Constraining the early universe with dark matter

Mitsuru Kakizaki (LAPTH)

November 18, 2008

LCWS08 @ University of Illinois, Chicago

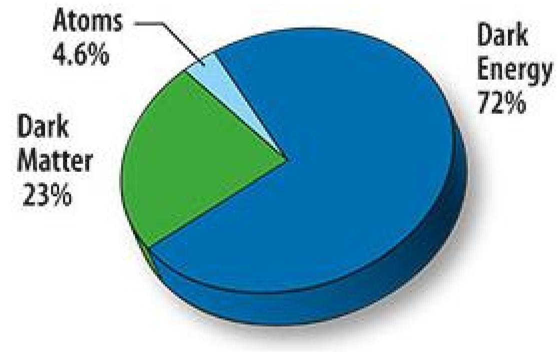
In collaboration with

- Manuel Drees (Bonn Univ.)
- Hoernisa Iminniyaz (Univ. of Xinjiang)
- Suchita Kulkarni (Bonn Univ.)

Refs:

- PRD76 103524 (2007)
- work in progress

1. Motivation



[http://wmap.gsfc.nasa.gov]

- Observations of
  - cosmic microwave background
  - structure of the universe
  - etc.

→ Non-baryonic dark matter: $\Omega_{\rm DM} h^2 = 0.1143 \pm 0.0034$

Physics beyond the standard model (SM) of particle physics necessary

- Weakly interacting massive particles (WIMPs) χ are good candidates for dark matter (DM)

The predicted thermal relic abundance naturally explains the observed dark matter abundance: $\Omega_{\chi,\rm standard} h^2 \sim 0.1$

- Neutralino (LSP); 1st KK mode of the B boson (LKP); etc.

Investigation of early universe using DM abundance

- The abundance of thermal relics (e.g. DM) is determined by the Boltzmann equation: $\dot{n}_\chi + 3Hn_\chi = -\langle\sigma_{\rm eff}v\rangle(n_\chi^2 - n_{\chi,{\rm eq}}^2)$

  (and the initial temperature: T_0)

Numerical calculation needed in evaluating the relic density in many cases

$\Rightarrow$ **Analytic methods should be developed in various scenarios**

- The (effective) cross section $\sigma_{\rm eff}$ can be determined from collider and DM detection experiments

$\Rightarrow$ **We can test the standard CDM scenario and investigate conditions of very early universe: $T_0, H, \cdots$**

$\Omega_{\chi,{\rm standard}}h^2 < \Omega_{\rm DM}h^2$ $\Leftarrow$ Non-thermal production, modified H , ...

$\Omega_{\chi,{\rm standard}}h^2 > \Omega_{\rm DM}h^2$ $\Leftarrow$ Low T_R , modified H, Late production of entropy, ...

Outline

This work

- Dark matter = thermal WIMPs
  → constraints on the reheating temperature and on modifications of the Hubble parameter
- Analytic treatment that connects the hot and cold relic solutions and late entropy production by semi-relativistic relics

1. Motivation
2. Standard calculation of WIMP abundance
3. Constraints on the very early universe from WIMP dark matter
4. Abundance of semi-relativistic relics and its application
5. Summary

2. Standard calculation of WIMP abundance

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- High temperature $(T \geq T_F)$:
  - Thermal equilibrium was maintained:

$$\Gamma = n_\chi \langle \sigma v \rangle > H = R/\dot{R}$$

- χ decoupled when non-relativistic in RD era:

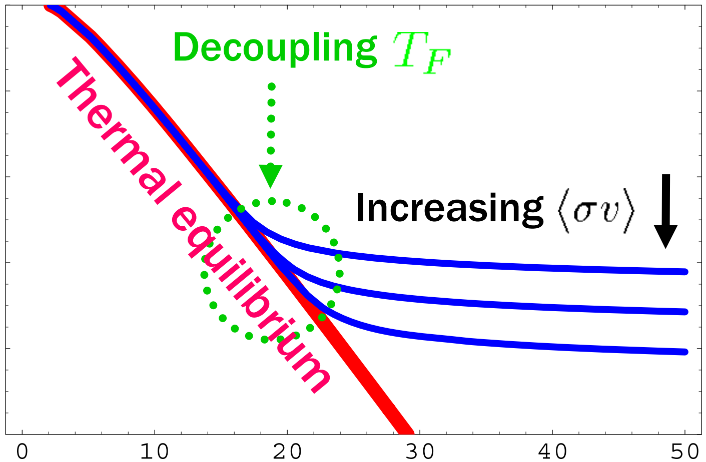
$$H = \frac{\pi T^2}{M_{\rm Pl}} \sqrt{\frac{90}{g_*}}$$

$$\langle \sigma v \rangle = a + 6b/x + \mathcal{O}(1/x^2),$$

$$n_{\chi,\rm eq} = g_\chi \; (m_\chi T/2\pi)^{3/2} e^{-m_\chi/T}$$

Co-moving number density

$$Y_\chi = n_\chi/s$$



$$\Omega_\chi h^2 = 2.7 \times 10^8 \; Y_\chi \left(\frac{m_\chi}{1 \text{ GeV}} \right)$$

