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Resolution of dual readout calorimeters

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Abstract

We discuss various corrections to improve the resolution of dual readout hadronic calorimeters.

1 Introduction

We assume to have a sampling calorimeter where some of the measurement layers measure the ionization (such as with a scintillating material) while other layers measure the cherenkov light produced. Both measurements count the number of charged tracks present, however they do it with differnt cut-offs. In particular the cherenkov layers are mostly sensitive to the electromagnetic component since electrons are more likely to be above threshold for the generation of cherenkov light. This dual measurement of the energy in principle allows us to correct for the fluctuation of the electromagnetic fraction, f_{em}, and therefore to improve the energy resolution. In the following we use some of the formalism and concepts discussed in ref. [1].

2 Formalizing the problem

Let s and q be the response of the scintillator and cherenkov layers after normalizing their response with electrons. That is on average $< s >=< q >= E$ for electron beams of energy E. In the case of hadronic showers the two measurements are in general different because of the different intrinsic response to the hadronic component of the shower. Making explicit the electromagnetic fraction contribution to the observed signals we have:

$$s = E(f + r_s(1 - f)) \tag{1}$$
$$q = E(f + r_q(1 - f)) \tag{2}$$

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indeed in case of a purely electromagnetic shower $f = 1$ the responce is as in the case of the electron calibration. The parameters $r_s$and r_q are the ratios of the intrinsic responses to the hadronic and electromagnetic components of the shower (h/e) which are in general different for the two types of measurement. Given this dependence on the electromagnetic fraction s and q are correlated with a correlation given by the elimination of f in the equations (1, 2):

$$
\begin{aligned}
s &= \frac{1-r_s}{1-r_q} q + E\left(1 - \frac{1-r_s}{1-r_q}\right) && (3)\\
&= \lambda q + E(1-\lambda) && (4)
\end{aligned}
$$

where we have introduced the slope parameter $\lambda = (1-r_s)/(1-r_q)$. An example of this correlation obtained with a GEANT4 simulation is shown in fig. 1, where both the 2-D plot and the corresponding profile are shown. Overlaid on the profile plot we show the fit for the parameter λ using the functional form of eq. (4) and fixing the energy, E.

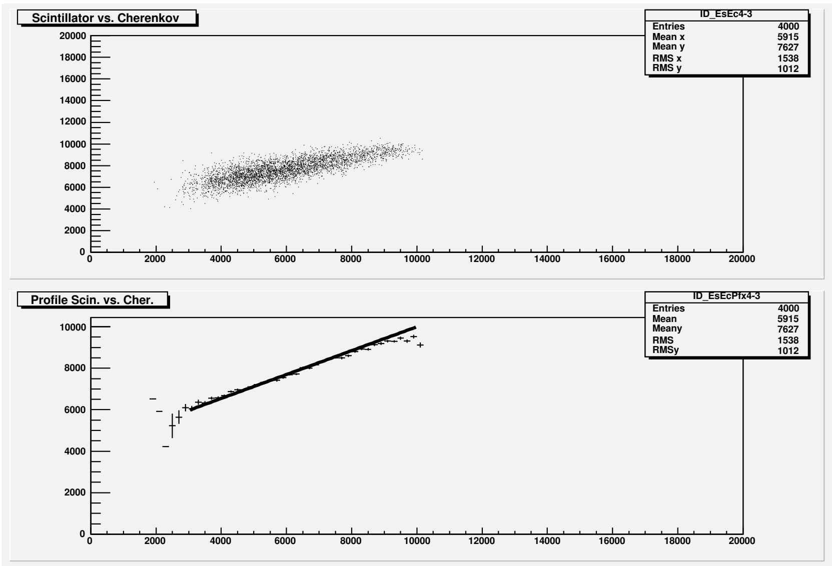

Figure 1: Correlation between the scintillator and cherenkov signals for 10 GeV pion showers.

Since there are some slight deviation from linearity in the low statistics regions at the lower and upper ends of the range, we restrict the fitting range to the interval $[0.3 * E, E]$. λ can also be calculated from from the correlation coefficients and the sigmas of the s and q distributions. If $\sigma_{qs} =< s - \bar{s})(q - \bar{q}) >$, $\sigma_s^2 =< (s - \bar{s})^2 >$ and $\sigma_q^2 =< (q - \bar{q})^2 >$ (where $\bar{s}$ and $\bar{q}$ are the mean values of s and q), then it is easy to show that: $\lambda = \sigma_{qs}/\sigma_q^2$. This second definition is equivalent to the first when no subrange is selected for the fit.

