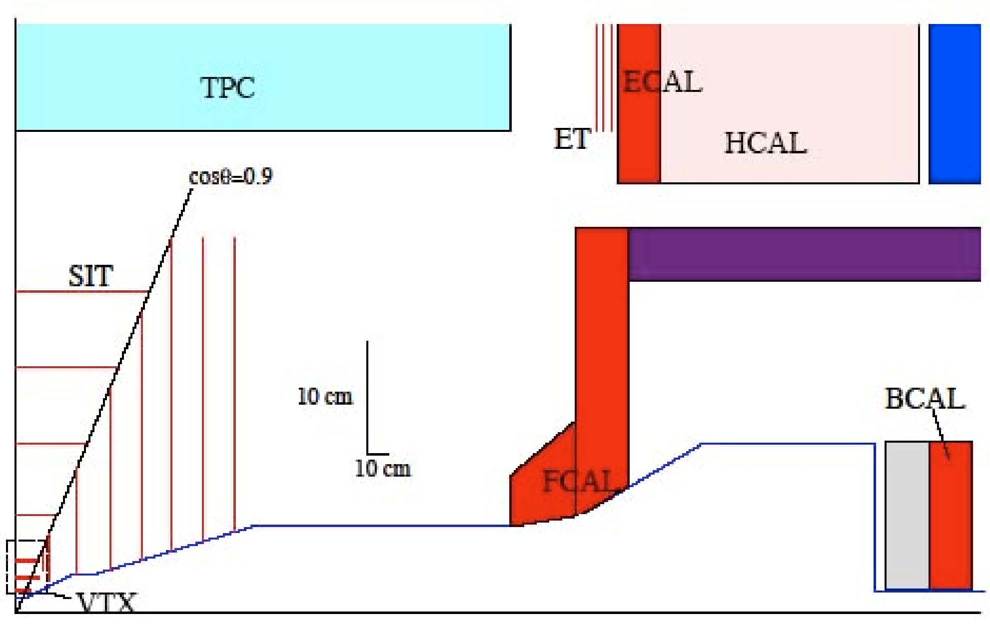
Heating of IR region

- Image current
- HOM heating

'GLD' IR



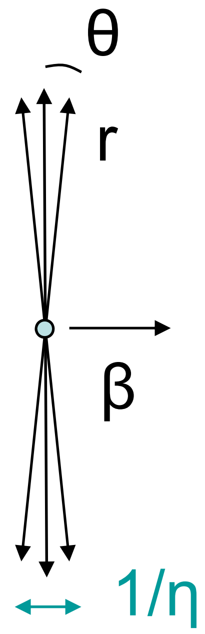
Fields of relativistic charge

- **Electric field**
  - Nearly perpendicular to velocity

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\frac{\gamma}{(\eta^2\theta^2+1)^{3/2}}\hat{r}$$

