

## The main goals of this study are :

- Explain why the measured vertical emittance is higher than the theoretical one.
- Quantify the wire scanner error effect on the emittance reconstruction method.
- Find a new emittance reconstruction method less sensitive to wire scanner error.  
(« Quadrupole Strength Variation » method is under study).

*In this presentation, only a 4d method is presented... a 6d analysis might be a possible upgrade of this study...*

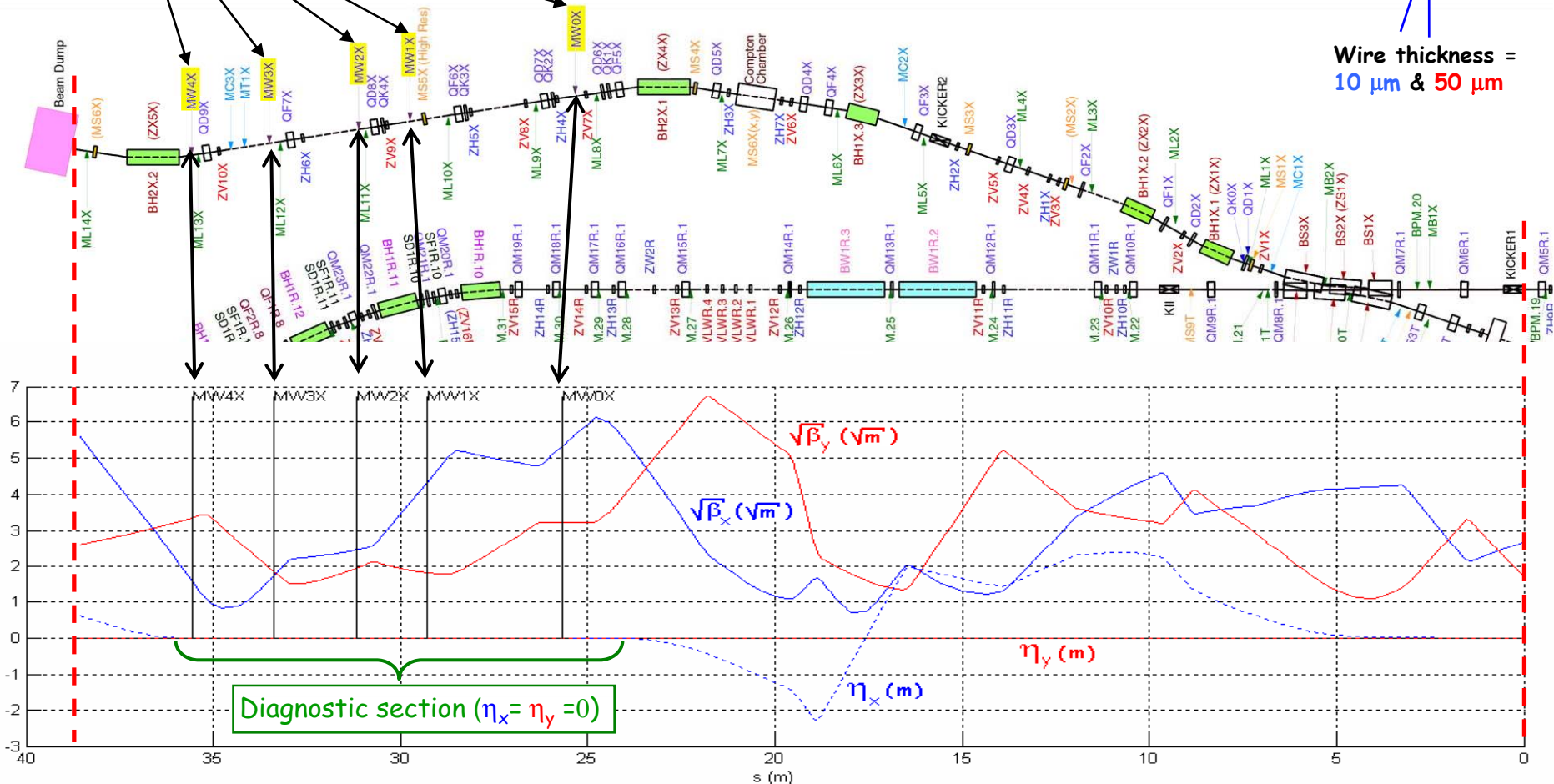
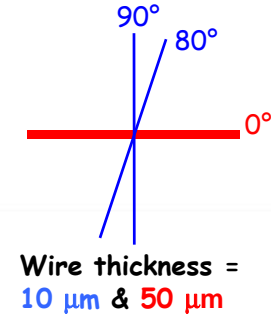
*Assumption :*

- *no jitter.*
- *no ground motion.*
- *perfect ATF-EXT line.*
- *no energy spread ( $E_0=1.3$  GeV)*

# ATF EXT line description & wire scanner position

5 positions for wire scanners

Each position is equipped with 3 wire scanners oriented at 0, 90 and 80° for x,y and 80° beam-size measurement.





