An estimate of NNNLO corrections to $t\bar{t}$ threshold X section

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TTP, Universität Karlsruhe

based on
M. Beneke, YK, A. Penin "Ultrasoft contribution to ...."
M.Beneke, YK, K. Schuller [hep-ph/0705.4518]

LCWS2007@DESY Hamburg

Our goal is QCD calculation of $\sigma_{t\bar{t}}$ with few % accuracy.

To meet experimental accuracy to extract precise top quark mass, we need the theory prediction with few % accuracy.

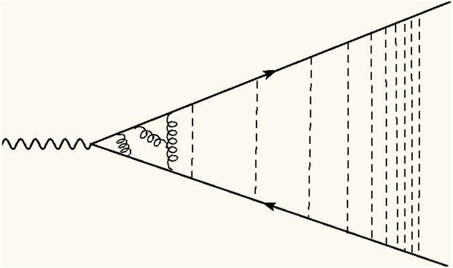
But the NNLO X section (Top WG Report) shows that more improvement is needed.

$\Updownarrow$

- Renormalization Group improvement $\Rightarrow$ Andre's talk
- Go to NNNLO (This talk)
- More ?

NNNLO threshold cross section

$$
\begin{aligned}
\sigma_{\mathrm{tot}} &= Im\langle T\widetilde{J}(q)\widetilde{J}(-q)\rangle \approx C_{\mathrm{3loop}}^2 \sum_n \frac{\Psi_n^*(0)\Psi_n(0)}{\sqrt{s}-2m_t-E_n+i\Gamma_t} \\
&= C_{\mathrm{3loop}}^2 \times \Big(\text{Potential Ins} + \text{Ultrasoft gluon}\Big)
\end{aligned}
$$



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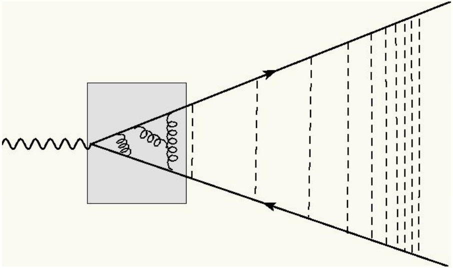

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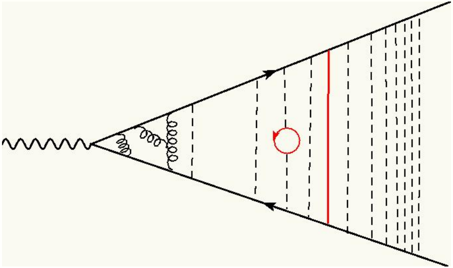
$$= C_{\rm 3loop}^2 \times \left(\text{Potential Ins} + \text{Ultrasoft gluon}\right)$$



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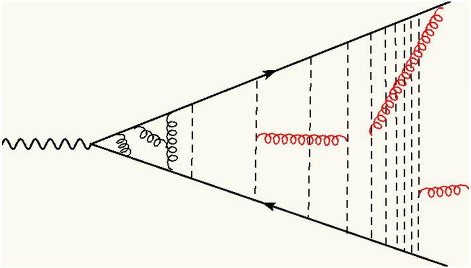
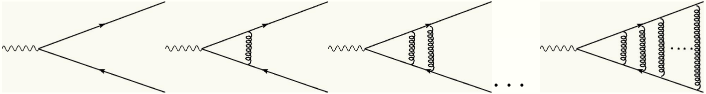


Part I

Quick Review of Effective Field Theory

Threshold cross section requires resummation of $\alpha_s/v \sim \mathcal{O}(1)$



- each gluon exchange yields Coulomb singularity, α_s/v

$\text{LO} \sim 1 + \frac{\alpha_s}{v} + (\frac{\alpha_s}{v})^2 + \cdots \sim \Sigma_n\, (\frac{\alpha_s}{v})^n$

$\text{NLO} \sim \Sigma_n\, \{\alpha_s,\, v\} \times (\frac{\alpha_s}{v})^n$

$\text{NNLO} \sim \Sigma_n\, \{\alpha_s^2,\, \alpha_s v,\, v^2\} \times (\frac{\alpha_s}{v})^n$

