi_{\mathbf{p}}^\dagger(x) \left\{ iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m_t} - \delta m_t \right\} \psi_{\mathbf{p}}(x)$$

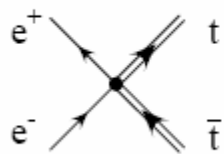


$$\mu_S^{2\epsilon} V(\nu) \psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}}$$



$$\mu_S^{2\epsilon} U_{\mu\nu}(\nu) \psi_{\mathbf{p}'}^\dagger A_q^\mu A_{q'}^\nu \psi_{\mathbf{p}}$$

external currents: (production & annihilation)



$$O_{\mathbf{p}} = C_{V,A}(\nu) \cdot [\bar{e}\gamma^i(\gamma_5)e][\psi_{\mathbf{p}}^\dagger \sigma^i \tilde{\chi}_{-\mathbf{p}}^*] + \dots \quad t\bar{t} (^3S_1)$$



# Cross Section at NNLL Order

Schematic:

$$\sigma_{\text{tot}} \propto \text{Im} \left[ \int d^4x e^{-i\hat{q}x} \langle 0 | T \mathbf{O}_{\mathbf{p}}^\dagger(0) \mathbf{O}_{\mathbf{p}'}(x) | 0 \rangle \right]$$

$$\propto \text{Im} \left[ (C_A(\nu)^2 + C_V(\nu)^2) G(0, 0, \sqrt{s}) \right]$$

$$\left( -\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3} + V(\mathbf{r}) - (\sqrt{s} - 2m - 2\delta m) - i\Gamma_t \right) G(\mathbf{r}, \mathbf{r}') = \delta^{(3)}(\mathbf{r} - \mathbf{r}')$$

fully known  
at NNLL order ✓

Manohar, Stewart; AH '99-'03  
Pineda, Soto '00-'01  
Peter '94, Schröder '98

NLL ✓

NNLL (matching) ✓ Benke et al; Czarnecki et al '99

NNLL (non-mixing) ✓ AH '03

NNLL (mixing)

spin-dependent (soft) Penin et al. '04

usoft  $1/m^2$  Stahlhofen, AH '05



# Cross Section at NNLL Order

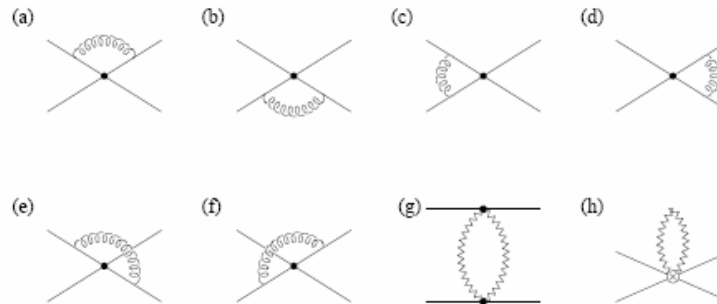
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LL running of  $V_1/m^2(\nu)$



$$\frac{\mathcal{V}_r(\nu)(\mathbf{p}^2 + \mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2(\nu)}{m^2} + \frac{\mathcal{V}_s(\nu)}{m^2} \mathbf{S}^2 + \frac{\mathcal{V}_\Lambda(\nu)}{m^2} \Lambda + \frac{\mathcal{V}_t(\nu)}{m^2} T$$



# Cross Section at NNLL Order

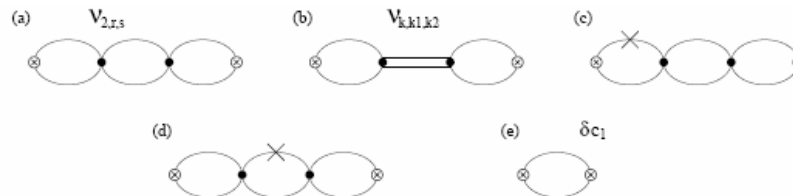
Schematic:

$$\sigma_{\text{tot}} \propto \text{Im} \left[ \int d^4x e^{-i\hat{q}x} \langle 0 | T \mathbf{O}_{\mathbf{p}}^\dagger(0) \mathbf{O}_{\mathbf{p}'}(x) | 0 \rangle \right]$$

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NLL running of  $C(\nu)$



$$\frac{d}{d \ln \nu} \ln C(\nu) = -\frac{\mathcal{V}_c(\nu)}{16\pi^2} \left( \frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2 \mathcal{V}_s(\nu) \right) + \frac{\mathcal{V}_k(\nu)}{2}$$

