Exploring the Phenomenology of the Noncommutative Standard Model with WW Scattering

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The Long History of Noncommutative Theories.

- Noncommutative (NC) physics first proposed to solve divergence problems in QFT. (Heisenberg; Pauli; Snyder)
- NC geometry became its own field of mathematics, (Connes)
- NCQFT shown to arise from low energy limit of string theory with a B field. (Seiberg, Witten)
- NC gauge theories are formulated; two versions of NCSM are developed. (Wess *et. al.*; Chaichian *et. al.*)

The Basics of Noncommutativity.

Fundamental assumption.

- $[x_\mu, x_\nu] = i\theta_{\mu\nu} \equiv i\frac{c_{\mu\nu}}{\Lambda^2}$. Λ is "NC scale."
- $c_{ij} \neq 0$ for $i, j \in \{1, 2, 3\}$ is called "space-space noncommutativity".
- $c_{0i} \neq 0$ is called "space-time noncommutativity."

Subtleties.

- For space-time NC, perturbation theory must be formulated carefully–see time-ordered perturbation theory (Chaichian *et. al.*).
- So-called UV-IR mixing makes renormalization tricky, but possible (Filk; Minwalla *et. al.*; Petriello; Wulkenhaar; Buric *et. al.*; Martin, *et. al.*; ...).

Expressing Noncommutative Fields in Terms of Commutative Fields.

Moyal Star Product.

- For $U(n)$ gauge groups we can simply replace products with Moyal Star Products.
- $\hat{f}(\hat{x})\hat{g}(\hat{y}) \rightarrow f(x) \star g(y) = f(x) \exp\left\{\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}\right\} g(y)$.

Seiberg-Witten Map (SWM).

- $SU(n)$ gauge groups don't close under $\star$-product.
- Can, however, impose consistency conditions on NC and normal gauge transformations.
- Doing so gives SWM, expanding NC quantities in terms of commuting quantities, order by order in $\theta_{\mu\nu}$.

NC Gauge Transformations

Same form as ordinary gauge tranformations

$$\hat{\psi} \to \hat{\psi}' = \exp(ig\hat{\lambda}\star)\hat{\psi} = \hat{\psi} + ig\hat{\lambda}\star\hat{\psi} + \frac{(ig)^2}{2!}\hat{\lambda}\star\hat{\lambda}\star\hat{\psi} + \mathcal{O}(\hat{\lambda}^3)\,,$$

$$\hat{A}_\mu \to \hat{A}'_\mu = \exp(ig\hat{\lambda}\star)\hat{A}_\mu \exp(-ig\hat{\lambda}\star) + \frac{i}{g}\exp(ig\hat{\lambda}\star)(\partial_\mu \exp(-ig\hat{\lambda}\star))$$

$$= \hat{A}_\mu + ig[\hat{\lambda} \stackrel{\star}{,} \hat{A}_\mu] + \frac{(ig)^2}{2!}[\hat{\lambda} \stackrel{\star}{,} [\hat{\lambda} \stackrel{\star}{,} \hat{A}_\mu]] + \partial_\mu\hat{\lambda} + ig[\hat{\lambda} \stackrel{\star}{,} \partial_\mu\hat{\lambda}] + \mathcal{O}(\hat{\lambda}^3),$$

To 1st Order in $\theta_{\mu\nu}$

$$\hat{A}_\mu(x) = A_\mu(x) + \frac{1}{4}\theta^{\rho\sigma}\{A_\sigma(x), \partial_\rho A_\mu(x) + F_{\rho\mu}(x)\} + \mathcal{O}(\theta^2)\,,$$

$$\hat{\psi}(x) = \psi(x) + \frac{1}{2}\theta^{\rho\sigma}A_\sigma(x)\partial_\rho\psi(x) + \frac{i}{8}\theta^{\rho\sigma}[A_\rho(x), A_\sigma(x)]\psi(x) + \mathcal{O}(\theta^2)$$

$$\hat{\lambda}(x) = \lambda(x) + \frac{1}{4}\theta^{\rho\sigma}\{A_\sigma(x), \partial_\rho\lambda(x)\} + \mathcal{O}(\theta^2)\,.$$

Noncommutative Standard Model (NCSM).