## 4d - reconstruction method based on 5 wire scanner measurements

If the 4d-linear transport matrix from point A to B is uncoupled

$$R = \begin{pmatrix} R_{11} & R_{12} & 0 & 0 \\ R_{21} & R_{22} & 0 & 0 \\ 0 & 0 & R_{33} & R_{34} \\ 0 & 0 & R_{43} & R_{44} \end{pmatrix}$$

$$\begin{aligned} \text{Then } \sigma_{11}^B &= (R_{11}^2) \sigma_1^A + (2R_{11}R_{12}) \sigma_2^A + (R_{12}^2) \sigma_3^A &= (\sigma_x^B)^2 &= (\text{x-beam size @ B})^2 \\ \sigma_{33}^B &= (R_{33}^2) \sigma_6^A + (2R_{33}R_{34}) \sigma_9^A + (R_{34}^2) \sigma_{10}^A &= (\sigma_y^B)^2 &= (\text{y-beam size @ B})^2 \\ \sigma_{13}^B &= (R_{11}R_{33}) \sigma_4^A + (R_{12}R_{33}) \sigma_5^A + (R_{11}R_{34}) \sigma_7^A + (R_{12}R_{34}) \sigma_8^A \end{aligned}$$

-> For « n » points (B,C,...Z), where the x beam size is measure, we have :

$$\begin{pmatrix} \sigma_{11}^B \\ \sigma_{11}^C \\ \dots \\ \sigma_{11}^Z \end{pmatrix} = \begin{pmatrix} R_{11}^{(A \rightarrow B)^2} & 2R_{11}^{(A \rightarrow B)} R_{12}^{(A \rightarrow B)} & R_{12}^{(A \rightarrow B)^2} \\ R_{11}^{(A \rightarrow C)^2} & 2R_{11}^{(A \rightarrow C)} R_{12}^{(A \rightarrow C)} & R_{12}^{(A \rightarrow C)^2} \\ \dots & \dots & \dots \\ R_{11}^{(A \rightarrow Z)^2} & 2R_{11}^{(A \rightarrow Z)} R_{12}^{(A \rightarrow Z)} & R_{12}^{(A \rightarrow Z)^2} \end{pmatrix} \begin{pmatrix} \sigma_1^A \\ \sigma_2^A \\ \sigma_3^A \end{pmatrix} = M_X \begin{pmatrix} \sigma_1^A \\ \sigma_2^A \\ \sigma_3^A \end{pmatrix}$$

If we know how to solve this linear system then, we can compute the horizontal emittance :

$$\mathcal{E}_x^A = \sqrt{\sigma_1^A \cdot \sigma_3^A - (\sigma_2^A)^2}$$

Similar analysis can be performed to estimate :  $\sigma_4$ , to  $\sigma_{10}$  ... and  $\mathcal{E}_y$  can be estimated.

## 4d - reconstruction method based on 5 wire scanner measurements

If  $n < 3$  No solution.

If  $n = 3$  and  $M_x^{-1}$  exist then the solution is known and unique.

If  $n > 3$  the system is « overestimated » and the Least Mean Square (LMS) method can be used to find the « LMS » solution. (in ATF  $n = 5$ )

First question : How sensitive to wire scanner errors this method is ?

Second question : What happen if the QM6 quadrupole strength is reduce by 20% ?

## First question : How sensitive to wire scanner errors this method is ?

Assumption : the relative error level on « measurements » at position MW0X, MW1X, MW2X, MW3X and MW4X is identical, and tested for values ranging between 0 and 30%

WS	Sigma_x nominal (micron)	10% error (micron)	20% error (micron)	30% error (micron)	Wire scanner thickness (micron)
MW0X	238.67	24	48	72	10
MW1X	192.77	19	39	58	10
MW2X	109.4	11	22	33	10
MW3X	77.04	8	15	23	10
MW4X	70.14	7	14	21	10

WS	Sigma_y nominal (microns)	10% error (micron)	20% error (micron)	30% error (micron)	Wire scanner thickness (micron)
MW0X	14.46	1	3	4	5
MW1X	8.11	1	2	2	5
MW2X	8.67	1	2	3	5
MW3X	8.21	1	2	2	5
MW4X	14.98	1	3	4	5

$$\sigma_{sim}^{X\%} = (1+r)\sigma_{ideal}$$

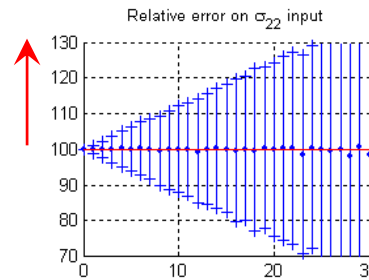
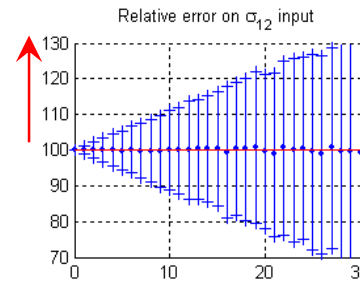
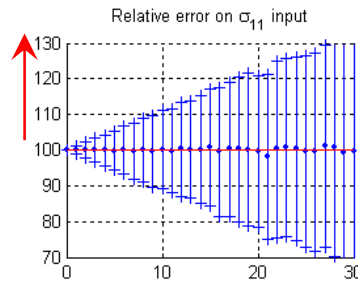
Where « r » comes from a gaussian distribution having zero mean and X/100 rms

