Analytic Formulation of Spatial Resolution for a MPGD-Readout TPC

-- Fundamental Limits on Spatial Resolution --

Important outcome from KEK beam tests (Asia+Europe+North America)

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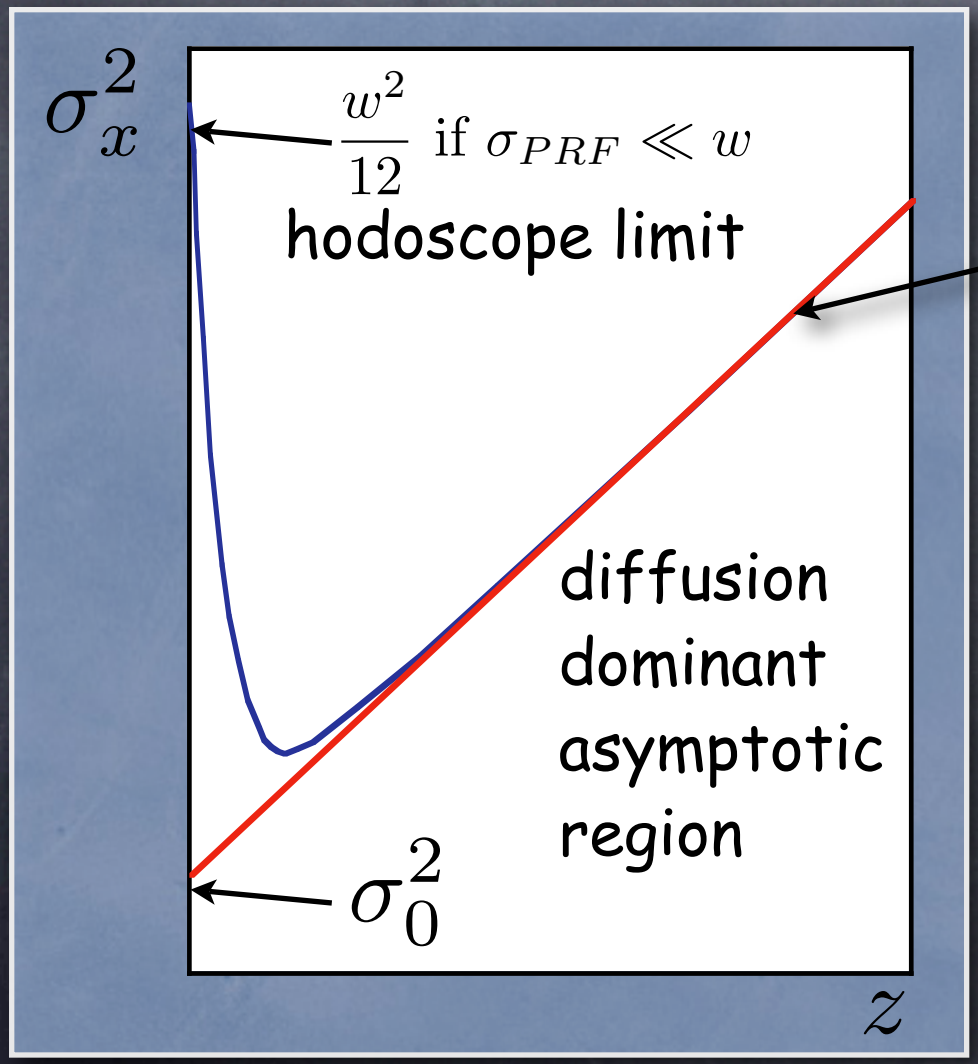
on behalf of people participating in the KEK beam tests using MP-TPC

TPC Analysis Jamboree: 15 Mar., 2007

Two Mysteries

Generic behaviors of resolution with an MPGD endplate when the lateral avalanche spread is much smaller than the pad width

