Running and Decoupling in the MSSM

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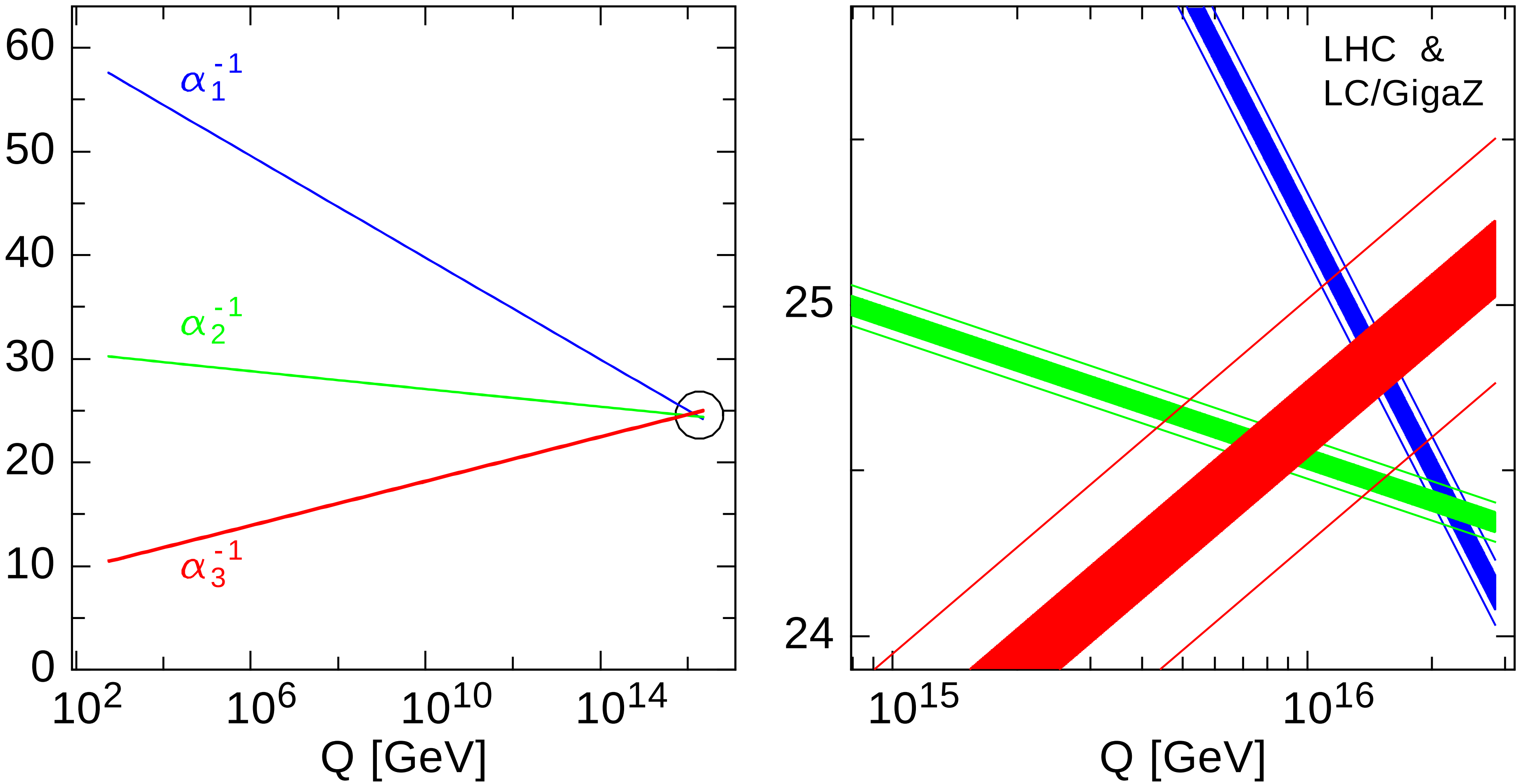
Outline

- Motivation
- Evaluation of $\alpha_s(M_{\mathrm{GUT}})$ (3-loop accuracy)
- Evaluation of $m_{\mathrm{b}}^{\overline{\mathrm{DR}}}(M_{\mathrm{SUSY}})$ (4-loop accuracy)
- Conclusions

Motivation

- MSSM: gauge couplings tend to unify at $M_{\mathrm{GUT}} \simeq 10^{16}$GeV

Allanach et al '04



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    1-loop threshold corrections *D. Pierce et al '96* $\Rightarrow$
    Public Codes: ISAJET *H. Baer et al '03*, SuSpect *A. Djouadi et al '03*
    SPHENO *W. Porod '01*, SOFTSUSY *W. Allanach '01*
  - 3-loop MSSM RGEs *I. Jack, D. R. T. Jones and C. G. North*
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    Running analysis: *I. Jack, D. R. T. Jones, A. F. Kord '04*

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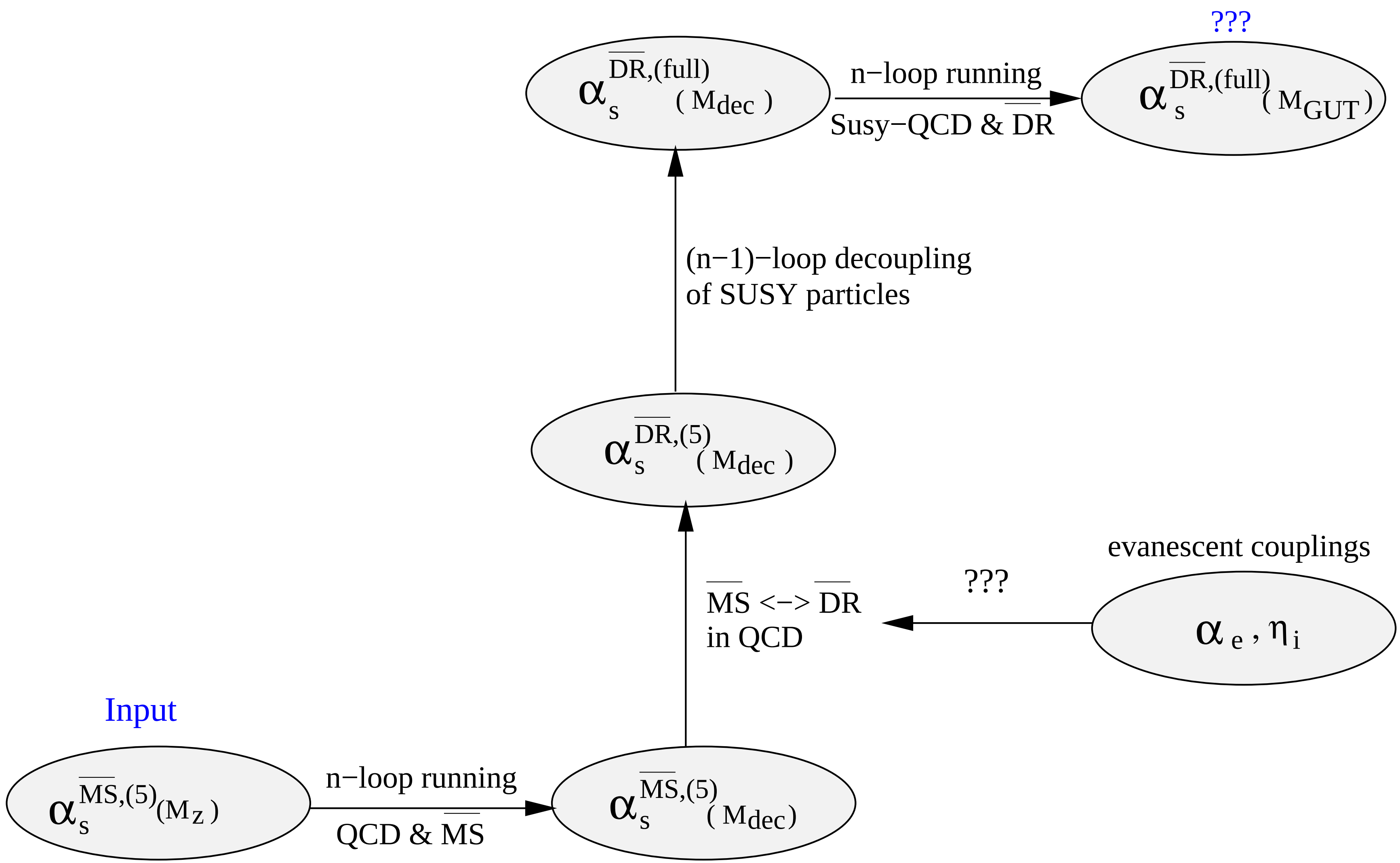
    Running analysis: *I. Jack, D. R. T. Jones, A. F. Kord '04*
  - Our aim: 3-loop RGEs for SUSY-QCD sector
    2-loop threshold corrections *R. Harlander, L. M., M. Steinhauser '05*
    $\Rightarrow \alpha_s^{\overline{\mathrm{DR}}}(M_{\mathrm{GUT}})$ with 3-loop accuracy

