CP-violating loop effects in the Higgs sector of the MSSM

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In collaboration with *T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, K. Williams*

- Introduction
- New results for Higgs masses, mixings and decays in the MSSM with complex phases
- Numerical analysis
- Conclusions

Introduction

MSSM Higgs potential contains two Higgs doublets:

$$V_H = m_1^2 H_{1i}^* H_{1i} + m_2^2 H_{2i}^* H_{2i} - \epsilon^{ij} (m_{12}^2 H_{1i} H_{2j} + {m_{12}^2}^* H_{1i}^* H_{2j}^*)$$

$$+ \frac{1}{8}(g_1^2 + g_2^2)(H_{1i}^* H_{1i} - H_{2i}^* H_{2i})^2 + \frac{1}{2} g_2^2 |H_{1i}^* H_{2i}|^2$$

$$\begin{pmatrix} H_{11} \\ H_{12} \end{pmatrix} = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}$$

$$\begin{pmatrix} H_{21} \\ H_{22} \end{pmatrix} = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}$$

Complex phases $\arg(m_{12}^2)$, ξ can be rotated away

$\Rightarrow$ Higgs sector is $\mathcal{CP}$-conserving at tree level

$\mathcal{CP}$ violation in the Higgs sector

Five physical states; tree level: $h^0, H^0, A^0, H^\pm$

Complex parameters enter via (often large) loop corrections:

– μ: Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings

– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– $m_{\tilde{g}}$: gluino mass

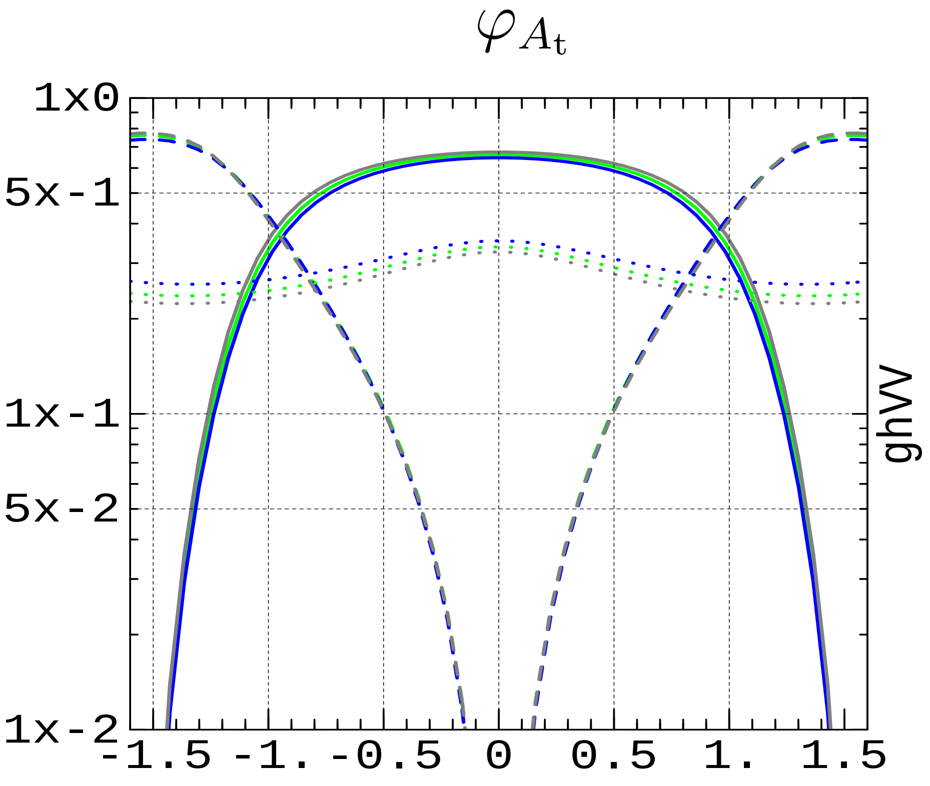
$\Rightarrow$ $\mathcal{CP}$-violating mixing between neutral Higgs bosons h_1, h_2, h_3

Lowest-order Higgs sector has two free parameters

$\Rightarrow$ choose $\tan\beta \equiv \frac{v_2}{v_1}$, $M_{\mathrm{H}^\pm}$ as input parameters

Impact of complex phases

Example: g^2_{hVV} for h_1, h_2, h_3: [*M. Frank, S. Heinemeyer, W. Hollik, G. W. '03*]



full: h_1, dashed: h_2, dotted: h_3

Parameters:

$M_{\rm SUSY} = 500$ GeV,

$M_2 = 500$ GeV,

$\mu = 2000$ GeV,

$|A_t| = 1000$ GeV,

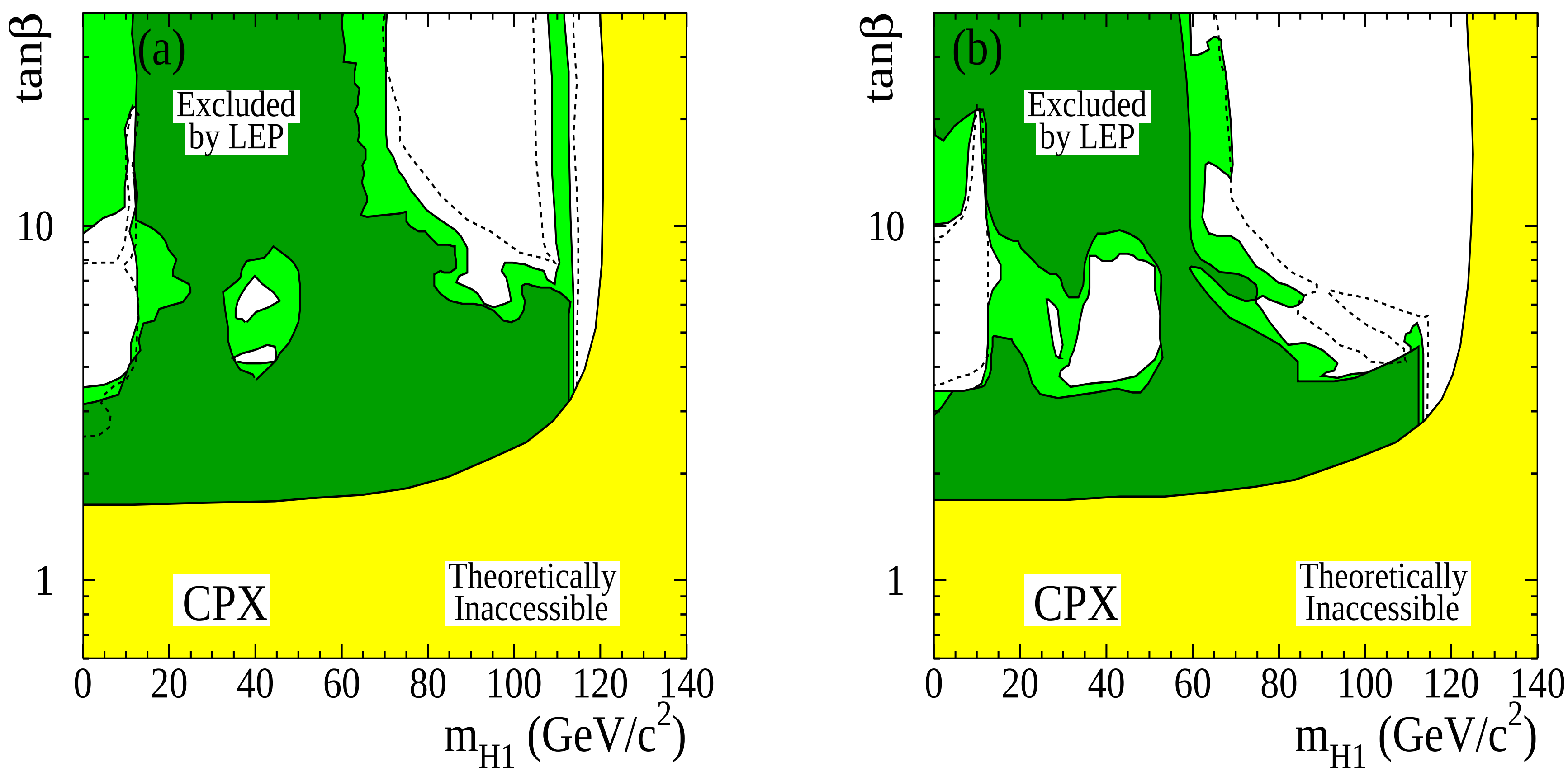
$M_{H^\pm} = 150$ GeV, $\tan\beta = 5$

$\Rightarrow$ Complex phases can have large effects on Higgs couplings

$\mathcal{CP}$-violating case (CPX scenario): LEP exclusion bounds

[*LEP Higgs Working Group '06*]

$m_\mathrm{t} = 169.3$ GeV $\qquad$ $m_\mathrm{t} = 174.3$ GeV



⇒ no lower limit on M_{h_1}: light SUSY Higgs not ruled out!

sensitive dependence on m_t

Reason for “CPX holes”

- Suppressed coupling of light Higgs, h_1, to gauge bosons over wide regions of parameter space

- Second-lightest Higgs, h_2, may be within LEP reach (with reduced VVh_2 coupling), h_3 beyond LEP reach

- Large $\mathrm{BR}(h_2 \to h_1 h_1) \Rightarrow$ difficult final state

$\Rightarrow$ Precise prediction for $\mathrm{BR}(h_2 \to h_1 h_1)$ needed for analysis of Higgs exclusion bounds

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- Future:

  Want to confront predictions with experimental measurements

  LHC: $\Delta M_{\mathrm{h}}^{\mathrm{exp}} \approx 0.2 \text{ GeV}$, ILC: $\Delta M_{\mathrm{h}}^{\mathrm{exp}} \approx 0.05 \text{ GeV}$ for a light SM-like Higgs

Theoretical uncertainties in the predictions for the observables in the MSSM Higgs sector

- Uncertainty from unknown higher-order corrections:

  Example: $\Delta M_{\rm h}^{\rm theo} \approx 2\text{–}3\ \mathrm{GeV}$, [*G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02*]
  [*S. Heinemeyer, W. Hollik, G. W. '06*]

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- Parametric uncertainty induced by the experimental errors of the input parameters

  Dominant effect: experimental error of $m_{\rm t}$

  $\Rightarrow$ ILC will yield improvement by an order of magnitude

  exp. error on $m_{\rm t}$: $\approx 1~{\rm GeV} \xrightarrow{\text{ILC + GigaZ}} 0.1~{\rm GeV}$

New results for Higgs masses, mixings and decays in the MSSM with complex phases

Complete one-loop results for masses and mixings with complex parameters
[*M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. W. '06*]

Two-loop $\mathcal{O}(\alpha_t \alpha_s)$ corrections
[*S. Heinemeyer, W. Hollik, H. Rzehak, G. W. '07*]

Complete one-loop results for $\Gamma(h_2 \to h_1 h_1)$, $\Gamma(h_i \to f\bar{f})$ with complex parameters
[*K. Williams, G. W. '07*]

Results obtained in Feynman-diagrammatic approach, previous results were based on renormalisation-group improved effective potential approach ($\to$ program *CPsuperH*)
[*A. Pilaftsis, C. Wagner '99*], [*M. Carena, J. Ellis, A. Pilaftsis, C. Wagner '00*], [*J. Lee et al. 03*]

Higher-order corrections in the MSSM Higgs sector with $\mathcal{CP}$-violating phases

Mixing between h, H, A

$\Rightarrow$ loop-corrected masses obtained from propagator matrix

$$\Delta_{hHA}(p^2) = -\left(\hat{\Gamma}_{hHA}(p^2)\right)^{-1}, \quad \hat{\Gamma}_{hHA}(p^2) = i\left[p^2 \mathbb{1} - \mathrm{M_n}(p^2)\right]$$

where

$$\mathrm{M_n}(p^2) = \begin{pmatrix} m_{\mathrm{h}}^2 - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hH}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ -\hat{\Sigma}_{hH}(p^2) & m_{\mathrm{H}}^2 - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ -\hat{\Sigma}_{hA}(p^2) & -\hat{\Sigma}_{HA}(p^2) & m_{\mathrm{A}}^2 - \hat{\Sigma}_{AA}(p^2) \end{pmatrix}$$