Having measured the correlation between q and s we now proceed to see how we can use it to improve the resolution. We investigate two possibilities:

1. we use the measurements of q and s to extract the electromagnetic fraction f and then define a corrected scintillation signal, $s' = s/(f + r_s(1-f))$. After performing all the calculations we find that s' depends only on λ and is given by the simple formula:

$$s' \;=\; \frac{s-\lambda q}{1-\lambda} \tag{5}$$

this is what we call the energy independent correction because λ has a rather soft variation with energy;

2. we add back to the scintillator signal the amount needed to get an average energy of E according to the $s-q$ plot already studied:

$$s' \;=\; s+\lambda(E-q) \tag{6}$$

this is what we call energy dependent correction, because the value of the energy enters explicitly in the correction.

3 Simulation results

The results of the simulation for various relative thickness of scintillation and cherenkov layers is shown in table 1 for 10 GeV pions. For various choices of scintillator and cherenkov layer thickness we calculate the relative resolution of the scintillator and cherenkov peaks (shown in columns 2 and 3); we then calculate the parameter λ in the two ways described earlier: 1) from a fit to the s vs. q plot, 2) from the correlation coefficient between s and q. We label these two ways to determine λ as “fit” and “stat” in the table. For each choice of λ we then apply both the energy dependent and the energy independent corrections.

We learn a lot from the table:

a. the energy dependent corrections are a factor of two or more better in resolution. This leads to the idea to use them with some guess for the beam energy and then iterate: that is define a corrected scintillator energy from the sequence: $s_{n+1} = s + \lambda(s_n - q)$. It is interesting to notice that the limit of this sequence gives $s' = (s-\lambda q)/(1-\lambda)$ which is precisely the energy independent definition.

b. the energy resolution improvement with energy independent corrections is rather modest and works only for large scintillator sampling fractions. We can calculate the energy resolution analytically if we use the statistical method to get λ: $\sigma^2_{s'} = (\sigma^2_s - \lambda^2\sigma^2_q)/(1-\lambda)^2$. For different sampling fractions λ does not change much, so in order to have a good resolution we want a small σ_s (i.e. large scintillator

Scintillator Thickness (mm)	Cherenkov Thickness (mm)	Scint. only	Cher. only	En. Ind. (λ from fit)	En. Dep. (λ from fit)	En. Dep. (λ from stat.)	En. Ind. (λ from stat)
3	2	11.90	25.63	8.81	3.66	3.67	8.25
3	10	12.80	25.59	12.55	5.25	5.30	11.84
3	20	14.02	25.59	16.32	6.79	6.87	15.40
3	40	17.07	25.57	22.98	9.95	10.06	22.23
10	2	11.84	25.81	8.91	3.71	3.71	8.23
10	10	12.24	25.72	10.66	4.46	4.49	9.95
10	20	12.98	25.65	13.48	5.66	5.72	12.61
10	40	14.64	25.62	18.29	7.69	7.80	17.19
20	2	11.82	26.09	9.45	3.97	3.97	8.61
20	10	12.14	25.86	10.58	4.43	4.46	9.76
20	20	12.59	25.76	12.29	5.18	5.23	11.45
20	40	13.88	25.73	16.45	6.98	7.08	15.31
30	2	11.80	26.43	10.06	4.24	4.22	8.92
30	10	11.99	26.18	10.52	4.44	4.46	9.55
30	20	12.42	25.99	12.22	5.16	5.21	11.17
30	40	13.31	25.85	15.00	6.46	6.55	13.92
40	2	11.78	26.70	10.30	4.35	4.32	8.99
40	10	11.95	26.38	10.90	4.61	4.62	9.71
40	20	12.32	26.12	12.16	5.19	5.23	11.03
40	40	13.27	26.00	15.06	6.36	6.45	13.73

Table 1: Relative calorimeter resolution in % in various configurations for 10 GeV pions.

sampling fraction) and a large σ_q; the latter however does not seem to be too affected by the sampling fraction as shown in table 1. We observe a similar behaviour for 20 GeV pions, but no improvement for 1 GeV pions in any of the configurations tested. This is consistent with the results obtained by the Fermilab group.

In table 2, we show the values of λ obtained for 1, 10 and 20 GeV pions in the various configurations. We can see that there is some energy dependence, while the dependence on the configuration is rather small.

We can fit the dependence of the relative energy resolution, σ_E/E, as function of $1/\sqrt{E}$, which is expected to be approximately linear, for all the configurations studied. In table 3 we show the summary of all the slopes and constant terms obtained. The results are interesting and are worth a few comments:

a. the uncorrected scintillator measurement has a slope which increases as the sampling fraction decreases (as expected), but this is partially compensated by a decrease of the constant term;