Wess *et. al.* hep-ph/0111115, ...

Strategy.

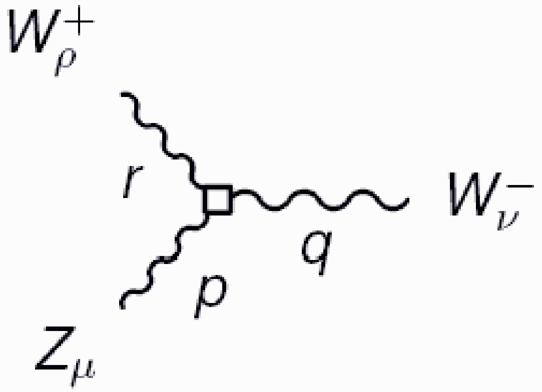
- Take SM Lagrangian, and replace all fields and coordinates with NC counterparts.
- Using $\star$-product and SWM, expand $\mathcal{L}$ in terms of commuting quantities, order-by-order in $\theta_{\mu\nu}$.

Result.

- No new particles (besides SM ones) appear in theory.
- At each order in θ, get many new interaction terms.
- Ambiguity in SWM for gauge kinetic terms gives three new free parameters κ_1, κ_2, κ_3.

All our calculations will be to first order in θ.

Feynman Rules. Example:



$$= -\frac{em_W^2}{2} f^A_{\mu\nu\rho}(p) + \frac{em_Z^2}{4} f^Z_{\mu\nu\rho}(p,q,r)$$

$$+2e \sin 2\theta_W K_{WWZ} \Theta_{\mu\nu\rho}(p,q,r) \,,$$ where

$$f^A_{\mu\nu\rho}(p) \equiv \theta_{\mu\nu} p_\rho + \theta_{\mu\rho} p_\nu + g_{\mu\nu}(\theta \cdot p)_\rho - g_{\nu\rho}(\theta \cdot p)_\mu + g_{\rho\mu}(\theta \cdot p)_\nu \,,$$

$$f^Z_{\mu\nu\rho}(p,q,r) \equiv \theta_{\mu\nu}(p-q)_\rho + \theta_{\nu\rho}(q-r)_\mu + \theta_{\rho\mu}(r-p)_\nu$$

$$- 2g_{\mu\nu}(\theta \cdot r)_\rho - 2g_{\nu\rho}(\theta \cdot p)_\mu - 2g_{\rho\mu}(\theta \cdot q)_\nu \,,$$

$$\Theta_{\mu\nu\rho}(p,q,r) \equiv \theta_{\mu\nu}\,(p \cdot r\, q_\rho - q \cdot r\, p_\rho) + (\theta \cdot p)_\mu\,(q \cdot r\, g_{\nu\rho} - q_\rho r_\nu)$$

$$- (\theta \cdot p)_\nu\,(q \cdot r\, g_{\rho\mu} - q_\rho r_\mu) - (\theta \cdot p)_\rho\,(q \cdot r\, g_{\mu\nu} - q_\mu r_\nu)$$

$$+ p \times q\,(r_\mu g_{\nu\rho} - r_\nu g_{\rho\mu}) + (\text{cyclic perms. of } \{p,q,r\} \text{ and } \{\mu,\nu,\rho\}) \,,$$

and $K_{WWZ} \equiv g^2 \kappa_2 / 2c_W^2$.

Partial-Wave Unitarity in WW scattering.

