Combination of multiple samples

- New readout schemes (e.g. SiPM) allow possibility of multiple sampling of showers
  - Verify effect of finer readout segmentation
  - Analysis a-la-Wigmans (NIM A537 (2005)) suggests that linear combination of measurements could be adequate

$$\begin{aligned} Q &= E(f + r_q(1-f)) &= E(r_q + (1-r_q)f) \\ S &= E(f + r_s(1-f)) &= E(r_s + (1-r_s)f) \end{aligned}$$

- Solving for E and f:

$$\begin{aligned} E &= \frac{S(1-r_q)-Q(1-r_s)}{r_s-r_q} \\ &= S + \frac{(S-Q)(1-r_s)}{r_s-r_q} \end{aligned}$$

A linear technique

- Definitions:
  - Assume shower is sampled in N locations
  - Let x_i be the response of i^{th} sample
    - Each event generates a vector of samples **x**
    - means of x_i are $\mu_i = \langle x_i \rangle$ ($\boldsymbol{\mu} = \langle \mathbf{x} \rangle$ in vector notation)
  - E is the beam energy generating the shower

- Statement of problem:
  - Find vector **w** such that:
    - $\boldsymbol{\mu}^t \mathbf{w} = E$
    - $\sigma^2_E = \langle (\mathbf{x}^t \mathbf{w} - E)^2 \rangle$ is minimized

Solution (1)

- Rework quantity to minimize:

$$
\begin{aligned}
\sigma_E^2 &= \left\langle (\vec{x}^t \vec{w} - E)^2 \right\rangle \\
&= \vec{w}^t [\vec{x}\vec{x}^t] \vec{w} - E^2 \\
&= \vec{w}^t [\vec{x}\vec{x}^t] \vec{w} - \vec{w}^t [\vec{\mu}\vec{\mu}^t] \vec{w} = \vec{w}^t C \vec{w}
\end{aligned}
$$

  - Where C is the covariance matrix of the samples:

$$
C = \begin{pmatrix} < x_1^2 > & < x_1 x_2 > & \cdots & < x_1 x_N > \\ < x_2 x_1 > & < x_2^2 > & \cdots & < x_2 x_N > \\ \cdots & \cdots & \cdots & \cdots \\ < x_N x_1 > & < x_N x_2 > & \cdots & < x_N^2 > \end{pmatrix} - \vec{\mu}\vec{\mu}^t
$$

Solution (2)

- Use Lagrange multiplier λ to account for total energy constraint term in minimization
  - Quantity to minimize: $\chi^2 = \vec{w}^t C \vec{w} + 2\lambda(\vec{w}^t \vec{\mu} - E)$
  - Solution:

$$\frac{1}{2}\frac{\partial \chi^2}{\partial \vec{w}} = C\vec{w} + \lambda\vec{\mu} = 0$$

$$\vec{w} = -\lambda C^{-1}\vec{\mu} \quad \text{replacing in energy constraint:}$$

$$-\lambda \vec{\mu}^t C^{-1} \vec{\mu} = E$$

$$\lambda = \frac{-E}{\vec{\mu}^t C^{-1} \vec{\mu}}$$

finally:

$$\vec{w} = E \frac{C^{-1}\vec{\mu}}{\vec{\mu}^t C^{-1} \vec{\mu}}$$

Preliminary results

- Results on GEANT4 simulations:
  - Problem: many files unusable (corruption in AFS copy)
  - No improvement at low momentum (1-2 GeV)
  - 10% improvement found for 10 GeV pions
    - $\sigma_E/E = 12.2\% \rightarrow 10.6\%$
  - No gain using finer segmentation
    - All resolution improvement obtained by using sum of Cherenkov and sum of ionization – additional segmentation does not change the resolution

Conclusions

- Linear optimization of resolution found
  - Depends on first and second moments (with correlations) of input distributions, but no assumption is made on the shape of such distributions
- Optimal weight vector can be obtained directly from data
- Questions to be addressed:
  - Optimization stability with different energy and particle type
  - Is current parameterization optimal?
    - We should do better!
    - Need guidance from calorimeter experts