$\Rightarrow$ Higgs propagators: $\Delta_{ii}(p^2) = \dfrac{i}{p^2 - m_i^2 + \hat{\Sigma}_{ii}^{\mathrm{eff}}(p^2)}$

Higher-order corrections in the MSSM Higgs sector with $\mathcal{CP}$-violating phases

$$\hat{\Sigma}^{\rm eff}_{ii}(p^2) = \hat{\Sigma}_{ii}(p^2) - i\,\frac{2\hat{\Gamma}_{ij}(p^2)\hat{\Gamma}_{jk}(p^2)\hat{\Gamma}_{ki}(p^2) - \hat{\Gamma}^2_{ki}(p^2)\hat{\Gamma}_{jj}(p^2) - \hat{\Gamma}^2_{ij}(p^2)\hat{\Gamma}_{kk}(p^2)}{\hat{\Gamma}_{jj}(p^2)\hat{\Gamma}_{kk}(p^2) - \hat{\Gamma}^2_{jk}(p^2)}$$

Complex pole $\mathcal{M}^2$ of each propagator is determined from

$$\mathcal{M}_i^2 - m_i^2 + \hat{\Sigma}^{\rm eff}_{ii}(\mathcal{M}_i^2) = 0,$$

where

$$\mathcal{M}^2 = M^2 - iM\Gamma,$$

Expansion up to first order in Γ around M^2:

$$M_i^2 - m_i^2 + \text{Re}\,\hat{\Sigma}^{\rm eff}_{ii}(M_i^2) + \frac{\text{Im}\,\hat{\Sigma}^{\rm eff}_{ii}(M_i^2)\,\left(\text{Im}\,\hat{\Sigma}^{\rm eff}_{ii}\right)'(M_i^2)}{1 + \left(\text{Re}\,\hat{\Sigma}^{\rm eff}_{ii}\right)'(M_i^2)} = 0$$

Wave function normalisation for amplitudes with external Higgs bosons

Correct on-shell properties of the S matrix $\Leftrightarrow$ finite wave-function normalisation factors

$$\sqrt{\hat{Z}_i}\left(\Gamma_i \;+\; \hat{Z}_{ij}\Gamma_j \;+\; \hat{Z}_{ik}\Gamma_k + \ldots\right)$$

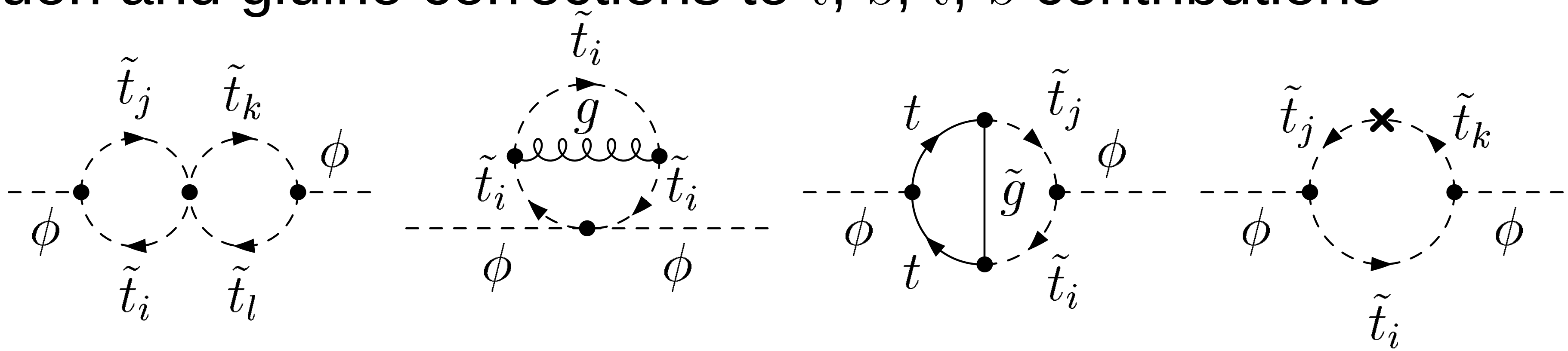
where

$$\hat{Z}_i = \frac{1}{1 + \left(\text{Re}\,\hat{\Sigma}_{ii}^{\text{eff}}\right)'(M_i^2)}$$

$$\hat{Z}_{ij} = \frac{\hat{\Sigma}_{ij}(M_i^2)\left(M_i^2 - m_k^2 + \hat{\Sigma}_{kk}(M_i^2)\right) - \hat{\Sigma}_{jk}(M_i^2)\hat{\Sigma}_{ki}(M_i^2)}{\hat{\Sigma}_{jk}^2(M_i^2) - \left(M_i^2 - m_j^2 + \hat{\Sigma}_{jj}(M_i^2)\right)\left(M_i^2 - m_k^2 + \hat{\Sigma}_{kk}(M_i^2)\right)}$$

Leading two-loop QCD corrections

Gluon and gluino corrections to t, b, $\tilde{\mathrm{t}}$, $\tilde{\mathrm{b}}$ contributions



Leading $\mathcal{O}(\alpha_\mathrm{t}\alpha_s)$ corrections: 2-loop contrib. evaluated in limit of vanishing gauge couplings, external momentum: $p^2 \to 0$

Renormalisation:

- 2-loop renormalisation in the Higgs sector,

  independent parameters: $M_{\mathrm{H}^\pm}$, $\tan\beta$

- 1-loop renormalisation in the $\tilde{\mathrm{t}}$, $\tilde{\mathrm{b}}$ sector, need also renormalisation of complex phase φ_{A_t}