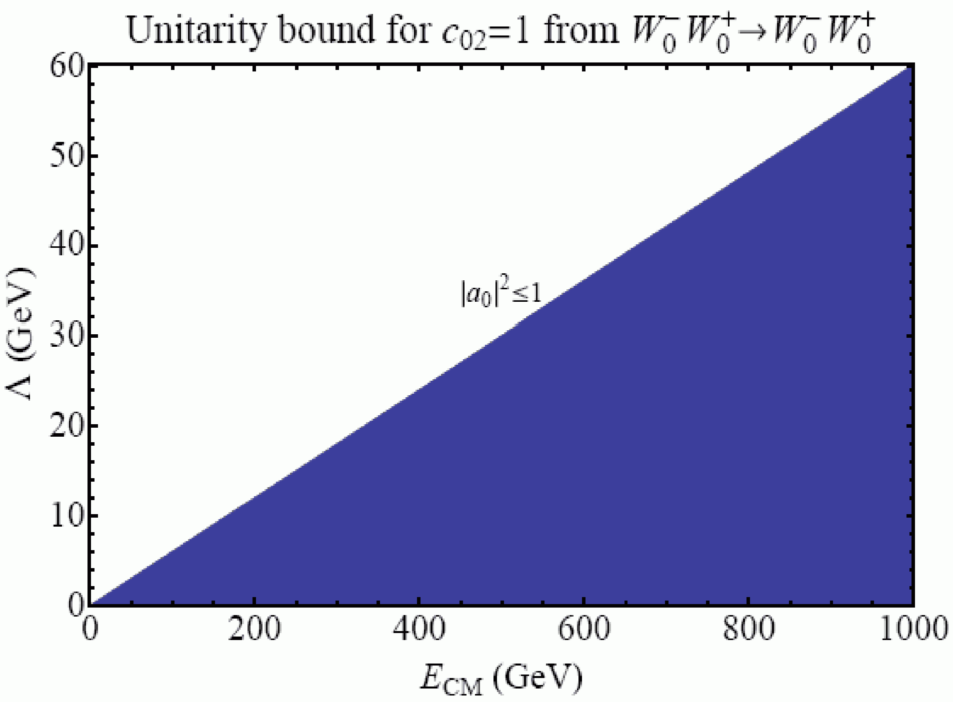
Partial-Wave Unitarity.

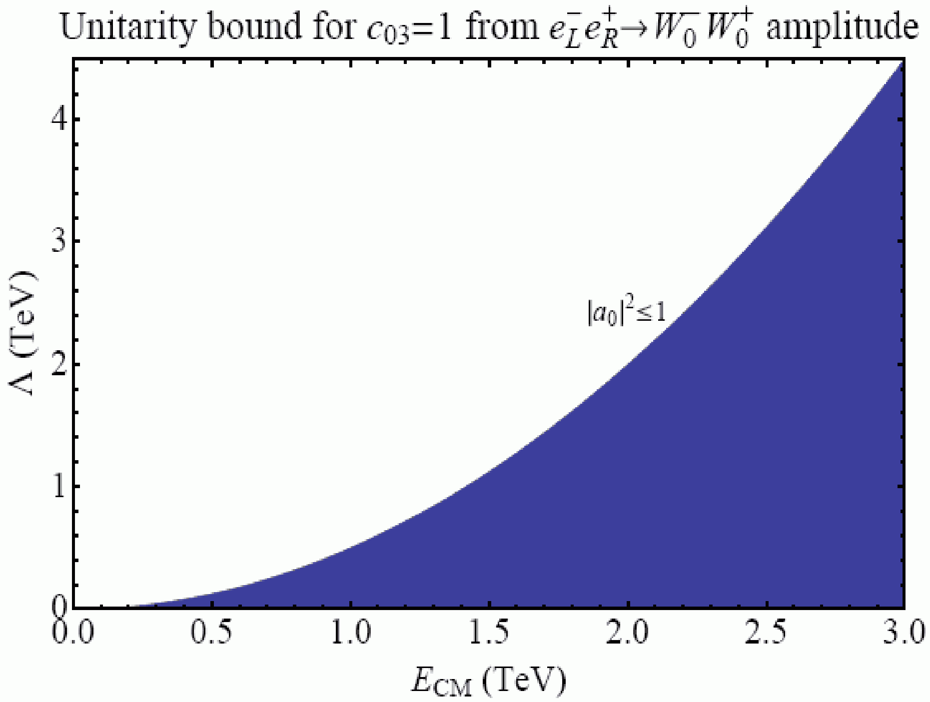
- Decompose a scattering amplitudes as $A = 16\pi \sum_{l=0}^{\infty} a_l (2l+1) P_l(\cos\theta)$.
- The partial waves must obey $\mathrm{Re}(a_l) \leq 1/2\,, \quad 0 \leq \mathrm{Im}(a_l) \leq 1\,, \quad \text{and} \quad |a_l|^2 \leq 1\,.$

