Evolution of ILC parameter optimization with center-of-mass energy (for constant beam intensity)

Total power $P_{electrical}$
Beamstrahlung emission δ_{BS}
Normalised vertical emittance $\varepsilon_{n,y}$

Number of bunches per train n_b
Number of particles per bunch N_e
Power transfer efficiency η
Collision frequency f
Bunch RMS sizes $\sigma_{x,y,z}$
Pinch enhancement factor H_D
Optical envelope function β
(at IP → interpreted as depth of field)

$$L \sim \frac{n_b N_e^2 f}{4\pi\sigma_x\sigma_y} H_D$$

$$\delta_{BS} \sim \frac{N_e^2 E_{cm}}{\sigma_z(\sigma_x+\sigma_y)^2}$$

$$\sigma_z < \beta_y \quad \text{(depth of field)}$$

$$\sigma^2 = \varepsilon_n \beta / \gamma$$

→

$$L \sim \eta \frac{P_{electrical}}{E_{CM}} \sqrt{\frac{\delta_{BS}}{\varepsilon_{n,y}}} H_D$$

$$x',y'_{(defl.)} \sim 2N_e r_e \, x,y / \gamma \, \sigma_{x,y} (\sigma_x + \sigma_y)$$

$$x',y'_{(max.\ defl.)} \sim 1 / \gamma\sigma_x \sim 1 / \sqrt{\gamma\beta_x}$$

$$\sigma_\theta^2 = \varepsilon_n / \beta\gamma \text{ (natural angular size)}$$

1. Scaling without changing any beam parameters

$L \sim E_{cm}$

$\delta_{BS} \sim E_{cm}^{2}$

$\Rightarrow$ IP angular divergence (final doublet quad aperture) $\sim E_{cm}^{-0.5}$

	L	δ_{BS}	FD aperture	σ_z, $\beta_{x,y}$
		(ratios to values at 500 GeV CM)		
500 (nom.)	1	1	1	1
350 (top)	0.7	0.5	1.2	1
250 (HZ)	0.5	0.25	1.4	1
100 (Z)	0.2	0.04	2.2	1

2. Changing beam parameters to maintain luminosity

$L \sim 1$

$\delta_{BS} \sim 1$

e.g. by changing σ_z , β_x and β_y as ~ E_{cm}

$\Rightarrow$ IP angular divergence (final doublet quad aperture) ~ E_{cm}^{-1}

	L	δ_{BS}	FD aperture	σ_z, $\beta_{x,y}$
		(ratios to values at 500 GeV CM)		
500 (nom.)	1	1	1	1
350 (top)	1	1	1.5	0.7
250 (HZ)	1	1	2	0.5
100 (Z)	1 (unrealistic)	1	5	0.2
100 (Z)	0.28 (difficult)	0.08	2.7	0.7(as for top)
100 (Z)	0.2	0.04	2.2	1 (no change)