- Low temperature $(T \leq T_F)$:
  - WIMP production negligible :

$$\Omega_{\chi,\rm standard} h^2 \simeq 0.1 \times \left(\frac{a + 3b/x_F}{10^{-9} \text{ GeV}^{-2}} \right)^{-1} \left(\frac{x_F}{22} \right) \left(\frac{g_*}{90} \right)^{-1/2} \sim \Omega_{\rm DM} h^2$$

3. Constraints on the very early universe from WIMP DM

- **Equilibrium case:** $\sigma \nearrow \Rightarrow \Omega h^2 \searrow$; $\Omega_\chi h^2$ is independent of T_0

  Out-of-equilibrium case: $\sigma \nearrow \Rightarrow \Omega h^2 \nearrow$; $T_0 = m_\chi / x_0 \nearrow \Rightarrow \Omega h^2 \nearrow$

[Giudice,Kolb,Riotto(2001), Gelmini,Gondolo(2006), ...]

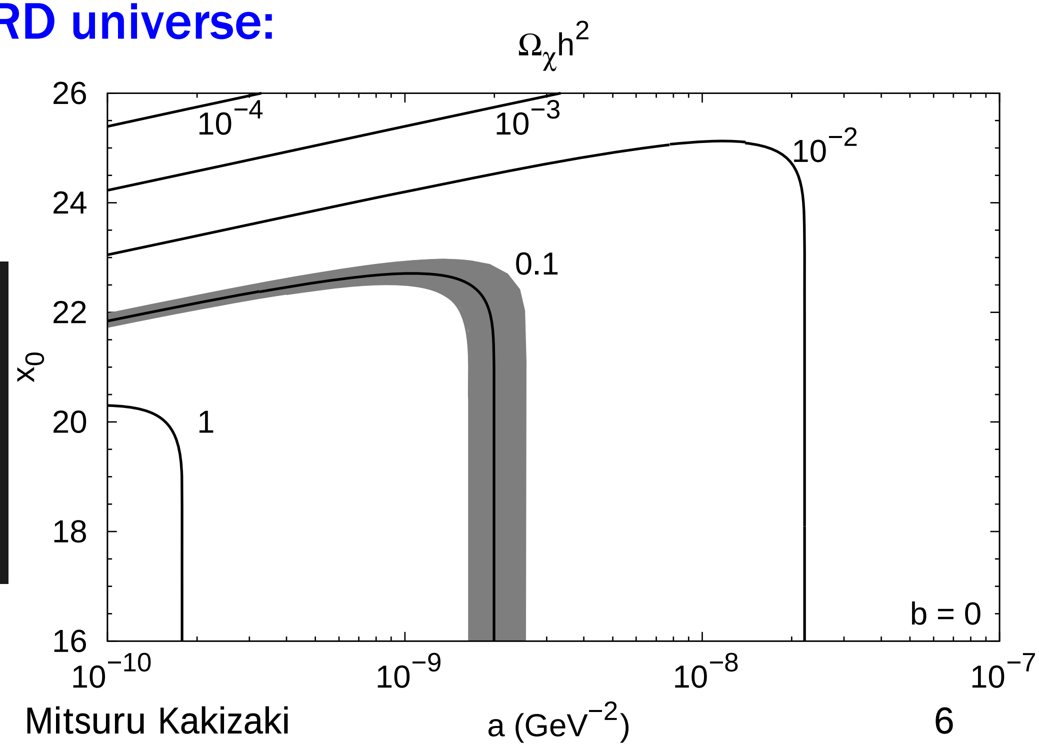
- **Thermal relic abundance in the RD universe:**

$0.8 < \Omega_{\rm DM} h^2 < 0.12$

Requirement that $\Omega_\chi h^2 \simeq 0.1$

$\Downarrow$

Lower bound on the initial temperature: $T_0 > m_\chi / 23$



Modified expansion rate

- Various cosmological models predict a non-standard early expansion

  [e.g. Scherrer et al.,PRD(1985); Salati,PLB(2003); Fernengo et al.,PRD(2003); Chung et al., PRD (1999); ...]

