Status of the Event Generator WHIZARD – SUSY Simulations at the ILC and Radiative Corrections

Jürgen Reuter

University of Freiburg

ALCPG/GDE 07, Fermilab, October 23, 2007

The need for Multi-Particle Event Generators

New collider environments more complicated

Very complicated signal/background processes

New physics:

- DM: Conserved discrete parity: pair production, decay chains
- Complicated, quasi-degenerate spectrum at the Terascale
- High-multiplicity final states

ILC allows for precision measurements at least at per cent-level

Need for Multi-Particle Event Generators

JR, Snowmass 05; Hagiwara et al., 06; Hewett, 07; Kilian/Kobel/Mader/JR/Schumacher

- BSM processes do not factorize into $2 \to 2$ production/decay
- Interferences of several (partially) resonant diagram groves
- Off-shell effects violate Breit-Wigner approximation

Berdine/Kauer/Rainwater 07; Berdine/Kauer/JR/Rainwater

Sbottom production at the ILC

Hagiwara/.../JR, 06

- In contrast to the LHC: Electroweak production Cross sections for $\sqrt{s} = 800$ GeV
- More channels contribute to $e^+e^- \to b\bar{b}\tilde{\chi}_1^0\tilde{\chi}_1^0$:
  $e^+e^- \to Zh, ZH, Ah, HA, \tilde{\chi}_1^0\tilde{\chi}_2^0, \tilde{\chi}_1^0\tilde{\chi}_3^0, \tilde{\chi}_1^0\tilde{\chi}_4^0, \tilde{b}_1\tilde{b}_1^*, \tilde{b}_1\tilde{b}_2^*$ (412 diagrams)
- Irreducible SM background: $e^+e^- \to b\bar{b}\nu_i\bar{\nu}_i$ (WW fusion, Zh, ZZ) (47 diagrams)

Channel	$\sigma_{2\to2}$ [fb]	$\sigma \times$ BR [fb]	$\sigma_{\rm BW}$ [fb]
Zh	20.574	1.342	1.335
ZH	0.003	0.000	0.000
hA	0.002	0.001	0.000
HA	5.653	0.320	0.314
$\tilde{\chi}_1^0\tilde{\chi}_2^0$	69.109	13.078	13.954
$\tilde{\chi}_1^0\tilde{\chi}_3^0$	24.268	3.675	4.828
$\tilde{\chi}_1^0\tilde{\chi}_4^0$	19.337	0.061	0.938
$\tilde{b}_1\tilde{b}_1$	4.209	0.759	0.757
$\tilde{b}_1\tilde{b}_2$	0.057	0.002	0.002
Sum		19.238	22.129
Exact			19.624
w/ISR			22.552

Channel	$\sigma_{2\to2/3}$ [fb]	$\sigma \times$ BR [fb]	$\sigma_{\rm BW}$ [fb]
ZZ	202.2	12.6	13.1
Zh	20.6	1.9	1.9
ZH	0.0	0.0	0.0
$Z\bar{\nu}\nu$	626.1	109.9	111.4
$h\bar{\nu}\nu$	170.5	76.5	76.4
$H\bar{\nu}\nu$	0.0	0.0	0.0
Sum		186.5	187.7
Exact			190.1
w/ISR			174.2

- Use widths to the same order as your process

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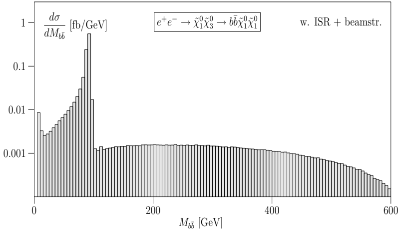
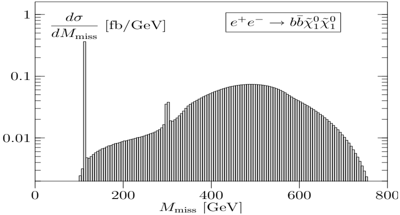
ILC Results

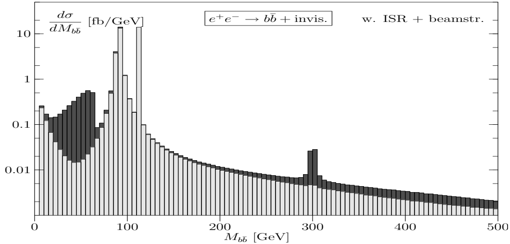
Off-shell decay $\tilde\chi_3^0 \to (\tilde b_1)_{off}\bar b \to b\bar b\tilde\chi_1^0$ gives broad continuum





ISR/beamstrahlung: corrections of same order (effects all p_{miss} observables)

$b\bar b$ invariant mass with SM background:



Cut out the resonances

$M_{b\bar b} < 150$ GeV

250 GeV $< M_{b\bar b} <$ 350 GeV

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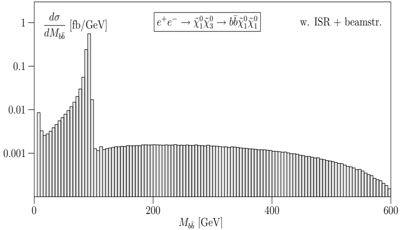

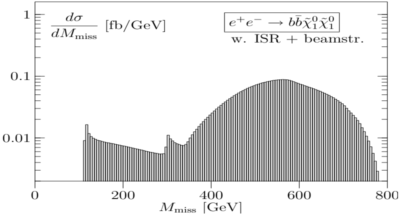

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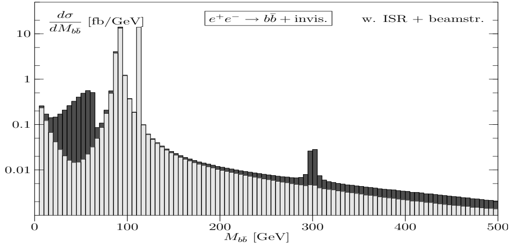

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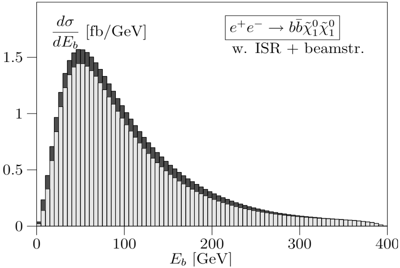
Sum	187.7	2.117
Exact	190.1	1.765
w/ISR	174.2	1.609

Channel	σ_{BW} [fb]	σ_{BW}^{cut} [fb]
Zh	1.335	0.009
HA	0.314	0.003
$\tilde\chi_1^0\tilde\chi_2^0$	13.954	0.458
$\tilde\chi_1^0\tilde\chi_3^0$	4.828	0.454
$\tilde\chi_1^0\tilde\chi_4^0$	0.938	0.937
$\tilde b_1\tilde b_1$	0.757	0.451
$\tilde b_1\tilde b_2$	0.002	0.001
Sum	22.129	**2.314**
Exact	19.624	**0.487**
w/ISR	22.552	**0.375**

$\tilde b_1 \to b\tilde\chi_1^0$ decay kinematics affected



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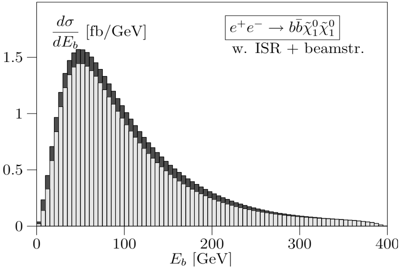

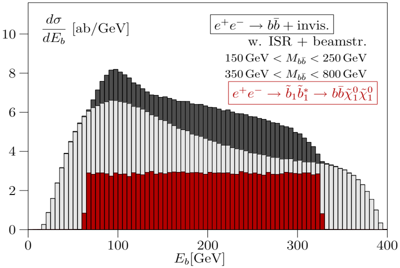

The Multi-Particle Generator WHIZARD

Kilian/Ohl/JR, 07

Very high level of Complexity:

- $e^+e^- \to t\bar{t}H \to b\bar{b}b\bar{b}jj\ell\nu$ (110,000 diagrams)
- $e^+e^- \to ZHH \to ZWWWW \to bb + 8j$ (12,000,000 diagrams)
- $pp \to \ell\ell + nj, n = 0, 1, 2, 3, 4, \ldots$ (2,100,000 diagrams with 4 jets + flavors)
- $pp \to \tilde{\chi}_1^0\tilde{\chi}_1^0 bbbb$ (32,000 diagrams, 22 color flows, $\sim 10,000$ PS channels)
- $pp \to VVjj \to jj\ell\ell\nu\nu$ incl. anomalous TGC/QGC
- Test case $gg \to 9g$ (224,000,000 diagrams)

Current versions:

WHiZard 1.51 / O'Mega 000.011beta Ω $\to$ joint version:

WHIZARD 1.99 release date: somehow this or next week

one grand unified package (incl. VAMP, Circe, Circe 2, WHiZard, O'Mega)

New web address:

`http://whizard.event-generator.org`

- Standard reference for 1.99 + upcoming versions:

Kilian/Ohl/JR, 0708.4233

Major upgrade this fall/winter: WHIZARD 2.0.0

Technical details about WHIZARD

Status of WHIZARD 1.99: Installation

- Download tar-ball from `http:whizard.event-generator.org`
- unpack, do `configure`, `make install` that's it!
- OK, granted: specify locations of external packages and O'Caml language (part of many Linux distributions, `http://caml.inria.fr`)

WHIZARD is written in Fortran 90/95. Compiler status?