$\text{NNNLO} \sim \Sigma_n\, \{\alpha_s^3,\, \alpha_s^2 v,\, \alpha_s v^2,\, v^3\} \times (\frac{\alpha_s}{v})^n$

Potential NRQCD: systematic threshold resummation

- Integrate out Hard (Caswell-Lepage('86))

$$\mathcal{L}_{\rm NRQCD} = \psi^\dagger\left(iD_0 + \frac{\vec{D}^2}{2m_t}\right)\psi + [\psi \to \chi] + \cdots$$

- Integrate Soft/Pot (Pineda-Soto('98), Luke-Manohar-Rothstein('99))

$$\mathcal{L}_{\rm pNRQCD} = \psi^\dagger\left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_t}\right)\psi + \int d\vec{r}\,[\psi^\dagger\chi]\,V_{pot}(r)\,[\chi^\dagger\psi]$$
$$+ig\,\psi^\dagger\left[A_{0,us} + \frac{\nabla\vec{A}_{us}}{m}\right]\psi - \frac{1}{4}F_{us}^2 + \cdots$$

- Remaining Mode is Ultra Soft gluon: $k \sim m(v^2, \vec{v}^2)$

Potentials are Wilson Coeff: $V_{pot}(r)\left[\psi^\dagger\chi\right](r)\left[\chi^\dagger\psi\right](0)$

- $V_{pot} = -\frac{C_F\alpha_s}{r} + \frac{C_2}{r^2} + C_3\delta(\mathbf{r}) + \cdots$
- Corr to the Coulomb potential

$$\widetilde{V}_C = -\frac{4\pi C_F\alpha_s(\mathbf{q})}{\mathbf{q^2}} \times \left[1 + \frac{\alpha_s(\mathbf{q})}{4\pi}a_1 + \left(\frac{\alpha_s(\mathbf{q})}{4\pi}\right)^2 a_2 + \left(\frac{\alpha_s(\mathbf{q})}{4\pi}\right)^3\left[a_3 + 8\pi^2 C_A^3\left(\frac{1}{3\epsilon} + \ln\frac{\mu_{US}^2}{\mathbf{q}^2}\right)\right]\right]$$

  - a_2 Schröder('99); $a_{3,pade}$ Chishtie-Elias (01)
  - ADM IR Div; Appelquist-Dine-Muzinich ('78)

    Brambilla-Pineda-Soto-Vairo('99), Kniehl-Penin-Smirnov-Steinhauser(02)

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$\rightarrow 1/\epsilon$ ADM Divergence is renormalized

- a_2 Schröder('99); $a_{3,pade}$ Chishtie-Elias (01)
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  Brambilla-Pineda-Soto-Vairo('99), Kniehl-Penin-Smirnov-Steinhauser(02)

If honestly "integrating out" the modes to get EFT

Result is .....

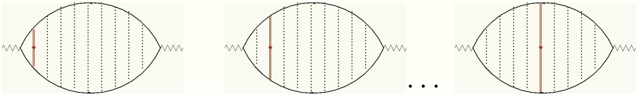
- Potentials are distributions
  $\delta V_\delta(\mathbf{r}) : \alpha_s \left(\frac{\mu^2}{\mathbf{q}^2}\right)^\epsilon \to \frac{\epsilon\alpha_s}{r^3}(\mu r)^{2\epsilon}$
- Potentials get $1/\epsilon$
  $\delta V_C^{(3)} \sim \frac{1}{\epsilon}\frac{C_A^3\alpha_s^4}{\mathbf{q}^2}\left(\frac{\mu^2}{\mathbf{q}^2}\right)^{3\epsilon} +$ finite term

Our strategy :

Point1: perform the calc in mom-space with DimReg honestly.
Point2: add and subtract $\Delta V_C = -\frac{1}{\epsilon}\frac{C_A^3\alpha_s^4}{\mathbf{q}^2}$,
(renormalization/reshuffling of $1/\epsilon$)
Point3: just be careful not to make a mistake

Part II

Insertion of N^3LO potentials



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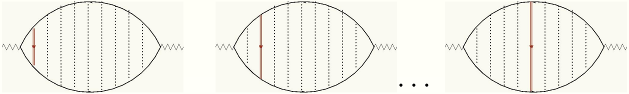