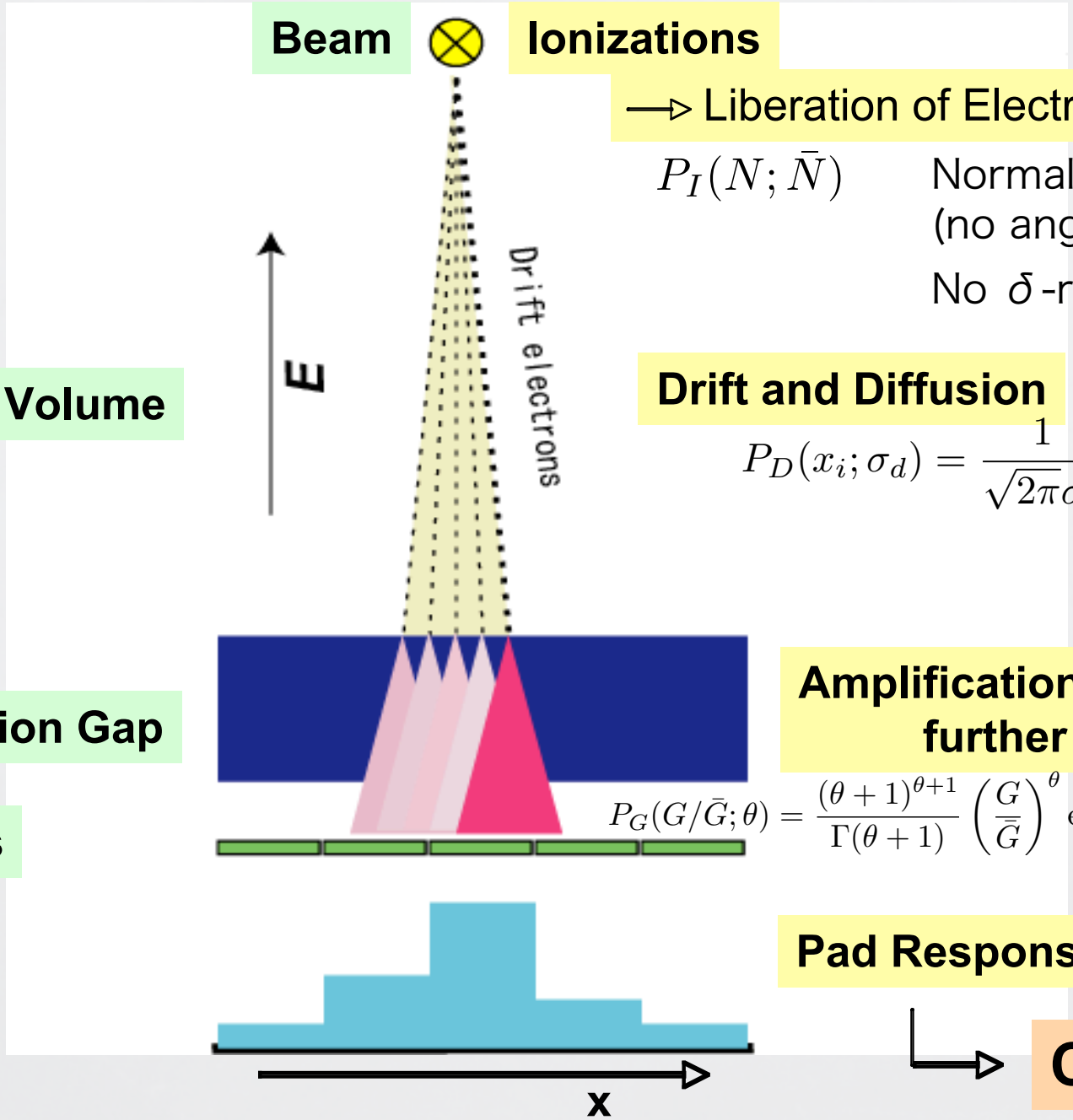


$$\sigma_x^2 = \sigma_0^2 + C_d^2 z / N_{eff}$$

- Why $N_{eff} < \langle N \rangle$?
- What is the origin of $\sigma_0 \equiv \sigma_x(z=0)$?

It's there even if electronic noise is negligible!

Fundamental Process



Ionizations

⟶ Liberation of Electrons

$P_I(N;\bar{N})$ Normal incidence (no angle effect)

No δ-ray

Drift and Diffusion

$$P_D(x_i;\sigma_d) = \frac{1}{\sqrt{2\pi}\sigma_d}\exp\left(-\frac{x_i^2}{2\sigma_d^2}\right)$$

$$\sigma_d = C_d\sqrt{z}$$

Amplification and further Diffusion

$$P_G(G/\bar{G};\theta) = \frac{(\theta+1)^{\theta+1}}{\Gamma(\theta+1)}\left(\frac{G}{\bar{G}}\right)^{\theta}\exp\left(-(\theta+1)\left(\frac{G}{\bar{G}}\right)\right)$$

Pad Response

⟶ **Coordinate**

Ionization Statistics

Ideal Readout Plane: Coordinate = Simple C.O.G.

PDF for Center of gravity of N electrons

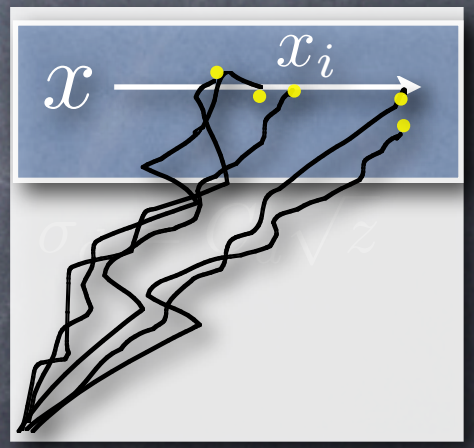
$$P(\bar{x}) = \sum_{N=1}^{\infty} P_I(N;\bar{N}) \prod_{i=1}^{N} \left(\int dx_i P_D(x_i;\sigma_d) \right) \delta\left(\bar{x} - \frac{1}{N} \sum_{i=1}^{N} x_i \right)$$

Ideal readout plane

Gaussian diffusion

$$P_D(x_i;\sigma_d) = \frac{1}{\sqrt{2\pi}\sigma_d} \exp\left(-\frac{x_i^2}{2\sigma_d^2} \right)$$

$$\sigma_d = C_d \sqrt{z}$$



$$\sigma_{\bar{x}}^2 \equiv \int d\bar{x}\, P(\bar{x})\, \bar{x}^2 = \sigma_d^2 \left\langle \frac{1}{N} \right\rangle \equiv \sigma_d^2 \frac{1}{N_{eff}}$$

$$N_{eff} \equiv 1/\langle 1/N \rangle < \langle N \rangle$$

Gas Gain Fluctuation

Coordinate = Gain-Weighted Mean

PDF for Gain-Weighted Mean of N electrons

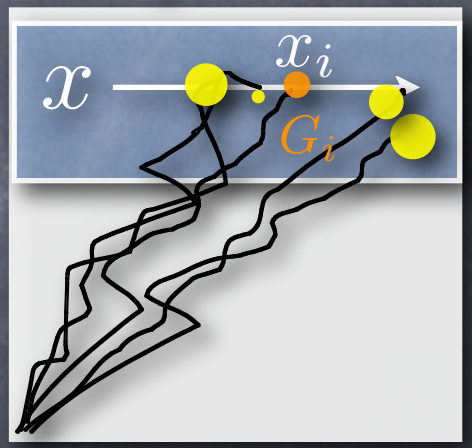
$$P(\bar{x}) = \sum_{N=1}^{\infty} P_I(N;\bar{N}) \prod_{i=1}^{N} \left(\int dx_i P_D(x_i;\sigma_d) \int d(G_i/\bar{G})\, P_G(G_i/\bar{G};\theta) \right) \delta\left(\bar{x} - \frac{\sum_{i=1}^{N} G_i\, x_i}{\sum_{i=1}^{N} G_i} \right)$$

Gaussian diffusion as before

Gas gain fluctuation (Polya)

Gain-weighted mean

$$\theta = \begin{cases} 0 & : \text{ exp.} \\ \infty & : \delta\text{-fun} \end{cases}$$



$$P_G(G/\bar{G};\theta) = \frac{(\theta+1)^{\theta+1}}{\Gamma(\theta+1)} \left(\frac{G}{\bar{G}}\right)^{\theta} \exp\left(-(\theta+1) \left(\frac{G}{\bar{G}}\right) \right)$$