- works w/ (almost) all commercial compiler: Intel, Lahey, NAG, Pathscale
- Portland has a severe compiler bug
- compiles with g95
- compiles with gfortran 4.3.0 (will be part of new Linux SuSe 11.0, Debian 4.1, ...)
- lots of Fortran2003 features coming (No need for reprogramming in C++)

Basic facts:

- Helicity amplitudes
- Iterative adaptive multi-channel phase space (viable for $2 \to 10$)
- Unweighted events (formats: binary, HEPEVT, ATHENA, LHA, STDHEP)
- Graphical analysis tool

Implemented Physics Content

Structured beams:

For Tevatron/LHC: PDFs from LHAPDF (or PDFLIB)

For ILC physics:

- ISR (implemented: Skrzypek/Jadach, Kuraev/Fadin)
- arbitrarily polarized beams
- beamstrahlung (CIRCE), photon collider spectra (CIRCE 2)
- external (user-defined) beam spectra can be read in

Supported Physics Models:

- Test models: QED, QCD
- SM
- Littlest/Simplest Little Higgs, Little Higgs Models with T parity
- Moose models: 3-site model
- MSSM, NMSSM, extended SUSY models, incl. gravitinos (SLHA)
- Graviton resonances, Extra dimensions
- Noncommutative Standard Model
- Higher-dimensional operators, SM effective field theory extensions
- Anomalous triple and quartic gauge couplings
- K-matrix/Padé unitarization, unitarized resonances Kilian/JR

Comparison of Automated Tools for Perturbative Interactions in SuperSymmetry

cf. `http://whizard.event-generator.org/susy_comparison.html`

$\tau^+\tau^- \to X$

Process	status	Madgraph/Helas 0.5 TeV	Madgraph/Helas 2 TeV	Whizard/O'Mega 0.5 TeV	Whizard/O'Mega 2 TeV	Sherpa/A'Megic 0.5 TeV	Sherpa/A'Megic 2 TeV
$\tilde{\tau}_1\tilde{\tau}_1^*$	●	257.57(7)	79.63(4)	257.32(1)	79.636(4)	257.30(1)	79.638(4)
$\tilde{\tau}_2\tilde{\tau}_2^*$	●	46.55(1)	66.86(2)	46.368(2)	66.862(3)	46.372(2)	66.862(3)
$\tilde{\tau}_1\tilde{\tau}_2^*$	●	95.50(3)	19.00(1)	94.637(3)	19.0015(8)	94.645(5)	19.000(1)
$\tilde{\nu}_\tau\tilde{\nu}_\tau^*$	●	502.26(7)	272.01(8)	502.27(2)	272.01(1)	502.30(3)	272.01(1)
$\tilde{\chi}_1^0\tilde{\chi}_1^0$	●	249.94(2)	26.431(1)	249.954(9)	26.431(1)	249.96(1)	26.431(1)
$\tilde{\chi}_1^0\tilde{\chi}_2^0$	●	69.967(3)	9.8940(3)	69.969(2)	9.8940(4)	69.968(3)	9.8937(5)
$\tilde{\chi}_1^0\tilde{\chi}_3^0$	●	17.0387(3)	0.7913(1)	17.0394(1)	0.79136(2)	17.040(1)	0.79137(5)
$\tilde{\chi}_1^0\tilde{\chi}_4^0$	●	7.01378(4)	1.50743(3)	7.01414(6)	1.5075(5)	7.0141(4)	1.50740(8)
$\tilde{\chi}_2^0\tilde{\chi}_2^0$	●	82.351(7)	18.887(1)	82.353(3)	18.8879(9)	82.357(4)	18.8896(1)
$\tilde{\chi}_2^0\tilde{\chi}_3^0$	●	—	1.7588(1)	—	1.75884(5)	—	1.7588(1)
$\tilde{\chi}_2^0\tilde{\chi}_4^0$	●	—	2.96384(7)	—	2.9640(1)	—	2.9639(1)
$\tilde{\chi}_3^0\tilde{\chi}_3^0$	●	—	0.046995(4)	—	0.0469966(9)	—	0.046999(2)
$\tilde{\chi}_3^0\tilde{\chi}_4^0$	●	—	8.5852(4)	—	8.55857(3)	—	8.5856(4)
$\tilde{\chi}_4^0\tilde{\chi}_4^0$	●	—	0.26438(2)	—	0.264389(5)	—	0.26437(1)
$\tilde{\chi}_1^+\tilde{\chi}_1^-$	●	185.09(3)	45.15(1)	185.093(6)	45.147(2)	185.10(1)	45.151(2)
$\tilde{\chi}_2^+\tilde{\chi}_2^-$	●	—	26.515(1)	—	26.5162(6)	—	26.515(1)
$\tilde{\chi}_1^+\tilde{\chi}_2^-$	●	—	4.2127(4)	—	4.21267(9)	—	4.2125(2)
h^0h^0	●	0.3533827(3)	0.0001242(2)	0.35339(2)	0.00012422(3)	0.35340(2)	0.000124218(6)
h^0H^0	●	—	0.005167(4)	—	0.0051669(3)	—	0.0051671(3)
H^0H^0	●	—	0.07931(3)	—	0.079301(6)	—	0.079311(4)
A^0A^0	●	—	0.07975(3)	—	0.079758(6)	—	0.079744(4)
Zh^0	●	59.591(3)	3.1803(8)	59.589(3)	3.1802(1)	59.602(3)	3.1829(2)
ZH^0	●	2.8316(3)	4.671(5)	2.83169(9)	4.6706(3)	2.8318(1)	4.6706(2)
ZA^0	●	2.9915(4)	4.682(5)	2.99162(9)	4.6821(3)	2.9917(2)	4.6817(2)
A^0h^0	●	—	0.005143(4)	—	0.0051434(3)	—	0.0051440(3)
A^0H^0	●	—	1.4880(2)	—	1.48793(9)	—	1.48802(8)
H^+H^-	●	—	5.2344(6)	—	5.2344(2)	—	5.2345(3)