Luke, Manohar, Rothstein '99, Pineda '01  
Stewart, AHH '02



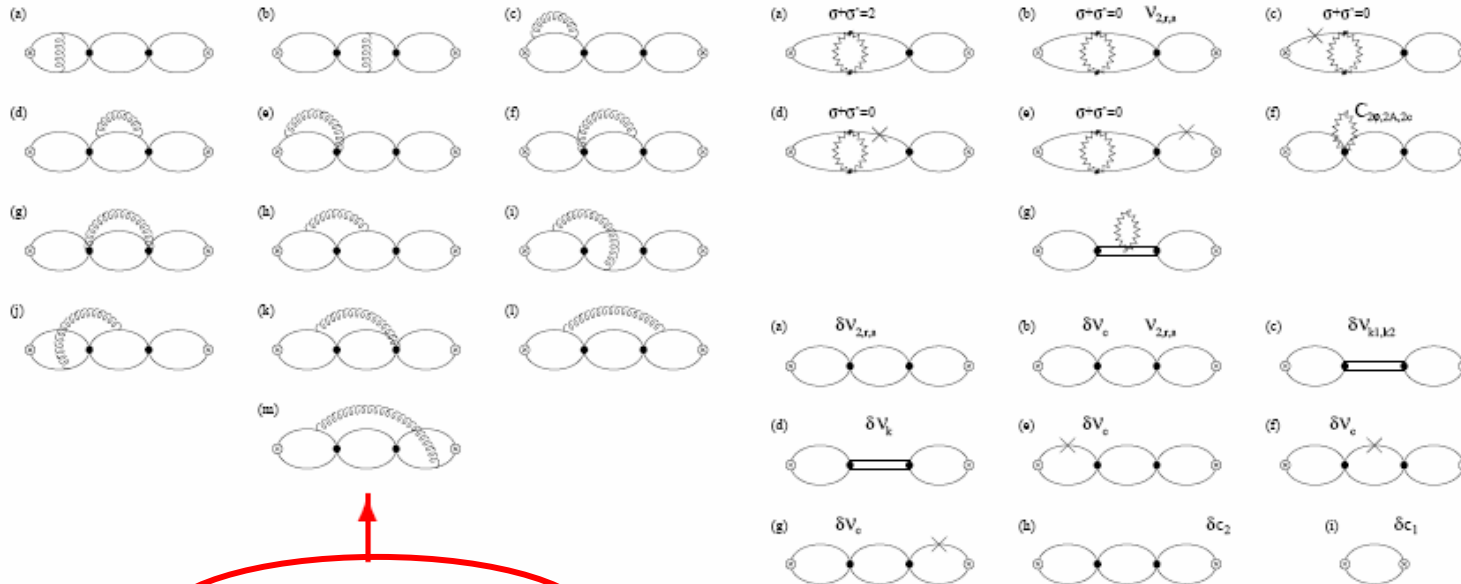
# Cross Section at NNLL Order

## NNLL running of $C(\nu)$

(a) next-order running of couplings in NLL anom. dim.  $\rightarrow$  w.i.p.

(b) genuine 3-loop anomalous dimension

AHH '03



ultrasoft corrections  
dominate by factor 10



# Cross Section at NNLL Order

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NNLL running of  $C(\nu)$

- (a) next-order running of couplings in NLL anom. dim.  $\rightarrow$  w.i.p.
- (b) genuine 3-loop anomalous dimension AHH '03

$$\frac{\delta z_c^{\text{NNLL},1}}{\epsilon} = \frac{\text{const}}{\epsilon} + \frac{\text{const}}{\epsilon} \times \ln \left( \frac{m_t \mu_U}{\mu_S^2} \right)$$

$\Rightarrow$  consistency under renormalization requires:

$$\text{scale correlation: } \mu_U \propto \mu_S^2 / m_t$$

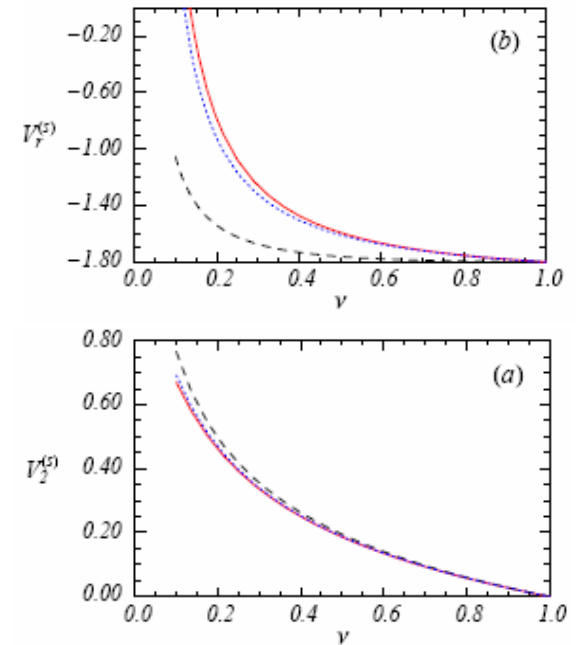
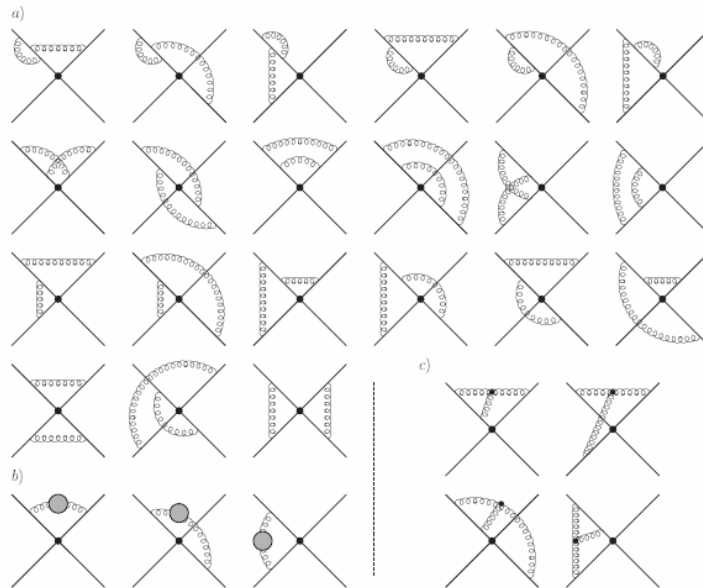


# Cross Section at NNLL Order

NNLL usoft running of  $C(\nu)$  (mixing contributions)

Stahlhofen, AH 2006

→ spin-independent  $\mathcal{O}(1/m^2)$  potentials



→ spin-independent  $\mathcal{O}(1/m|\vec{k}|)$  potentials w.i.p.