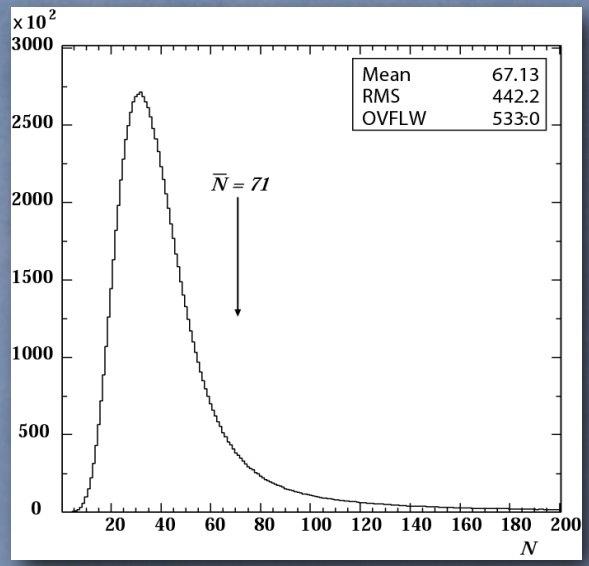
$$\sigma_{\bar{x}}^2 \equiv \int d\bar{x}\, P(\bar{x})\, \bar{x}^2 = \sigma_d^2 \left\langle \frac{1}{N} \right\rangle \left\langle \left(\frac{G}{\bar{G}}\right)^2 \right\rangle \equiv \sigma_d^2 \frac{1}{N_{eff}}$$

$$N_{eff} = \left[\left\langle \frac{1}{N} \right\rangle \left\langle \left(\frac{G}{\bar{G}}\right)^2 \right\rangle \right]^{-1} = \frac{1}{\langle \frac{1}{N} \rangle} \left(\frac{1+\theta}{2+\theta} \right) < \langle N \rangle$$

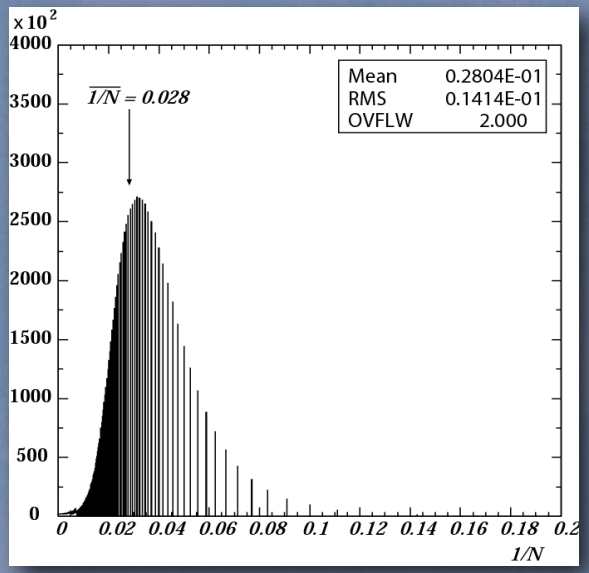
Sample Calc. for Neff

For 4 GeV pion and pad row pitch of 6mm in pure Ar

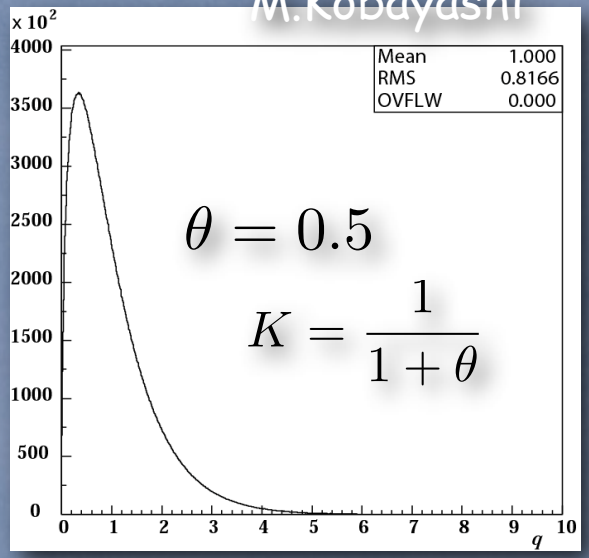
M.Kobayashi



Distribution of *N*
(<*N*> = 71)



Distribution of 1/*N*
(<1/*N*> = 0.028)



Distribution of Q
(*K* = 0.67)

$$\left\langle \left(\frac{G}{\bar{G}}\right)^2 \right\rangle = 1 + \left(\frac{\sigma_G}{\bar{G}}\right)^2 \equiv 1 + K$$

$$N_{eff} = \left[\left\langle \frac{1}{N} \right\rangle \left\langle \left(\frac{G}{\bar{G}}\right)^2 \right\rangle \right]^{-1} = 21 < \langle N \rangle = 71$$

Finite Size Pads

Coordinate = Charge Centroid

Charge on Pad j

Electronic noise

$$\langle \Delta Q^2 \rangle = \sigma_E^2$$

$$Q_j = \sum_{i=1}^{N} G_i \cdot f_j(\tilde{x} + \Delta x_i) + \Delta Q_j,$$

Normalized response fun. for pad j

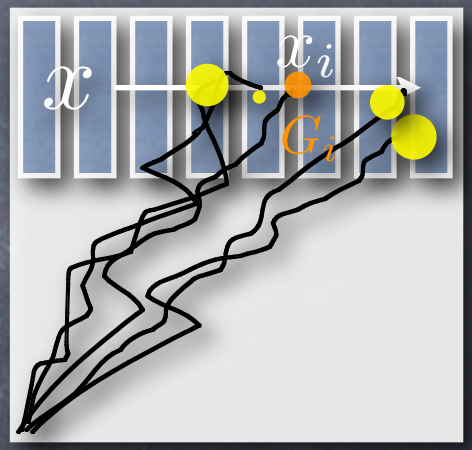
$$\sum_j f_j(\tilde{x} + \Delta x_i) = 1$$

track position

$$x_i = \tilde{x} + \Delta x_i$$

diffusion

$$\langle \Delta x^2 \rangle = \sigma_d^2 = C_d^2 z$$



Pad pitch

Charge Centroid

$$\bar{x} = \sum_j Q_j \, (wj) / \sum_j Q_j$$

PDF for Charge Centroid

$$\begin{aligned} P(\bar{x};\tilde{x}) \;=\; & \sum_{N=1}^{\infty} P_I(N;\bar{N}) \prod_{i=1}^{N} \left(\int d\Delta x_i P_D(\Delta x_i;\sigma_d) \int d(G_i/\bar{G})\, P_G(G_i/\bar{G};\theta) \right) \\ & \times \prod_j \left(\int d\Delta Q_j \; P_E(\Delta Q_j;\sigma_E) \; \int dQ_j \; \delta\left(Q_j - \sum_{i=1}^{N} G_i \cdot f_j(\tilde{x} + \Delta x_i) - \Delta Q_j \right) \right) \\ & \times \delta\left(\bar{x} - \frac{\sum_j Q_j \, (wj)}{\sum_j Q_j} \right) \end{aligned}$$