  ⇨ Predicted WIMP relic abundances are also changed

- When WIMPs were in full thermal equilibrium, in terms of the modification parameter $A(x) = H_{\rm st}(x)/H(x)$ the relic abundance is

$$\Omega_\chi h^2 = 0.1 \left(\frac{I(x_F)}{8.5 \times 10^{-10}\ {\rm GeV}^{-2}} \right)^{-1}$$

$$I(x_F) = \int_{x_F}^{\infty} dx \frac{\sqrt{g_*}\langle\sigma v\rangle A(x)}{x^2}; \quad x_F = \ln \left[\sqrt{\frac{45}{\pi^5}} \xi m_\chi M_{\rm Pl} g_\chi \frac{\langle\sigma v\rangle A(x)}{\sqrt{x g_*}} \right] \Bigg|_{x=x_F}$$

If $A(x) = 1$, $x_F = x_{F,\rm st}$ and we recover the standard formula

This formula is capable of predicting the final relic density correctly

Constrains on modifications of the Hubble parameter

- In terms of $z \equiv T/m_\chi = 1/x$

  we need to know $A(z)$ only for $z_{\rm BBN} = 10^{-5} - 10^{-4} \le z \le z_F \sim 1/20 \ll \mathcal{O}(1)$

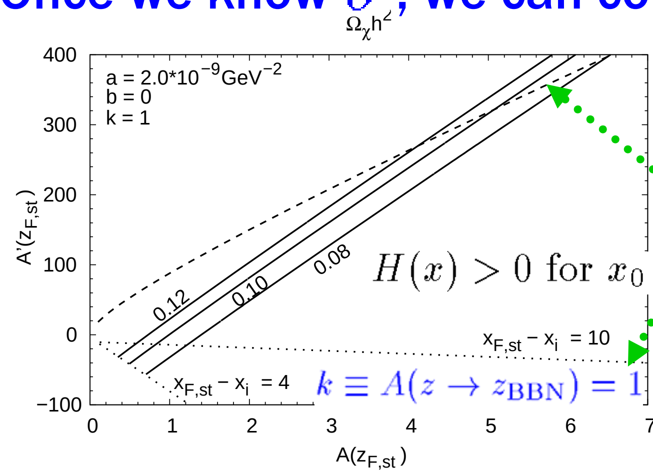
➡ This suggests a parametrization of $A(z)$ in powers of $(z - z_{F,\rm st})$:

$$A(z) = A(z_{F,\rm st}) + (z - z_{F,\rm st})A'(z_{F,\rm st}) + \frac{1}{2}(z - z_{F,\rm st})^2 A''(z_{F,\rm st})$$

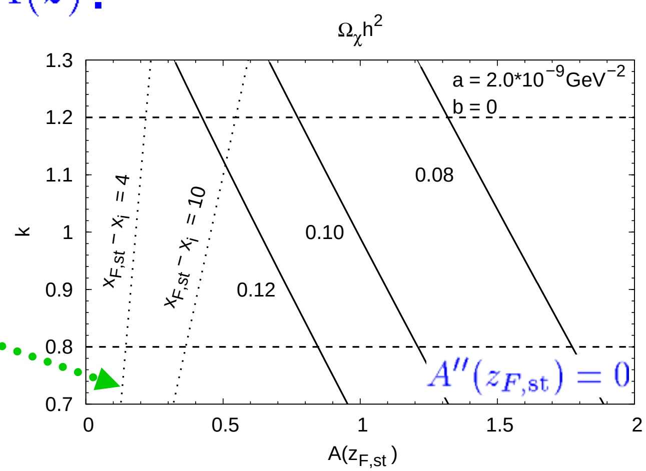
subject to the BBN limit: $0.8 \le k \equiv A(z \to z_{\rm BBN}) \le 1.2$

x_i :Maximal temperature where

- Once we know σ, we can constrain $A(z)$:



$H(x) > 0$ for $x_0 < x < \infty$



$\Omega_\chi h^2$ depends on all $H(T)$ ➡ Larger allowed region for $H(T_F)$

4. Abundance of semi-relativistic relics and entropy production

- Precise evaluation of the abundance of particles that freeze out when they are semi-relativistic $(x_F \sim 3)$ is complicated

Goal: simple analytic treatment that describes the transition from non-relativistic to relativistic relics

- Assume the Maxwell-Boltzmann distribution:

$$Y_{\chi,\mathrm{eq}} \equiv \frac{n_{\chi,\mathrm{eq}}}{s} = 0.115 \frac{g_\chi}{g_{*s}} x^2 K_2(x)$$

($K_n(x)$: modified Bessel function)