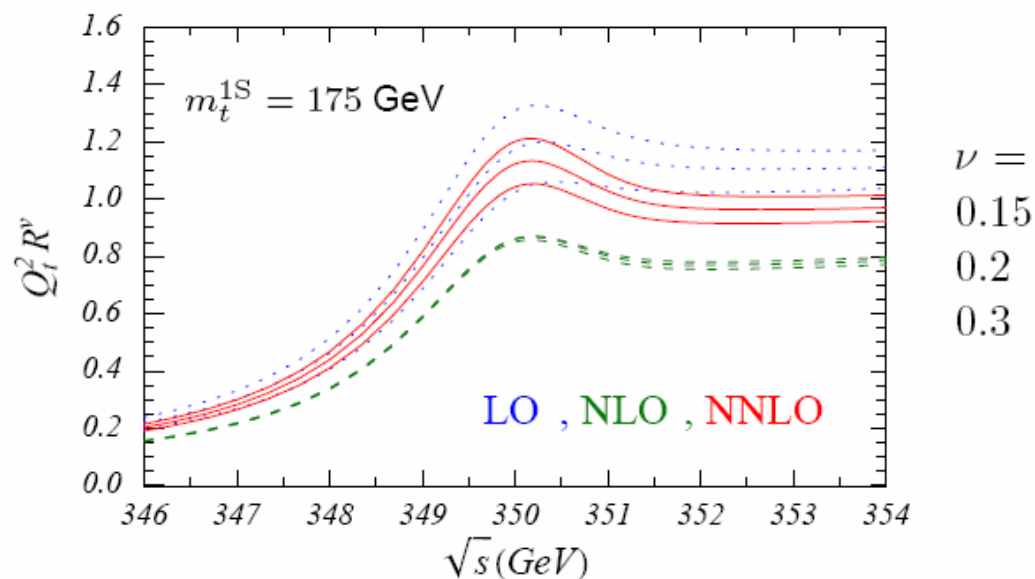




# Cross Section at NNLO Order

1S mass - fixed order approach

Teubner,AH; Melnikov, Yelkovski;Yakovlev;  
Beneke,Signer,Smirnov; Sumino, Kiyo



- peak position stable (threshold masses: 1S, PS, ...)
- large sensitivity to factorization/renormalization scale setting
- NNNLO partial results: Penin et al. '02 '05, Beneke et al. '05, Eiras et al. '05, '06  
**Marquard, Steinhauser et al '06**

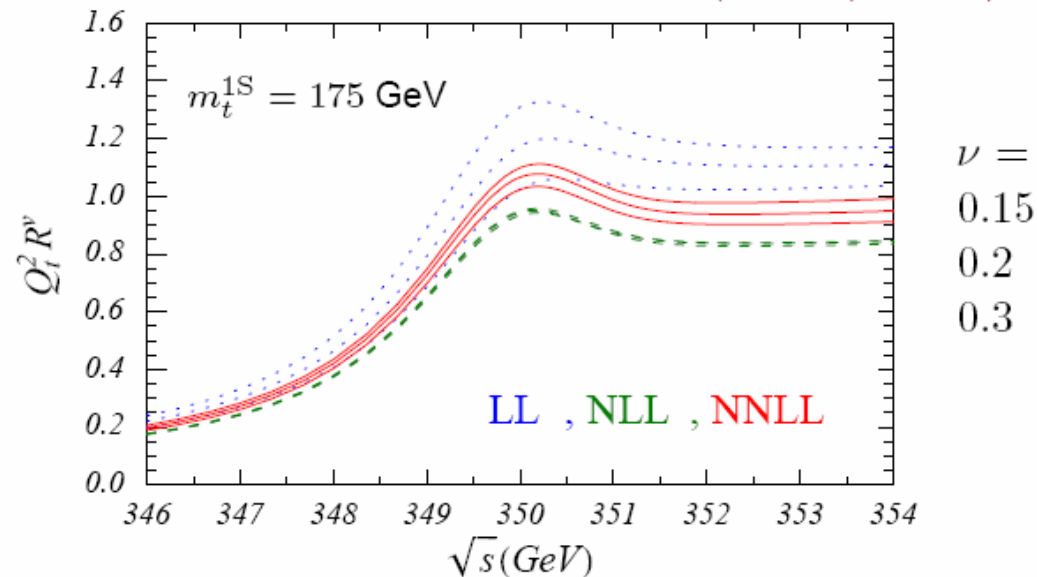
→ talk by Y. Kiyo



# Cross Section at NNLL Order

1S mass - RG-improved, with NNLL non-mixing terms

Manohar, Stewart, Teubner, AH



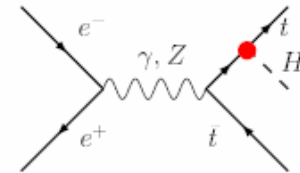
- RGI expansion shows better convergence
  - theory error:  $\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim \pm 6\%$       **goal: 3%**
- full NNLL (mixing) running of  $C(\nu)$  required → w.i.p.



# Reminder: $t\bar{t}H$ Production

$$e^+e^- \rightarrow t\bar{t}H$$

→ top-Yukawa coupling



- Theory Status:  $\sigma(e^+e^- \rightarrow t\bar{t}H)$

Born ✓

1-loop ew. ✓

$\mathcal{O}(\alpha_s)$  fixed-order ✓

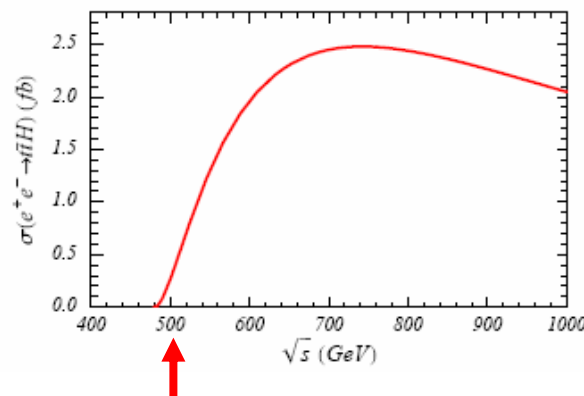
[Gaemers et al., Djouadi et al.]

[Denner et al., Belanger et al., You et al.]