Full Analytic Formula

$$\sigma_{\bar{x}}^2 \equiv \int_{-1/2}^{+1/2} d\left(\frac{\tilde{x}}{w}\right) \int d\bar{x}\, P(\bar{x};\tilde{x})\,(\bar{x}-\tilde{x})^2 = \int_{-1/2}^{+1/2} d\left(\frac{\tilde{x}}{w}\right) \left[[A] + \frac{1}{N_{eff}}[B]\right] + [C]$$

- Purely geometric term

$$[A] = \left(\sum_j (jw)\,\langle f_j(\tilde{x}+\Delta x)\rangle - \tilde{x}\right)^2$$

- Diffusion, gas gain fluctuation & finite pad pitch term

$$[B] = \sum_{j,k} jkw^2\,\langle f_j(\tilde{x}+\Delta x) f_k(\tilde{x}+\Delta x)\rangle - \left(\sum_j jw\,\langle f_j(\tilde{x}+\Delta x)\rangle\right)^2$$

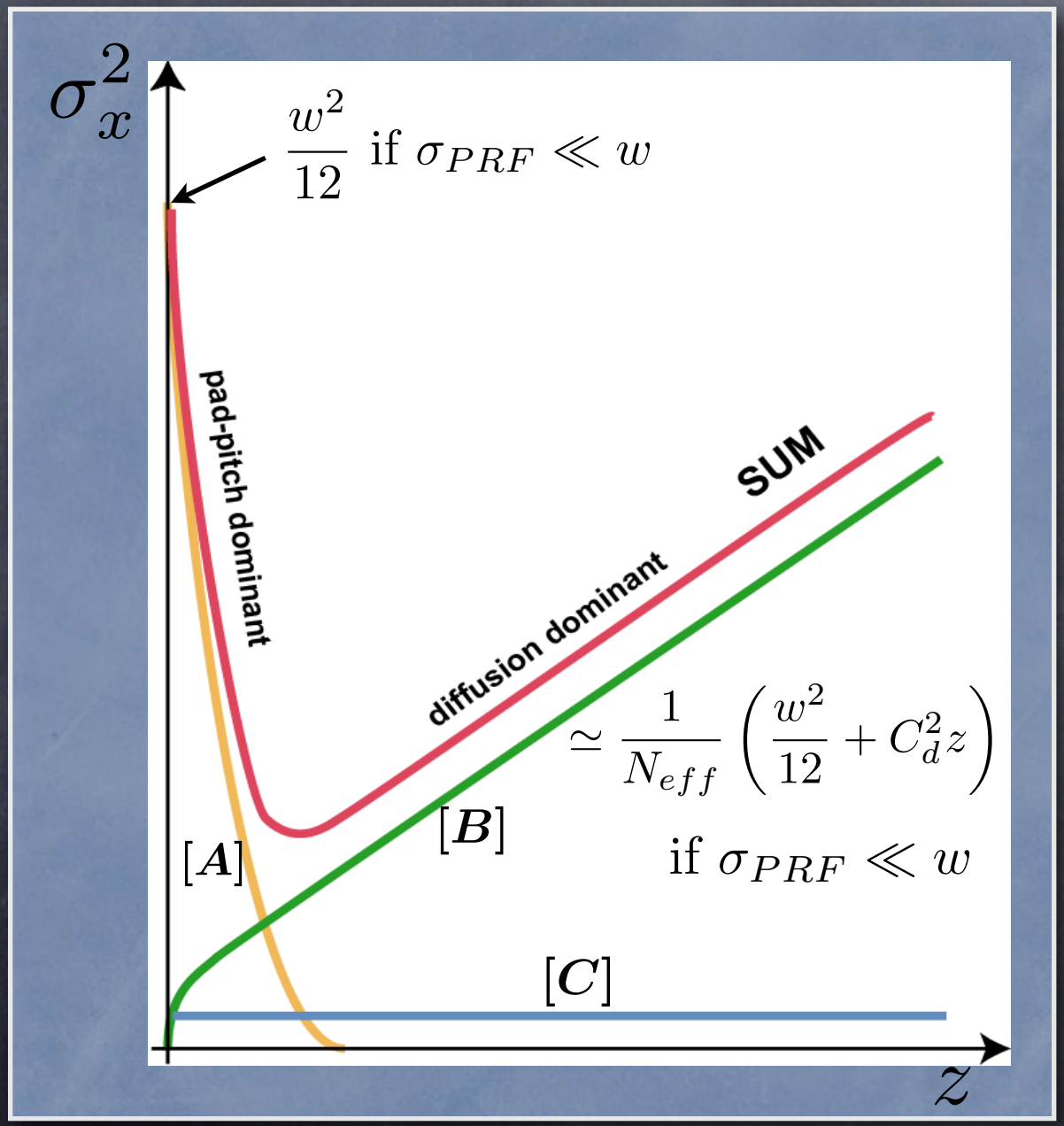
$$\langle f_j(\tilde{x}+\Delta x) f_k(\tilde{x}+\Delta x)\rangle \equiv \int d\Delta x P_D(\Delta x;\sigma_d)\, f_j(\tilde{x}+\Delta x)\, f_k(\tilde{x}+\Delta x)$$

$$\langle f_j(\tilde{x}+\Delta x)\rangle \equiv \int d\Delta x P_D(\Delta x;\sigma_d)\, f_j(\tilde{x}+\Delta x)$$

- Electronic noise term

$$[C] = \left(\frac{\sigma_E}{\bar{G}}\right)^2 \left\langle \frac{1}{N^2} \right\rangle \sum_j (jw)^2$$