Upcoming Features

WHIZARD version 2.0.0 coming out this fall/winter

- (More) Automatized installation tool
- New syntax for defining cuts, scales and analyses: allows for arbitrary functions of kinematical variables
- fancier (and faster) color structures from O'Mega
- WHIZARD uses O'Mega info for better/faster phase space generation
- Cascade decays (apply with great care!!!)
  WHIZARD calls itself recursively, breaks double decay chains down into subprocesses
- Leading order (QCD) parton shower

  (so only fragmentation/hadronization and PDFs by external routines)
- Dark matter relic density calculator
- Support for `ROOT` data format
- TAUOLA interface

All points close to finalization;
Major restructuring of the code

Upcoming Features / Future Features

$\longrightarrow$

Future features, 2008ish

- NLO parton shower with correct matching to hard matrix elements
- New manual
- Graphical User Interface (partially already there)
- Standardized interface to FeynArts/FormCalc/LoopTools
- Full-fledged parallelization (partially under way)
- Own algebraic tool for deriving Feynman rules from Lagrangians
- Web interface

Status of SUSY NLO calculations (for ILC processes)

1) Consistent renormalization procedure ($\overline{\text{DR}}$) — Stöckinger, 2005
2) Higgs observables
   - effective potential approx. + RGE — Carena/Garcia/Nierste/Wagner, 1999
   - full 1-loop calculation — Degrassi et al., 2005; Heinemeyer et al., 2004-6
   - leading 2-loop pieces — Rzehak et al., 2005
3) Charginos and Neutralinos
   - full 1-loop: renormalization/spectrum — Fritzsche/Hollik, 2004; Eberl/Majerotto/Öller, 2004
   - pair production and 2-body decays — Fritzsche/Hollik, 2004; Eberl/Majerotto/Öller, 2004
   - 3-body decays — Kovaric/Rolbiecki et al.
4) Sfermions
   - 1-loop: renormalization and mass spectrum — Hollik/Rzehak, 2005
   - $e^+e^- \to \tilde{q}\tilde{q}^*, \tilde{\ell}\tilde{\ell}^+$ — Arhrib/Hollik,Kovaric et al., Freitas et al., 2002-2004
   - 2-body decays — Guasch/Hollik/Solà, 2004
5) Unified framework for codes/calculations: SPA convention — SPA, 2005
6) Full matrix elements, Off-shell and Interference effects
   - all particles, all processes, all colliders — JR et al., 2005; Hagiwara et al., 2006

Classification of NLO corrections

- Loop corrections to SUSY production and decay processes
- nonfactorizable, maximally resonant photon exchange between production and decay
- real radiation of photons
- off-shell kinematics for the signal process
- irreducible background from all other SUSY processes
- reducible, experimentally indistinguishable SM background processes

Multi-pole approximation, justified from EW SM processes Denner et al., 0006307, 0502063, 0604011.