$W^+W^- \to W^+W^-$ and $e^+e^- \to W^+W^-$ in NCSM.

- Amplitudes depend on $\theta_{\mu\nu}$ and κ_2.
- Use partial-wave unitarity conditions on amplitudes to bound these parameters.
- Amplitudes are ϕ-dependent; maximize wrt ϕ to get "worst-case scenario" and apply conditions above.
- $e_L^- e_R^+ \to W_0^+ W_0^-$ gives strongest bound.

Unitarity bounds on NCSM.

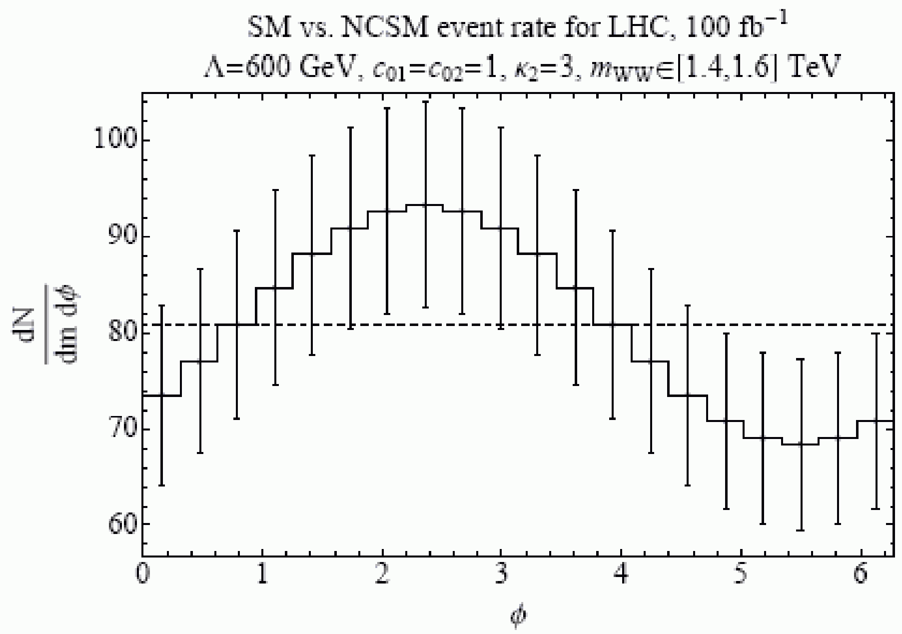


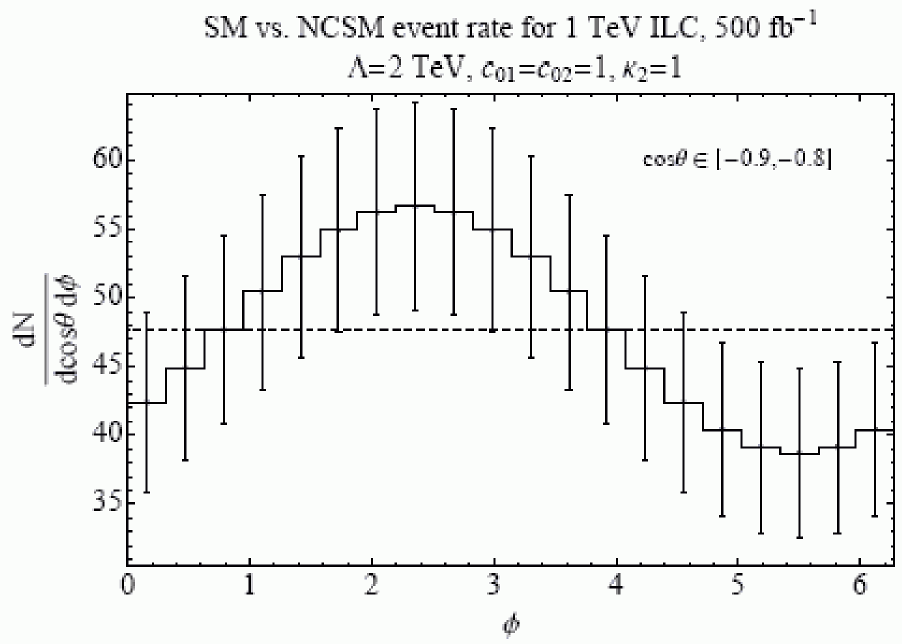


WW scattering at colliders.

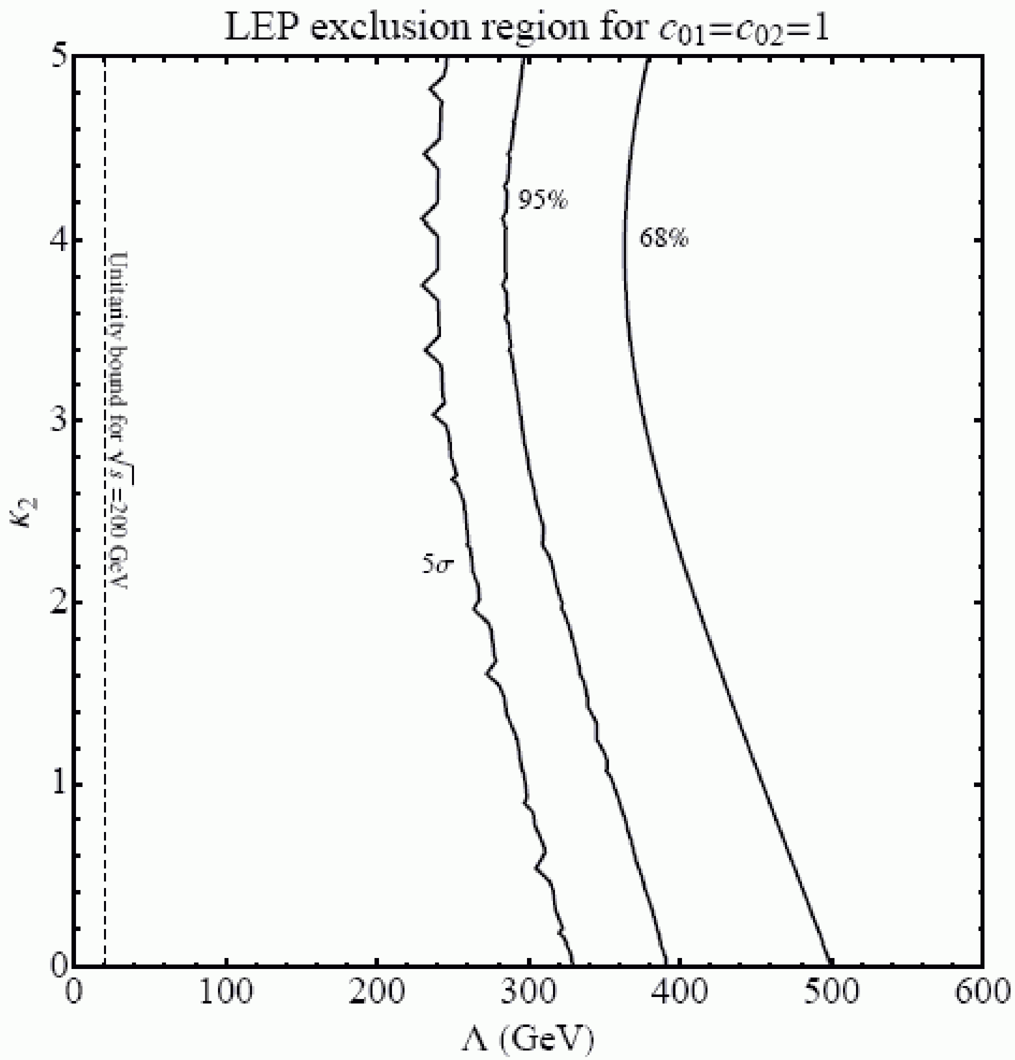
- Helicity-summed observables $\propto (c_{01} \sin\phi - c_{02} \cos\phi)/\Lambda^2$.
    - At LEP-II we look at $d\sigma/d\cos\theta d\phi$.
    - At LHC we look $d\sigma/dm_{WW} d\phi$.
- At ILC, distinguishing W helicities can give sensitivity to other parameters.
    - We use $d\sigma/d\cos\theta d\phi$ and $dA_{LR}/d\cos\theta d\phi$ to get search reach in $\kappa_2 - \Lambda$ plane.
    - We look at $d\sigma/d\cos\theta d\phi$ for different W helicities to determine $c_{\mu\nu}$, Λ, and κ_2.