Stop and sbottom mass matrices

$$\mathbf{M}_{\tilde{q}} = \begin{pmatrix} M_L^2 + m_q^2 + M_Z^2 c_{2\beta}(T_q^3 - Q_q s_\mathrm{w}^2) & m_q X_q^* \\ m_q X_q & M_{\tilde{q}_R}^2 + m_q^2 + M_Z^2 c_{2\beta} Q_q s_\mathrm{w}^2 \end{pmatrix}$$

with

$$X_q = A_q - \mu^* \kappa \;, \qquad \kappa = \{\cot\beta, \tan\beta\} \quad \text{for} \quad q = t, b$$

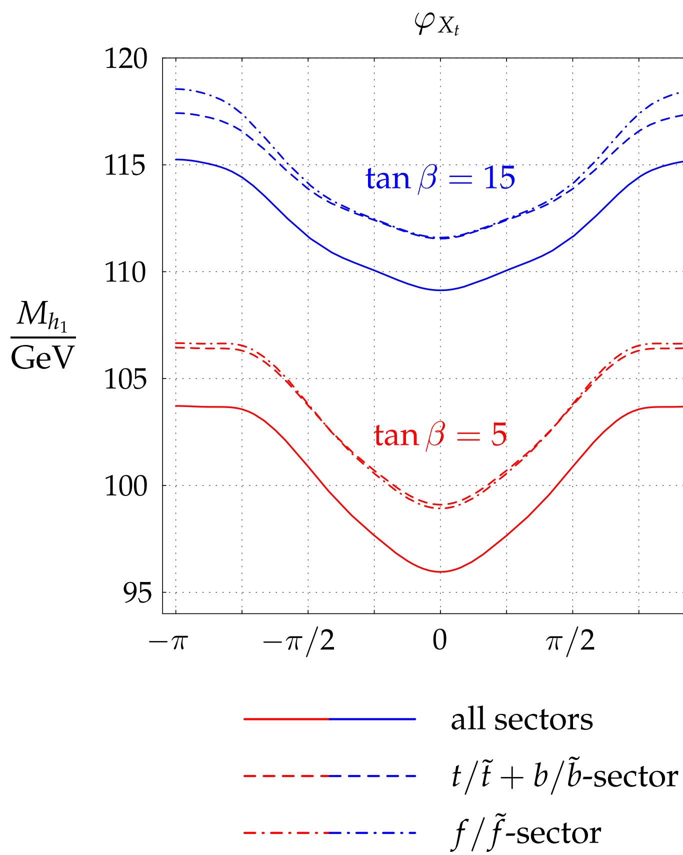
$\Rightarrow$ Mass eigenvalues $m_{\tilde{q}_1}^2$, $m_{\tilde{q}_2}^2$, mixing angle $\theta_{\tilde{q}}$, phase $\varphi_{\tilde{q}}$

Mass eigenvalues $m_{\tilde{q}_1}^2$, $m_{\tilde{q}_2}^2$ depend only on $|X_q|$

Effect of varying φ_{X_t} on the prediction for the lightest Higgs mass, one-loop corrections

Impact of the different sectors of the MSSM ($M_{H^\pm} = 150$ GeV):

[*M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. W. '06*]



$\Rightarrow$ Variation of φ_{X_t} leads to shift in M_{h_1} of up to 8 GeV

Shift of ≈ 3 GeV from corrections beyond $f/\tilde{f}$ loops

Impact of complex phases at the two-loop level

$\mathcal{O}(\alpha_t \alpha_s)$ corrections depend only on the phase combinations

$$\mu\, A_t\; \left(m_{12}^2\right)^* \quad \text{and} \quad A_t\, M_3^*$$

Phase of m_{12}^2 has been rotated away (see above)

$\Rightarrow$ Analyse the dependence on the phases of A_t (X_t) and M_3

Variation of φ_{A_t} for fixed μ, $\tan\beta$

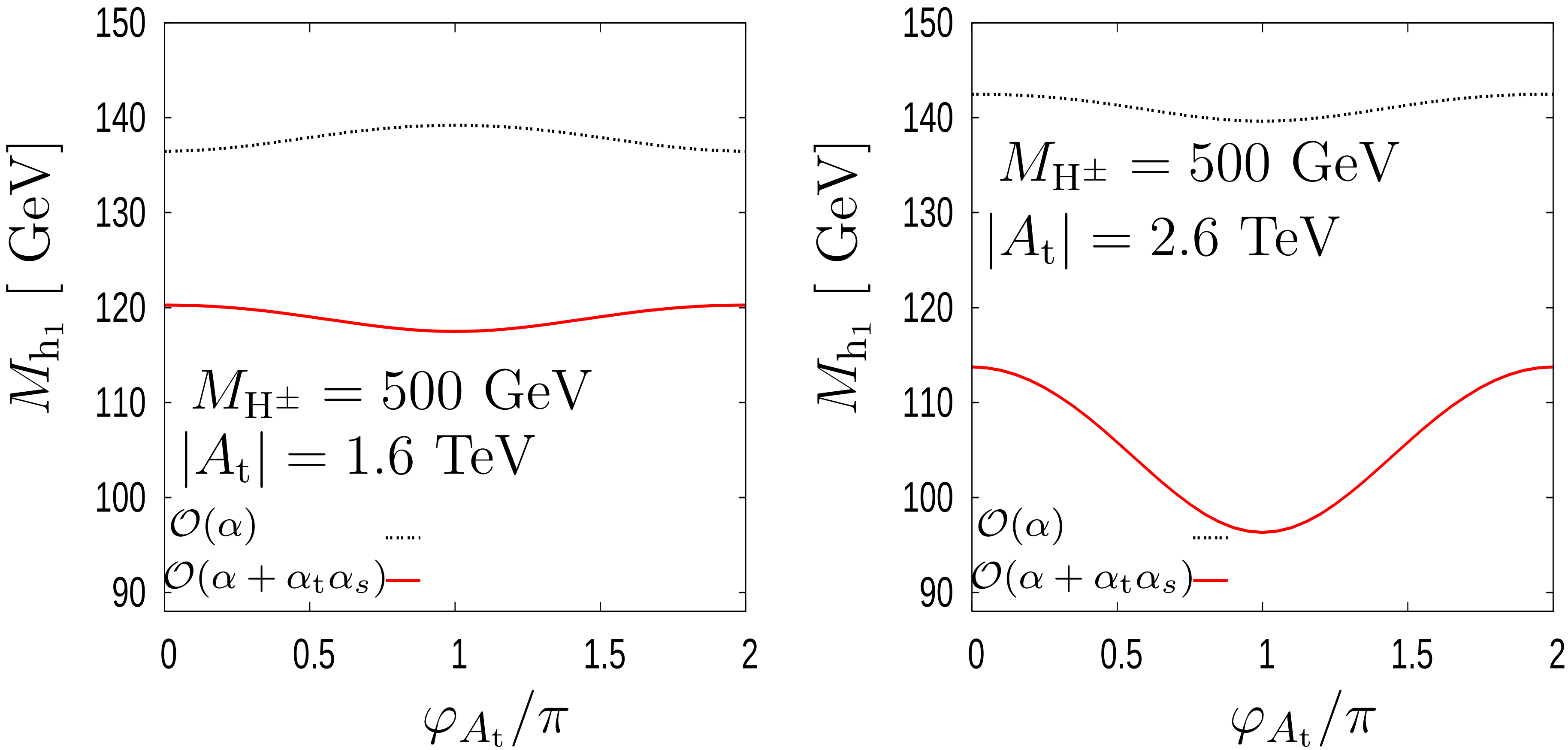
$\Rightarrow$ change of $|X_t|$ $\Rightarrow$ change of stop masses