Interpretation



[A] Purely geometric term (S-shape systematics from finite pad pitch): rapidly disappears as Z increases

[B] Diffusion, gas gain fluctuation & finite pad pitch term: scales as $1/N_{eff}$, for delta-fun like PRF asymptotically:

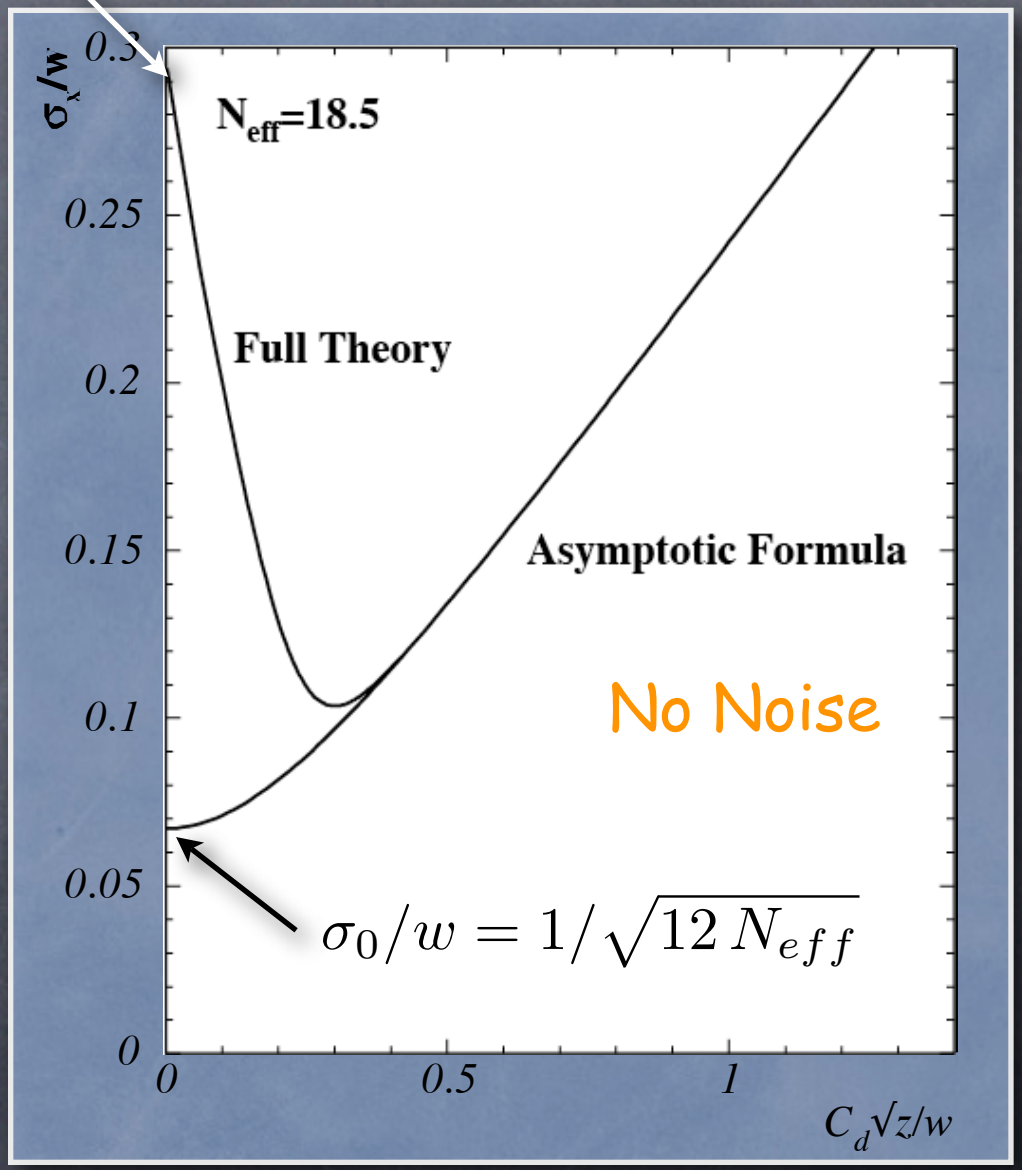
$$\sigma_{\bar{x}}^2 \simeq \frac{1}{N_{eff}}\left(\frac{w^2}{12} + C_d^2 z\right)$$