The minimum relative error level for y-beam size measurement is at least equal to 13% (D/4~1.25 microns)

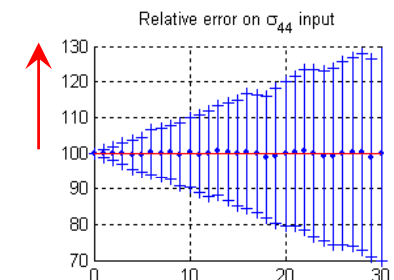
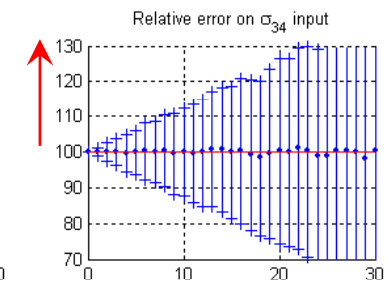
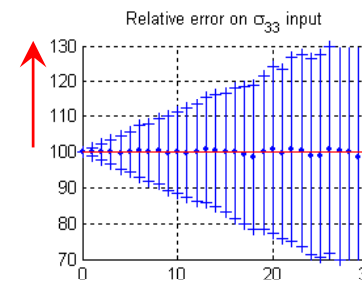
# First question : How sensitive to wire scanner errors this method is ?

For each error level,  
1000 « measure »  
have been tested.

The plot shows the  
mean and +/- 1 rms

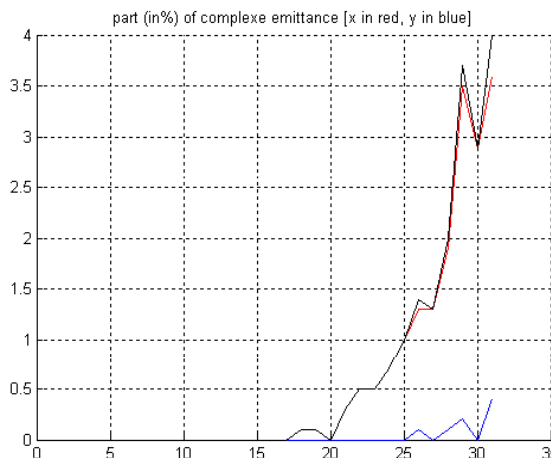
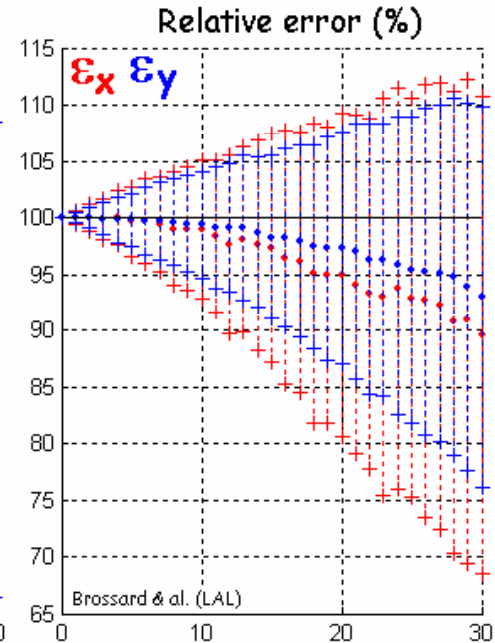
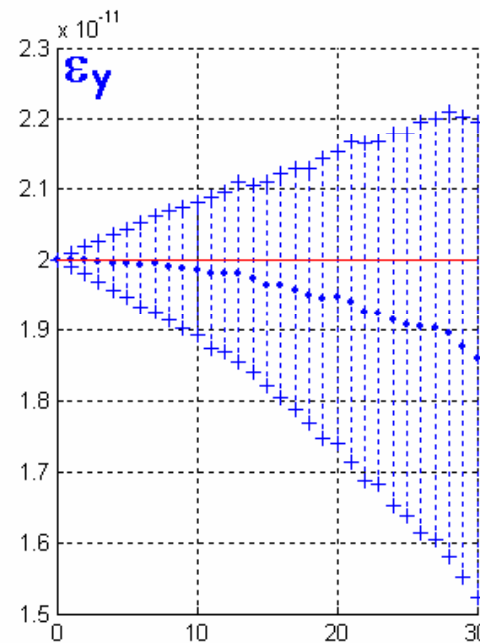
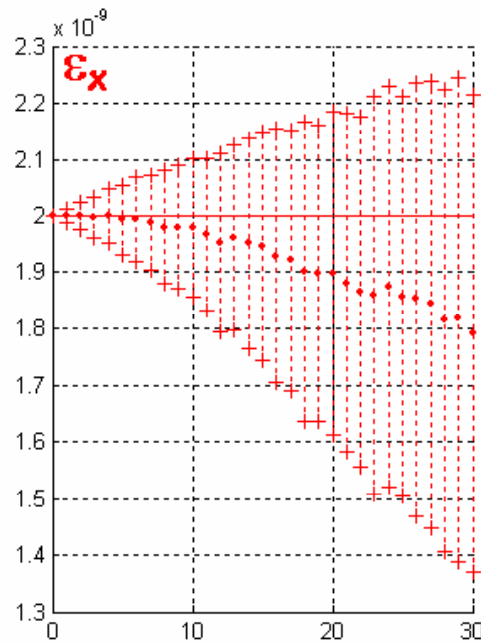
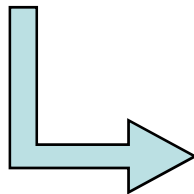