Scintillator Thickness (mm)	Cherenkov Thickness (mm)	λ-fit 1GeV	λ-stat 1 GeV	λ-fit 10 GeV	λ-stat 10 GeV	λ-fit 20 GeV	λ-stat 20 GeV
3	2	0.4915	0.5559	0.5848	0.5468	0.5808	0.5602
3	10	0.4868	0.5514	0.5815	0.5435	0.5783	0.5614
3	20	0.4816	0.5465	0.5836	0.5448	0.5782	0.5594
3	40	0.4928	0.5149	0.5824	0.5545	0.5780	0.5641
10	2	0.4888	0.5172	0.5836	0.5386	0.5795	0.5552
10	10	0.4852	0.5172	0.5814	0.5391	0.5787	0.5520
10	20	0.4728	0.4991	0.5802	0.5366	0.5792	0.5551
10	40	0.4676	0.4742	0.5792	0.5365	0.5781	0.5581
20	2	0.4803	0.4661	0.5790	0.5262	0.5769	0.5430
20	10	0.4760	0.4758	0.5804	0.5324	0.5779	0.5476
20	20	0.4756	0.4647	0.5777	0.5327	0.5775	0.5495
20	40	0.4538	0.4307	0.5753	0.5267	0.5756	0.5531
30	2	0.4682	0.4229	0.5771	0.5109	0.5725	0.5346
30	10	0.4755	0.4438	0.5765	0.5196	0.5762	0.5418
30	20	0.4691	0.4368	0.5767	0.5203	0.5756	0.5447
30	40	0.4402	0.3932	0.5685	0.5181	0.5661	0.5415
40	2	0.4614	0.3759	0.5763	0.5028	0.5714	0.5205
40	10	0.4668	0.4106	0.5755	0.5087	0.5756	0.5358
40	20	0.4506	0.3969	0.5721	0.5117	0.5725	0.5412
40	40	0.4523	0.3553	0.5762	0.5157	0.5746	0.5304

Table 2: Correlation parameter λ for various configurations and energies.

b. the energy independent correction usually degrades the slope except for very low scintillator sampling fractions;

c. the energy independent correction improves the constant term, but only as long as the scintillator sampling fraction does not get too small. In fact the constant term for this corrected data grows with scintillator sampling fraction;

d. the energy dependent correction improves always the slope and eliminates the constant term.

It is interesting to note for comparison that the result obtained with the DREAM calorimeter on the test beam [1]. The sampling fraction of the scintillating fibers in DREAM is ~2% and the slope and constant term are 49% and 7% before correction, and 41% and 4.2% after correction. They get a better improvement of the slope at comparable slopes and also a simulataneous improvement of the constant term. The source of this better performance relative to our simulation should be better understood.

Scint. Thick. mm	Cher. Thick. mm	Scintillator Slope %	Scintillator C. term %	En. Ind. Corr. Slope %	En. Ind. Corr. C. term %	En. Dep. Corr. Slope %	En. Dep. Corr. C. term %
3	2	16.49	6.40	23.76	0.61	11.18	0.12
3	10	24.18	4.99	33.57	1.15	17.65	-0.24
3	20	32.46	3.94	39.21	3.46	22.93	-0.06
3	40	47.42	2.26	44.24	7.90	28.22	1.44
10	2	16.02	6.48	22.68	0.90	11.47	0.07
10	10	19.81	5.80	29.33	0.78	15.09	-0.16
10	20	24.78	5.06	34.61	1.73	19.60	-0.36
10	40	35.04	3.68	40.93	4.42	26.75	-0.37
20	2	15.81	6.52	21.95	1.44	12.25	0.06
20	10	18.01	6.15	26.22	1.24	14.50	-0.14
20	20	21.55	5.65	30.74	1.83	17.74	-0.24
20	40	29.15	4.61	36.95	3.62	24.44	-0.49
30	2	15.66	6.55	20.89	2.02	12.72	0.14
30	10	17.29	6.29	24.47	1.71	14.31	-0.04
30	20	19.86	5.92	27.89	2.18	16.81	-0.08
30	40	26.26	4.97	34.34	3.21	23.34	-0.60
40	2	15.60	6.55	20.44	2.37	13.71	0.02
40	10	16.82	6.37	23.09	2.19	14.47	0.01
40	20	18.92	6.12	26.02	2.63	16.99	-0.14
40	40	23.90	5.52	31.05	3.78	22.19	-0.42

Table 3: Slopes and constant terms from linear fits to relative errors versus the inverse of the square root of the energy in GeV.

4 Conclusions

In conclusion we have looked at ways to improve the resolution by merging scintillation and cherenkov light signals in a dual readout calorimeter. Current estimates indicate that a 30% improvement in resolution is feasible, but at the price of a large scintillator sampling fraction which may be difficult to obtain experimentally. The next step is to investigate is configurations with more passive absorber, but it is hard to believe that reducing the information the resolution can improve!

Alternate methods exploiting the depth segmentation may be needed to improve the resolution beyond this point and for a more "friendly" range of scintillator sampling fractions.

Comparisons of the energy dependence of the resolutions obtained in this simulation with data from the DREAM calorimeter suggests a different behaviour, which should be better understood.

References

[1] N. Akchurin *at al.*, Nucl. Instr. and Meth. A **537**, (2005) 537-561.

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