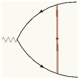

$$= 1/\epsilon + f_0(p) + O(\epsilon) f_1(p) + ...$$

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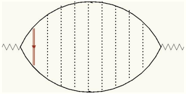
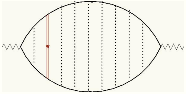
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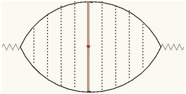
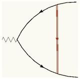
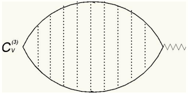




$$= 1/\epsilon + f_0(p) + O(\epsilon) f_1(p) + \dots$$



$$\delta C_V^{(3)} G_C^{DimReg}$$

structure of the potentials

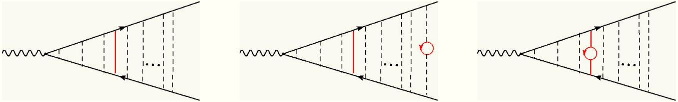
order of α_s	tree	1loop	2loop	3loop
$1/r$	○	○	○	○ (red)
$1/r^2$	None	○	○ (red)	✓
$1/r^3$	○	○ (red)	✓	✓
$\cdots$				

- ○ (red) are the N^{3}LO:$r \sim 1/(m\alpha_s) \Rightarrow \alpha_s^a \times (1/r)^b \sim \alpha_s^{a+b}$
- Counter term is added to divergent potential

e.g. $\delta V_{C,R}^{(3)} \equiv \frac{1}{\epsilon}\frac{c_A^3\alpha_s^4}{q^2}\left(\frac{\mu^2}{q^2}\right)^{3\epsilon} - \frac{1}{\epsilon}\frac{c_A^3\alpha_s^4}{q^2}$

$$\Delta V_C \equiv \frac{1}{\epsilon}\frac{c_A^3\alpha_s^4}{q^2}$$

Pot insertions to the wave functions



$$
\begin{aligned}
&\delta_3 |\Phi(0)|^2_{nonC} \\
&= |\Phi(0)|^2_C \times \frac{\alpha_s^3}{\pi} \Bigg\{ \left[\frac{C_A\, {C_F}^2}{8} + \frac{{C_F}^3}{12} \right] {L_m}^2 + \left[\frac{C_A\, {C_F}^2}{2} + \frac{{C_F}^3}{3} \right] L_m\, L_p + \Bigg[C_A\, {C_F}^2 \left(-\frac{5}{9} + \frac{1}{n} - \frac{S_1}{2} \right) \\
&+ {C_F}^3 \left(\frac{1}{12} + \frac{2}{3\,n} - \frac{S_1}{3} \right) + \frac{{C_F}^2\, T_F}{15} \Bigg] L_m + \Bigg[\frac{4\, {C_A}^2\, C_F}{3} + \frac{37\, C_A\, {C_F}^2}{12} + \frac{7\, {C_F}^3}{6} + \Big(2\, C_A\, C_F \\
&+ \frac{4\, {C_F}^2}{3} \Big) \beta_0 \Bigg] {L_p}^2 + \Bigg[C_A\, {C_F}^2 \left(\frac{226}{27} - \frac{5}{3\,n^2} + \frac{37}{3\,n} + \frac{8\,\log(2)}{3} - \frac{37\, S_1}{6} \right) + {C_A}^2\, C_F \Big(\frac{145}{18} + \frac{16}{3\,n} \\
&+ \frac{4\,\log(2)}{3} - \frac{8\, S_1}{3} \Big) + {C_F}^3 \left(-\frac{3}{2} + \frac{14}{3\,n} - \frac{7\, S_1}{3} \right) + \frac{2\, {C_F}^2\, T_F}{15} - \frac{109\, C_A\, C_F\, n_l\, T_F}{36} - \frac{59\, {C_F}^2\, n_l\, T_F}{27} \\
&+ \Big\{ {C_F}^2 \left(\frac{16}{3} - \frac{75}{16\,n^2} + \frac{10}{3\,n} - \frac{n\,\pi^2}{9} - \frac{4\, S_1}{3} + \frac{2\, n\, S_2}{3} \right) + C_A\, C_F \Big(\frac{15}{8} + \frac{5}{n} - \frac{n\,\pi^2}{6} - 2\, S_1 \\
&+ n\, S_2 \Big) \Big\} \beta_0 \Bigg] L_p + C^{nonC}_{\Psi,3} \Bigg\}.
\end{aligned}
$$