The maximum relative errors  
(define by the rms value) on  
 $\sigma_{11}$ ,  $\sigma_{12}$ ,  $\sigma_{22}$ ,  $\sigma_{33}$ ,  $\sigma_{34}$  and  $\sigma_{44}$   
values are approximatively  
between -30% and +30%



# First question : How sensitive to wire scanner errors this method is ?

Impact on horizontal and vertical emittance reconstruction ...



→ This method leads to underestimate vertical and horizontal emittance.

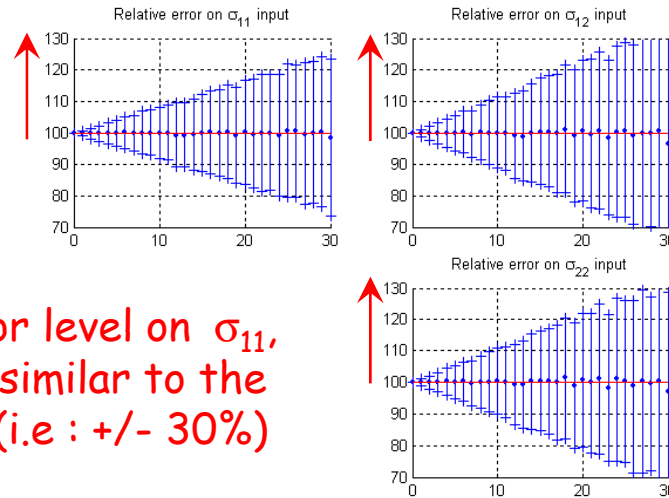
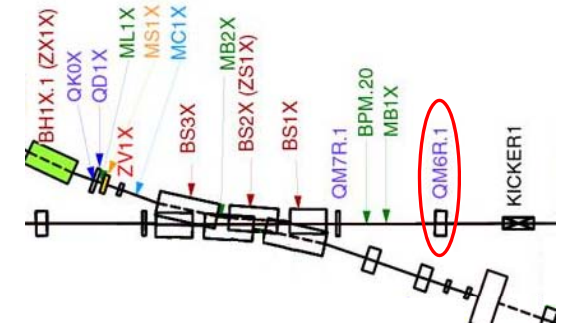
The underestimation of horizontal emittance is higher than the vertical one.

A part of the 1000 tested « measurements » lead to a unphysical results.  
In the current analysis, this rejection level is always lower than 4%.

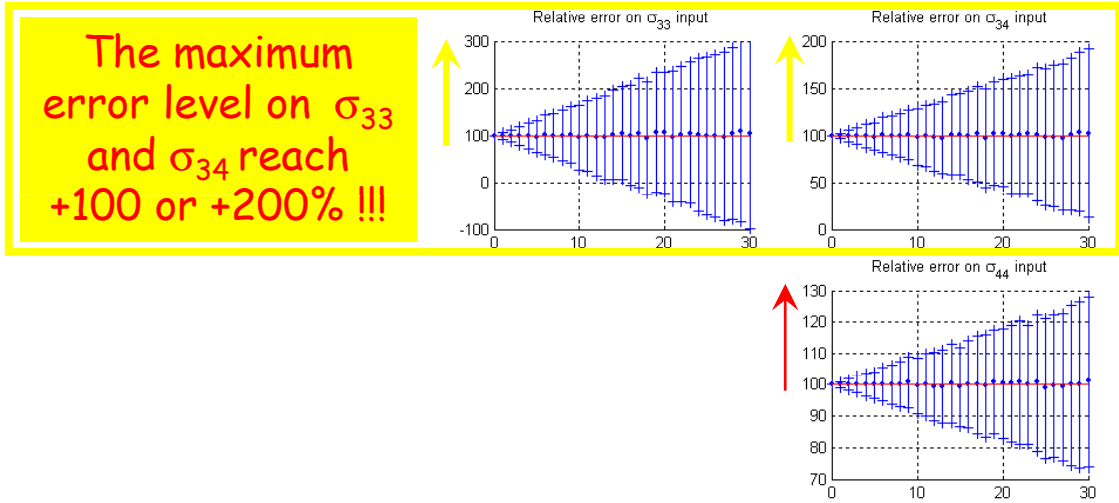


# Second question : What happen if the QM6 quadrupole strength is reduce by 20% ?

QM6 is the first bending magnet (with quadrupole part) after the kicker.  
The quad. strength of this element might have been over-estimated...(TBC)