Variation of φ_{X_t}

$\Rightarrow$ change of A_t, stop masses stay the same

Dependence of prediction for M_{h_1} on φ_{A_t}: one-loop vs. two-loop

[*S. Heinemeyer, W. Hollik, H. Rzehak, G. W. '07*] $\mu = 1$ TeV, $\tan\beta = 10$



$\Rightarrow$ Two-loop corrections significantly enhance the effects of the complex phase φ_{A_t}, sizable effects for large $|A_\mathrm{t}|$

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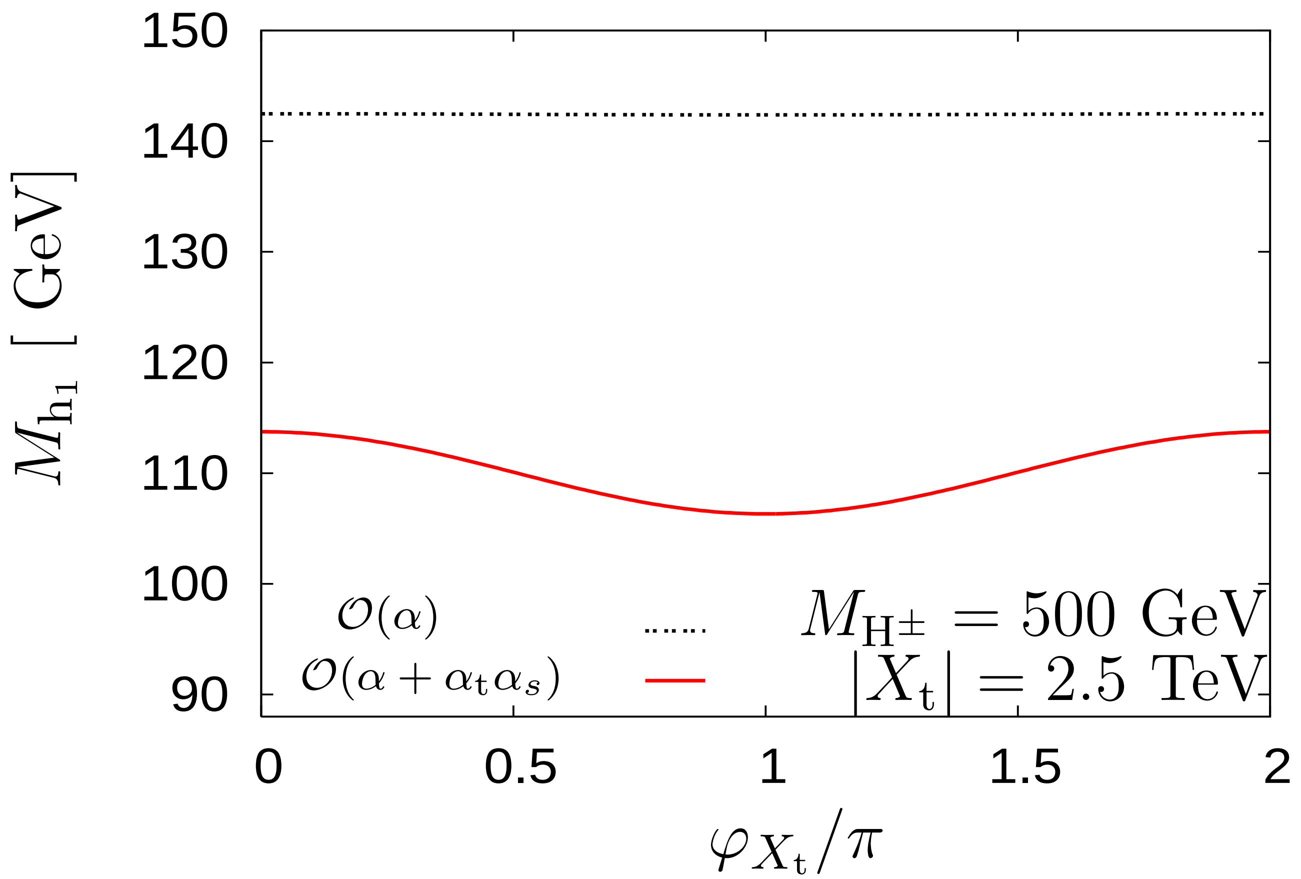