Thermal average of cross section σ :

$$\langle \sigma v \rangle = \frac{1}{8 m_\chi^4 T K_2^2(m_\chi/T)} \int_{4m_\chi^2}^{\infty} \mathrm{d}s \; \sigma (s - 4m_\chi^2) \sqrt{s} \; K_1(\sqrt{s}/T)$$

Ansatz for approximate cross sections

- Consider neutrinos as stable relic:
- Annihilation cross section:

$$\sigma v^{\text{Dirac } \nu} = \frac{G_F^2 s}{16\pi}$$

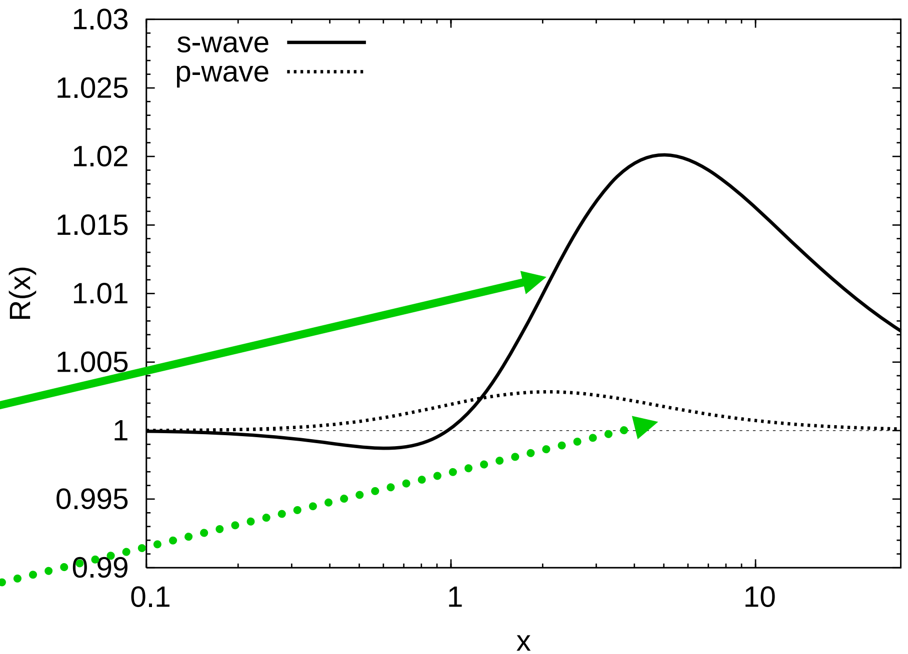
$$\sigma v^{\text{Majorana } \nu} = \frac{G_F^2 s v^2}{16\pi}$$

- Ansatz for the thermally-averaged annihilation cross section:

$$\langle \sigma v \rangle_{\text{app}}^{\text{Dirac}} = \frac{G_F^2 m_\chi^2}{16\pi} \left(\frac{12}{x^2} + \frac{5+4x}{1+x} \right)$$

$$\langle \sigma v \rangle_{\text{app}}^{\text{Majorana}} = \frac{G_F^2 m_\chi^2}{16\pi} \left(\frac{12}{x^2} + \frac{3+6x}{(1+x)^2} \right)$$

- $\langle \sigma v \rangle_{\text{app}} / \langle \sigma v \rangle_{\text{exact MB}}$:



The approx. cross sections reproduce the exact results with accuracy of a few %

Approximate abundance of semi-relativistic relics

- Define the freeze–out temperature by

$$\Gamma(x_F) = H(x_F)$$

where

$$\Gamma(x_F) = n_{\chi,\mathrm{eq}}(x_F)\langle\sigma v\rangle(x_F)$$

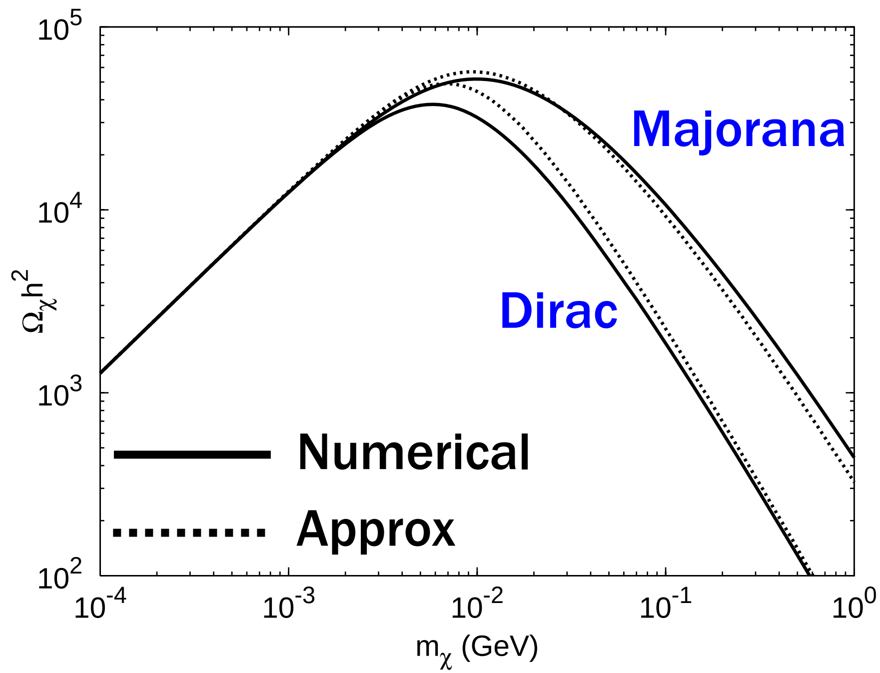
(different from
the standard definition of x_F)

- Assume the relic abundance does not change after decoupling

➡ Final abundance:

$$Y_{\chi,\infty} = Y_{\chi,\mathrm{eq}}(x_F)$$

- Comparison between the numerical and approx solutions



Applications of semi-relativistic relics

- As DM candidates

  Hypothetical semi-relativistic relics should decouple before BBN

  $$\Rightarrow m_\chi \sim T_F > T_{\rm BBN} \simeq {\rm MeV} \Rightarrow \Omega_\chi h^2 > 10^3$$

  The relic abundance is too high!

- As source of large entropy production [Steinhardt, Turner (1983)]

  Out-of-equilibrium decay of relic particles produces entropy

  Ratio of the final to initial entropy: $\dfrac{S_f}{S_i} = g_*^{1/4} \dfrac{m_\chi Y_{\chi,i} \tau_\chi^{1/2}}{M_{\rm Pl}^{1/2}} \propto \Omega_\chi h^2$

  $\Rightarrow$ **Semi-relativistic relics can produce significant entropy!**

Example: sterile neutrino

- Consider a sterile neutrino mixed with an active neutrino (mixing angle: θ)
- Decay rate of the sterile neutrino:

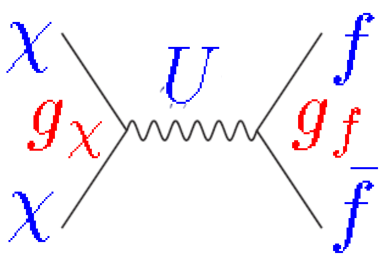
$$\Gamma_\chi = \frac{G_F^2 m_\chi^5}{192\pi^3}\sin^2\theta, \qquad \frac{G_F m_\chi^3}{16\pi}\sin^2\theta$$

(for small m_χ) (for large m_χ)

should be large enough not to spoil BBN

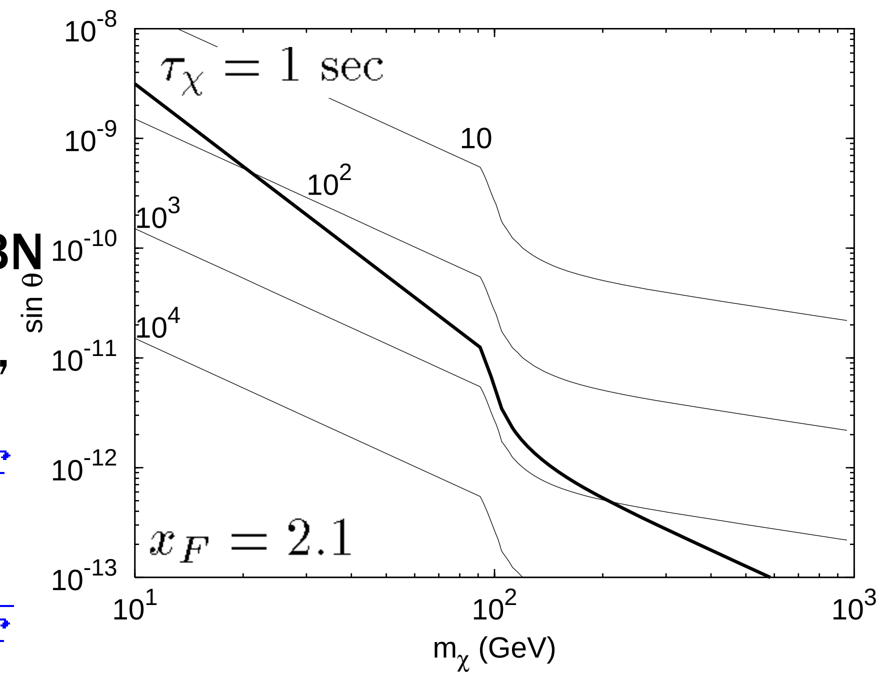
- By introducing a new heavy particle, U, large pair annihilation can be induced:

$$\sigma v = \frac{s v^2}{12\pi}\frac{g_\chi^2 g_f^2}{M_U^4}$$



→ $x_F \sim 3$ possible

- Entropy production S_f/S_i by the decay of semi-relativistic sterile neutrinos



5. Summary

- Using the DM relic density we can probe very early universe at around $T \sim m_\chi/20 \sim \mathcal{O}(10)\ \mathrm{GeV}$ (well before BBN $T_{\mathrm{BBN}} \sim \mathcal{O}(1)\ \mathrm{MeV}$)

- $\Omega_{\chi,\mathrm{thermal}} h^2 = \Omega_{\mathrm{DM}} h^2$

  $\Rightarrow$ Lower bound on the reheat temperature: $T_R > m_\chi/23$

- The sensitivity of $\Omega_{\chi,\mathrm{thermal}} h^2$ on $H(T_F)$ is weak

- We find an approximate analytic formula for the abundance of semi-relativistic relics

- Semi-relativistic relics are useful for producing a large amount of entropy

Backup slides

2. Standard calculation of the WIMP relic abundance

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- Conventional assumptions for WIMPs as DM particle:
  - $\chi = \bar{\chi}$, single production of χ is forbidden
- WIMP abundance n_χ is determined by the Boltzmann eq.:

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\mathrm{eq}}^2)$$

$H = \dot{R}/R$: Hubble expansion parameter

$\langle \sigma v \rangle$: thermal average of the annihilation cross section $\sigma(\chi\chi \to \mathrm{SM\ particles})$ times relative velocity v

$n_{\chi,\mathrm{eq}}$: equilibrium number density

- Introduce $Y_{\chi(,\mathrm{eq})} = \dfrac{n_{\chi(,\mathrm{eq})}}{s}, x = \dfrac{m_\chi}{T}$

$$\Rightarrow \quad \frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\langle \sigma v \rangle s}{Hx} (Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2)$$

3. Low-temperature scenario

[c.f. Giudice,Kolb,Riotto(2001), Gelmini,Gondolo(2006)]

- T_R : Reheat temperature

The initial abundance is assumed to be negligible: $Y_\chi(x_0) = 0\,, \quad x_0 = \dfrac{m_\chi}{T_R}$

- Zeroth order approximation:

$T_R < T_F$ ⇒ χ annihilation is negligible:

$$\frac{dY_0}{dx} = 0.028\, g_\chi^2 g_*^{-3/2} m_\chi M_{\mathrm{Pl}} \mathrm{e}^{-2x} x \left(a + \frac{6b}{x}\right)$$

⇒ The solution is proportional to the cross section:

At late times,

$$\boxed{Y_0(x \gg x_0) \simeq 0.014\, g_\chi^2 g_*^{-3/2} m_\chi M_{\mathrm{Pl}} \mathrm{e}^{-2x_0} x_0 \left(a + \frac{6b}{x_0}\right)}$$

This solution should be smoothly connected to the standard result

First order approximation

- Add a correction term describing annihilation to Y_0 : $Y_1 = Y_0 + \delta \ (\delta < 0)$
- As long as $|\delta| \ll Y_0$, the evolution equation for δ is

$$\frac{d\delta}{dx} = -1.3 \sqrt{g_*} m_\chi M_{\rm PL} \left(a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}$$