Collider Observables.





LEP-II search reach.

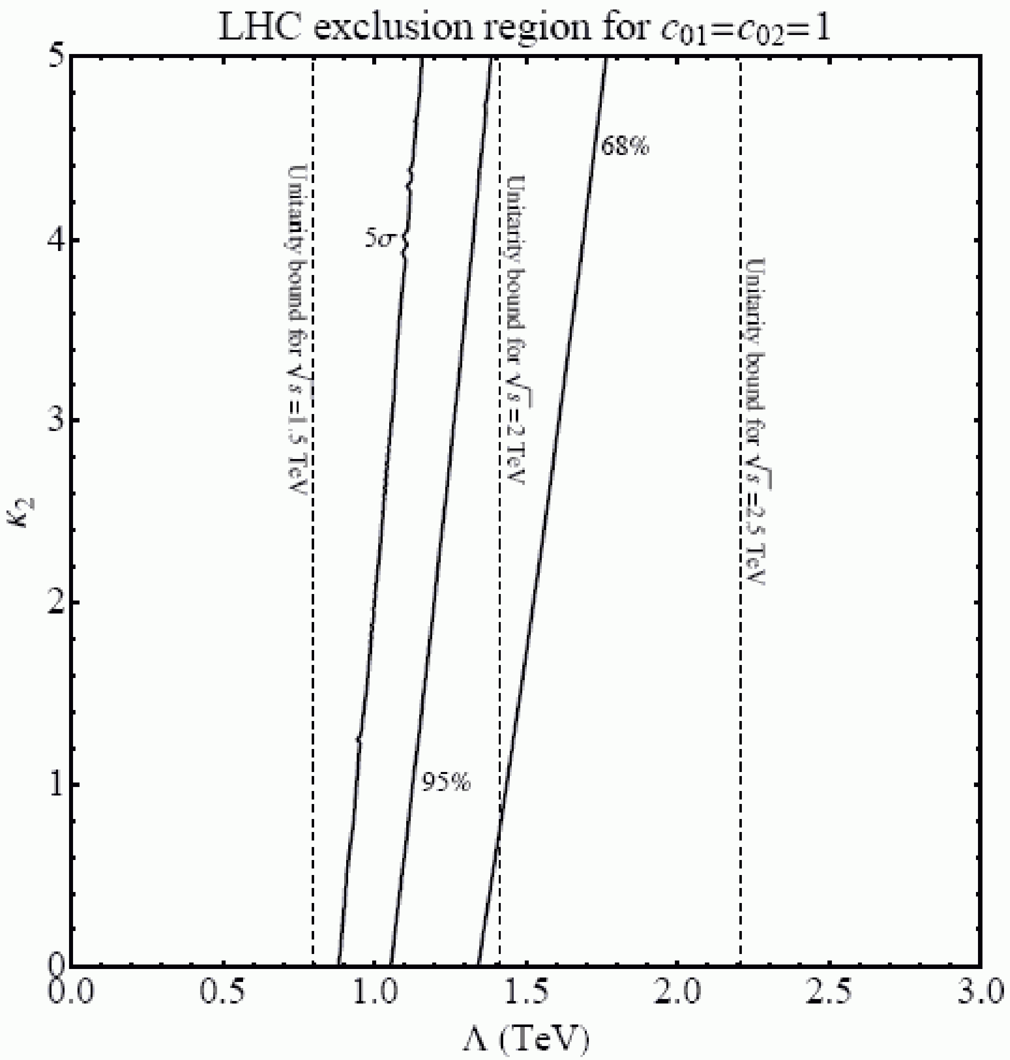


Observables summed over all final state helicities.