$$\eta \equiv \beta\gamma \sim 5\times 10^5$$

  - Spread at r~10cm: 0.2 μm (σ_z=300 μm)
    assume spread=0



- **Magnetic field**

$$\vec{B} = \frac{1}{c}\vec{\beta}\times\vec{E}$$

  - β=1: Energy of E ~ Energy of B

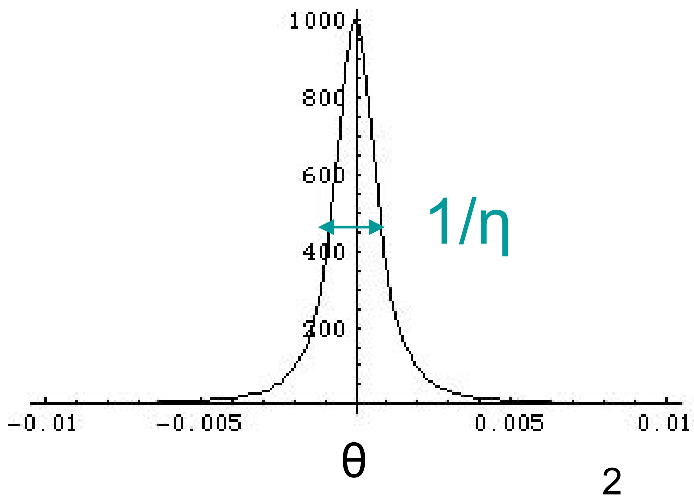
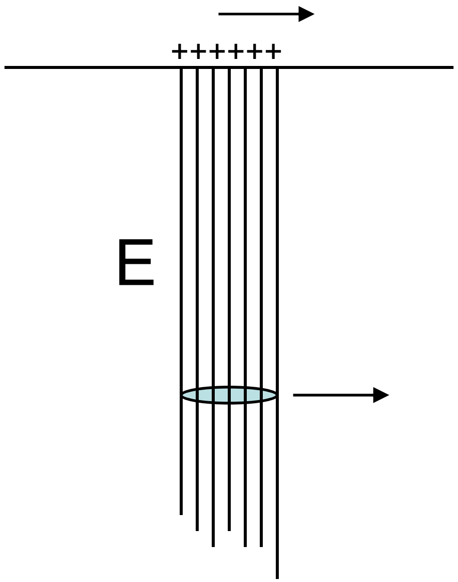


Image current I

Skin depth

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \quad (\sigma : \text{conductivity}, \omega : \text{frequency})$$



- Effective wavelength:

$$\lambda = 2\pi\sigma_z, \quad \omega = \frac{2\pi c}{\lambda}$$

- Cu pipe, $2\text{X}10^{10}$/bunch, σ_z=300μm.

$$\delta = 0.18\mu m$$

Image current II

- Assume the bunch current flows uniformly in depth δ, radius r, and length $4\sigma_z$. Current density j:

$$j = \rho c = \frac{qc}{2\pi r \delta 4\sigma_z}$$ (q: bunch charge)

- Heat by one bunch passing L(m) of beampipe

$$h_{1b} = \frac{j^2}{\sigma}(2\pi r \delta 4\sigma_z)(\frac{L}{c})$$

- Heat per second over L(m) of beampipe (radius r)

$$W = \frac{h_{1b}}{t_{sp}} = \frac{q^2 L}{8\sqrt{2}\pi r t_{sp}}(\frac{c}{\sigma_z})^{\frac{3}{2}}\sqrt{\frac{\mu}{\sigma}}$$ (t_{sp}: bunch spacing)

Image current III

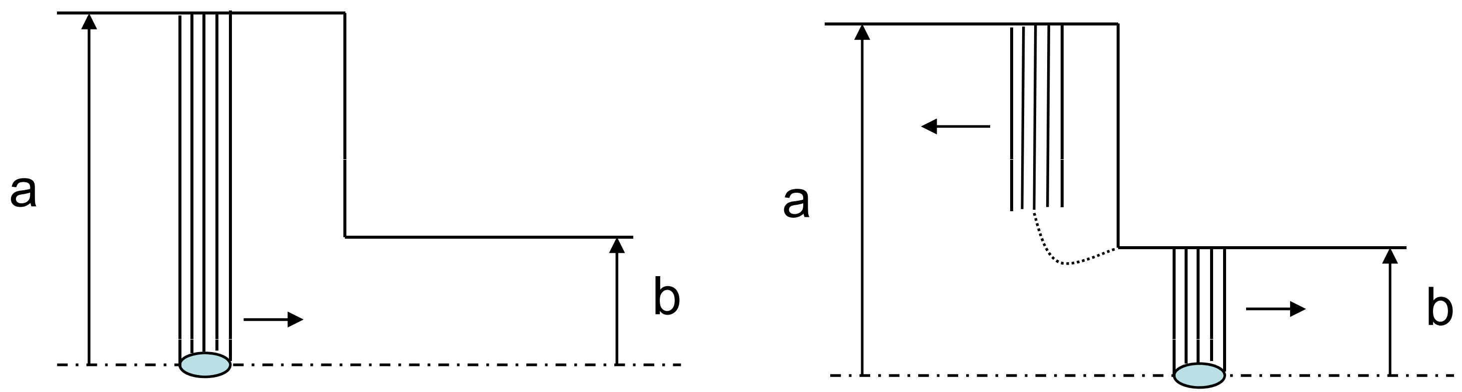
- With the duty factor of 1/200, t_{sp}=308ns,
  - Image heating = 0.075 W (per m of beampipe)
- Formula by Saito-san

$$W = \frac{\Gamma(\frac{4}{3})\, q^2 L}{\sqrt{2}(2\pi)^2 r t_{sp}} \left(\frac{c}{\sigma_z}\right)^{\frac{3}{2}} \sqrt{\frac{\mu}{\sigma}}$$