[C] Electronic noise term: Z-independent, scales as $\langle 1/N^2 \rangle$

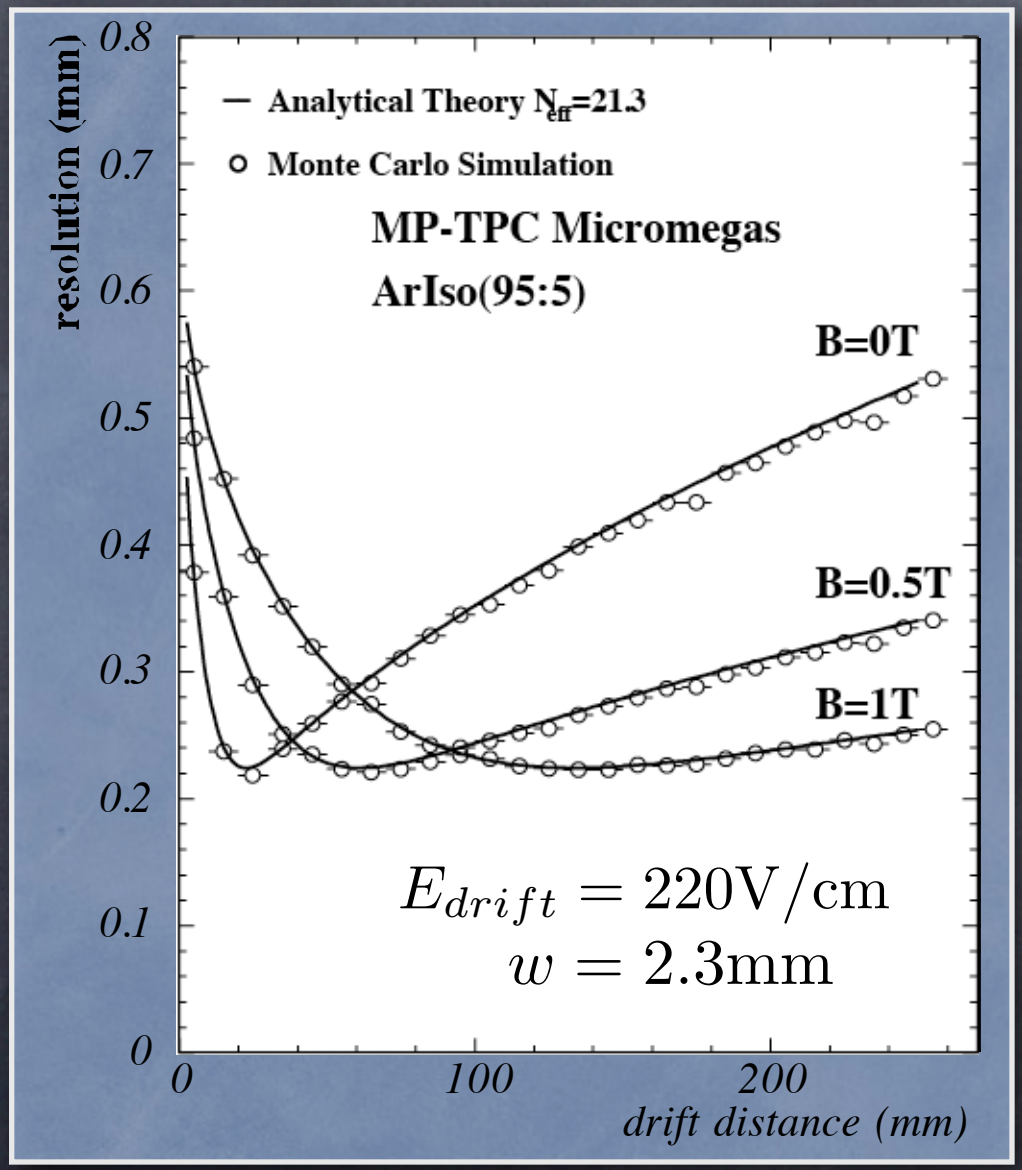
Application to MM

- For delta-function like PRF, σ_x/w depends only on σ_d/w and N_{eff}
- Full formula has a fixed point $(0, 1/\sqrt{12})$
- Full formula enters asymptotic region at $\sigma_d/w \simeq 0.4$
- Full formula has a minimum of $\sigma_x/w \simeq 0.1$ at $\sigma_d/w \simeq 0.3$

$(0, 1/\sqrt{12})$: hodoscope limit



Comparison with MC



- Theory reproduces the Monte Carlo simulation very well !
- We can estimate the resolution analytically

drift distance

$$\sigma_x = \sigma_x(z; w, C_d, N_{eff}, [f_j])$$

pad pitch

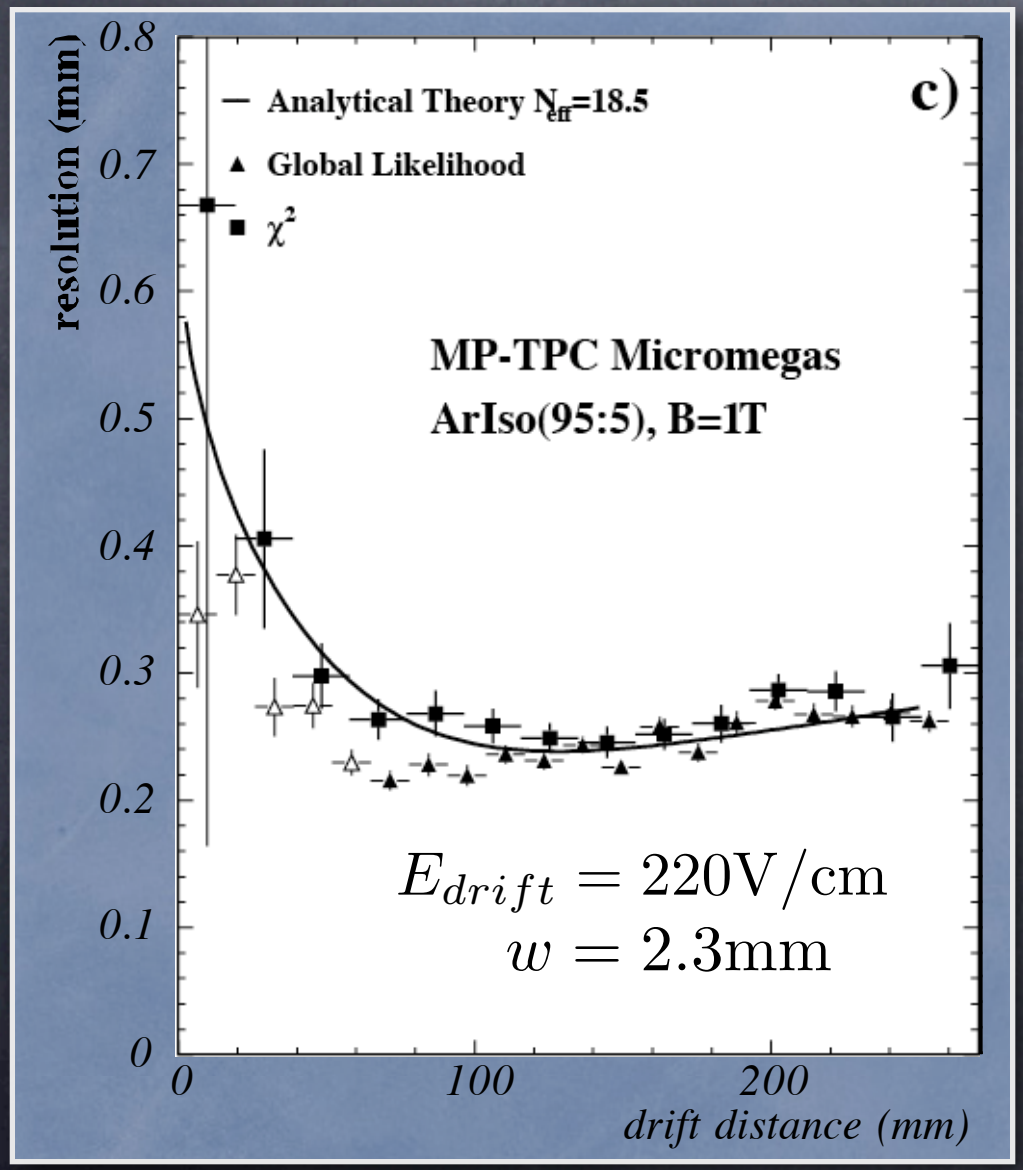
diffusion const.