(Beneke-YK-Schuller, hep-ph/0705.4518)

Const of pot ins to the wave functions

$$
\begin{aligned}
C^{nonC}_{\Psi,3} &= {C_A}^2\, C_F \left[\frac{3407}{432} + \frac{133}{9\,n} - \frac{5\,\pi^2}{18} + \left(\frac{187}{108} + \frac{8}{3\,n} \right) \ln 2 - \frac{8\,\ln^2 2}{9} + \left(-\frac{145}{18} - \frac{16}{3\,n} \right.\right. \\
&\left.\left. - \frac{4\,\ln 2}{3} \right) S_1 + \frac{4\,{S_1}^2}{3} - \frac{4\,S_2}{3} \right] + {C_F}^3 \left[-\frac{137}{36} + \frac{35}{12\,n^2} - \frac{25}{6\,n} - \frac{49\,\pi^2}{432} + \left(\frac{3}{2} - \frac{14}{3\,n} \right) S_1 + \frac{7\,{S_1}^2}{6} \right. \\
&\left. - \frac{7\,S_2}{6} \right] + C_A\, {C_F}^2 \left[\frac{7061}{486} - \frac{321}{32\,n^2} + \frac{1475}{108\,n} + \left(-\frac{50}{81} + \frac{1}{9\,n} \right) \pi^2 + \left(\frac{353}{54} + \frac{16}{3\,n} \right) \ln 2 - \frac{16\,\ln^2 2}{9} \right. \\
&\left. + \left(-\frac{226}{27} - \frac{1}{n^2} - \frac{37}{3\,n} - \frac{8\,\ln 2}{3} \right) S_1 + \frac{37\,{S_1}^2}{12} + \left(-\frac{37}{12} - \frac{2}{3\,n} \right) S_2 \right] + {C_F}^2\, T_F \left[\frac{1}{15} + \frac{4}{15\,n} - \frac{2\,S_1}{15} \right] \\
&+ {C_F}^2\, n_l\, T_F \left[-\frac{3391}{486} + \frac{125}{24\,n^2} - \frac{118}{27\,n} + \frac{5\,\pi^2}{648} - \frac{2\,\ln 2}{27} + \frac{59\,S_1}{27} \right] + C_A\, C_F\, n_l\, T_F \left[-\frac{361}{108} - \frac{109}{18\,n} \right. \\
&\left. + \frac{49\,\ln 2}{108} + \frac{109\,S_1}{36} \right] + \left\{ C_A\, C_F \left[\frac{7}{24} - \frac{1}{4\,n} + \left(-\frac{91}{144} - \frac{5\,n}{24} \right) \pi^2 + \left(\frac{3}{2} + \frac{5\,n}{4} \right) S_2 + S_1 \left(-\frac{3}{8} - \frac{1}{2\,n} \right.\right.\right. \\
&\left.\left. + \frac{n\,\pi^2}{6} - n\,S_2 \right) + 2\,n\,S_3 - n\,S_{2,1} \right] + {C_F}^2 \left[\frac{1027}{648} + \frac{25}{24\,n^2} + \frac{19}{6\,n} + \left(-\frac{35}{108} + \frac{5}{16\,n} - \frac{11\,n}{27} \right) \pi^2 \right. \\
&\left.\left. + \left(1 - \frac{15}{8\,n} + \frac{22\,n}{9} \right) S_2 + S_1 \left(-\frac{10}{9} - \frac{45}{16\,n^2} - \frac{1}{3\,n} + \frac{n\,\pi^2}{9} - \frac{2\,n\,S_2}{3} \right) + \frac{4\,n\,S_3}{3} - \frac{2\,n\,S_{2,1}}{3} \right] \right\} \beta_0 \\
&+ \frac{1}{\epsilon} \left[\epsilon\, C_F^2 \left(\frac{v^m_{1,\epsilon}}{8} + \frac{v^q_{1,\epsilon}}{12} + \frac{w_{1,\epsilon}}{12} \right) - \epsilon\, \frac{C_F\, b_{2,\epsilon}}{6} \right].
\end{aligned}
$$