Running of α_s

- Input parameter: $\alpha_s^{\overline{\rm MS},(5)}(M_Z) \Rightarrow$ Output parameter: $\alpha_s^{\overline{\rm DR},({\rm full})}(M_{\rm GUT})$



Running of α_s

- Precision calculations in MSSM require a manifestly SUSY and gauge invariant Regularization scheme $\Rightarrow$ DRED
- Mass-independent Renormalization scheme
  Decoupling Theorem does not hold $\Rightarrow$ threshold effects should be added *by hand*
  - SUSY models with severely split mass spectrum
    Multi-Scale Approach: each particle decoupled at its own threshold
  - SUSY models with roughly degenerate mass spectrum
    Common Scale Approach: all SUSY particles decoupled at

$$\mu \simeq M_{\mathrm{SUSY}}$$

! implemented in almost all currently available codes

DRED Framework

Quasi-4-dim. space (Q4S): $4 = d \oplus 4 - d$

- *Quasi*-4-dim metric tensor: $G_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu}$
- Dirac matrices in Q4S: $\Gamma_\mu = \gamma_\mu + \tilde{\gamma}_\mu$
- space-time coordinates continued from 4 to $d \leq 4$ dim.
- the number of field components unchanged
  - 4-dim gluon field: $A_\mu^a = V_\mu^a + S_\mu^a$,

$$\begin{aligned} V_\mu^a &= g_{\mu\nu}\, A_\nu^a = && d\text{- dim. vector} \\ S_\mu^a &= \tilde{g}_{\mu\nu}\, A_\nu^a = && \varepsilon \text{ scalar} \end{aligned}$$

under gauge transformations

Renormalization

$$\mathcal{L}_{\rm B} = \mathcal{L}_{\rm B}^{d} + \mathcal{L}_{\rm B}^{\varepsilon}$$

- $\mathcal{L}_{\rm B}^{d}$ same as in DREG
- $\mathcal{L}_{\rm B}^{\varepsilon}$ new contribution due to ε-scalars

$$\mathcal{L}_{\rm B}^{d} = -\frac{1}{4}G^{a,ij}G^{a}_{ij} - \frac{(\partial^i V_i^a)^2}{2(1-\xi)} + \mathcal{L}^{d}_{\rm ghost,B} + i\bar\psi^\alpha\gamma^i D_i^{\alpha\beta}\psi^\beta$$

$$\mathcal{L}_{\rm B}^{\varepsilon} = \frac{1}{2}(D_i^{ab}S_\sigma^b)^2 - g\bar\psi\tilde\gamma_\sigma T^a\psi S_\sigma^a - \frac{1}{4}g^2 f^{abc}f^{ade}S_\sigma^b S_{\sigma'}^c S_\sigma^d S_{\sigma'}^e$$

- each term in $\mathcal{L}_{\rm B}^{\varepsilon}$ invariant under gauge transformations
  - no reason that Yukawa-type $\bar\psi\psi S$ and $\bar\psi\psi V$ vertices renormalize the same way [except for SUSY theories !]
  - $f - f$ structure not preserved under renormalization

Renormalization(2)

$$\mathcal{L}^{\varepsilon} = \frac{1}{2} Z_3^{\varepsilon} (\partial_i S_\sigma)^2 + Z^{\varepsilon\varepsilon V} g f^{abc} \partial_i S_\sigma^a V^{b,i} S_\sigma^c$$
$$+ Z^{\varepsilon\varepsilon VV} g^2 f^{abc} f^{ade} V_i^b S_\sigma^c V^{d,i} S_\sigma^e - Z_1^{\varepsilon} g_e \bar{\psi} T^a \tilde{\gamma}^\sigma \psi S_\sigma^a$$
$$- \frac{1}{4} \sum_{r=1}^{p} Z_{1,r}^{4\varepsilon} \eta_r H_r^{abcd} S_\sigma^a S_{\sigma'}^c S_\sigma^b S_{\sigma'}^d$$

- Evanescent Yukawa-type g_e and $\boldsymbol{p}$ quartic couplings η_r
- a possible choice of H^{abcd} for $SU(3)$ case

$$H_1 = \frac{1}{2} \left(f^{ace} f^{bde} + f^{ade} f^{bce} \right)$$
$$H_2 = \frac{1}{2} \delta^{ab} \delta^{cd} \qquad H_3 = \frac{1}{2} (\delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc})$$

QCD β- and γ_m-functions within DRED

- Dimensional Reduction $\oplus$ Minimal Subtraction $\overline{\mathrm{DR}}$

$$\beta_s^{\overline{\mathrm{DR}}} = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi} \quad \cdots \qquad \gamma_m^{\overline{\mathrm{DR}}} = \frac{\mu^2}{m^{\overline{\mathrm{DR}}}} \frac{\mathrm{d}}{\mathrm{d}\mu^2} m^{\overline{\mathrm{DR}}}$$

$$\begin{aligned}
\beta_s^{\overline{\mathrm{DR}}}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) &= -\sum_{i,j,k,l,m} \beta_{ijklm}^{\overline{\mathrm{DR}}} \left(\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m \\
\beta_e(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) &= -\sum_{i,j,k,l,m} \beta_{ijklm}^{e} \left(\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m \\
\beta_{\eta_r}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) &= -\sum_{i,j,k,l,m} \beta_{ijklm}^{\eta_r} \left(\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m \\
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\end{aligned}$$