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implemented in Sherpa, Smadgraph, WHIZARD thoroughly checked

Hagiwara et al., 0512260; JR et al., 0512012

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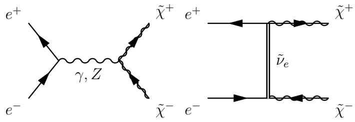
SUSY (NLO) Simulations for the ILC

Example: NLO Chargino Production at the ILC

For the rest: always SPS1a'
SUGRA-scenario with (sgn $\mu = 1$)

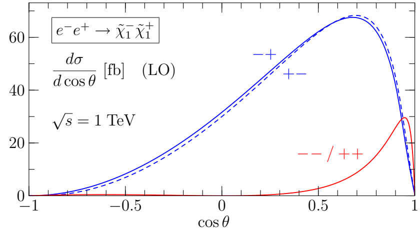
$$m_0 = 70\,\text{GeV}$$
$$m = 250\,\text{GeV}$$
$$\tan\beta = 10$$
$$A_0 = -300\,\text{GeV}$$

SPS1a'-preferred decay (2-step cascade):
$\tilde{\chi}_1^+ \to \tilde{\tau}_1 \nu_\tau \to \tau^+ \tilde{\chi}_1^0 \nu_\tau$



Chargino masses and widths:

	M	Γ	Γ/M
$\tilde{\chi}_1^+$	183.7 GeV	0.077 GeV	0.00042
$\tilde{\chi}_2^+$	415.4 GeV	3.1 GeV	0.0075



$\cos\theta$ angle between e^- and $\tilde{\chi}_1^-$

- Born helicity amplitudes known analytically Choi et al., 9812236, 0002033
- Implemented in narrow width approx. in many programs
- Full (tree-level) processes in Sherpa, SMadgraph, WHIZARD

- No massless t-channel particles $\Rightarrow$ neglect m_e for phase space
- to clarify notation

$$\sigma_{\text{Born}}(s) = \int d\Gamma_2 \, |\mathcal{M}_{\text{Born}}(s, \cos\theta)|^2$$

Virtual Corrections

Virtual corrections from SUSY and SM particles: self energies, vertex corrections, box diagrams (as usual)

(Semi-)automatized calculation with `FeynArts/FormCalc`

Hahn et al., 9807565, 0012260, 0105349 ; Fritzsche, 05; Fritzsche/Hollik, 0407095

Independent check of numerical results Öller/Eberl/Majerotto, 0504109

Regulators:

- Electron mass m_e for collinear photon radiation
- Fictitious photon mass λ for infrared divergencies

Interference of Born and virtual corrections

$$\sigma_{\text{virt}}(s, \lambda^2, m_e^2) = \int d\Gamma_2 \left[2\text{Re}\left(\mathcal{M}_{\text{Born}}(s)^* \, \mathcal{M}_{\text{1-loop}}(s, \lambda^2, m_e^2)\right)\right]$$

Eliminate dependence on λ by

- neglecting power corrections in λ
- Adding real (1st order) photon radiation with $E_\gamma < \Delta E_\gamma$
- Correction (terms $\propto \log \Delta E_\gamma$) is shifted into soft-photon factor

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- Fictitious photon mass λ for infrared divergencies

Interference of Born and virtual corrections

$$\sigma_{\text{virt}}(s,\lambda^2,m_e^2) = \int d\Gamma_2 \left[2\text{Re} \left(\mathcal{M}_{\text{Born}}(s)^* \, \mathcal{M}_{\text{1-loop}}(s,\lambda^2,m_e^2) \right) \right]$$

Soft-photon factor:

$$f_{\text{soft}} = -\frac{\alpha}{2\pi} \sum_{i,j=e^\pm,\widetilde{\chi}^\pm} \int_{|\mathbf{k}|\le\mathbf{\Delta E}_\gamma} \frac{d^3k}{2\omega_k} \, \frac{(\pm) p_i p_j \, Q_i \, Q_j}{(p_i k)(p_j k)}$$

Real and Collinear Photons

"Virtual + Soft"

$$\sigma_{\text{v+s}}(s, \Delta E_\gamma, m_e^2) = \int d\Gamma_2 \left[f_{\text{soft}}(\tfrac{\Delta E_\gamma}{\lambda}) \, |\mathcal{M}_{\text{Born}}(s)|^2 + 2\text{Re} \left(\mathcal{M}_{\text{Born}}(s)^* \, \mathcal{M}_{\text{1-loop}}(s, \lambda^2, m_e^2) \right) \right]$$

for simulation choose $\Delta E_\gamma \leq \Delta E_\gamma^{\text{exp}}$

Real radiation (i.e. the process $e^- e^+ \to \tilde{\chi}_1^+ \tilde{\chi}_1^- \gamma$):

$$\sigma_{2\,\to\,3}(s, \Delta E_\gamma, m_e^2) = \int_{\Delta E_\gamma} d\Gamma_3 \, |\mathcal{M}_{2\to 3}(s, m_e^2)|^2 .$$