$$\delta_3|\Psi_1(0)|^2_{nC} = \frac{(m\alpha_s C_F)^3}{8\pi}\,\frac{\alpha_s^3}{\pi}\Big[\left(149.3 - 6.9n_f\right)L_p^2 + 0.9L_m^2 + 3.5L_p\,L_m + \left(449.8 - 21.9n_f\right)L_p + 0.8L_m + \left(-149.7 - 3.1n_f\right) + \delta_\epsilon\Big],$$

with unknown $\delta_\epsilon = \frac{1}{\epsilon_{\rm UV}}\left[\epsilon\, C_F^2\left(\frac{v_{1,\epsilon}^m}{8} + \frac{v_{1,\epsilon}^q}{12} + \frac{w_{1,\epsilon}}{12}\right) - \epsilon\,\frac{C_F}{6}\,b_{2,\epsilon}\right].$

"*Estimated*" δ_ϵ is less than 10% of the corr.

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- $\delta\widetilde{V}_{1/m} = \frac{4\pi^3 C_F\alpha_s}{mq}\left[\frac{\alpha_s}{4\pi}\,b_1(\epsilon)\left(\frac{\mu^2}{\mathbf{q^2}}\right)^\epsilon + \left(\frac{\alpha_s}{4\pi}\right)^2 4b_2(\epsilon)\left(\frac{\mu^2}{\mathbf{q^2}}\right)^{2\epsilon} + \cdots\right]$
- $b_2(\epsilon) = b_2(0) + \epsilon\, b_{2,\epsilon} + \cdots$

  ($b_2(0)$ is known Kniehl-Penin-Steinhauser-Smirnov (2002))
- Assumption: $b_{2,\epsilon} \sim \pm 2b_2(0)$
- 1-loop $\mathcal{O}(\epsilon)$ parameters: $v_{1,\epsilon}, w_{1,\epsilon}$ were calculated by S. Wüster, (Dipl Th., 2003)

$$\delta_3|\Psi_1(0)|^2_{nC} = \frac{(m\alpha_s C_F)^3}{8\pi}\frac{\alpha_s^3}{\pi}\Big[\left(149.3-6.9n_f\right)L_p^2+0.9L_m^2+3.5L_p\,L_m + \left(449.8-21.9n_f\right)L_p+0.8L_m+\left(-149.7-3.1n_f\right)+\delta_\epsilon\Big],$$

with unknown $\delta_\epsilon = \frac{1}{\epsilon_{\rm UV}}\left[\epsilon\, C_F^2\left(\frac{v^m_{1,\epsilon}}{8}+\frac{v^q_{1,\epsilon}}{12}+\frac{w_{1,\epsilon}}{12}\right)-\epsilon\,\frac{C_F}{6}\,b_{2,\epsilon}\right].$

"*Estimated*" δ_ϵ is less than 10% of the corr.

The non-Coulomb Corr to the toponium wave func ($\delta_\epsilon = 0$):

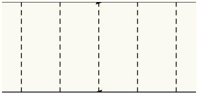
- $\delta_3|\Psi_1(0)|^2_{nC}/|\Psi_1^{(0)}(0)|^2 = -0.14$ at $\mu = 32.6$ GeV
- $\delta_3|\Psi_1(0)|^2_{nC}/|\Psi_1^{(0)}(0)|^2 = 0.36$ at $\mu = 175$ GeV

The strong scale dependence $\Leftrightarrow$ US, Wilson coeff scale dependencies.

Part III

Ultra Soft gluon exchange

Dress the LO diagram by ultrasoft gluon in $\forall$ way.

