

Combination of multiple samples

- ❖ New readout schemes (e.g. SiPM) allow possibility of multiple sampling of showers
 - Verify effect of finer readout segmentation
 - Analysis a-la-Wigmans (NIM A537 (2005)) suggests that linear combination of measurements could be adequate

$$\begin{aligned}
 Q &= E(f + r_q(1 - f)) = E(r_q + (1 - r_q)f) \\
 S &= E(f + r_s(1 - f)) = E(r_s + (1 - r_s)f)
 \end{aligned}$$

■ Solving for E and f:

$$\begin{aligned}
 E &= \frac{S(1 - r_q) - Q(1 - r_s)}{r_s - r_q} \\
 &= S + \frac{(S - Q)(1 - r_s)}{r_s - r_q}
 \end{aligned}$$

A linear technique

➤ Definitions:

- Assume shower is sampled in N locations
- Let x_i be the response of i^{th} sample
 - Each event generates a vector of samples \mathbf{x}
 - means of x_i are $\mu_i = \langle x_i \rangle$ ($\boldsymbol{\mu} = \langle \mathbf{x} \rangle$ in vector notation)
- E is the beam energy generating the shower

➤ Statement of problem:

- Find vector \mathbf{w} such that:
 - $\boldsymbol{\mu}^t \mathbf{w} = E$
 - $\sigma^2_E = \langle (\mathbf{x}^t \mathbf{w} - E)^2 \rangle$ is minimized

Solution (1)

❖ Rework quantity to minimize:

$$\begin{aligned}\sigma_E^2 &= \langle (\vec{x}^t \vec{w} - E)^2 \rangle \\ &= \vec{w}^t [\vec{x} \vec{x}^t] \vec{w} - E^2 \\ &= \vec{w}^t [\vec{x} \vec{x}^t] \vec{w} - \vec{w}^t [\vec{\mu} \vec{\mu}^t] \vec{w} = \vec{w}^t C \vec{w}\end{aligned}$$

➤ Where C is the covariance matrix of the samples:

$$C = \begin{pmatrix} \langle x_1^2 \rangle & \langle x_1 x_2 \rangle & \cdots & \langle x_1 x_N \rangle \\ \langle x_2 x_1 \rangle & \langle x_2^2 \rangle & \cdots & \langle x_2 x_N \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle x_N x_1 \rangle & \langle x_N x_2 \rangle & \cdots & \langle x_N^2 \rangle \end{pmatrix} - \vec{\mu} \vec{\mu}^t$$

Solution (2)

❖ Use Lagrange multiplier λ to account for total energy constraint term in minimization

➤ Quantity to minimize: $\chi^2 = \vec{w}^t C \vec{w} + 2\lambda(\vec{w}^t \vec{\mu} - E)$

➤ Solution: $\frac{1}{2} \frac{\partial \chi^2}{\partial \vec{w}} = C \vec{w} + \lambda \vec{\mu} = 0$
 $\vec{w} = -\lambda C^{-1} \vec{\mu}$ replacing in energy constraint:
 $-\lambda \vec{\mu}^t C^{-1} \vec{\mu} = E$
 $\lambda = \frac{-E}{\vec{\mu}^t C^{-1} \vec{\mu}}$

finally:

$$\vec{w} = E \frac{C^{-1} \vec{\mu}}{\vec{\mu}^t C^{-1} \vec{\mu}}$$

Preliminary results

❖ Results on GEANT4 simulations:

- Problem: many files unusable (corruption in AFS copy)
- No improvement at low momentum (1-2 GeV)
- 10% improvement found for 10 GeV pions
 - $\sigma_E/E = 12.2\% \rightarrow 10.6\%$
- No gain using finer segmentation
 - All resolution improvement obtained by using sum of Cherenkov and sum of ionization – additional segmentation does not change the resolution

Conclusions

❖ Linear optimization of resolution found

- Depends on first and second moments (with correlations) of input distributions, but no assumption is made on the shape of such distributions

❖ Optimal weight vector can be obtained directly from data

❖ Questions to be addressed:

- Optimization stability with different energy and particle type
- Is current parameterization optimal?
 - We should do better!
 - Need guidance from calorimeter experts