QCD: 3-loop $\beta_s^{\overline{\mathrm{DR}}}$-function

- up to 2-loop order $\beta_s^{\overline{\mathrm{DR}}} = \beta_s^{\overline{\mathrm{MS}}}$
- explicit 3-loop computation *R. Harlander, P. Kant, L. M., M. Steinhauser '06*
  comprises Yukawa like evanescent coupling $\boldsymbol{\alpha_e}$

$$\begin{aligned}\beta_{\mathbf{s}}^{\overline{\mathrm{DR}},\mathbf{3l}}(\alpha_{\mathbf{s}}^{\overline{\mathrm{DR}}},\alpha_{\mathbf{e}}) \;=\; & \left[\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right]^3 \frac{\alpha_e}{\pi}\frac{3}{16}C_F^2 T n_f + \left[\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\frac{\alpha_e}{\pi}\right]^2 C_F T n_f \left[\frac{C_A}{16} - \frac{C_F}{8} - \frac{T n_f}{16}\right] \\ & - \left[\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right]^4 \left[\frac{3115}{3456}C_A^3 - \frac{1439}{1728}C_A^2 T n_f + \frac{1}{32}C_F^2 T n_f \right. \\ & \left. - \frac{193}{576}C_A C_F T n_f + \frac{79}{864}C_A T^2 n_f^2 + \frac{11}{144}C_F T^2 n_f^2\right]\end{aligned}$$

- 4-loop order $\beta_s^{\overline{\mathrm{DR}}}$ *R. Harlander, T. Jones, P. Kant, L. M., M. Steinhauser '06*
  contains also quartic ε-scalar couplings $\boldsymbol{\eta_i}\,,(i=1,2,3)$

Conversion from $\overline{\mathrm{MS}}$ to $\overline{\mathrm{DR}}$

- Requirement: evanescent couplings should decouple from physical observables
- known through 3-loop *R. Harlander, T. Jones, P. Kant, L. M., M. Steinhauser '06*
- n-loop conversion relation needed for (n+1)-loop running analysis
- 2-loop conversion relation $\alpha_s^{\overline{\mathrm{MS}}} \leftrightarrows \alpha_s^{\overline{\mathrm{DR}}}$, $\alpha_s^{\overline{\mathrm{DR}}} = f(\alpha_s^{\overline{\mathrm{MS}}}, \alpha_e)$

$$\frac{\alpha_s^{\overline{\mathrm{DR}},(n_f)}}{\alpha_s^{\overline{\mathrm{MS}},(n_f)}} = 1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\frac{C_A}{12} + \left[\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\right]^2 \frac{11}{72}C_A^2 - \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\frac{\alpha_e}{\pi}\frac{1}{8}C_F T n_f$$

- $\alpha_e \neq \alpha_s^{\overline{\mathrm{DR}}}$ proves equivalence of **DRED** and **DREG** at 3-loops

Decoupling of SUSY particles

- SUSY-QCD & DRED: SUSY preserved $\Rightarrow$ **one** coupling $\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu)$

$$\alpha_e^{(\mathrm{full})}(\mu) = \alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu) = \eta_1^{(\mathrm{full})}(\mu)$$

$$\eta_2^{(\mathrm{full})}(\mu) = \eta_3^{(\mathrm{full})}(\mu) = 0$$

$$\beta_s = \beta_e = \beta_{\eta_1} \quad \text{and} \quad \beta_{\eta_2} = \beta_{\eta_3} = 0$$

- QCD($n_f = 5$): low-energy effective theory of SUSY-QCD

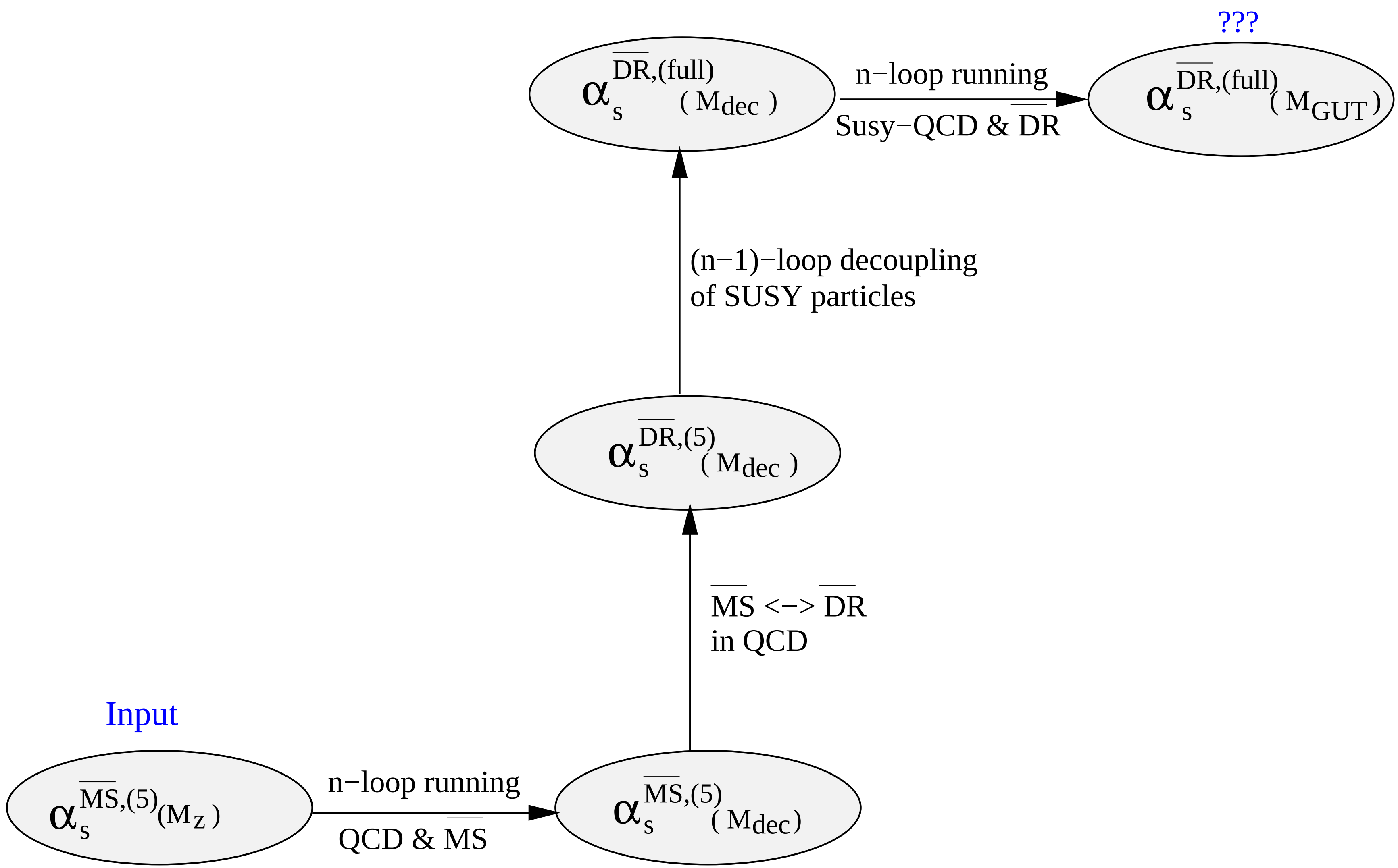
$\Rightarrow$ integrate out all SUSY-particles and top-quark at $\mu = \mu_{\mathrm{dec}}$

$$\alpha_s^{\overline{\mathrm{DR}},(n_f)}(\mu_{\mathrm{dec}}) = \zeta_s^{(n_f)}\,\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu_{\mathrm{dec}})$$

$$\alpha_e^{q,(5)}(\mu_{\mathrm{dec}}) = \zeta_e^q\,\alpha_e^{(\mathrm{full})}(\mu_{\mathrm{dec}})$$