pad response function

δ-fun. for MM: $\sigma_{PRF} \simeq 12\mu\mathrm{m}$

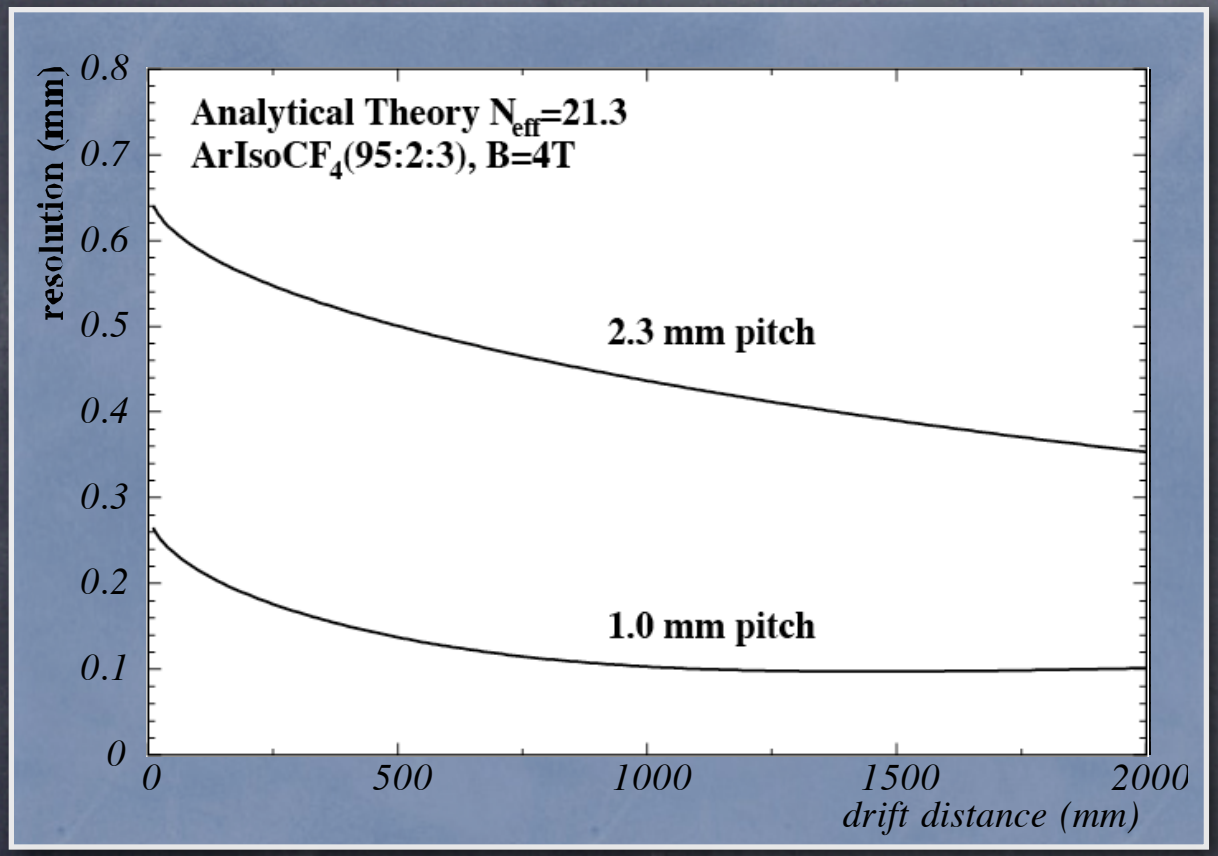
gauss. for GEM: $\sigma_{PRF} \simeq 350\mu\mathrm{m}$

Comparison with Measurements



- Theory reproduces the data well
- Underestimation in the data of σ_x at short drift distance is due to track bias
- Global likelihood method eliminates S-shape systematics at short distance when possible