➡ The solution is proportional to σ^3

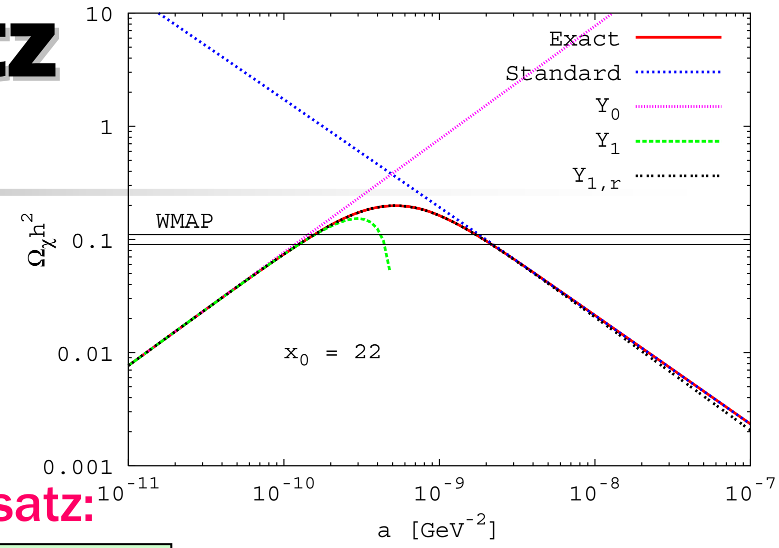
At late times,

$$\boxed{\delta(x \gg x_0) \simeq -2.5 \times 10^{-4} \ g_\chi^4 g_*^{-5/2} m^3 M_{\rm Pl}^3 {\rm e}^{-4x_0} x_0 \left(a + \frac{3b}{x_0} \right) \left(a + \frac{6b}{x_0} \right)^2}$$

- $|\delta|$ soon dominates over Y_0 for not very small cross section

➡ Y_1 fails to track the exact solution

Re-summed ansatz



- It is noticed that $Y_0 \propto \sigma > 0,\ \delta \propto \sigma^3 < 0$

  For large cross section,
  $Y_\chi(x \to \infty)$ should be $\propto 1/\langle \sigma v \rangle$

- This observation suggests the re-summed ansatz:

$$\boxed{Y(x) = Y_0 + \delta = Y_0 \left(1 + \frac{\delta}{Y_0}\right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}}$$

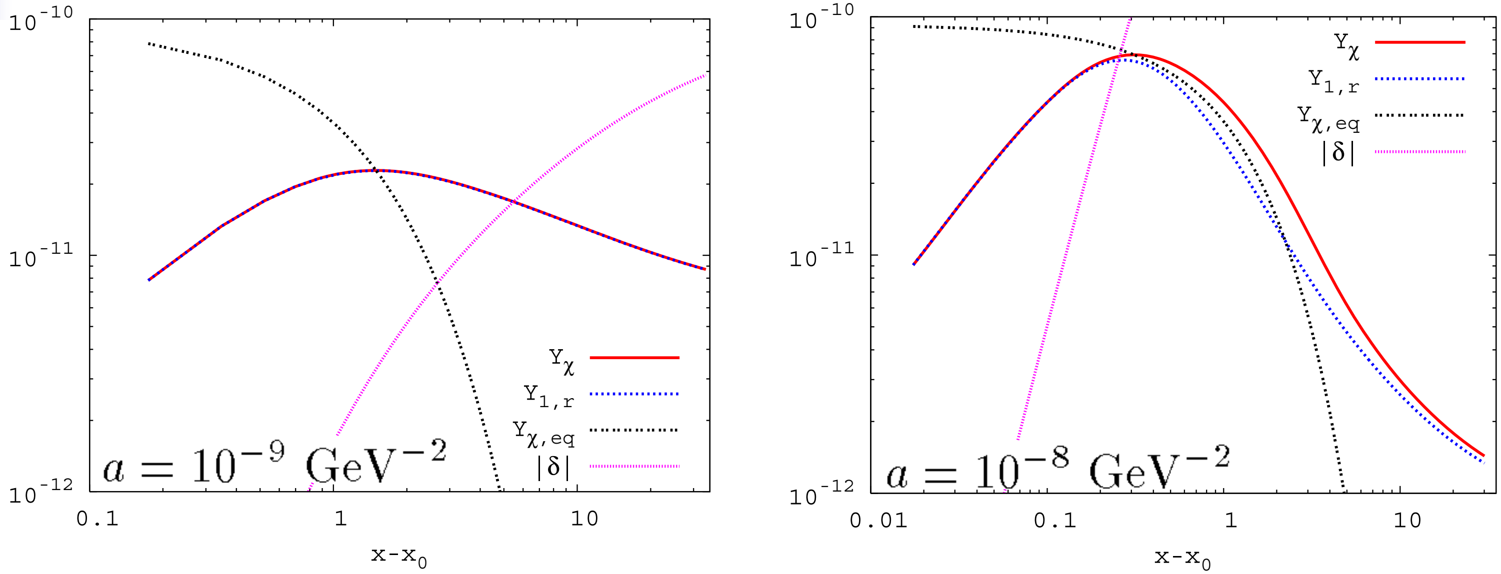
- For $|\delta| \gg Y_0$, $\boxed{Y_{1,r}(x) \simeq -\frac{Y_0^2}{\delta} \propto \frac{1}{\sigma}}$

$x_0 \to x_F$ Standard formula

At late times, $Y_{1,r}(x \to \infty) = \dfrac{x_0}{1.3\sqrt{g_*} m_\chi M_{\rm Pl}(a + 3b/x_0)}$