$\Rightarrow$ One-loop: very weak dependence on φ_{X_t}

Two-loop: large change in phase dependence

Reason for the large impact of the phase in the two-loop contribution

Leading one-loop result in the limit $M_{\mathrm{H}^\pm} \gg M_{\mathrm{Z}}$ depends only on the absolute value $|X_{\mathrm{t}}| \equiv |A_{\mathrm{t}} - \mu^* / \tan\beta|$

$\Leftrightarrow$ only combination $\varphi_{A_{\mathrm{t}}} + \varphi_\mu$ enters

$\Rightarrow$ weak dependence of one-loop result on $\varphi_{X_{\mathrm{t}}}$

dependence on $\varphi_{A_{\mathrm{t}}}$ mainly through $|X_{\mathrm{t}}|$

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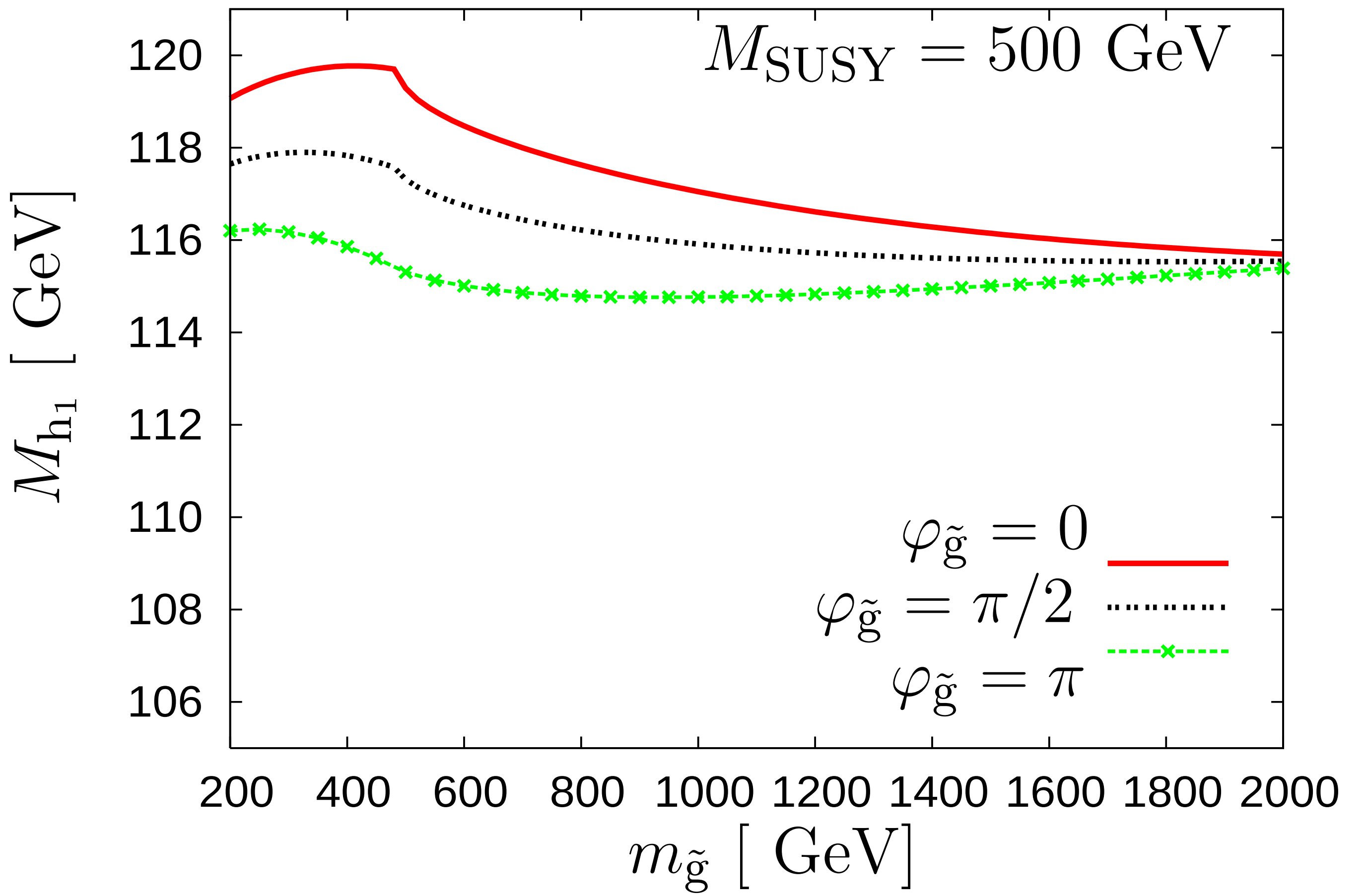
Two-loop level:

$\Rightarrow$ Gluino contributions introduce dependence on phase combination $A_{\mathrm{t}}\, M_3^*$

$\Rightarrow$ Large modification of phase dependence

Effect of the gluino phase on the prediction for M_{h_1}

[*S. Heinemeyer, W. Hollik, H. Rzehak, G. W. '07*]