→ 0.056 W (per m of beampipe)

HOM loss I

- Iris step r = a to b



EM energy bounced back = HOM loss

HOM loss II

- Assume gaussian line charge

$$\zeta(x)dx = \frac{q}{\sqrt{2\pi}\sigma_z} e^{-\frac{x^2}{2\sigma_z^2}} dx$$

- Energy in B = Energy in E. u: energy density

$$u = \frac{1}{2}(\varepsilon_0 E^2 + \frac{B^2}{\mu_0^2}) = \varepsilon_0 E^2, \quad E(x) = \frac{\zeta(x)}{2\pi\varepsilon_0 r}$$

- Energy from r = a to b(per bunch): lost by hitting iris

$$\Delta E = \int_{-\infty}^{\infty} dx \int_a^b dr\, 2\pi r\, u = \frac{q^2 \log \frac{b}{a}}{4\pi^{3/2}\varepsilon_0 \sigma_Z}$$

HOM loss III

- Loss factor k :

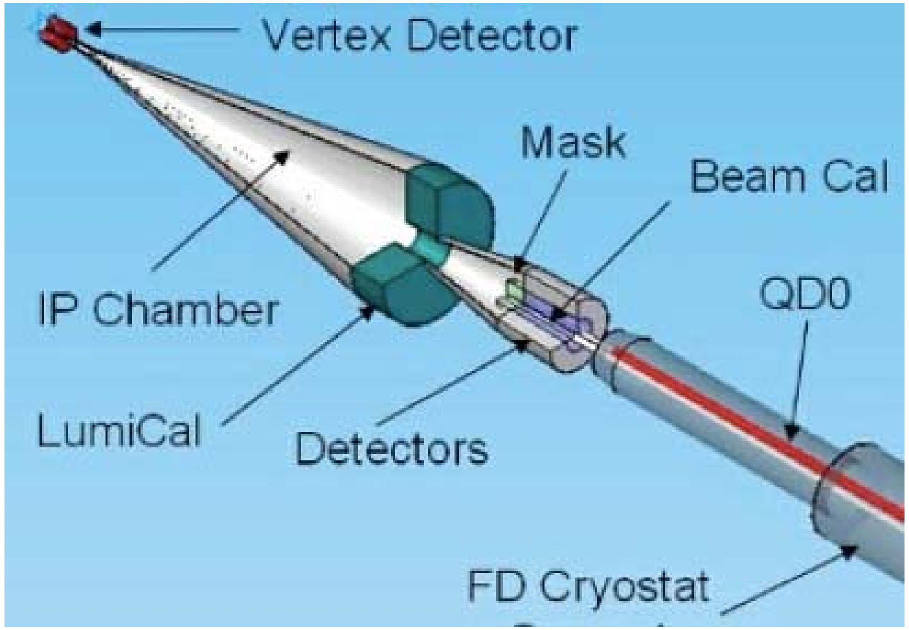
$$\Delta E = kq^2 \quad \rightarrow \quad k = \frac{\log \frac{b}{a}}{4\pi^{3/2}\varepsilon_0 \sigma_Z} \quad (V/C)$$

- HOM loss by an iris of a=1cm to b=10cm

$$P = n_{\text{bunch}} n_{\text{train}} kq^2 = 2820 \times 5kq^2$$
$$= 5.6W$$

- This is to turn to heat somewhere

HOM loss IV



SiD

- Two beams & FCAL/BCAL

5.6 x 2 x 2 ~ 22.4 W

Things to do

- Numerical calculation
- Estimation of distribution of energy deposit
  - HOM absorbers
- Include all items
  - BPM
  - Flanges
  - etc.
- Effects on final quads