- In the case where χ production is negligible but the initial abundance is sizable, $Y_{1,r}$ is exact

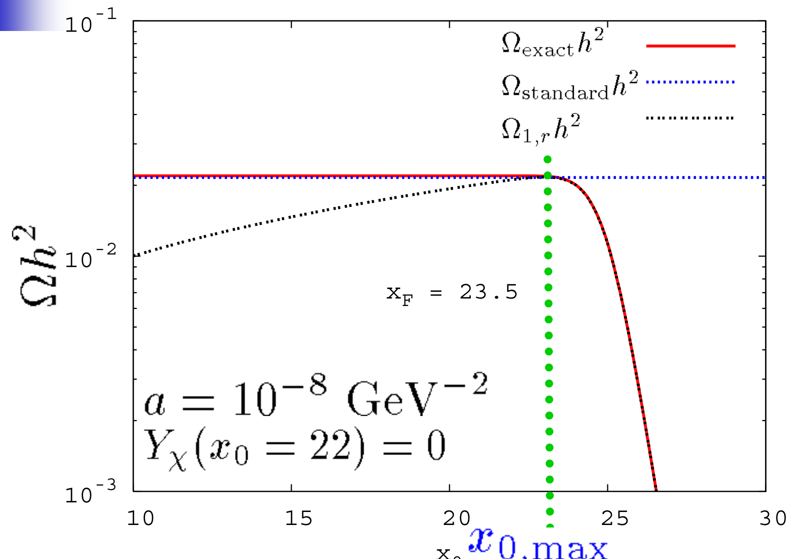
Evolution of solutions

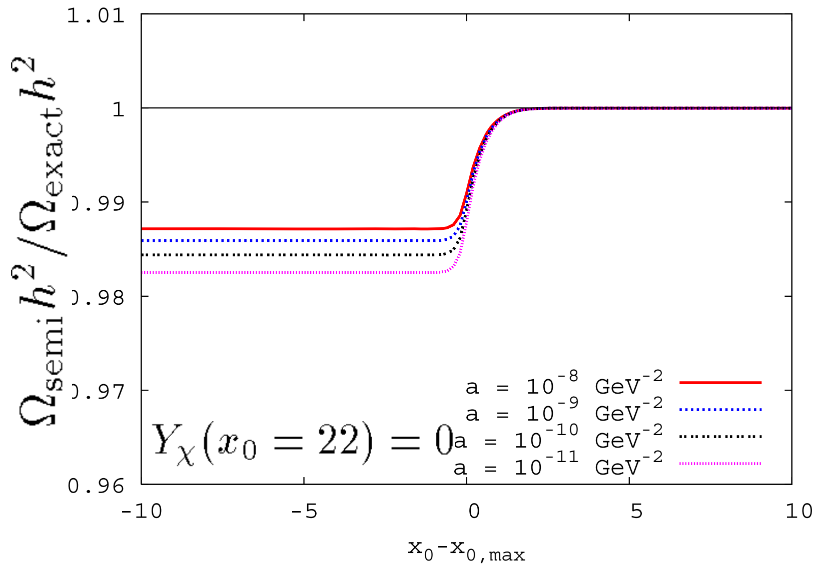


Y_χ : Exact result, $Y_{1,r}$: Re-summed ansatz, $b = 0,\ Y_\chi(x_0 = 22) = 0$

- The re-summed ansatz $Y_{1,r}$ describes the full temperature dependence of the abundance when equilibrium is not reached
- For larger cross section the deviation becomes sizable for $x - x_0 \sim 1$, but the deviation becomes smaller for $x \gg x_0$

Semi-analytic solution





- $Y_{1,r}(x_0, x \to \infty)\ (\propto \Omega_{1,r} h^2)$ has a maximum (left)
- New semi-analytic solution can be constructed: $\Omega_{\rm semi} h^2$ (right)

For $x_0 > x_{0,\rm max}$, use $Y_{1,r}(x_0)$; for $x_0 < x_{0,\rm max}$, use $Y_{1,r}(x_{0,\rm max})$

The semi-analytic solution $\Omega_{\rm semi} h^2$ reproduces the correct final relic density $\Omega_{\rm exact} h^2$ to an accuracy of a few percent

Hot relics

- Hot relics (decouple for $x_F < 3$):

  $Y_{\chi,\mathrm{eq}}(x)$ almost constant

  ⇒ Final abundance is insensitive to the freeze out temperature:

$$Y_{\chi,\infty} = Y_{\chi,\mathrm{eq}}(x_F) = \frac{45}{2\pi^4}\frac{g_\chi}{g_{*s}(x_F)}$$