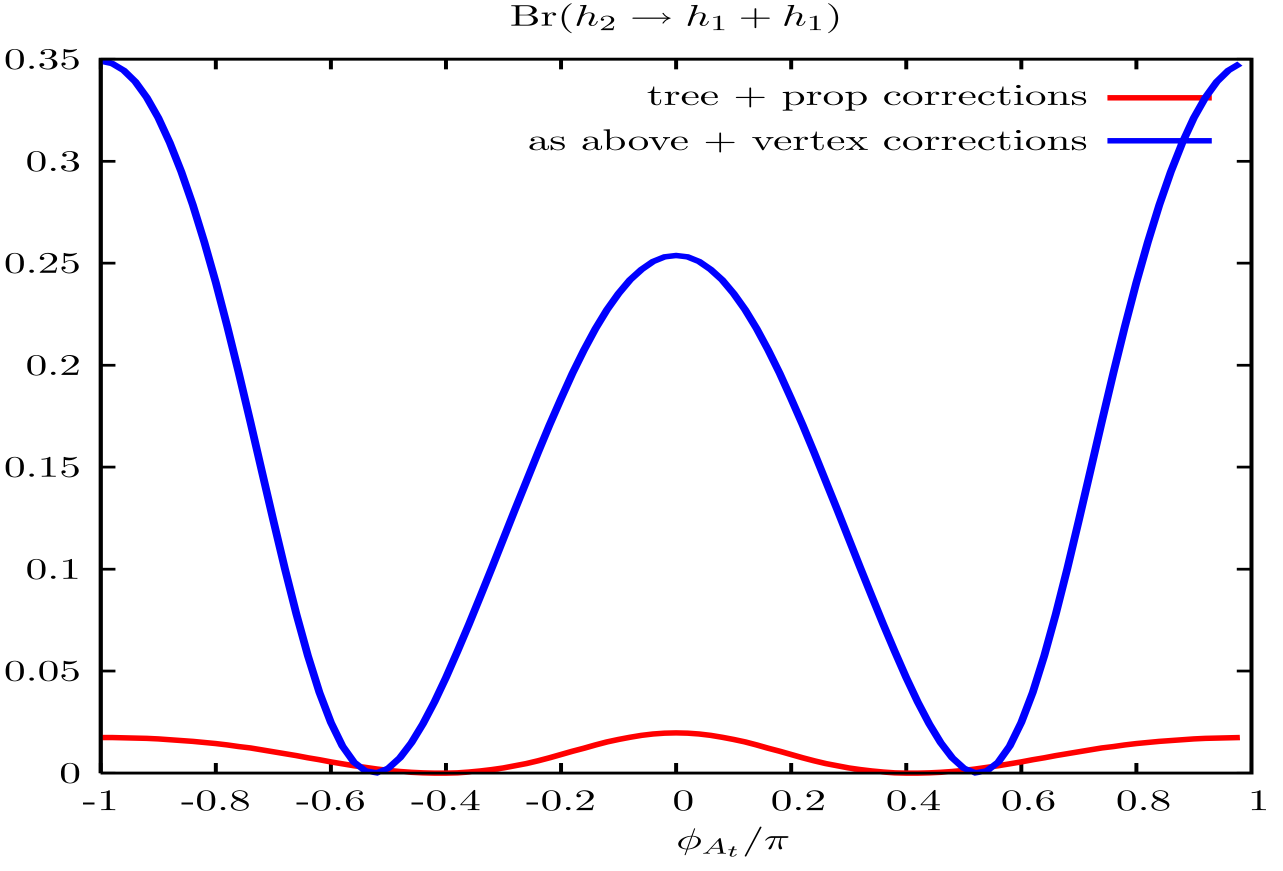


$\Rightarrow$ Sizable effects in the threshold region,

$m_{\tilde{g}} \approx m_{\tilde{t}_1} - m_t$, $m_{\tilde{g}} \approx m_{\tilde{t}_2} - m_t$

Dependence of $\mathrm{BR}(h_2 \to h_1 h_1)$ on φ_{A_t}: Impact of generic 1-loop vertex correction

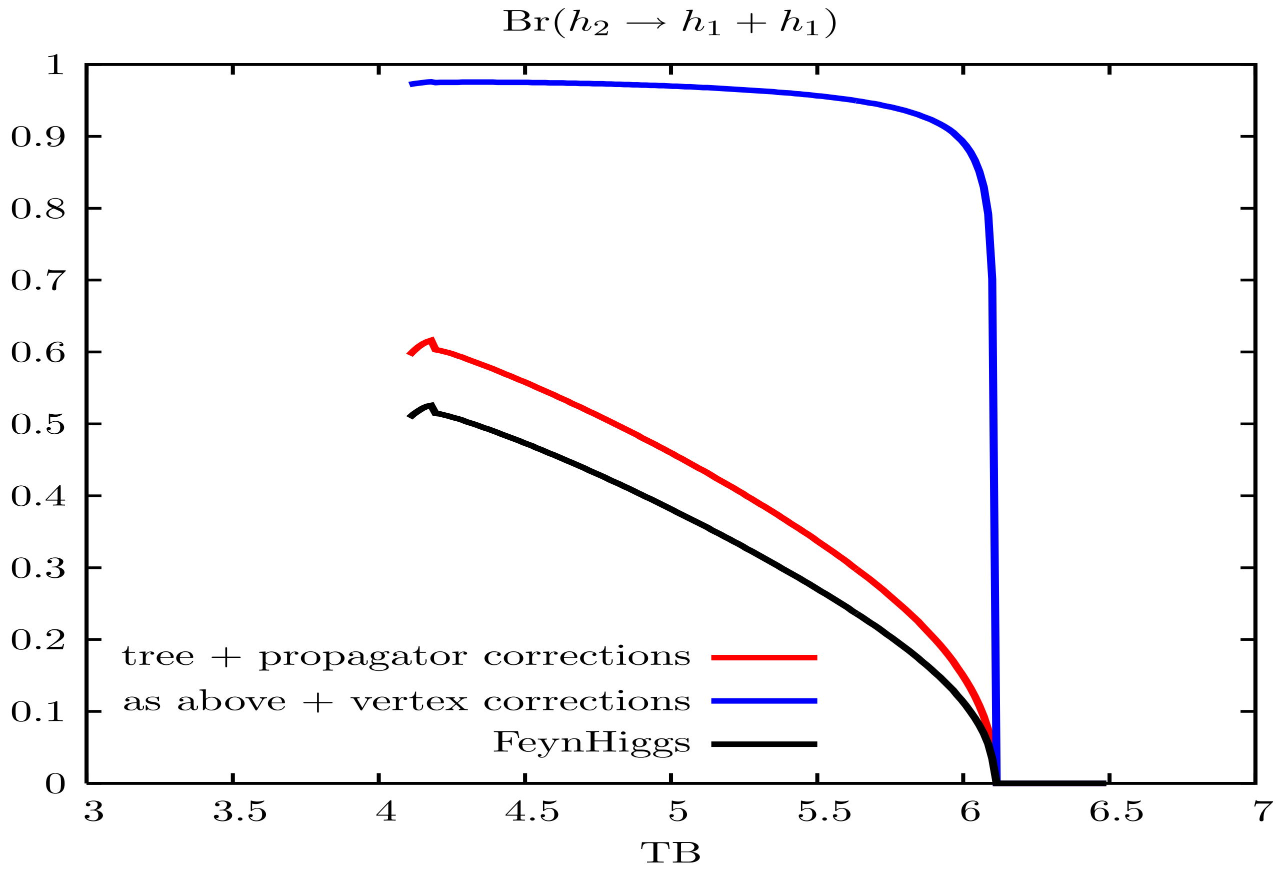
New result for $h_2 \to h_1 h_1$: complete 1-loop vertex contributions $+$ 2-loop masses $+$ 2-loop Z factors [*G. W., K. Williams '07*]



