Oide Effect in ILC line,

based on K. Oide, *Synchrotron-radiation limit on the focusing of electron beams*

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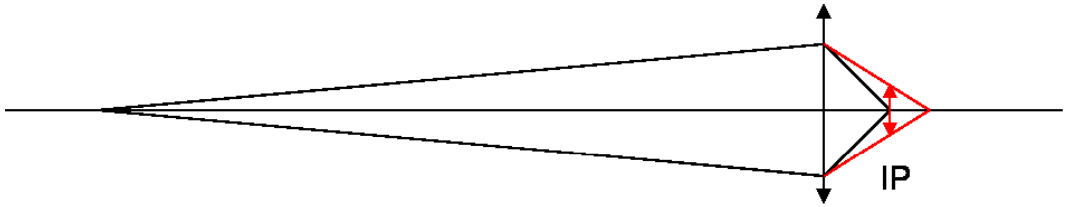
Origin of the Oide Effect

- Limitation of the beam size
- Synchrotron radiation



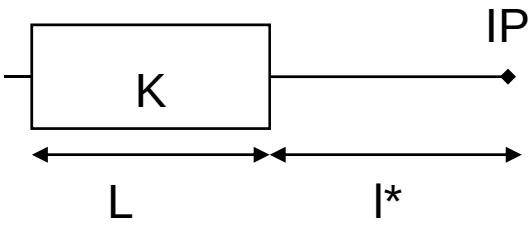
- Beam motion in a quadrupole

$$y''(s)+K(E)y(s)=0 \text{ with } K=\frac{ec}{E}\frac{\partial B}{\partial x}$$

- Photon energy $u << E_0$

$$y''(s)+K\left(1+\frac{u}{E_0}\right)y(s)=0$$

Beam position



g(s) : Green function associated to y

$$g(s) = \frac{\sin\sqrt{K}s}{\sqrt{K}} + l^* \cos\sqrt{K}s \quad \text{with} \quad \sqrt{K}l^* << 1$$

Beam position at the interaction point, for 1 photon emitted at s

$$y^* = y_0^* - \int_0^s g(s_1)\frac{u}{E_0} K y_0(s_1) ds_1$$

Beam position

Solution without synchrotron radiation

$$y_0(s_1) \approx -y_0^{'*} g(s_1)$$

Beam position at IP

$$y^* = y_0^* - \int_0^s \left(g(s_1)\right)^2 \frac{u}{E_0} K y_0^{'*} ds_1$$

Average energy loss per unit length

Radiated power

$$P = \frac{2}{3} r_e c E_0 \frac{\gamma^3}{|\rho|^2} \quad \text{with} \quad \frac{1}{\rho} = K y_0(s)$$

Energy loss per unit length

$$\frac{\langle u \rangle}{E_0} = \frac{2}{3} r_e \gamma^3 K^2 g(s)^2 y_0'^{*2}$$

$$\frac{\langle u^2 \rangle}{E_0^2} = \frac{55}{24\sqrt{3}} r_e \lambda_e \gamma^5 K^3 |g(s)|^3 |y_0'^{*}|^3$$

Beam size

Average beam position does not change.

But beam size is modified.

$$\sigma_y^* = \sqrt{\left\langle y^{*2} \right\rangle}$$

$$\sigma_y^{*2} = \beta_y^* \varepsilon_y + \frac{110}{3\sqrt{6\pi}} r_e \lambda_e \gamma^5 F(\sqrt{K}L, \sqrt{K}l^*) \left(\frac{\varepsilon_y}{\beta_y^*} \right)^{5/2}$$

$$F(\sqrt{K}L, \sqrt{K}l^*) \equiv \int_0^{\sqrt{K}L} \left| \sin\phi + \sqrt{K}l^* \cos\phi \right|^3 \left[\int_0^{\phi} (\sin\phi' + \sqrt{K}l^* \cos\phi')^2 d\phi' \right]^2 d\phi$$