[Dittmaier et al., Dawson et al.]

NLL large- $E_H$  QCD endpoint corrections ✓

[C. Farrell, AHH]



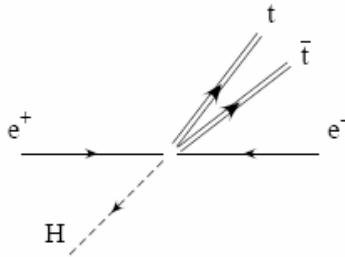
- tiny cross section for  $\sqrt{s} = 500$  GeV
- measurement of Yukawa coupling difficult
- $\delta Y_t / Y_t \sim 30\%$  feasible at  $\sqrt{s} = 500$  GeV [A. Juste, 2002]



# Reminder: ttH Production

Farrell, AH 2006

→ region of large Higgs energy

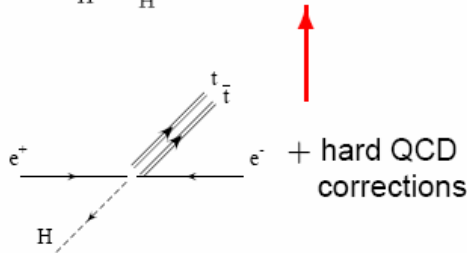


→  $t\bar{t}$  collinear

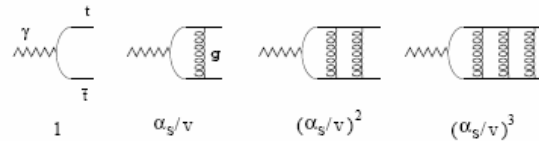
→ QCD effects localized in  $t\bar{t}$  system

⇒  $t\bar{t}$  dynamics non-relativistic

→  $\left(\frac{d\sigma}{dE_H}\right)_{E_H \approx E_H^{\max}} \sim C^2(\mu, \sqrt{s}, m_t, m_H) \times \text{Im}[G(0, 0, v, \mu)]$



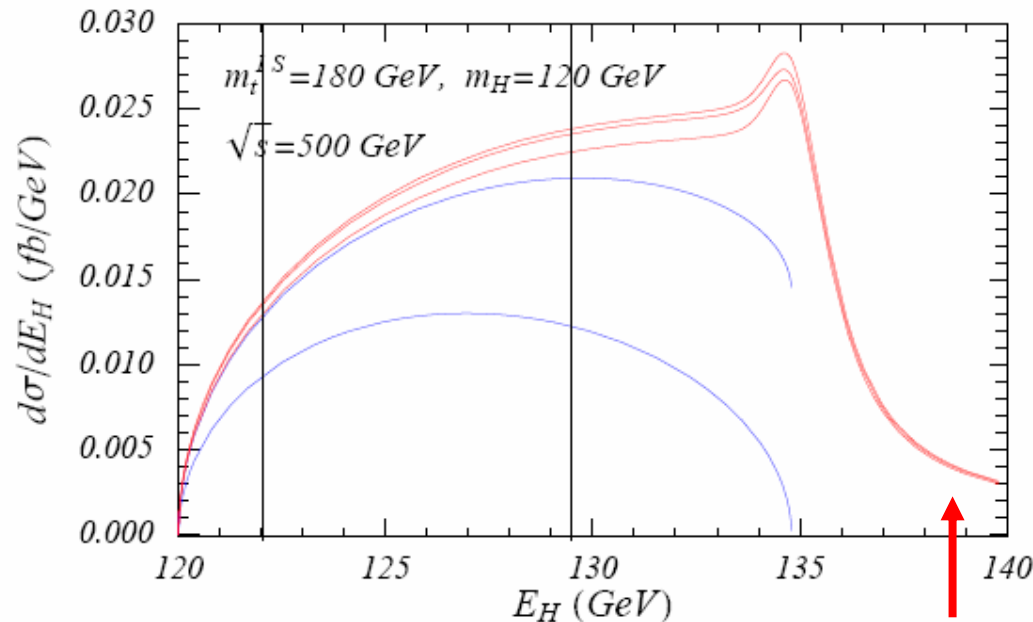
soft & ultrasoft QCD corrections



# Reminder: $t\bar{t}H$ Production

$$e^+e^- \rightarrow t\bar{t}H$$

→ NLL Higgs energy spectrum



Farrell, AHH

**Current work:**  
NLL finite lifetime effects

500 GeV:

- **factor 2 enhancement** over tree level from summation of  $(\alpha_s/v)^n$ ,  $(\alpha_s \ln v)^n$  terms
- essential for realistic studies for ILC (phase I)      **Juste '02, '06**

$$\Rightarrow (\delta\lambda_t/\lambda_t)_{500 \text{ GeV}}^{\text{ILC}} \sim 30\% \quad \rightsquigarrow \quad 10 - 15\%$$



# NRQCD (unstable quarks)

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“inclusive treatment”

- ⇒ Optical Theory: effective complex indices of refraction for absorptive processes
- ⇒ NRQCD: contributions from real  $Wb$  final states included in EFT matching conditions to QCD+ew. theory (=SM)
- complex matching conditions & anomalous dimensions
  - effective Lagrangian non-hermitian
  - total rates through the optical theorem
  - phase space matching
  - power counting maintained

Christoph Reisser, AH; Phys. Rev. D 71, 074022 (2005)

Christoph Reisser, AH; hep-ph/0604104



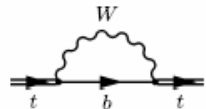
# NRQCD (unstable quarks)



quark bilinears:

( $\rightarrow \mathcal{L}_{\text{usoft}}$ )

time dilatation  
correction

$$\text{Im}\Sigma_t = \frac{1}{2}\Gamma_t$$



$$= i\Sigma_t \quad \Rightarrow \quad \delta\mathcal{L} = \psi_{\mathbf{p}}^\dagger \left[ i\frac{\Gamma_t}{2} \left( 1 - \frac{\mathbf{p}^2}{2m_t^2} \right) \right] \psi_{\mathbf{p}}$$