"Total" cross section (fixed order):

$$\sigma_{\text{tot}}(s, m_e^2) = \sigma_{\text{Born}}(s) + \sigma_{\text{v+s}}(s, \Delta E_\gamma, m_e^2) + \sigma_{2\,\to\,3}(s, \Delta E_\gamma, m_e^2)$$

should not depend on ΔE_γ, but power corrections only in $\sigma_{2\,\to\,3}$, not in $\sigma_{\text{v+s}}$

As usual, split $2 \to 3$ cross section:

$$\sigma_{2\,\to\,3}(s, \Delta E_\gamma, m_e^2) = \sigma_{\text{hard,non-coll}}(s, \Delta E_\gamma, \Delta\theta_\gamma) + \sigma_{\text{hard,coll}}(s, \Delta E_\gamma, \Delta\theta_\gamma, m_e^2)$$

$x = 1 - 2E_\gamma/\sqrt{s}$ electron energy fraction after radiation

Approximate collinear radiation by convoluting the Born cross section with a structure function

$$\begin{aligned}\sigma_{\text{hard,coll}}(s, \Delta E_\gamma, \Delta\theta_\gamma, m_e^2) &= \int_{\Delta E_\gamma, \Delta\theta_\gamma} d\Gamma_3 \, |\mathcal{M}_{2\to3}(s, m_e^2)|^2 \\ &= \int_0^{x_0} dx\, f(x; \Delta\theta_\gamma, \tfrac{m_e^2}{s}) \int d\Gamma_2 \, |\mathcal{M}_{\text{Born}}(xs, m_e^2)|^2.\end{aligned}$$

collinear structure functions (helicity conserving/flip): Böhm/Dittmaier, 1993

$$\begin{aligned} f^+(x) &= \tfrac{\eta}{4}\tfrac{1+x^2}{1-x} \\ f^-(x) &= \tfrac{\alpha}{2\pi}(1-x) \end{aligned} \qquad \eta := \frac{2\alpha}{\pi}\left[\log\left(\frac{s}{4m_e^2}(\Delta\theta_\gamma)^2\right) - 1\right]$$

Cutoff $\Delta E_\gamma \to x_0 = 1 - 2\Delta E_\gamma/\sqrt{s}$ (no power corrections in $\Delta\theta_\gamma$)

Simulation

Combining all parts:

$$\sigma_{\text{tot}}(s, m_e^2) = \int dx\, f_{\text{eff}}(x_1,\ x_2; \Delta E_\gamma, \Delta\theta_\gamma, \frac{m_e^2}{s}) \int d\Gamma_2\, |\mathcal{M}_{\text{eff}}(s, x_1,\ x_2; m_e^2)|^2 + \int_{\Delta E_\gamma, \Delta\theta_\gamma} d\Gamma_3\, |\mathcal{M}_{2\to 3}(s)|^2,$$

with

$$\begin{aligned} f_{\text{eff}}(x_1,\ x_2; \Delta E_\gamma, \Delta\theta_\gamma, \frac{m_e^2}{s}) &= \delta(1-x_1)\,\delta(1-x_2) \\ &+ \delta(1-x_1)\, f(x_2; \Delta\theta_\gamma, \frac{m_e^2}{s})\, \theta(x_0 - x_2) \\ &+ f(x_1; \Delta\theta_\gamma, \frac{m_e^2}{s})\, \delta(1-x_2)\, \theta(x_0 - x_1) \end{aligned}$$

$$\begin{aligned} |\mathcal{M}_{\text{eff}}(s, x_1,\ x_2; m_e^2)|^2 &= \left[1 + f_{\text{soft}}(\Delta E_\gamma, \lambda^2)\,\theta(x_1, x_2)\right]\ |\mathcal{M}_{\text{Born}}(s)|^2 \\ &+ 2\text{Re}\left[\mathcal{M}_{\text{Born}}(s)\, \mathcal{M}_{\text{1-loop}}(s, \lambda^2, m_e^2)\right]\, \theta(x_1 - x_0)\theta(x_2 - x_0) \end{aligned}$$

All corrections defined as a generalized structure function
$\Rightarrow$ suitable for implementation in an event generator

Technical Details and Failure of Approach

Generate Born + $2 \to 3$ by O'Mega, convolute Born with generalized structure function ("user-defined structure function" in WHIZARD)

Sampling δ-functions:

- splitting sampling region $[0, x_0] \cup [x_0, 1]$
- map first region as exactly as possible
- set $x = 1$ in the 2nd region (δ-functions)
- reweighting according to

$$w(x > x_0) : w(x < x_0) = 1 : \int_0^{x_0} dx\, f(x; \Delta\theta_\gamma, \frac{m_e^2}{s})$$

For fixed-order simulation avoid double-counting:
$f(x_1 < x_0, x_2 < x_0) \equiv 0$ (strictly here)

Numerical agreement: WHIZARD and fixed-order calculation

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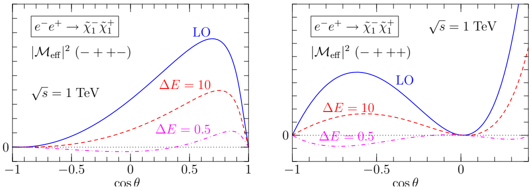
In the soft-photon region: negative event weights

- $2 \to 2$ and $2 \to 3$ runs separately
- Lowering the cutoff from $\Delta E_\gamma/\sqrt{s} < 10^{-2}$ to $\Delta E_\gamma/\sqrt{s} < 10^{-3}$: $2 \to 2$ NLO becomes negative, compensating the $2 \to 3$

Resumming photons



Experimental resolution drives one into negative weights region

Soft-collinear region: $E_\gamma < \Delta E_\gamma$, $\Delta\theta_\gamma < \theta_\gamma$: double logs

$\frac{\alpha}{\pi} \log \frac{E_\gamma^2}{s} \log \theta_\gamma$ invalidate perturbative series

In that region resummation of all orders is possible

$$\sigma_{\text{Born+ISR}}(s, \Delta\theta_\gamma, m_e^2) = \int dx\, f_{\text{ISR}}(x; \Delta\theta_\gamma, \frac{m_e^2}{s}) \int d\Gamma_2 \, |\mathcal{M}_{\text{Born}}(xs)|^2,$$

f_{ISR} includes all order soft-photon radiation (LLA), hard-collinear up to 3rd order Skrzypek/Jadach, 1991

For collinear photons cancellation of infrared divergencies built in, main source of negative weights removed

Matching with NLO

Kilian/JR/Robens,2006

Combine ISR-resummed LO with NLO, avoid double-counting

Subtract contribution of one soft photon (already in soft-photon factor)

$$f_{\text{soft,ISR}}(\Delta E_\gamma, \Delta\theta_\gamma, m_e^2) = \frac{\eta}{4}\int_{x_0}^{1} dx \left(\frac{1+x^2}{1-x}\right)_+ = \frac{\eta}{4}\left(2\ln(1-x_0) + x_0 + \frac{1}{2}x_0^2\right).$$

After this subtraction we have

$$|\widetilde{\mathcal{M}}_{\text{eff}}(\hat{s};\Delta E_\gamma, \Delta\theta_\gamma, m_e^2)|^2 = \left[1 + f_{\text{soft}}(\frac{\Delta E_\gamma}{\lambda}) - 2f_{\text{soft,ISR}}(\Delta E_\gamma, \Delta\theta_\gamma, \frac{m_e^2}{s})\right] |\mathcal{M}_{\text{Born}}(\hat{s})|^2 + 2\text{Re}\left[\mathcal{M}_{\text{Born}}(\hat{s})\, \mathcal{M}_{\text{1-loop}}(\hat{s}, \lambda^2, m_e^2)\right],$$

contains Born, virtual + soft contr. with LL part of virtual and soft-coll. removed

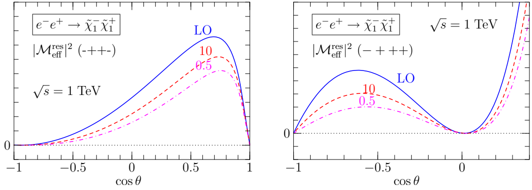
New “s+v” term (contains also soft/coll. corrections to Born/1-loop interference)

$$\sigma_{\text{v+s,ISR}}(s, \Delta E_\gamma, \Delta\theta_\gamma, m_e^2) = \int dx_1\, f_{\text{ISR}}(x_1; \Delta\theta_\gamma, \frac{m_e^2}{s}) \int dx_2\, f_{\text{ISR}}(x_2; \Delta\theta_\gamma, \frac{m_e^2}{s}) \int d\Gamma_2\, |\widetilde{\mathcal{M}}_{\text{eff}}(\hat{s}; \Delta E_\gamma, \Delta\theta_\gamma, m_e^2)|^2$$

Simulation

Kilian/JR/Robens,2006

Resummation approach eliminates problem of negative weights:



Only source for negative weights: soft-noncollinear region, does not cause problems