$\zeta_s^{(n_f)}$ and ζ_e^q decoupling coefficients for α_s and α_e

Evaluation of $\alpha_s(\mu_{\mathrm{GUT}})$ from $\alpha_s(M_Z)$



Evaluation of $\alpha_s(\mu_{\mathrm{GUT}})$ from $\alpha_s(M_Z)$

- Iterative Method :
  1. Start with a trial value for $\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu_{\mathrm{dec}})$
  2. Get $\alpha_s^{\overline{\mathrm{DR}},(5)}(\mu_{\mathrm{dec}})$ and $\alpha_e^{(5)}(\mu_{\mathrm{dec}})$ through decoupling relations
  3. Evaluate $\alpha_s^{\overline{\mathrm{MS}},(5)}(\mu_{\mathrm{dec}})$ and from that $\alpha_s^{\overline{\mathrm{MS}},(5)}(M_z)$
  4. Vary $\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu_{\mathrm{dec}})$ until $\alpha_s^{\overline{\mathrm{MS}},(5)}(M_z)$ fits the experimental value
- Practical phenomenological analyses: approximate formula

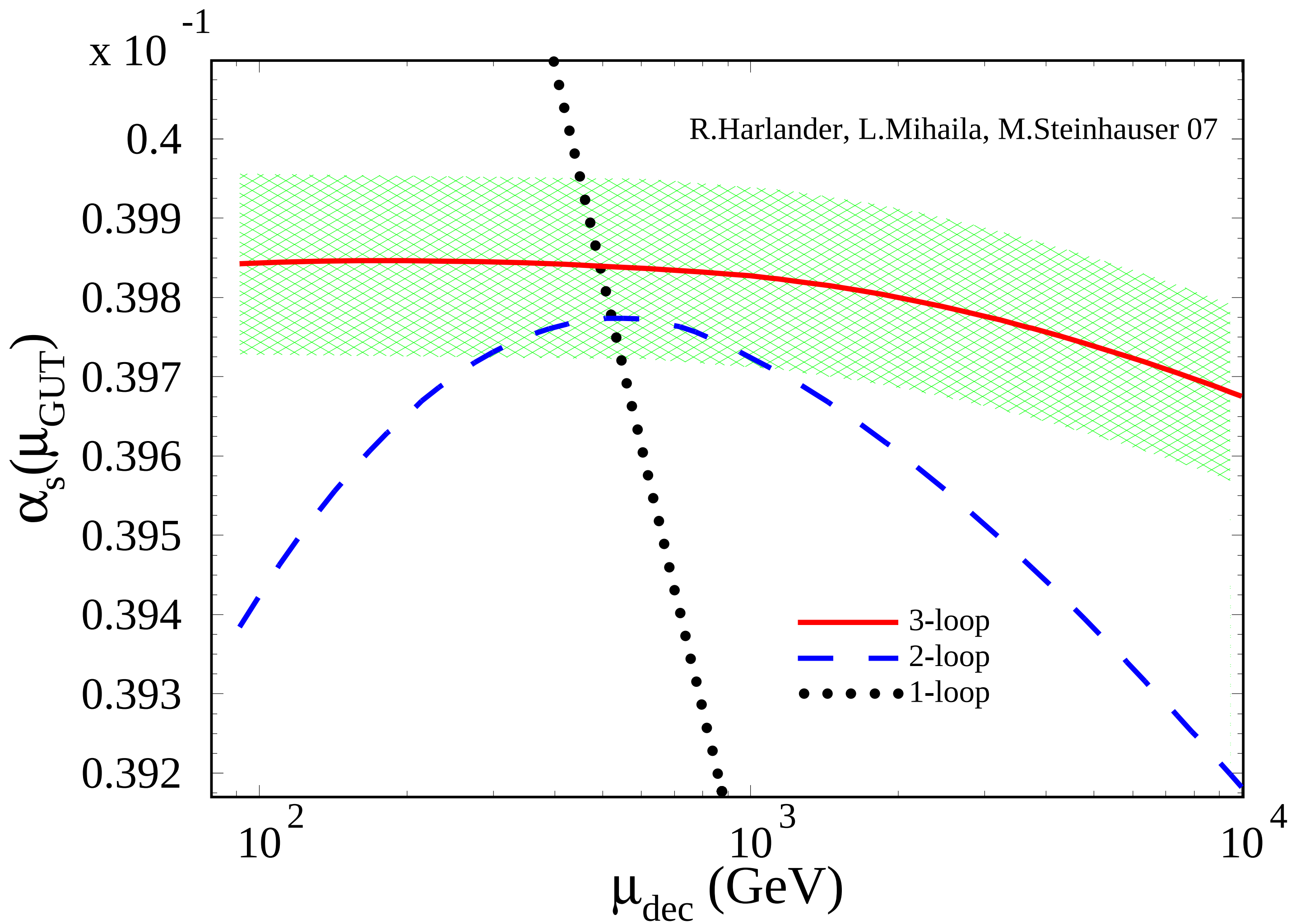
$$\begin{aligned}\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})} \quad &= \quad \alpha_s^{\overline{\mathrm{MS}},(n_f)}\Bigg\{1+\frac{\alpha_s^{\overline{\mathrm{MS}},(n_f)}}{\pi}\left(\frac{1}{4}-\zeta_{s1}^{(n_f)}\right)\\ &\quad + \left(\frac{\alpha_s^{\overline{\mathrm{MS}},(n_f)}}{\pi}\right)^2\left[\frac{11}{8}-\frac{n_f}{12}-\frac{1}{2}\,\zeta_{s1}^{(n_f)}+2\,(\zeta_{s1}^{(n_f)})^2-\zeta_{s2}^{(n_f)}\right]\Bigg\}\end{aligned}$$