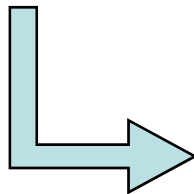
The maximum error level on  $\sigma_{11}$ ,  $\sigma_{12}$ ,  $\sigma_{22}$  and  $\sigma_{44}$  is similar to the previous analysis (i.e : +/- 30%)



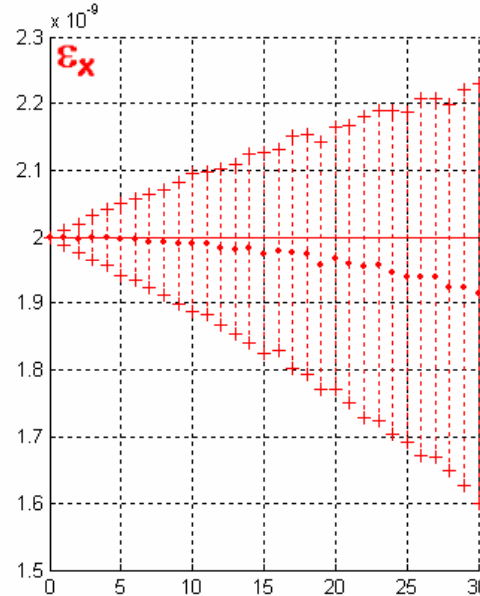
The maximum error level on  $\sigma_{33}$  and  $\sigma_{34}$  reach +100 or +200% !!!

# Second question : What happen if the QM6 quadrupole strength is reduce by 20% ?

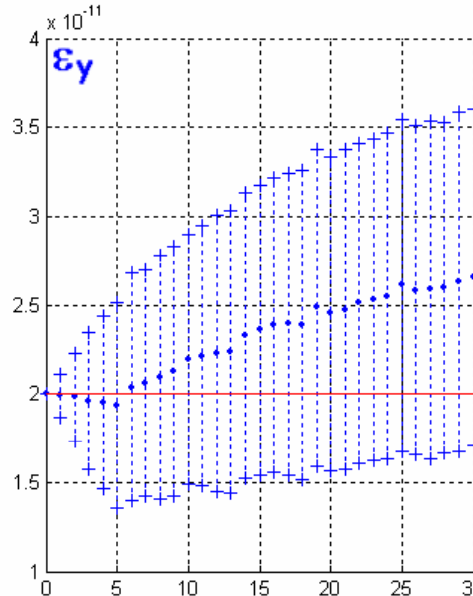
Impact on horizontal and vertical emittance reconstruction ...



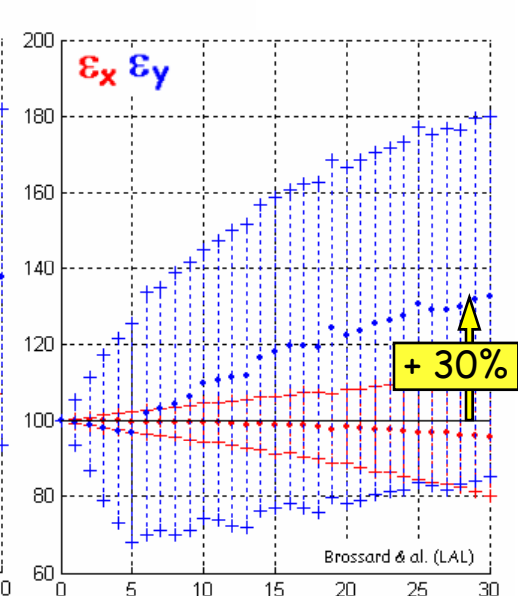
Absolute values (m.rad)



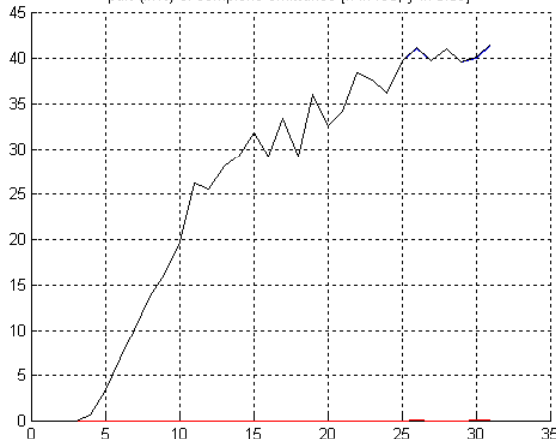
Absolute values (m.rad)



relative values (%)



part (in%) of complexe emittance [x in red, y in blue]