$\Rightarrow$ Very pronounced phase dependence, dominated by generic vertex corrections

$\mathrm{BR}(h_2 \to h_1 h_1)$ ***in CPX scenario (**$\varphi_{M_3} = 0$**)***

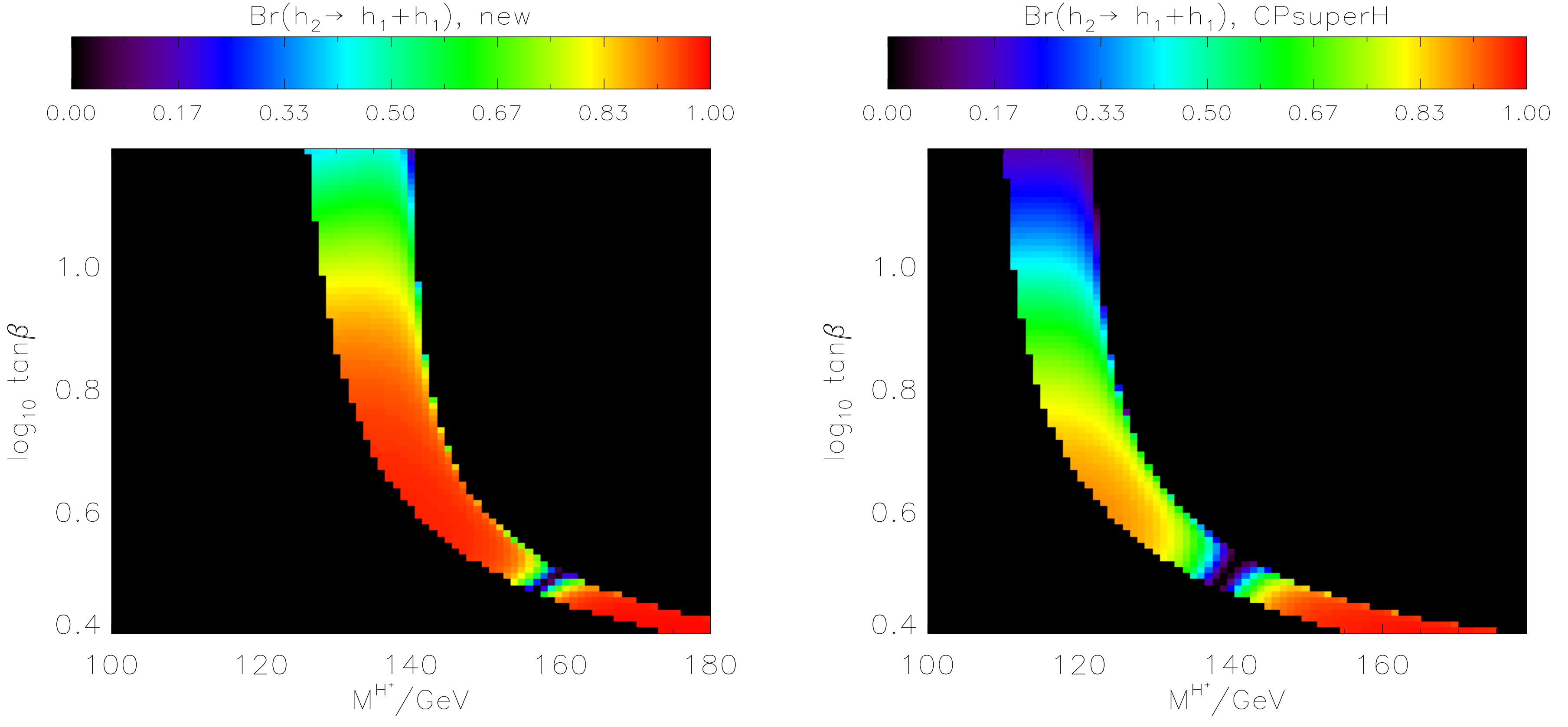
Full result vs. result with tree-level vertex vs. “improved Born approximation” used so far in *FeynHiggs*:
[*G. W., K. Williams ’07*]



$\Rightarrow$ Generic vertex corrections have large impact on BR

Comparison of $\mathrm{BR}(h_2 \to h_1 h_1)$*, CPX scen.,* $\varphi_{M_3} = 0$*:*
New diagramm. result (left) vs. CPsuperH (right)

Comparison takes into account conversion of $|A_\mathrm{t}|$ from on-shell scheme to $\overline{\mathrm{DR}}$ scheme [*G. W., K. Williams '07*]



$\Rightarrow$ Qualitative agreement, $\mathrm{BR}(h_2 \to h_1 h_1)$ enhanced

$\Rightarrow$ Confirmation of "CPX holes"

Conclusions

- New results in MSSM Higgs sector with complex param.:

  Complete one-loop results for masses, mixings, $\Gamma(h_2 \to h_1 h_1)$, $\Gamma(h_i \to f\bar{f})$ $+$ two-loop $\mathcal{O}(\alpha_t \alpha_s)$ corrections

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- Complex phases can have large impact on Higgs phenomenology:
  - 2-loop contrib. yield large enhancement of phase dep.
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- Outlook:
  - Implementation of new results into *FeynHiggs*
  - Detailed comparison with *CPsuperH*