numerical deviation from the *Two-Step Approach* $\leq 0.1\%$

Numerical results

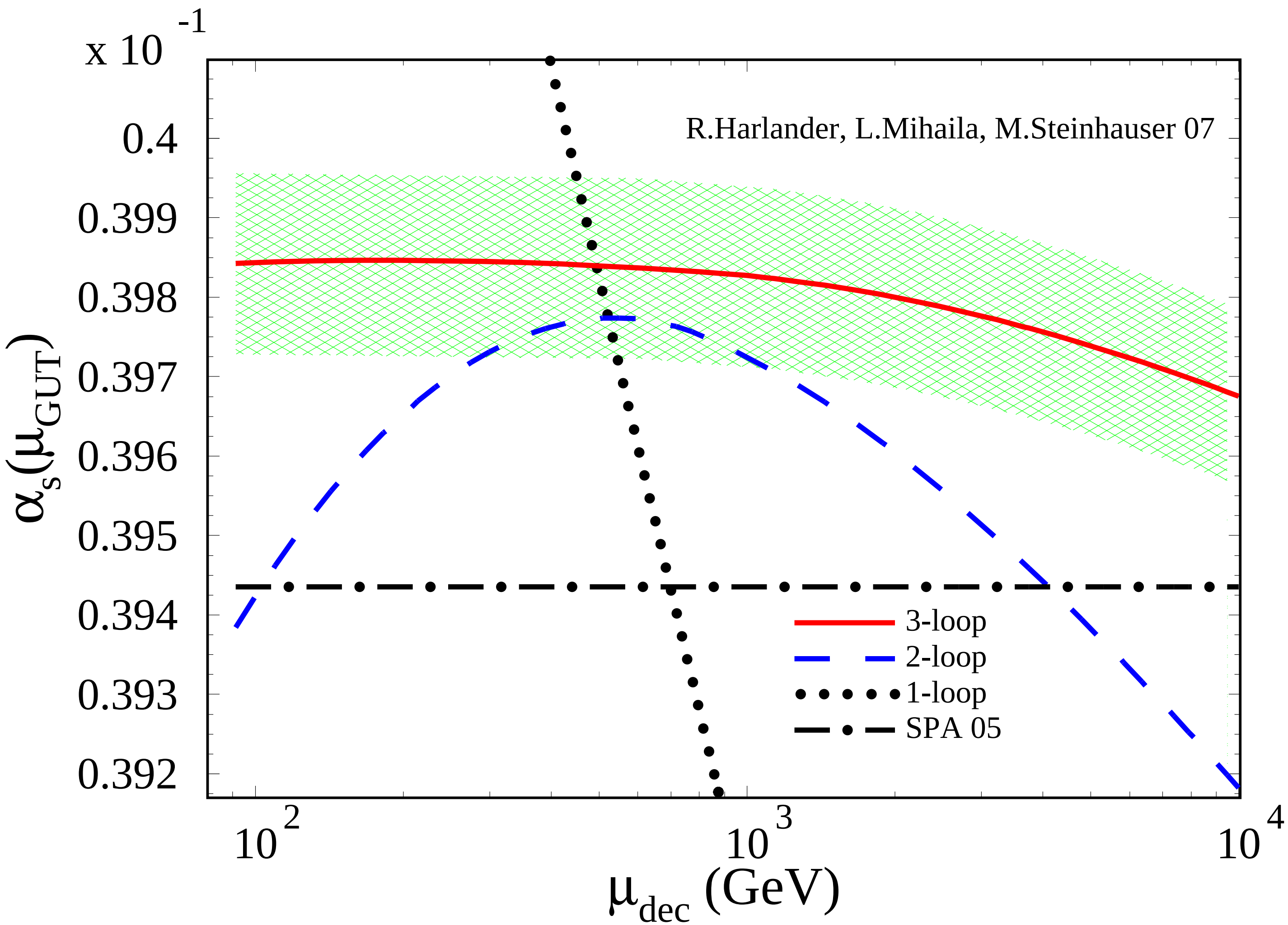
$$\alpha_s^{\overline{\rm MS},(5)}(M_Z) = 0.1189 \pm 0.001 \;\; \textit{Bethke}'\,06, \quad M_Z = 91.1876\ \text{GeV}$$

and $\tilde{M} = m_{\tilde{q}} = m_{\tilde{g}} = 1000$ GeV *SPS1a' '05*



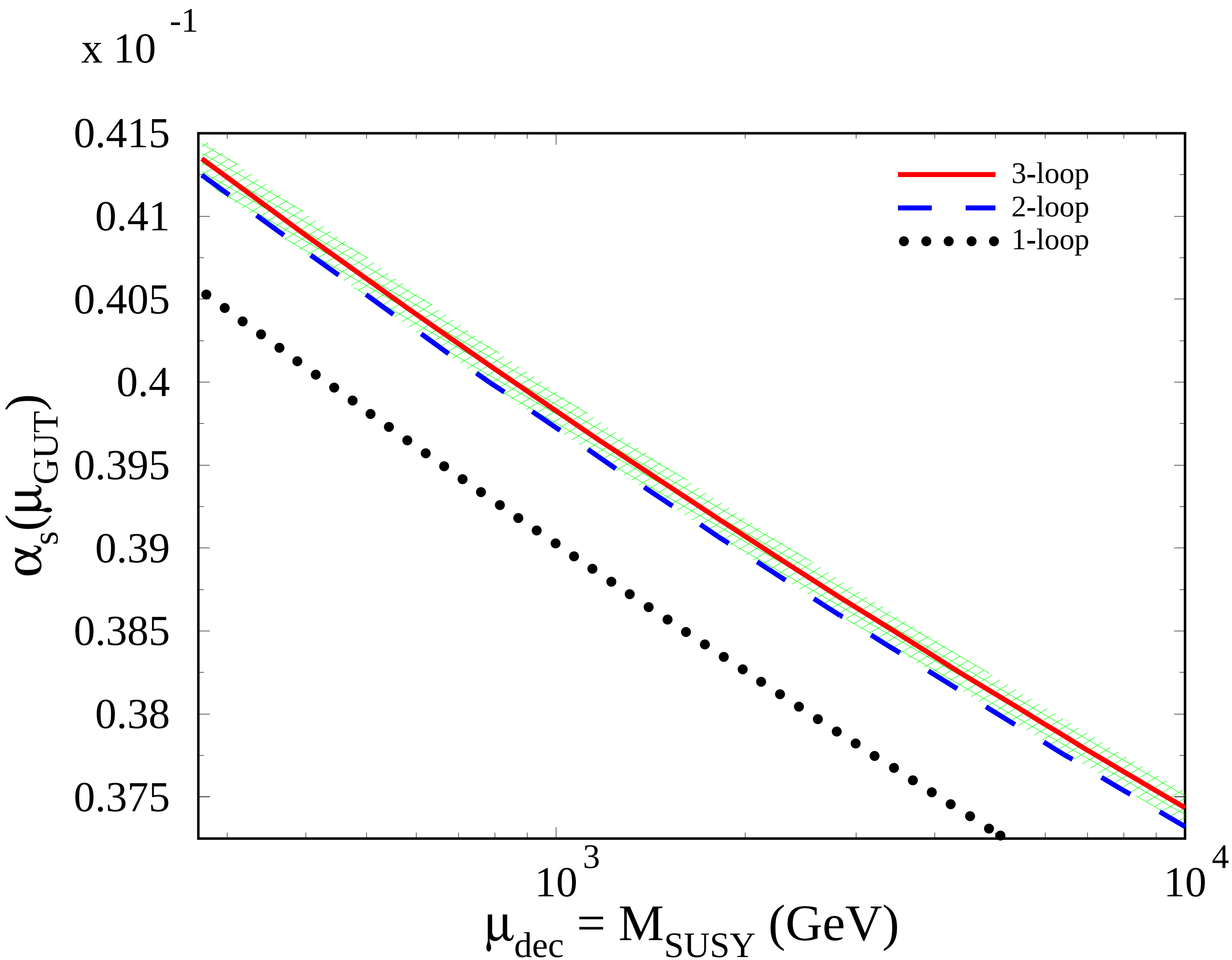
Numerical results

- Comparison with the Leading-Log Approximation *SPA-Convention'05* shows a big numerical deviation.