Final improvement:

- convoluting $2 \longrightarrow 3$ part with ISR structur function
- add $2 \longrightarrow 4$ part

$$\sigma_{\text{tot,ISR+}}(s, m_e^2) = \int dx_1\, f_{\text{ISR}}(x_1; \Delta\theta_\gamma, \frac{m_e^2}{s}) \int dx_2\, f_{\text{ISR}}(x_2; \Delta\theta_\gamma, \frac{m_e^2}{s})$$

$$\times \left(\int d\Gamma_2\, |\widetilde{\mathcal{M}}_{\text{eff}}(\hat{s}; \Delta E_\gamma, \Delta\theta_\gamma, m_e^2)|^2 + \int_{\Delta E_\gamma, \Delta\theta_\gamma} d\Gamma_3\, |\mathcal{M}_{2\to 3}(\hat{s})|^2 \right)$$

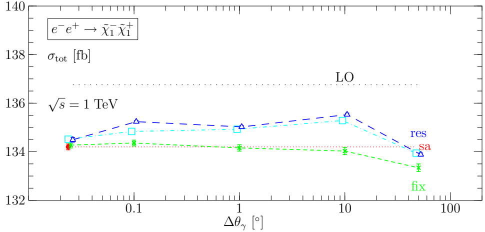
$$+ \int_{\Delta E_{\gamma,i}, \Delta\theta_{\gamma,i}} d\Gamma_4\, |\mathcal{M}_{2\to 4}(s)|^2$$

Choosing Cutoffs

- **Collinear (angular) cutoff**

  Collinear approximation breaks down at $\theta_\gamma > 10°$

  Higher-order effects for emission angles below 0.1°



- **Energy cutoff**

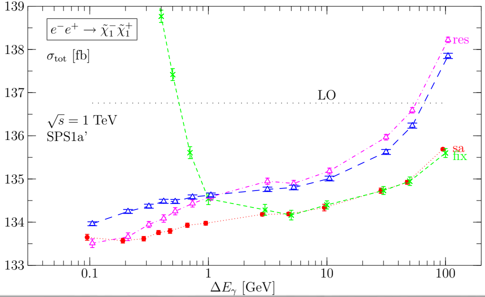
  Fixed order/semianalytic agree

  Small angles: interference term overshoots

  5 ‰ correction from higher order γ radiation

  ILC statist. fluctuation: 2.5 ‰

  $\Rightarrow \Delta E_\gamma \lesssim 0.5$ GeV



Results and Distributions

Kilian/JR/Robens,2006

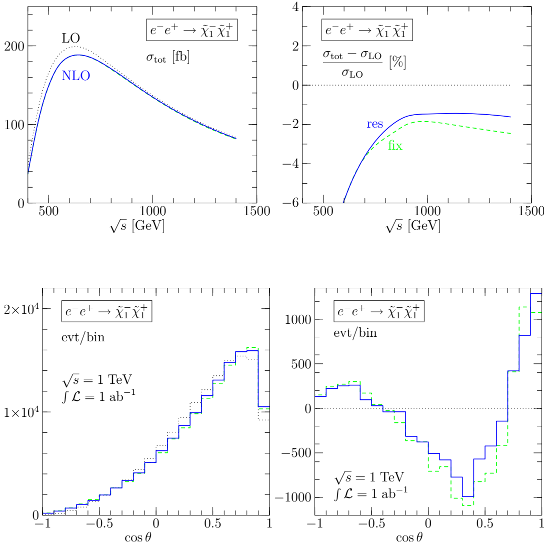
NLO corrections -5% (Xsec max.)

-2% (-1.5%) fixed-order (resummed) @ 1 TeV

Binned distribution of chargino scattering angle

Cutoffs: $\Delta\theta_\gamma = 1°$, $\Delta E_\gamma = 3\,\text{GeV}$ (fixed-order)

K-factor approach insufficient



Summary and Outlook

- Extended WHIZARD: 1st NLO SUSY MC Event Generator for the ILC
- All possible distributions available at NLO
- Matching of resummed soft-collinear photons and explicit NLO parts avoids negative weights
- Interface to `FeynArts`: all MSSM $2 \to 2$ processes for ILC available
- Open issues/Next step(s):
  - Include chargino decays Kalinowski/Kilian/Kovaric/JR/Robens/Rolbiecki
  - Resummation of Coulomb singularity: improved threshold behavior
  - Semiautomatized version / Program library

New version WHIZARD 1.99 $\rightarrow$ 2.0.0

`http://whizard.event-generator.org`

Functional cut/analysis syntax, more models, recursive cascades, improved phase space, parton shower, . . .

as usual: we're open to users wish list!