$\sqrt{s} = 200$ GeV
$L = 700\ pb^{-1}$

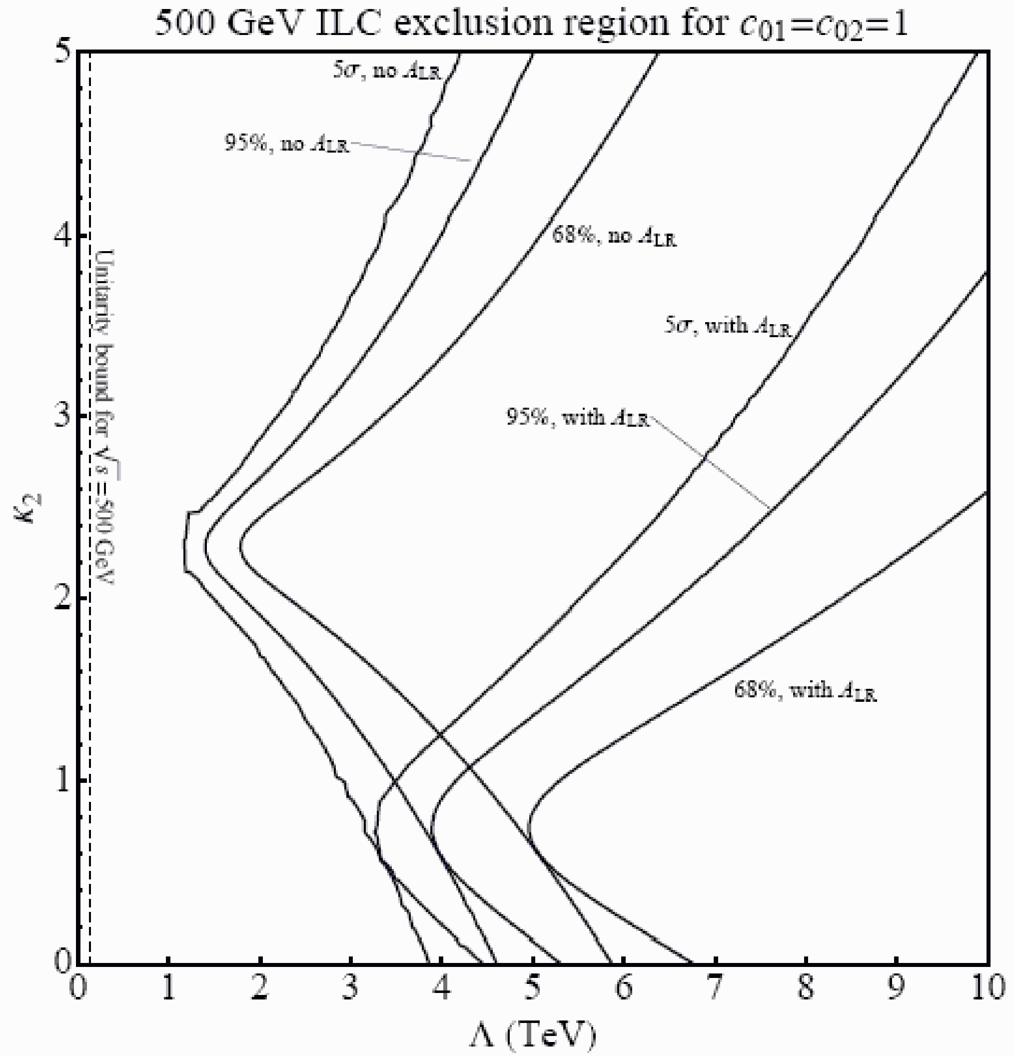
All values of κ_2

LHC search reach.