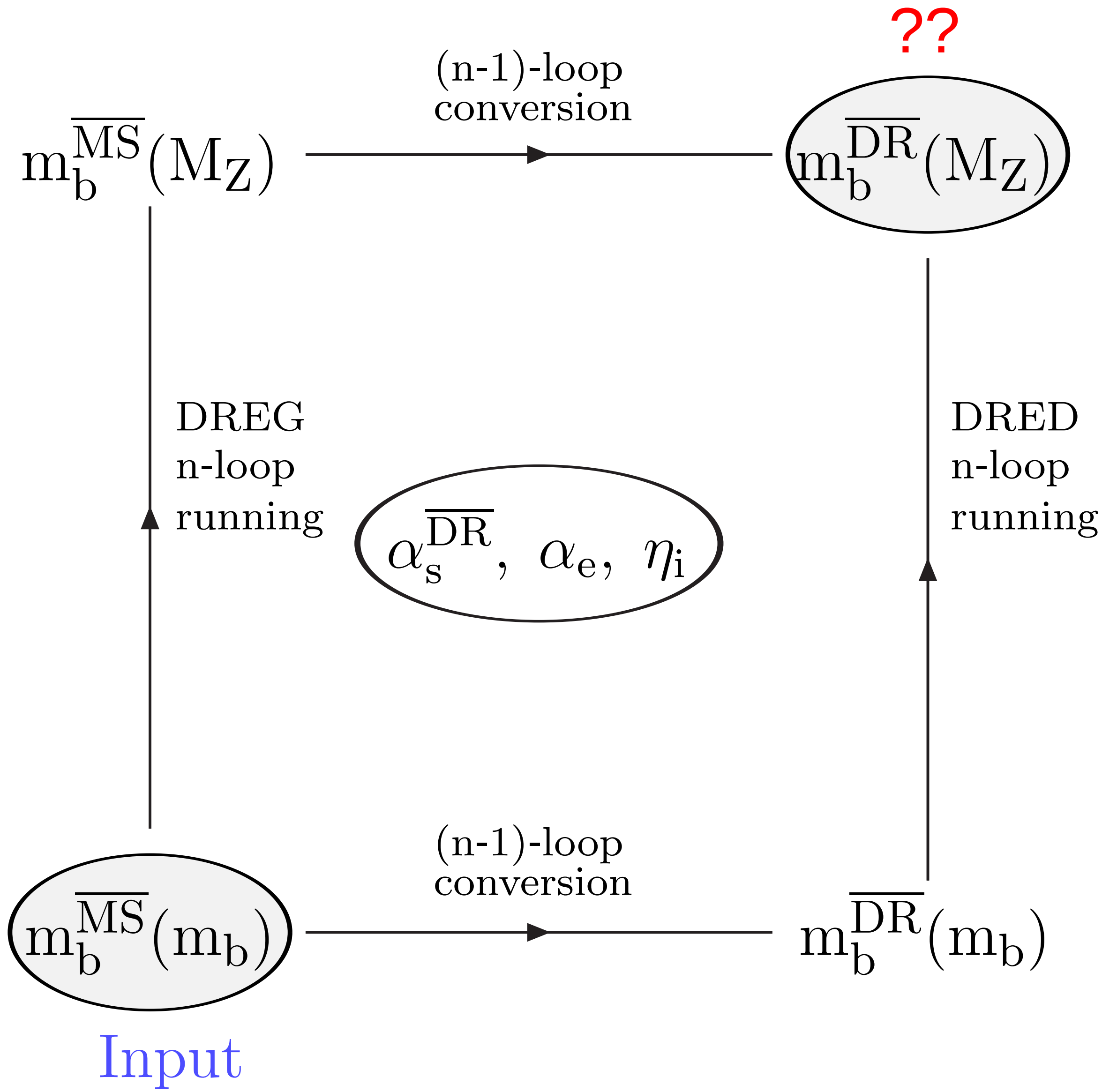
Numerical results

- Sensitivity of $\alpha_s(M_{\rm GUT})$ to SUSY-mass scale:



Evaluation of $m_b(\mu)$ in $\overline{\mathrm{DR}}$ scheme

- Yukawa sector of SUSY-GUT models $\Rightarrow m_{\mathrm{top}}, m_{\mathrm{bottom}}/m_{\mathrm{tau}}$
- SUSY models with large $\tan\beta$
  - SUSY mass spectrum and Higgs mass sensitive to bottom Yukawa coupling
  - relation between $Y_b(\mu)$ and $m^{\overline{\mathrm{DR}}}(\mu)$ affected by large SUSY radiative corrections
  - $m_b^{\overline{\mathrm{DR}}}(M_{\mathrm{SUSY}})$ input parameter $\Rightarrow$ need to be known with the highest possible accuracy
- Relate $m_b^{\overline{\mathrm{DR}}}(M_{\mathrm{SUSY}})$ directly with $m_b^{\overline{\mathrm{MS}}}(m_b)$
- $m_b^{\overline{\mathrm{MS}}}(m_b)$ known with 4-loop accuracy *J. H. Kühn, M. Steinhauser, C. Sturm '07*



Aim : check 2 ways at n = 1, 2, 3, 4-loops

Relation $m^{\overline{\mathrm{DR}}} \leftrightarrow m^{\overline{\mathrm{MS}}}$

- Extract $m_{\mathrm{b}}^{\overline{\mathrm{DR}}}(M_Z)$ from accurately determined $m_{\mathrm{b}}^{\overline{\mathrm{MS}}}(m_b)$

$$m_b^{\overline{\mathrm{DR}}}(\mu) = m_b^{\overline{\mathrm{MS}}}(\mu)\Big[1 + \delta_m^{(1l)}(\alpha_e) + \delta_m^{(2l)}(\alpha_s^{\overline{\mathrm{MS}}}, \alpha_e) \\ + \delta_m^{(3l)}(\alpha_s^{\overline{\mathrm{MS}}}, \alpha_e, \eta_i)\Big]\Big|_{\mu=\mu_S},$$

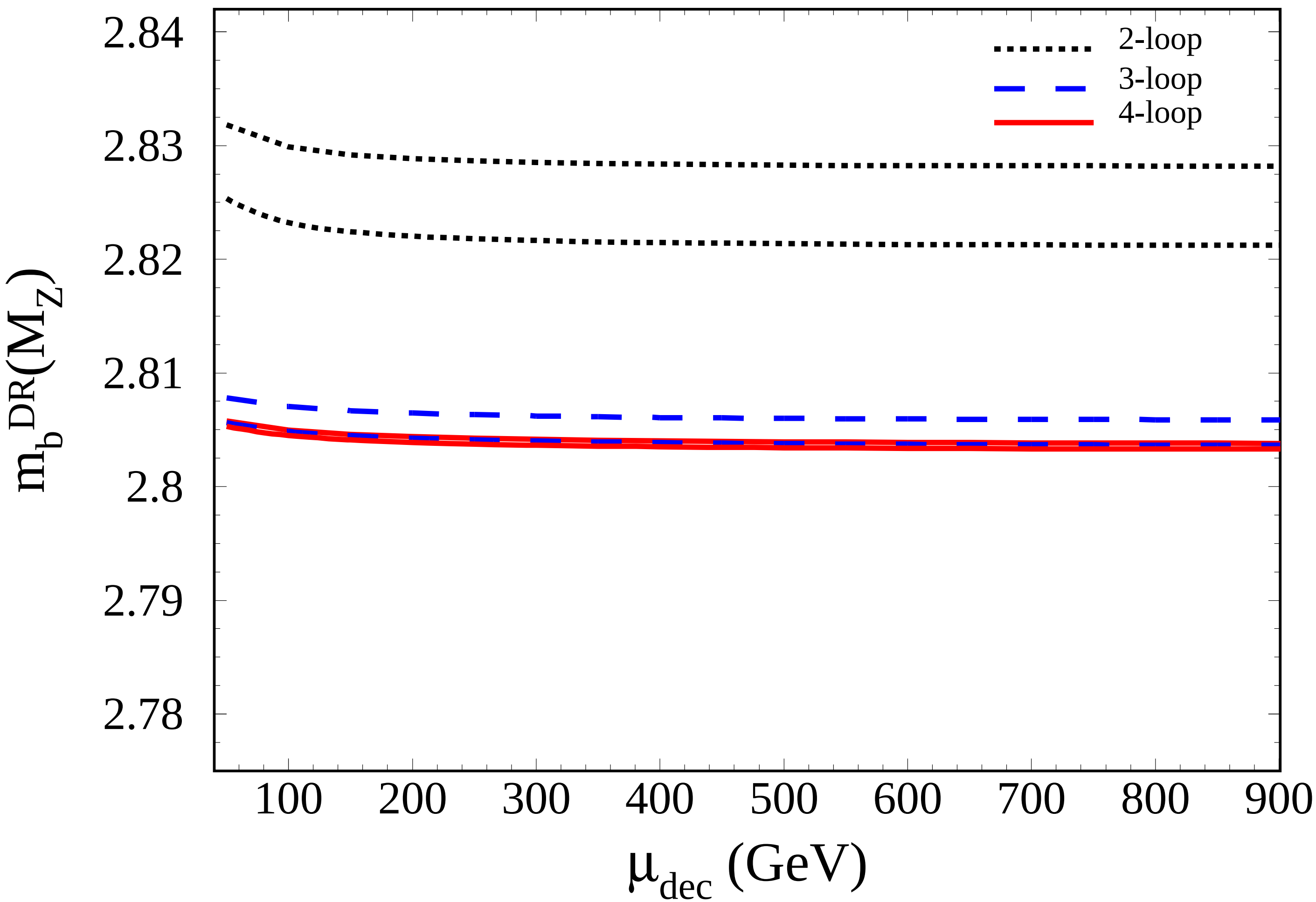
$\{\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \eta_i\}\big|_{\mu=\mu_S}$ have to be known.