- power counting:  $\Gamma_t \propto m_t g^2 \sim m_t v^2 \Rightarrow \boxed{g \sim g' \sim v \sim \alpha_s}$
- finite lifetime is LL order,  $E \rightarrow E + i\Gamma_t$  Fadin, Khoze
- NNLL time dilatation effect
- $E \rightarrow E + i\Gamma_t$  prescription does not work beyond LL order



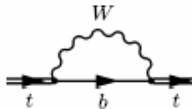
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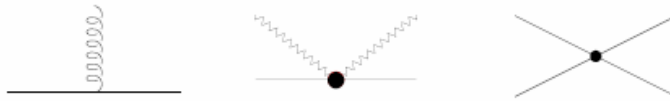
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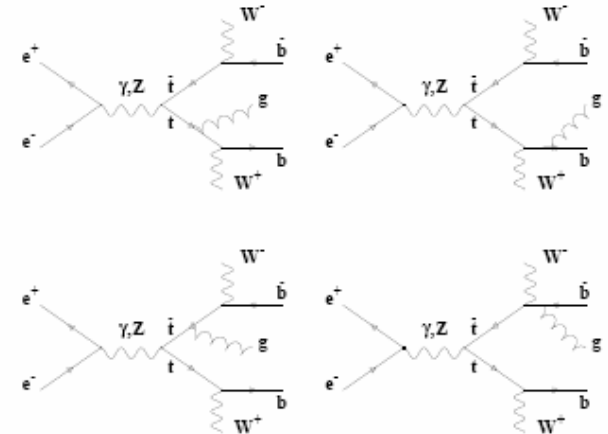


$$= i\Sigma_t \quad \Rightarrow \quad \delta\mathcal{L} = \psi_P^\dagger \left[ i\frac{\Gamma_t}{2} \left( 1 - \frac{\mathbf{p}^2}{2m_t^2} \right) \right] \psi_P$$


gluon interactions & potentials:



- electroweak corrections either beyond NNLL order or vanish due to gauge cancellations
- $\rightarrow$  ultrasoft gluon interference effects vanish at NLL and NNLL order (new !)



[Khoze et al., Melnikov et al.]

**Requirement: inclusive with respect to top and antitop decay products**

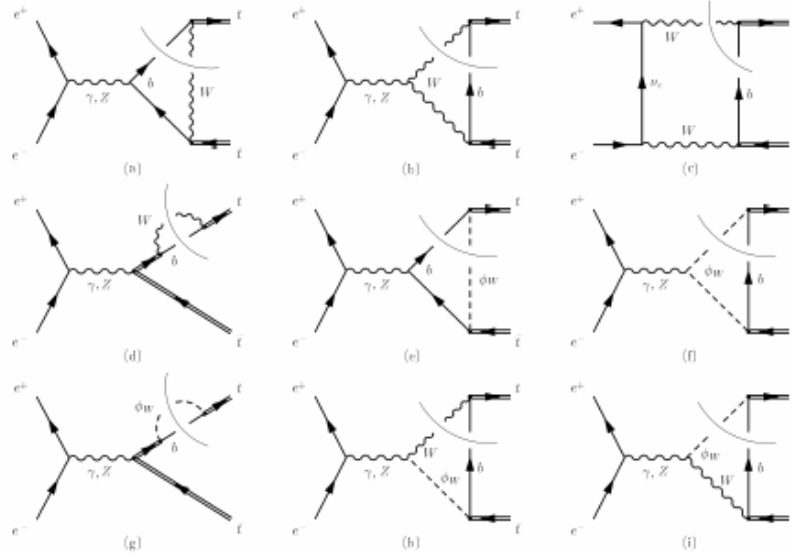
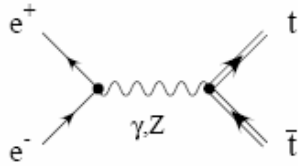




# NRQCD (unstable quarks)

## Currents:

- only bW-cuts included
- bW-cuts gauge invariant
- W treated as stable particle



$$O_P = \left[ C^{LL} + C^{NLL} + C^{NNLL} + iC_{abs}^{NNLL} \dots \right] \cdot \left( \begin{array}{c} e^+ \quad t \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ e^- \quad \bar{t} \end{array} \right) + \dots$$

$\text{Re}[C_{ew}^{NNLL}] \rightarrow$  weak contributions: C. Reisser, AH, hep-ph/0604104

Guth, Kühn 1992



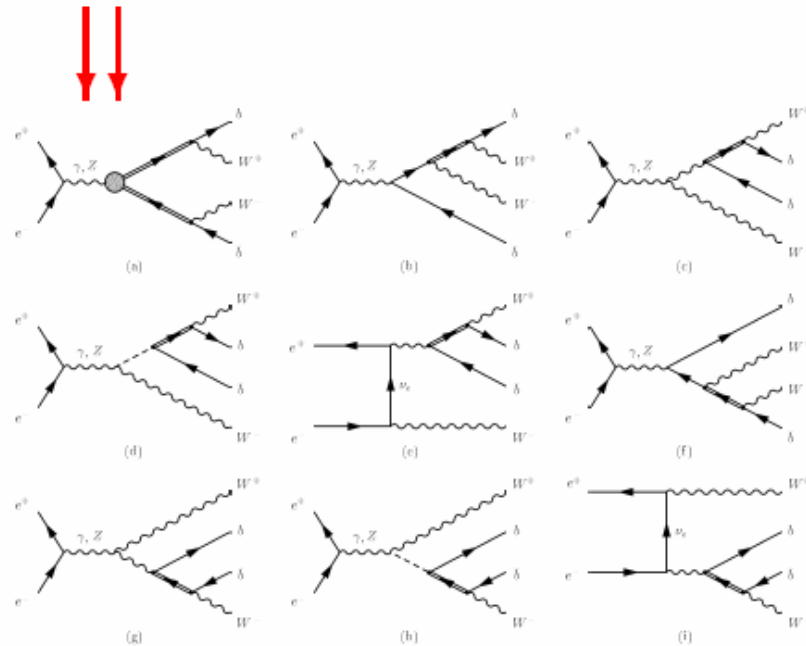
# NRQCD (unstable quarks)