$\cdots$ 

$$\delta\mathcal{L} = +ig\,\psi^\dagger\Big[A_{0,us} + \frac{\nabla \vec{A}_{us}}{m}\Big]\psi + \frac{C_A \vec{r}.\vec{A}_{us}}{r} \otimes [\psi^\dagger\chi][\chi^\dagger\psi]\cdots$$

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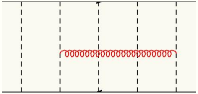
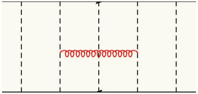
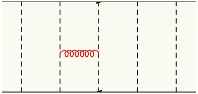
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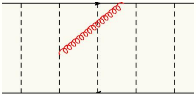
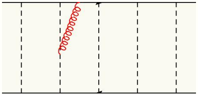
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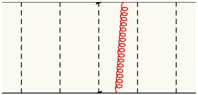
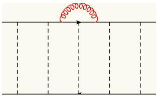
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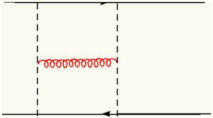
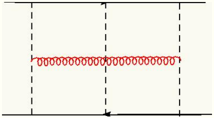
$1/\epsilon$ cancelation in static limit (Coulomb potential)





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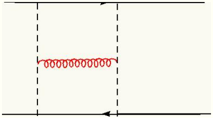

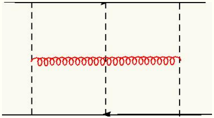

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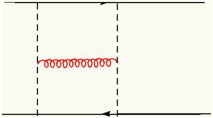

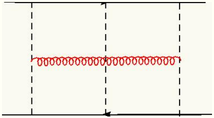

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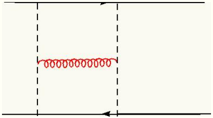

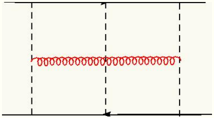

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UV divergence cancelation mechanism:

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\begin{aligned}
\delta V_C^{\mathrm{ADM}} &= \frac{1}{\epsilon}\frac{c_A^3\alpha_s^4}{\mathbf{q}^2} \\
\mathrm{H}_0 &= -\frac{\alpha_s^3 c_A^2}{\epsilon}\left(\frac{1}{mq} + \frac{2(p^2/m - E)}{\mathbf{q}^3} + \epsilon L_{Bethe}\right) + O(\epsilon) \\
\mathrm{H}_1 &= -\frac{1}{\epsilon}\frac{c_A^2(c_A - 2c_F)\alpha_s^4}{\mathbf{q}^2} + L_{Bethe} + O(\epsilon)
\end{aligned}
\tag{1}
$$

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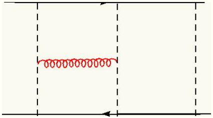

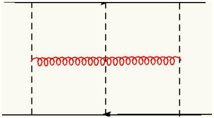

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\end{aligned} \qquad (1)$$

$$\text{eq. of motion} \quad : \quad \frac{(p^2/m - E)}{\mathbf{q}^3} \Rightarrow \frac{c_F\alpha_s}{r}$$

There are exceptions always...

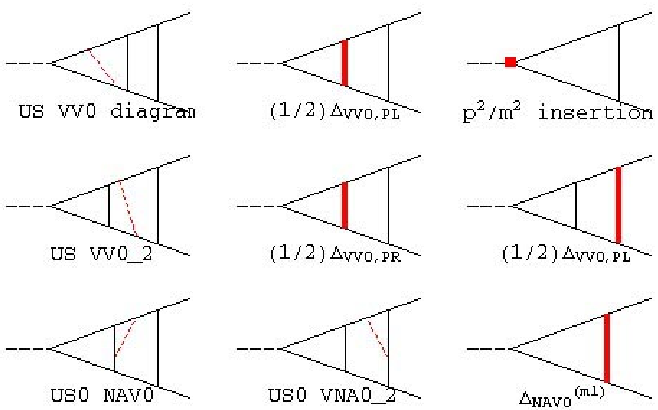
The $1/\epsilon$ cancelations happen only when potential loops are finite