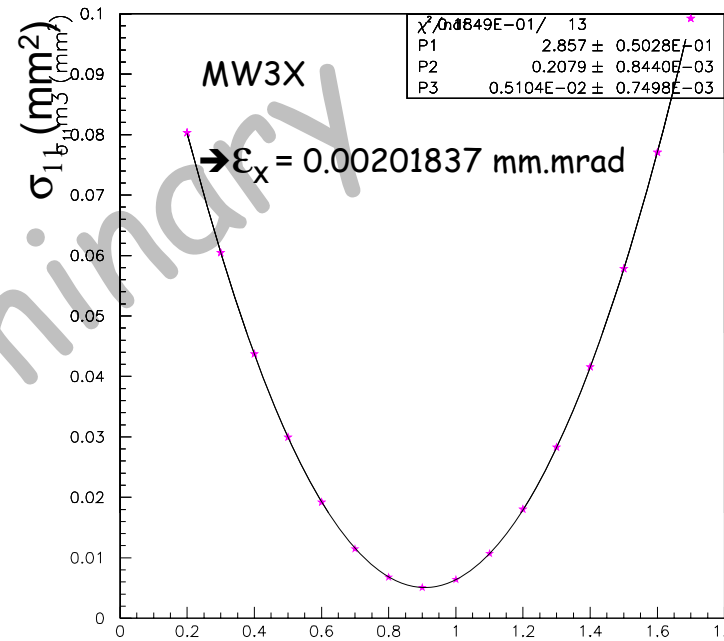
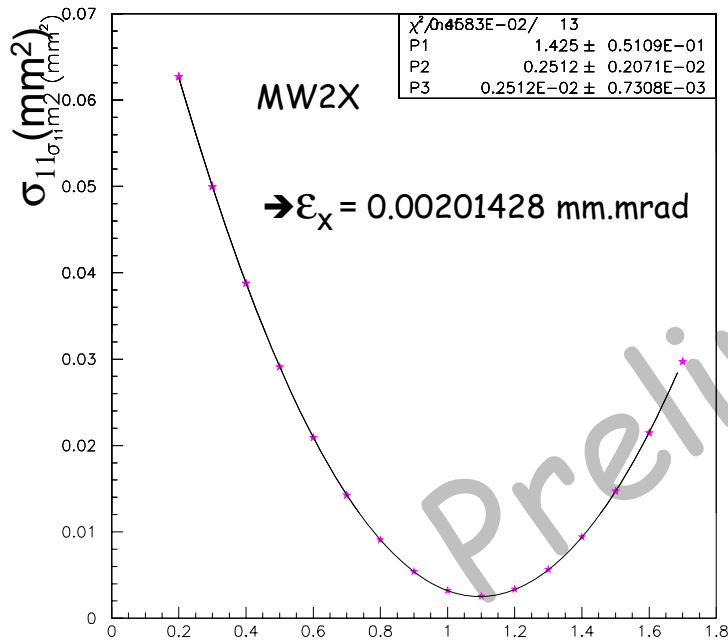
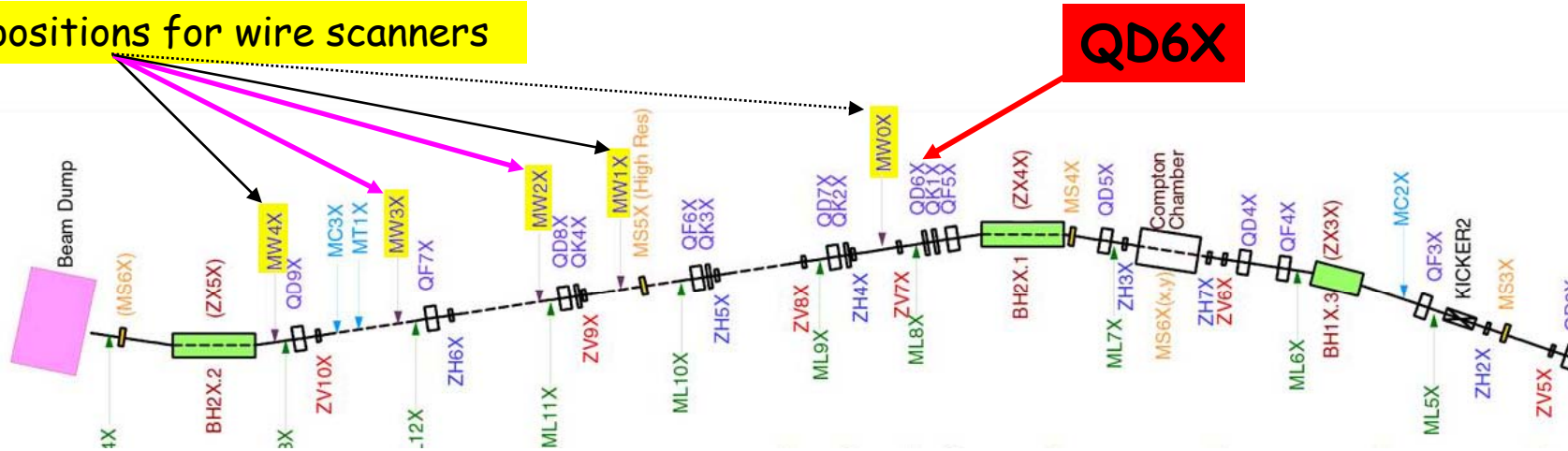
In presence of WS error, the

- horizontal emittance is slightly underestimated
- vertical emittance is greatly over-estimated.