Currents:

$$O_P = \left[ C^{LL} + C^{NLL} + C^{NNLL} + iC_{abs}^{NNLL} \dots \right] \cdot \left( \begin{array}{c} e^+ \\ e^- \end{array} \rightarrow \begin{array}{c} t \\ \bar{t} \end{array} \right) + \dots$$

$$\sigma_{tot} \propto \text{Im} [C(\nu)^2 G(0, 0, \sqrt{s} + i\Gamma_t)]$$

- accounts for **irreducible interference** contributions:  
resonant  $\leftrightarrow$  non-resonant  
 $W^+W^-b\bar{b}$  final states



# NRQCD (unstable quarks)

Currents:

$$O_p = \left[ C^{LL} + C^{NLL} + C^{NNLL} + iC_{abs}^{NNLL} \dots \right] \cdot \left( \begin{array}{cc} e^+ & t \\ e^- & \bar{t} \end{array} \right) + \dots$$

$$\sigma_{tot} \propto \text{Im} [C(\nu)^2 G(0, 0, \sqrt{s} + i\Gamma_t)]$$

$$O(v^3) \sim O(\alpha_s^3)$$

- $(\Delta\sigma_{tot}^\Gamma) \sim \alpha_s \Gamma_t \frac{1}{\epsilon} \Rightarrow$  NNLL logarithmic phase space UV divergences

→ NLL anom. dim. for  $(e^+e^-)(e^+e^-)$  operator →  $iC(\nu) \cdot \left( \begin{array}{cc} e^+ & e^- \\ e^- & e^+ \end{array} \right)$  ✓

→ matching for  $iC(\nu)$ :

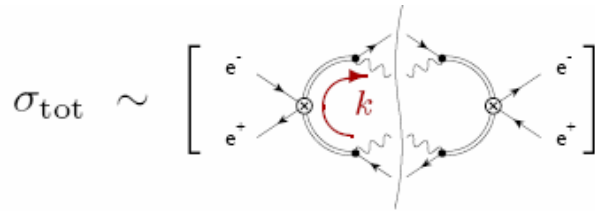
- physical  $W^+W^-b\bar{b}$  phase space → “Phase Space Matching” w.i.p.



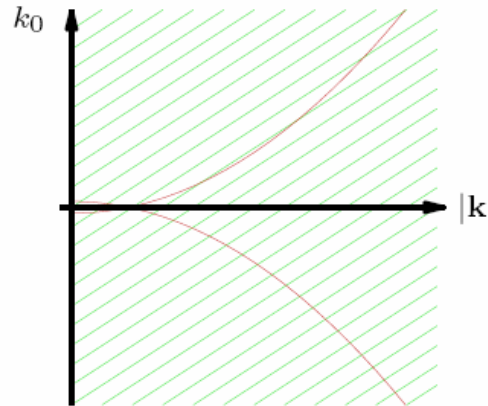
# Phase Space Matching

Reisser, Ruiz-Femenia, AH **w.i.p.**

- required for  $\sigma_{\text{tot}}$  from optical theorem in EFT
- finite imaginary matching conditions for all EFT operators
  - removal of unphysical EFT phase space contributions (violate unitarity)



$$\int_{-\infty}^{+\infty} dk_0 \int_0^{+\infty} d|\mathbf{k}| \frac{|\mathbf{k}|^2 \Gamma_t^2}{|k_0 - \frac{\mathbf{k}^2}{2m_t^2} + i\Gamma_t|^2 | -k_0 - \frac{\mathbf{k}^2}{2m_t^2} + i\Gamma_t|^2}$$