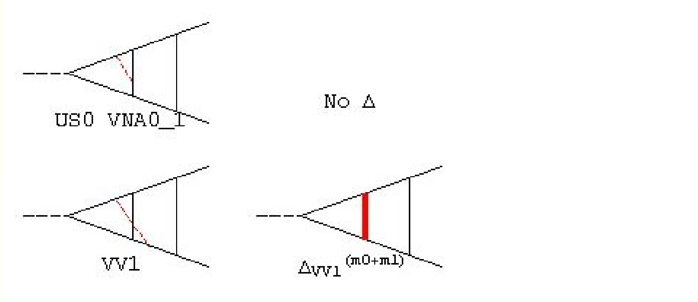
- ultra soft gluon near photon vertex is potential divergent
  $\Leftrightarrow$ external operator renormalization in QFT
- potential counter terms $\Leftrightarrow$ local $1/\epsilon$ structure
- Remaining $1/\epsilon$ taken care by $\delta C^{WilsonCoeff}_{Bare}$

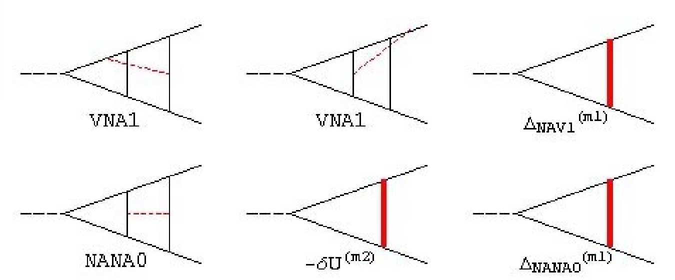
Kniehl-Penin-Steinhauser-Smirnov '01,'03, Manohar-Stewart '01, Hoang '04

We studied the structure of UV divergences, and found that dimensional regularization perfectly works:
Finite linear divergence in DimReg
$\Leftrightarrow$ bad high energy behavior in momentum
$\Rightarrow$ next loop integrals can be divergent
$\Rightarrow$ three-loop calc needed at most at N^{3}LO

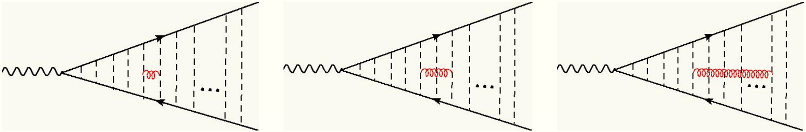
US exchange near γ vertex is exception




US0 VNA0_1
No Δ
VV1
$\Delta_{VV1}{}^{(m0+m1)}$


VNA1
VNA1
$\Delta_{NAV1}{}^{(m1)}$
NANA0
$-\delta U^{(m2)}$
$\Delta_{NANA0}{}^{(m1)}$

Result: Ultrasoft corrections



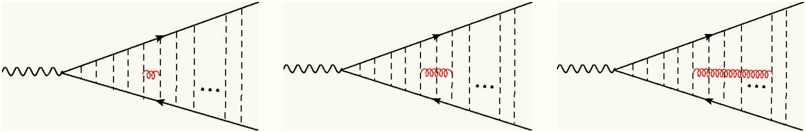
$$\frac{\delta|\psi_1^2|}{|\psi_{C,1}^2|} = \frac{\alpha_s^3}{\pi}\Big[(-66.9_0 - 3.05_1)\,\mathrm{L_{us}} - 58.8\mathrm{L_p}^2 - 57.4\mathrm{L_p}$$
$$-5.5\mathrm{L_m^2} + (8.7 + 43.7\mathrm{L_p})\mathrm{L_m} + 351.2\Big],$$

(Beneke-YK-Penin)

with $\mathrm{L_{us}} = \ln\frac{e^{5/6}\mu}{2m\alpha_s^2}$, $\mathrm{L_p} = \ln\frac{n\mu}{mC_F\alpha_s}$, $\mathrm{L_m} = \ln\frac{\mu}{m}$,

- The constant part: $351.2(\alpha_s^3/\pi) \sim 7$ %
- analytic log terms, and numerical constants ($n \leq 6$) obtained
- The X section was evaluated numerically (in preparation)

Result: Ultrasoft corrections



$$\frac{\delta E_1}{E_C} = \frac{\alpha_s^3}{\pi}\Big[\big(-42.81_0 - 1.784_1\big)\ln\big(\frac{\mu}{m\alpha_s^2}\big)$$
$$+88.86_0 + 3.783_1 + 0.04426_\infty + \text{pot terms}\Big]$$

(Beneke-YK-Penin)

- This gives $L_E = -78.20_0 - 3.310_1 - 0.0280_\infty = -81.54$ agrees with Kniehl-Penin(2000).