  - Log contributions absent (mass-independent schemes)
- 2-step approach for computing $\boldsymbol{m}_{\mathrm{b}}^{\overline{\mathrm{DR}}}(M_Z)$ *H. Baer et al ’02*
  - Running of $m_b(\mu)$ and conversion between $\overline{\mathrm{MS}} \leftrightarrow \overline{\mathrm{DR}}$
- Check if using QCD& $\overline{\mathrm{DR}}$ or QCD& $\overline{\mathrm{MS}}$ $\Rightarrow$ same result

Input parameters:

$\alpha_s^{\overline{\mathrm{MS}}}(M_Z) = 0.1189 \pm 0.001$ *S. Bethke '06* (green band)

$m_b^{\overline{\mathrm{MS}}}(m_b) = 4.164 \pm 0.025$ GeV *J. H. Kühn, M. Steinhauser, C. Sturm '07* (pink band)



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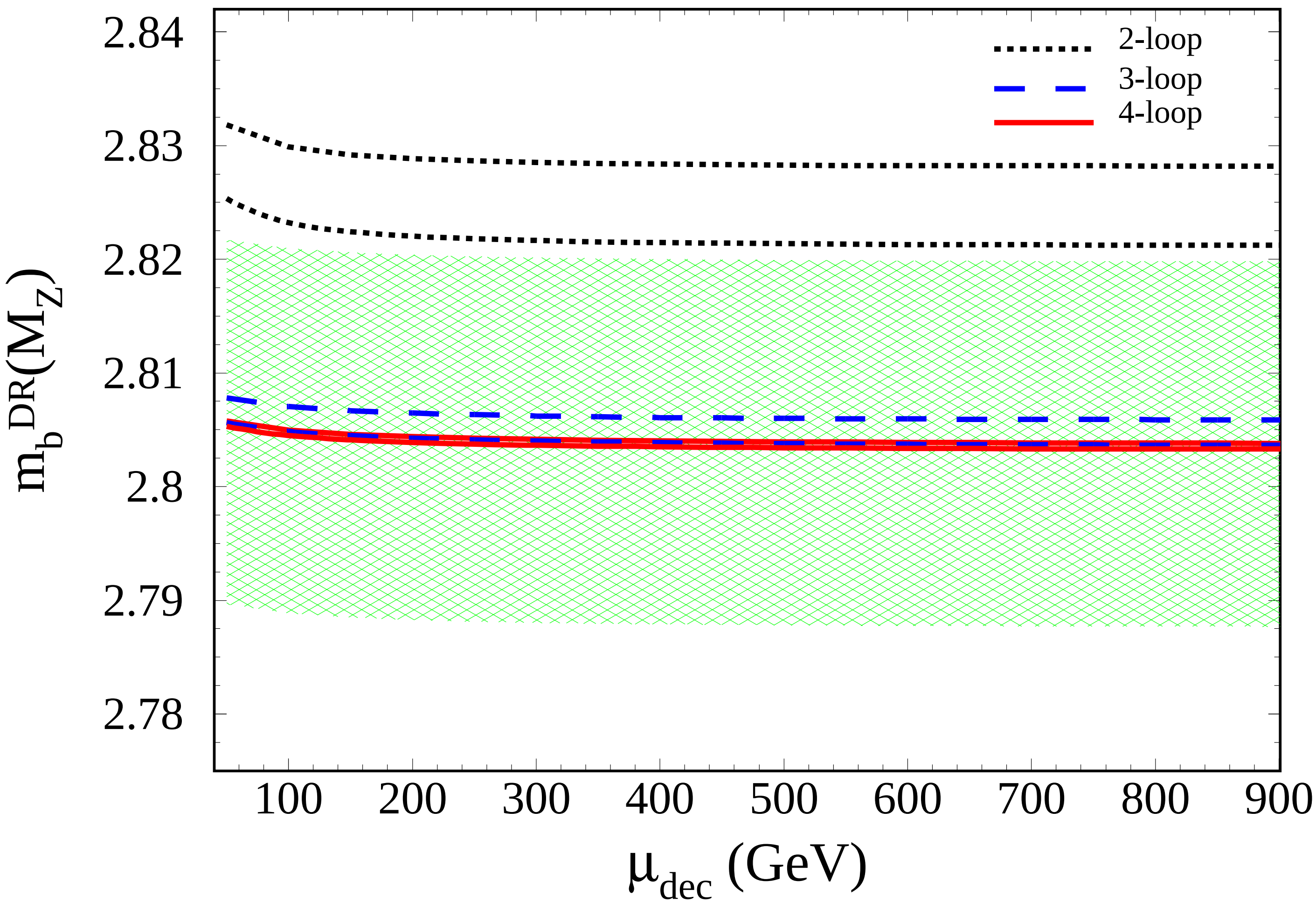