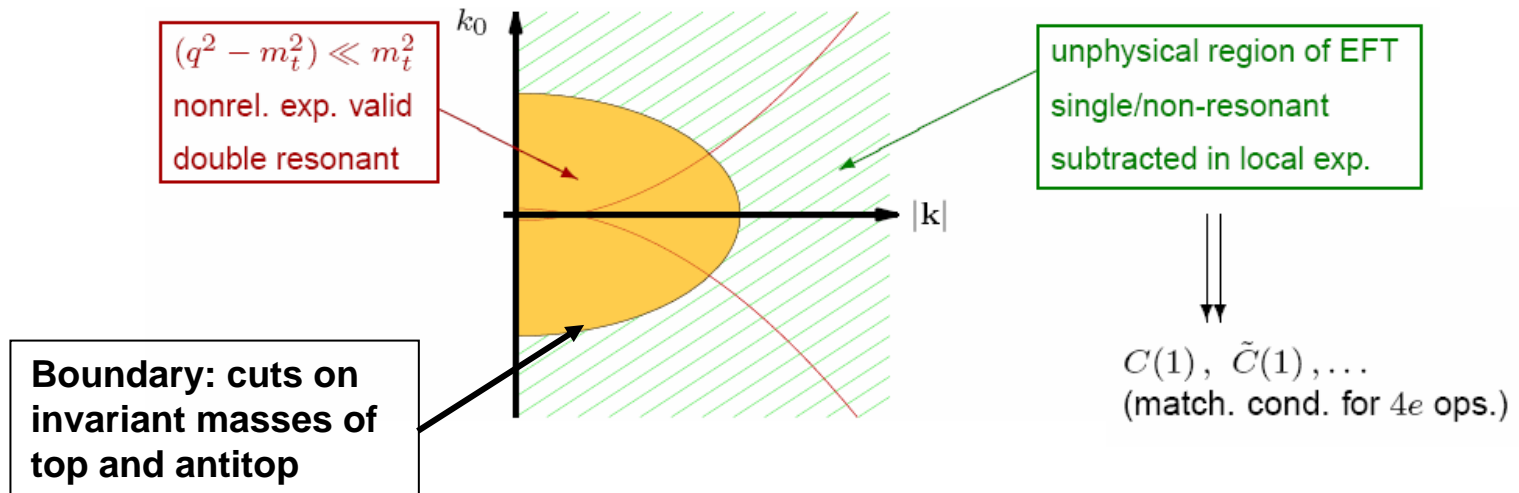


# Phase Space Matching

Reisser, Ruiz-Femenia, AH **w.i.p.**

- required for  $\sigma_{\text{tot}}$  from optical theorem in EFT
- finite imaginary matching conditions for all EFT operators
  - removal of unphysical EFT phase space contributions (violate unitarity)

$$\sigma_{\text{tot}} \sim \left[ \text{diagram of } e^-e^+ \text{ annihilation into } e^-e^+ \text{ via a loop} \right] + \text{Im} \left[ iC(\nu) \text{diagram} + i\tilde{C}(\nu) \frac{D^2}{m^2} \text{diagram} + \dots \right]$$



# Phase Space Matching

Reisser, Ruiz-Femenia, AH **w.i.p.**

- required for  $\sigma_{\text{tot}}$  from optical theorem in EFT
- finite imaginary matching conditions for all EFT operators
  - removal of unphysical EFT phase space contributions (violate unitarity)

$$\sigma_{\text{tot}} \sim \left[ \text{Diagram 1} \right] + \text{Im} \left[ \text{Diagram 2} + \text{Diagram 3} + \dots \right]$$

The equation shows the total cross-section  $\sigma_{\text{tot}}$  as a sum of a real part and an imaginary part. The real part is represented by a diagram of an electron-positron annihilation into a photon, which then produces an electron-positron pair. The imaginary part is represented by a series of diagrams showing the imaginary part of the forward scattering amplitude, with terms like  $iC(\nu)$  and  $i\tilde{C}(\nu)\frac{D^2}{m^2}$ .

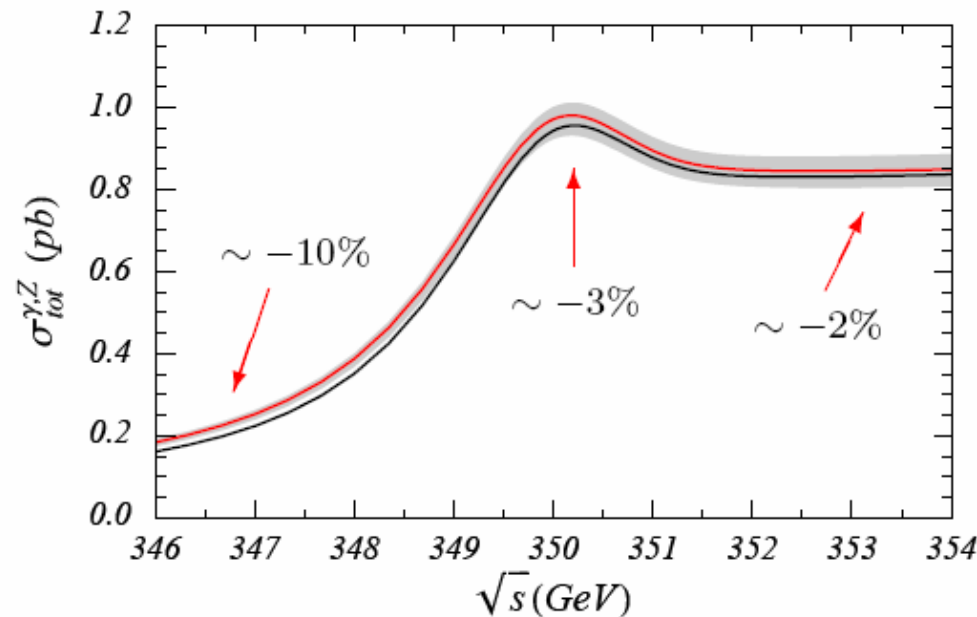
- “mild” powercounting breaking through use of momentum cuts
- mandatory for consistent predictions for P-, D-, etc. wave production (e.g. squark pairs) even at LO

## Current work:

- phase space matching at NLL and NNLL for S-wave production (NNLL insertions + ultrasoft gluon effects)
- P- and higher partial waves



# Effect for Total Cross Section



Reisser, AHH '05

- phase space logs & time-dilatation corrections
- finite lifetime corrections comparable to NNLL QCD corrections
- shift in the peak position: 30 – 50 MeV  $(\delta m_t^{\text{ex}} \approx 50 \text{ MeV})$



# Conclusions & Outlook

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- Status for total cross section (with invariant mass cuts):
    - LL (QCD+ew): ok
    - NLL (QCD): ok
    - NLL (ew): completed soon (affect line-shape)
    - NNLL (QCD): ok, except for prod. current (absolutely crucial for coupling measurements) dom. ultrasoft contributions completed soon
    - NNLL (ew): partially known, could be relevant in the off-shell region below  $Q=2m$
  - LO (QCD+ew): ok
  - NLO (QCD): ok
  - NNLO (QCD): ok
  - NNLO (QCD): completed soon
- ← Combination →
- Total cross section depends on experimental selection cuts: formally a NLL effect. (But numerical size depends on cuts.)
  - EW+finite lifetime effects generally small where cross section is large, but important for the  $Q < 2m$  tail region. (Could slightly affect mass determination. ↔ Luminosity spectrum).
  - More differential predictions are needed for more sophisticated selection cuts where additional theory input is required (e.g. contributions to distributions that cancel in the total cross section and are therefore usually neglected).