The rejection part increase... to reach 40% for error measurement of 30 %

# Quadrupole Strength Variation method - 1

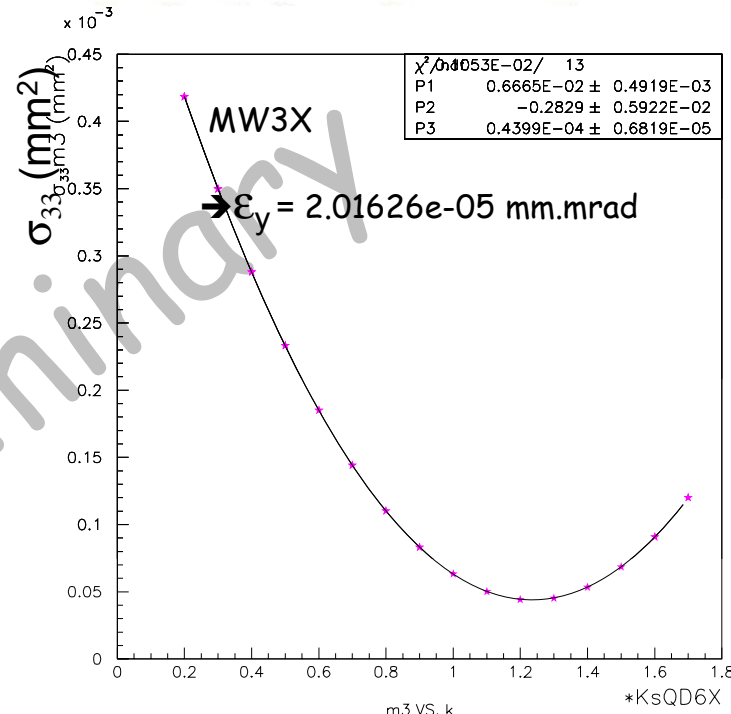
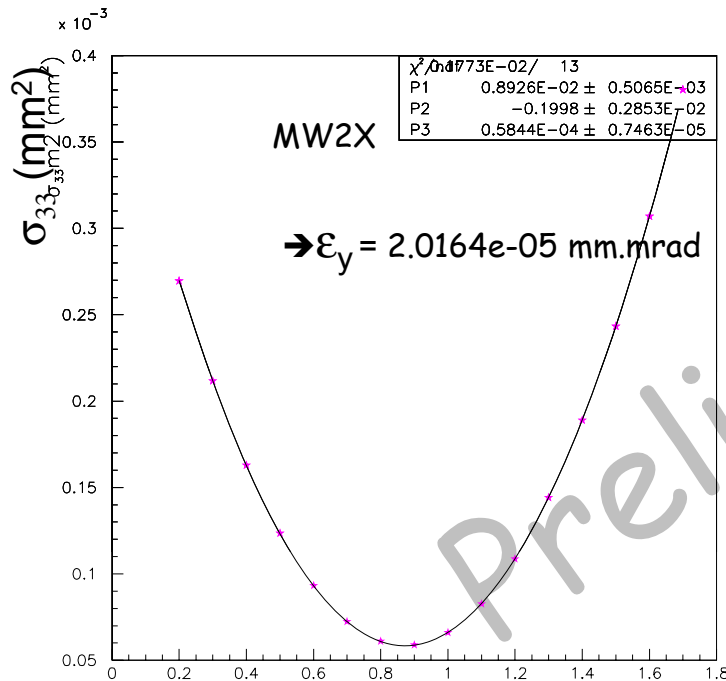
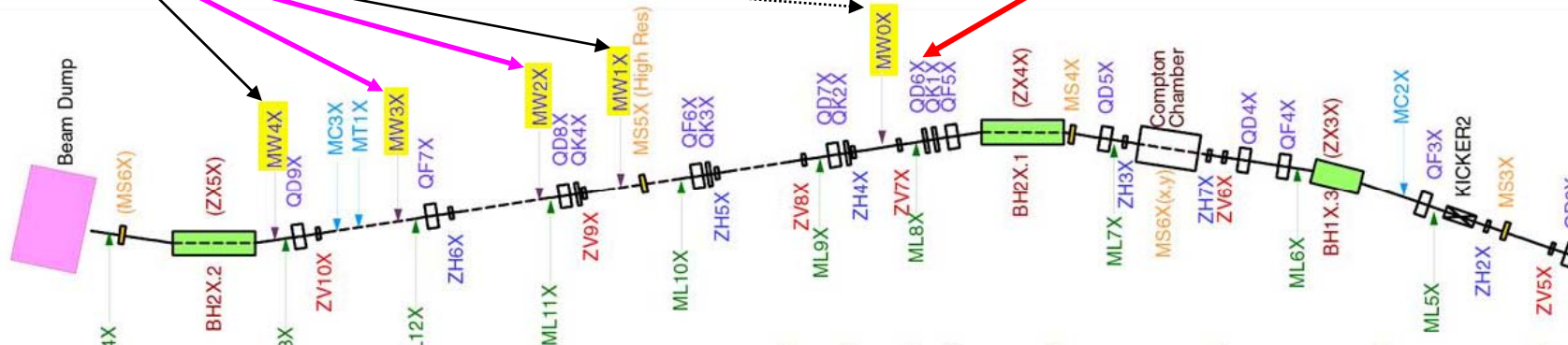
5 positions for wire scanners