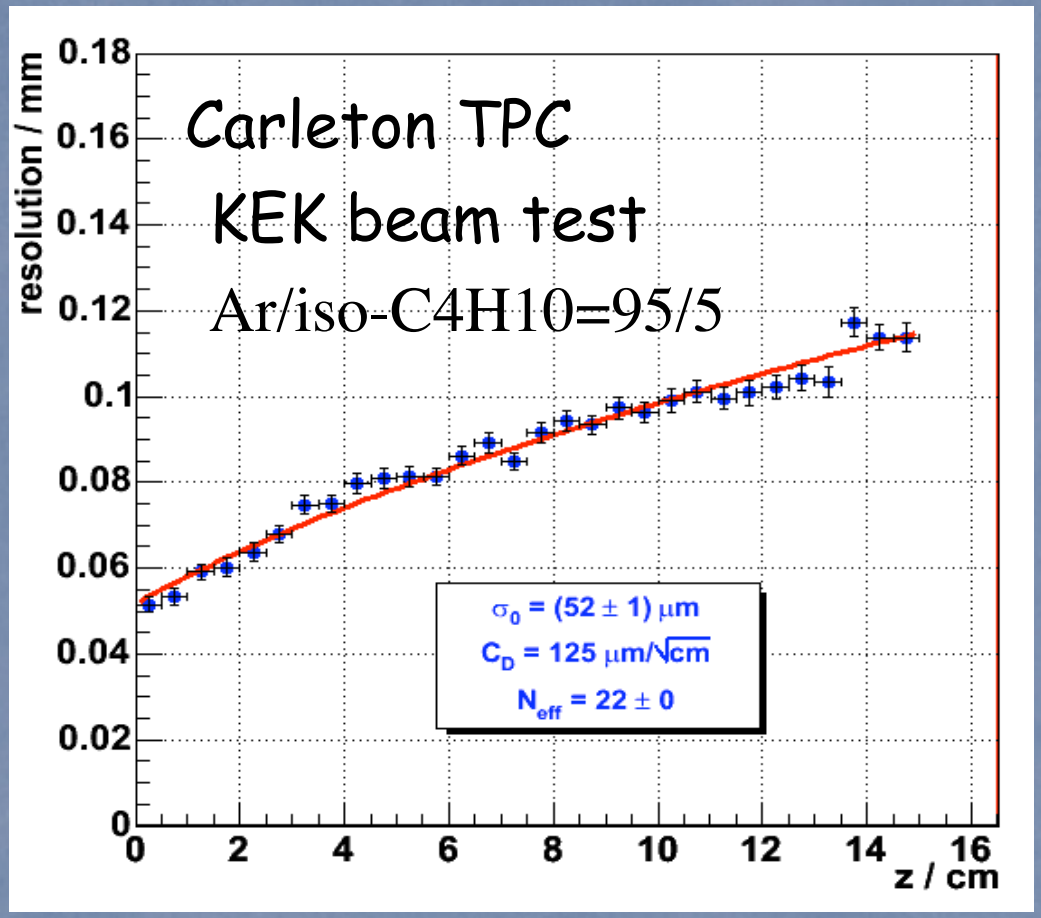
Extrapolation to LC TPC



- Need to reduce pad size relative to PRF
  - Resistive anode for MM
  - Digital pixel readout? ideal to avoid effect of gain fluctuation if possible
  - Defocusing + narrow (1mm) pad for GEM

→ Preliminary results seem promising for a resistive anode!

MM w/ Resistive Anode



At B=4T

$C_d = 25\mu\mathrm{m}/\sqrt{\mathrm{cm}}$ for $\mathrm{Ar/CH_4} = 91/9$

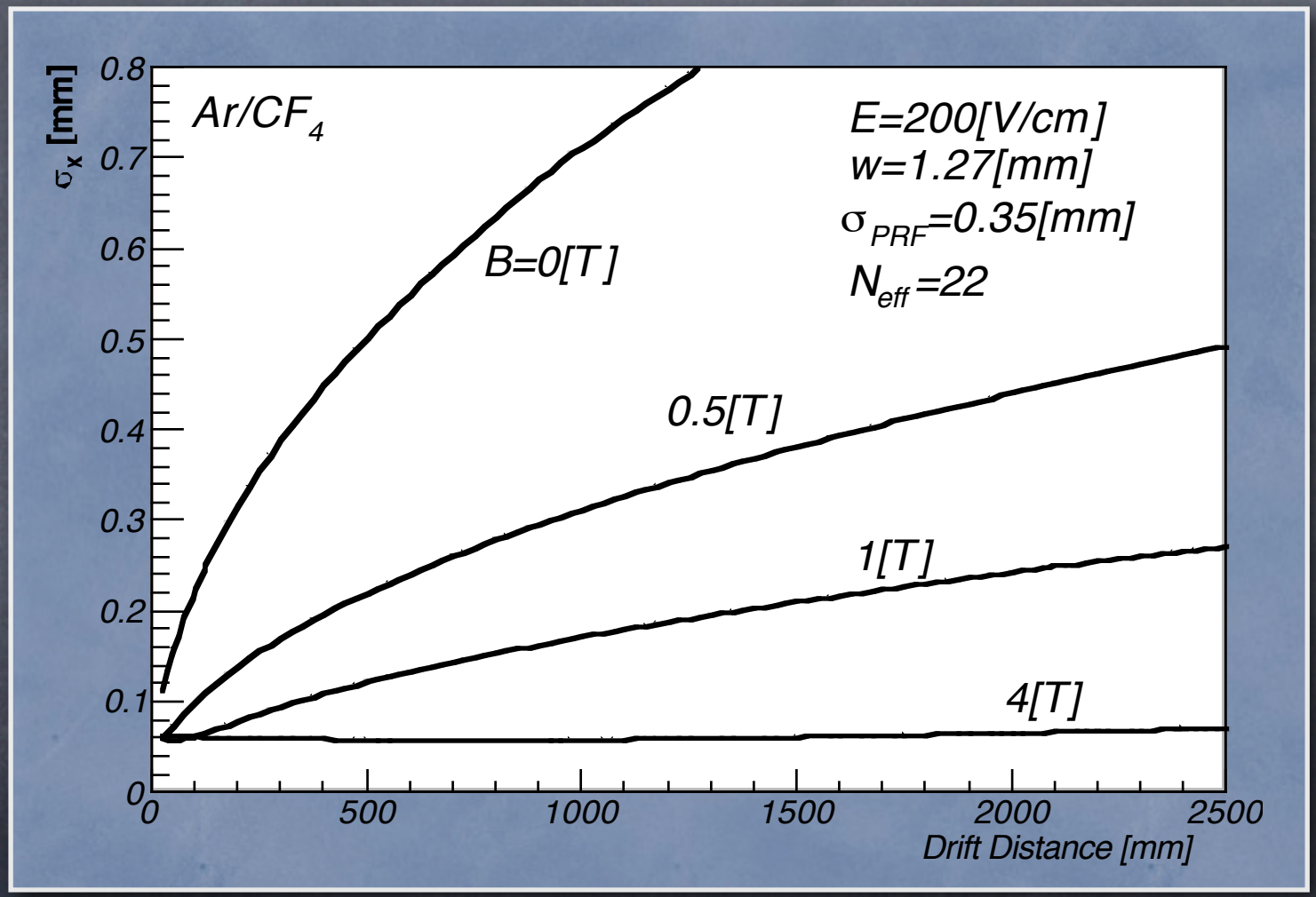
$C_d = 20\mu\mathrm{m}/\sqrt{\mathrm{cm}}$ for $\mathrm{Ar/CF_4} = 97/3$

Extrapolation to LC-TPC (4T)

$\sigma_x \simeq 100\mu\mathrm{m}$ at $2.5\mathrm{m}$

Extrapolation to LC TPC

Sample calculation for GEM with Ar/CF4



GEM in Ar/CF4: promising but needs R&D

Summary

- Efforts to understand KEK beam test data crystallized into an analytic formula for the spatial resolution of a MPGD readout TPC.
- We can now analytically estimate the spatial resolution

drift distance

$$\sigma_x = \sigma_x(z; w, C_d, N_{eff}, [f_j])$$

pad pitch

pad response function

diffusion const.

Effective No. track electrons

Theoretical basis for how to improve the spatial resolution!

Possible improvement of theory: angle effects