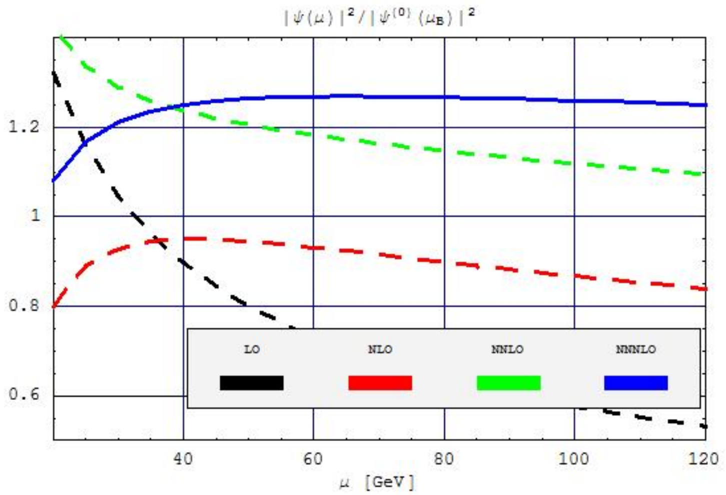
To reminde you, The X section formulae

$$\sigma_{\text{tot}} = Im\langle T\widetilde{J}(q)\widetilde{J}(-q)\rangle \approx C_{3\text{loop}}^2 \sum_n \frac{\Psi_n^*(0)\Psi_n(0)}{\sqrt{s} - 2m_t - E_n + i\Gamma_t}$$

$$= C_{3\text{loop}}^2 \times \Big(\text{Potential Ins} + \text{Ultrasoft gluon}\Big)$$

- All the correction to the wave function are combined
- Physical scale invariant "wave fubctuib" at the origin is $C^2\Psi_n^*(0)\Psi_n(0)$

wave func at the origin



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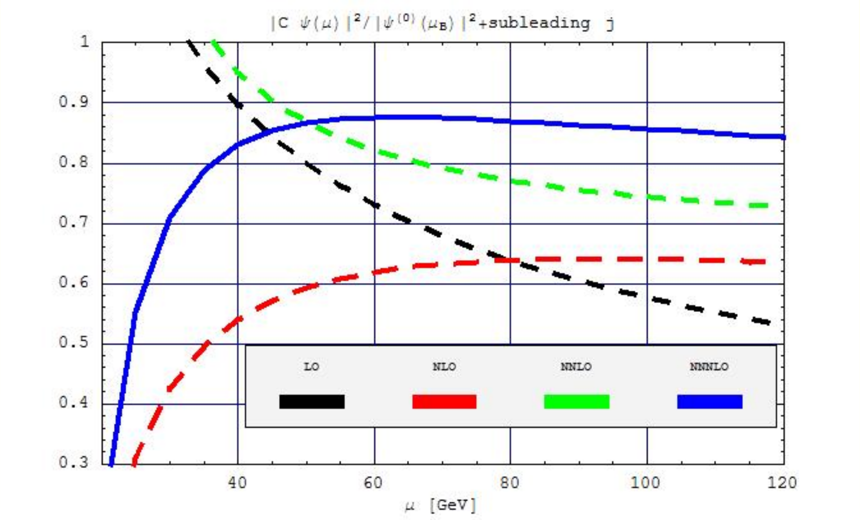

Summary

We have completed the NNNLO quarkonium wave function calculation (Pot Ins + US effect), parameterizing unknown $\mathcal{O}(\epsilon)$ potentials.

- All the logarithm were obtained analytically, and the US constant part numerically

(c.f. Kniehl-Penin-Steinhauser-Smirnov '01,'03, Manohar-Stewart '01, Hoang '04)

- The size of the corr is about 10 %, and the scale dependence becomes milder (5% variation if $40 < \mu < 120$ GeV)
- So we are waiting for C_v^{3loop}, and several input potential parameters

There are several issues to be discussed,

- What's the good scale choice for US corr? RG improvement?
- How large the continuum contribution?
- Interplay between electro-weak and QCD

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- Rather new aspect... and people started to think seriously