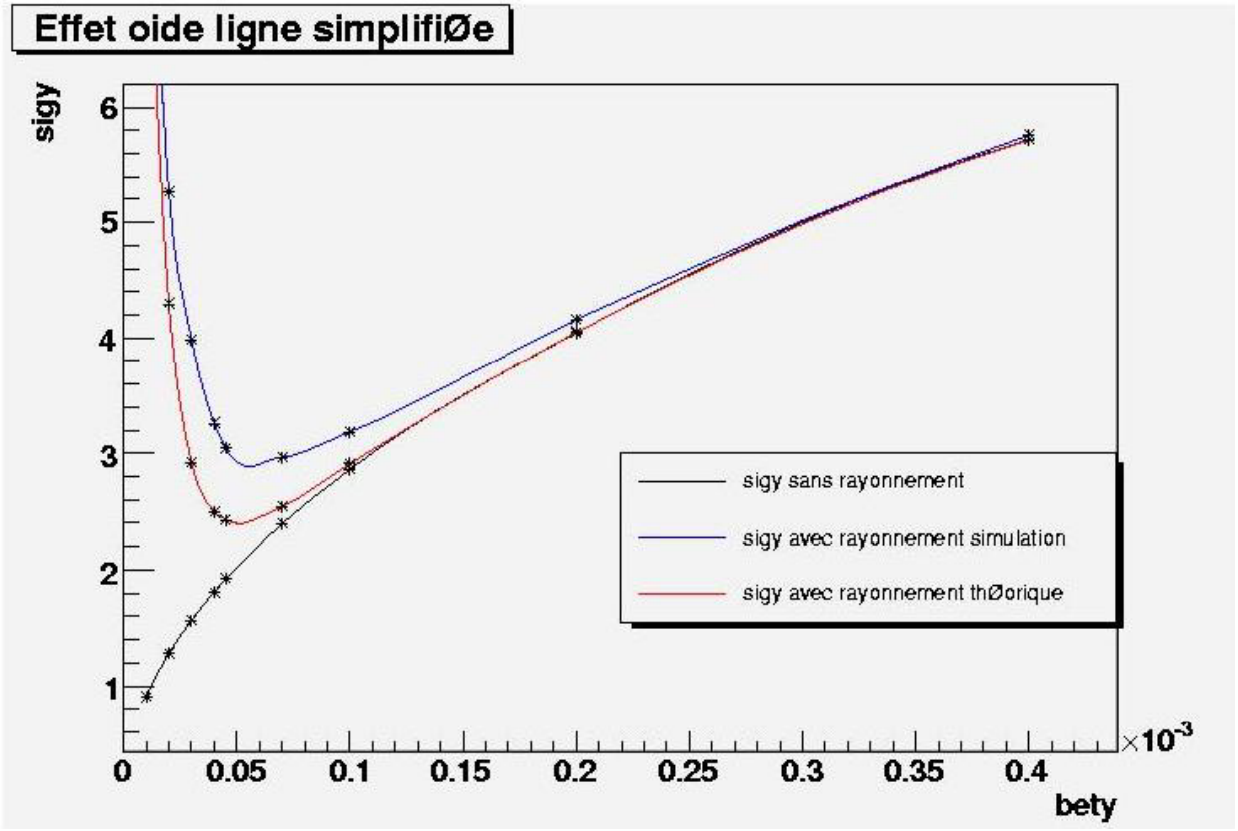
Minimal beam size

$$\sigma^*_{y\min} = \sqrt{\frac{7}{5}}\left[\frac{275}{3\sqrt{6\pi}} r_e \lambda_e F(\sqrt{K}L, \sqrt{K}l^*)\right]^{1/7} (\varepsilon_{Ny})^{5/7}$$