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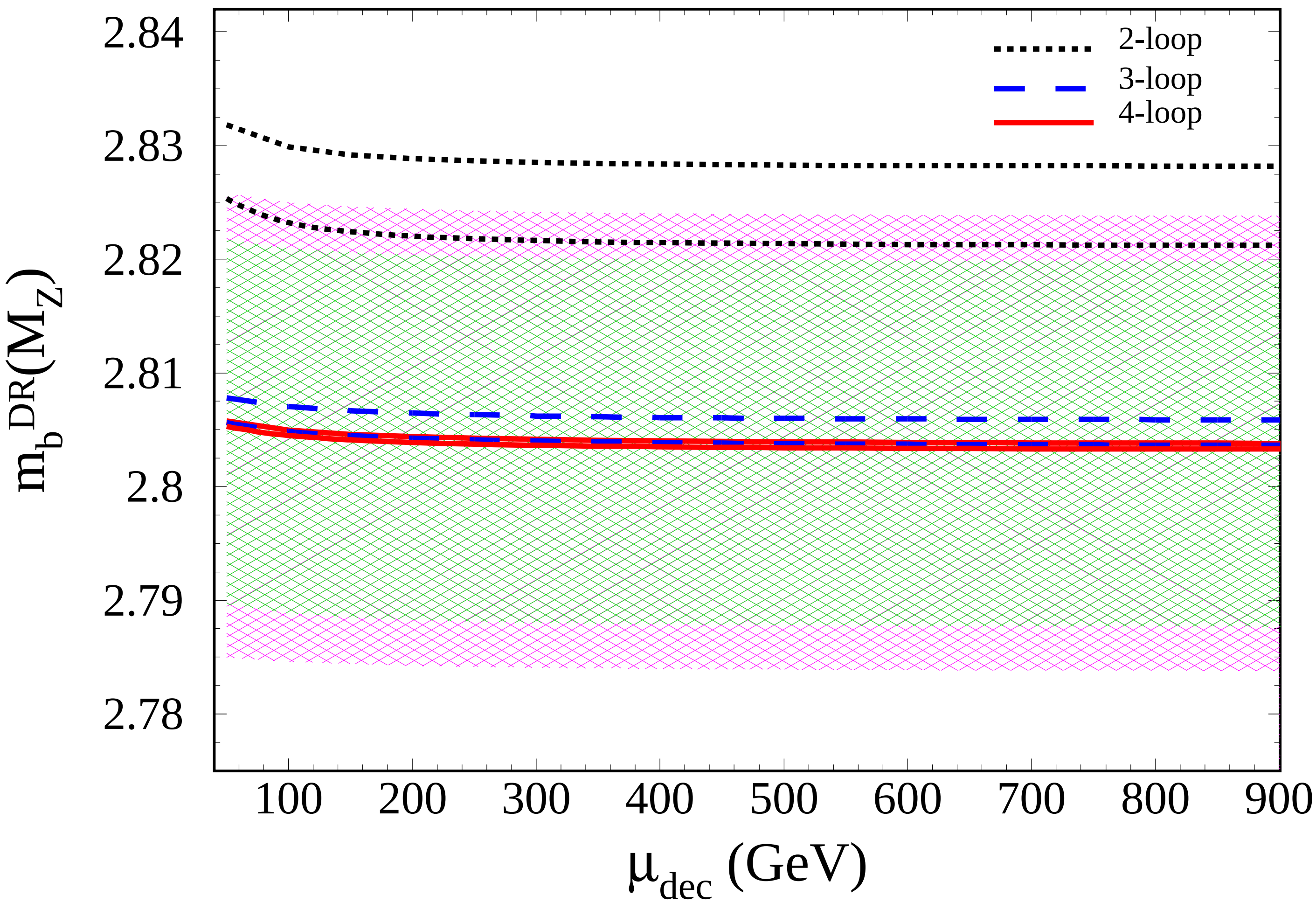
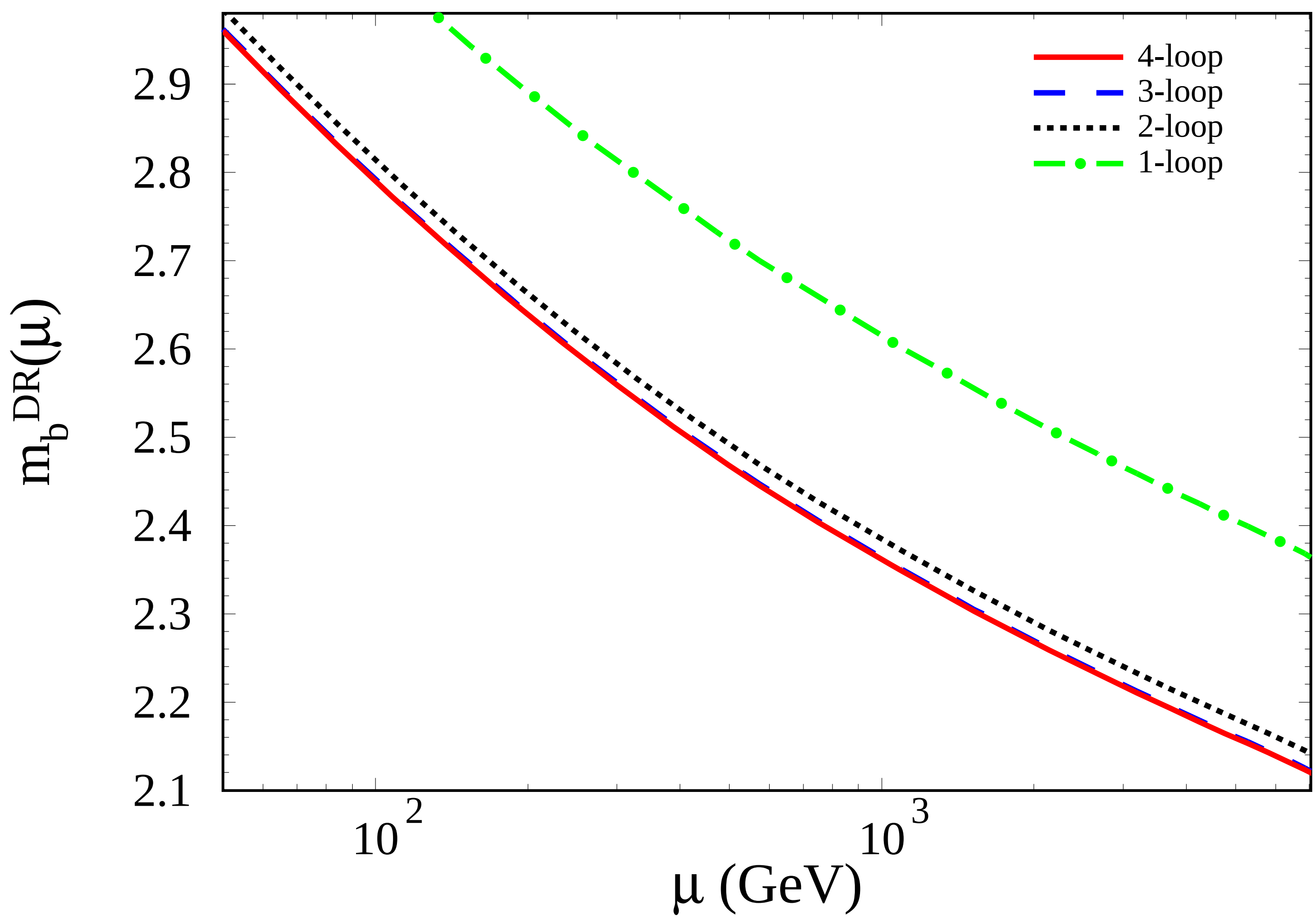


Running of $m_b^{\overline{\mathrm{DR}}}(\mu)$ in QCD& $\overline{\mathrm{DR}}$ with 4-loop accuracy



Conclusions

- A consistent approach to compute $\alpha_s^{\overline{\rm DR}}(M_{\rm GUT})$ and $m_b^{\overline{\rm DR}}(M_{\rm SUSY})$ with 3- and 4-loop accuracy is proposed
- The 3-loop effects comparable with the experimental accuracy for α_s and m_b
- Correct treatment of the evanescent couplings essential: 2- and 3-loop conversion from $\overline{\rm MS}$ to $\overline{\rm DR}$ schemes.
- 1-loop LL-approximation not adequate to precision analyses
- $\alpha_s^{\overline{\rm DR}}(M_{\rm GUT})$ very sensitive to SUSY-mass scale