# Quadrupole Strength Variation method - 2

5 positions for wire scanners

QD6X



## Conclusions and perspectives

- For the nominal ATF-EXT line, the LMS\* method based on the 5 existing wire scanners induced an underestimation of the vertical and horizontal emittance (function of WS error level).
- If the QM6 quadrupole strength is underestimated (by 20%) then the LMS method based on the 5 existing wire scanner induced a « small » horizontal emittance reduction and a « large » vertical emittance estimation (function of WS error level). For QM7 quadrupole strength underestimation an symmetric (x-y) effect is observed (see extra slide).
- Compare the error sensitivity of the LMS reconstruction method with a « quadrupole strength variation » method.

## Questions

- What are the real « quadrupole strength » of QM6 and QM7 (seen by the extracted bunch) ?
- Is it possible to determine WS positions leading to less error sensitive reconstruction method ?
- What is the sensitivity of « quadrupole strength variation » method ? (which quad ? which WS ?, new quad position ?, new WS position ? ...).
- Is it possible to realize more than 3 measurement per WS to reduce the error level ?
- others ideas ? ...

\* LMS : Least Mean Square

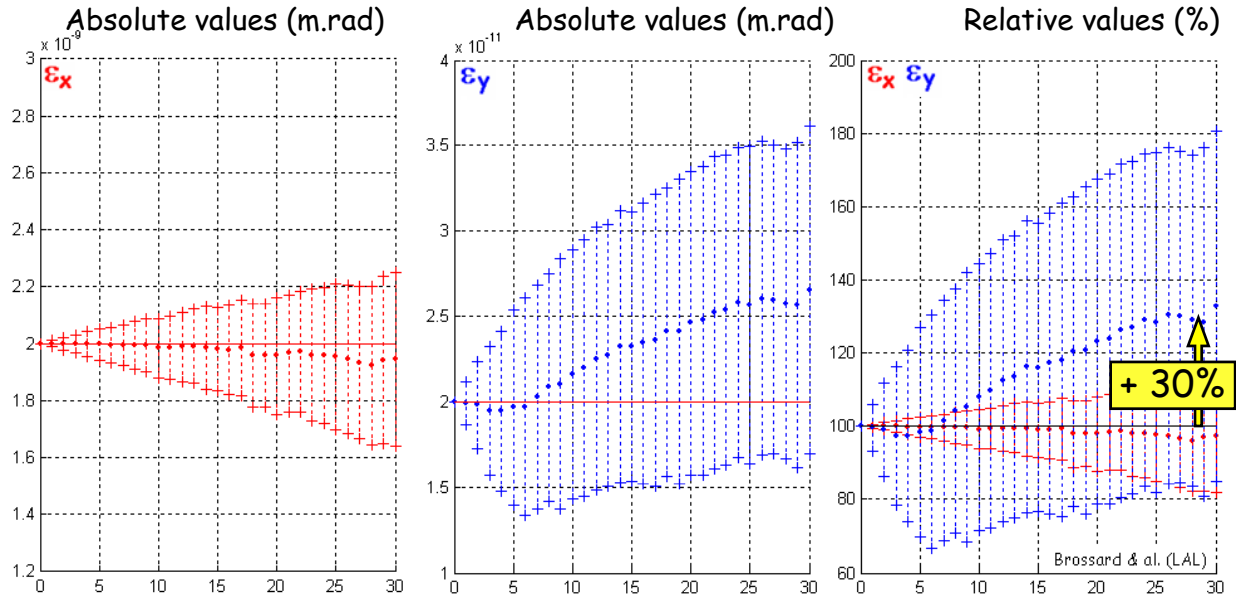
**Many thanks to Mark Woodley** for the input MAD file

(see : <http://www.slac.stanford.edu/~mdw/ATF/EXT.mad>)

## References :

ATF Internal reports : ATF-99-01, ATF-00-06, ATF-99-17, ATF-99-08, ATF-00-01...

QM6 quadrupole strength is reduce by 20% →



QM7 quadrupole strength is reduce by 20% →  
(see M. Alabau Pons & al. Talk)