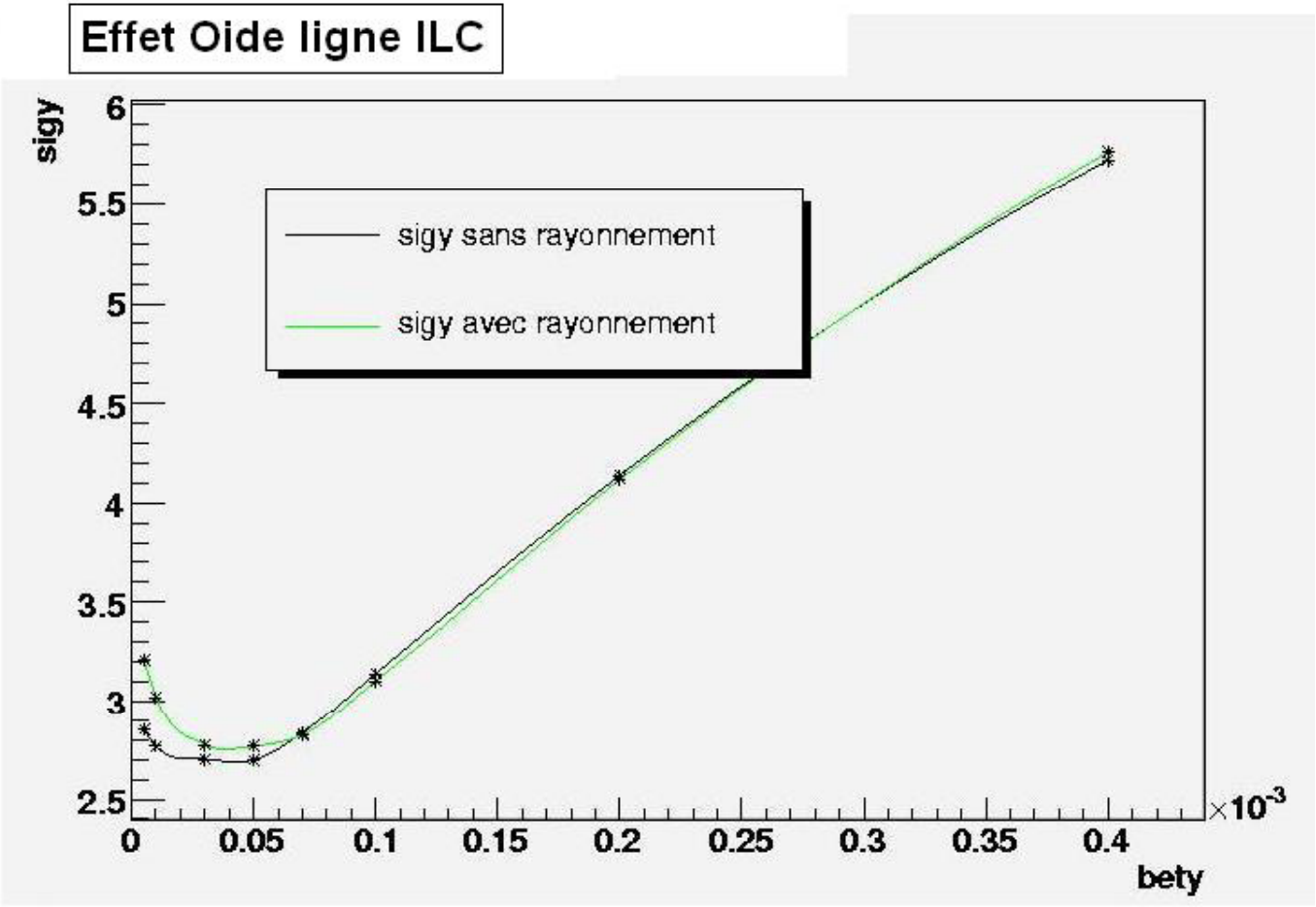
$$\beta^*_y = \left[\frac{275}{3\sqrt{6\pi}} r_e \lambda_e F(\sqrt{K}L, \sqrt{K}l^*)\right]^{2/7} \gamma(\varepsilon_{Ny})^{3/7}$$

Energy independance

Simulation on simplified line



ILC line simulation



Conclusion

- Beam size limit
- Not significant for ILC vertical size
- Not reached for ILC horizontal size