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# Backup Slides

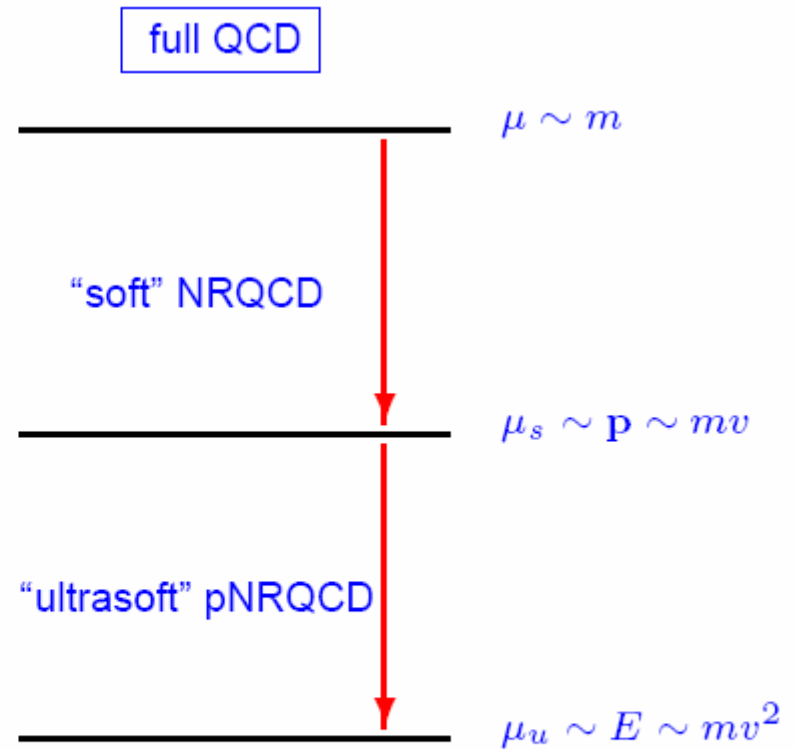


# Degrees of Freedom

pNRQCD:

$$m_t \gg \mathbf{p} \gg E$$

- successively integrated out d.o.f.'s
- sequence of effective theories
- soft matching scale ( $mv$ ) and ultrasoft scale ( $mv^2$ ) independent
- appropriate setup for static quarks
- top threshold:  
correlate  $\mu_u = \mu_s^2/m$  by hand  
→ large logs for matching at  $\mu_s$



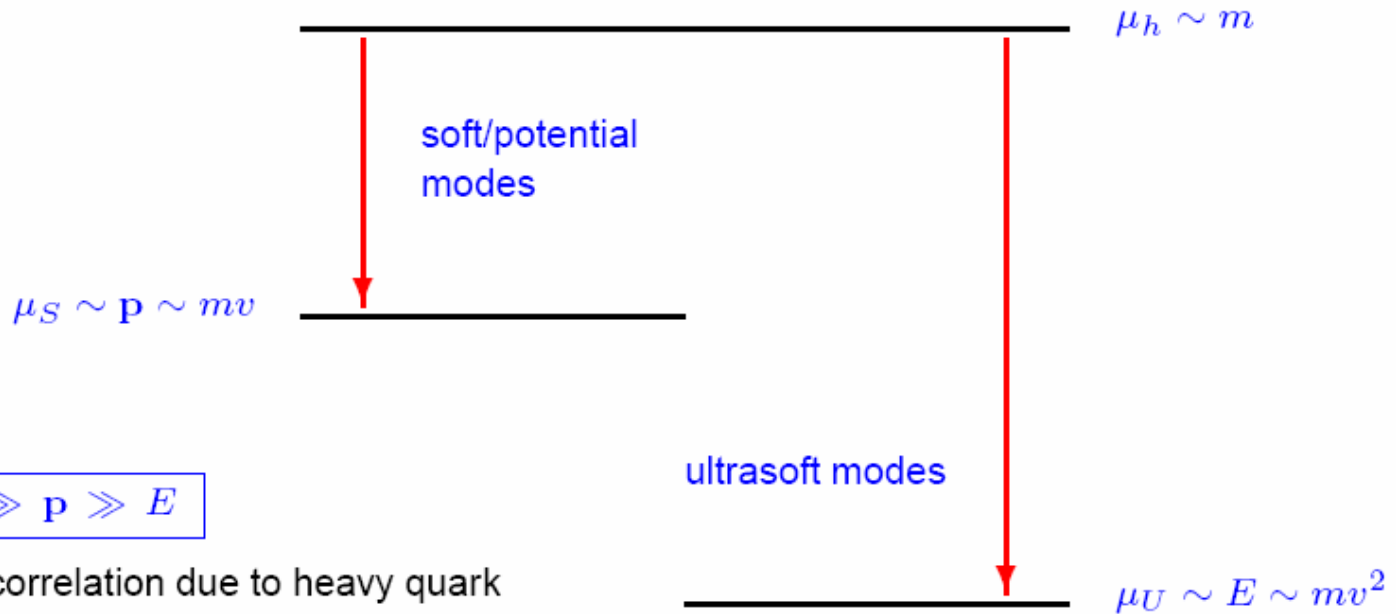
Pineda, Soto, Brambilla, Vairo



# Degrees of Freedom

vNRQCD:

full QCD



$$m_t \gg \mathbf{p} \gg E$$

and scale correlation due to heavy quark

equation of motion:  $E = \mathbf{p}^2/m$

Luke, Manohar, Rothstein, Stewart, A.H.

- soft and ultrasoft modes exist at the same time
- $\mu_U = \mu_S^2/m \rightarrow \mu_S = mv, \mu_U = mv^2$