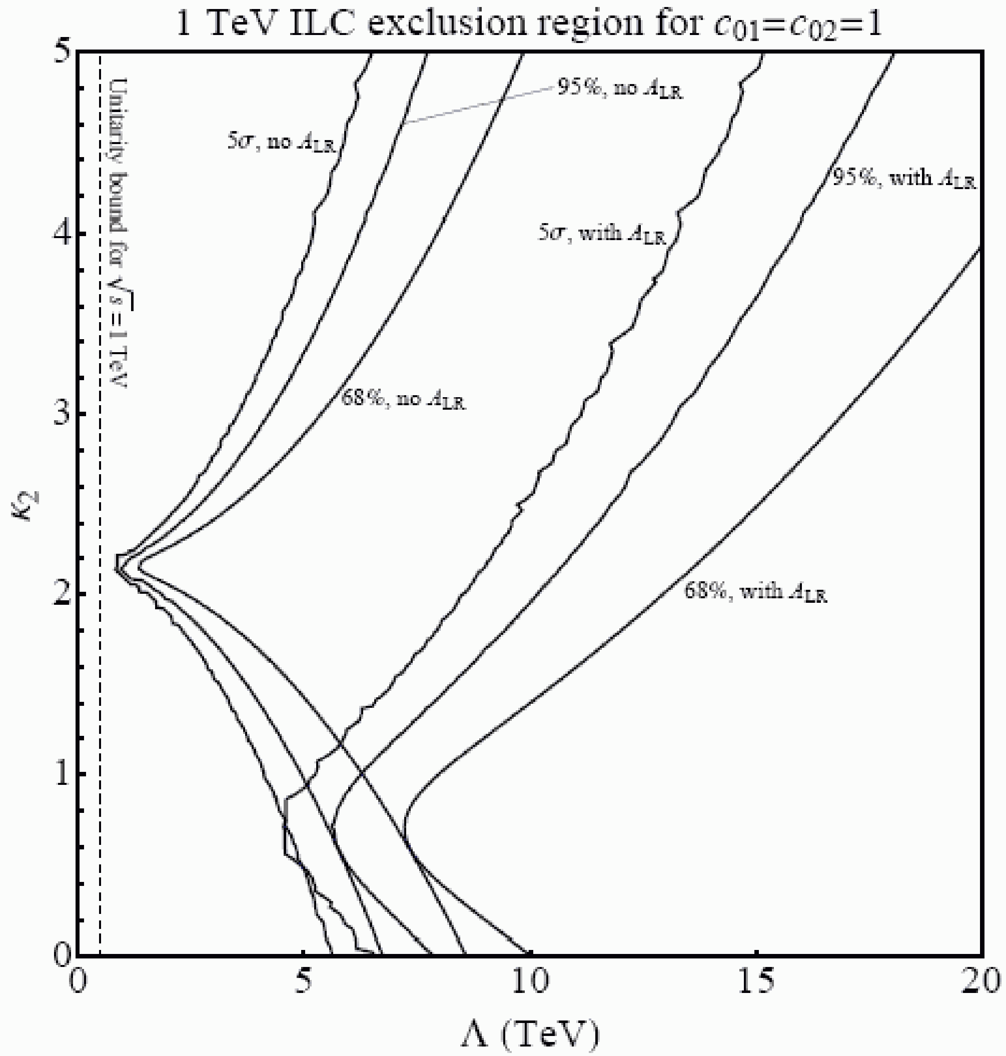
Using $\mathcal{L} = 100\ \text{fb}^{-1}$, and multiplying by branching ratio to semileptonic final states. Observables summed over all W helicities.

500 GeV ILC search reach.



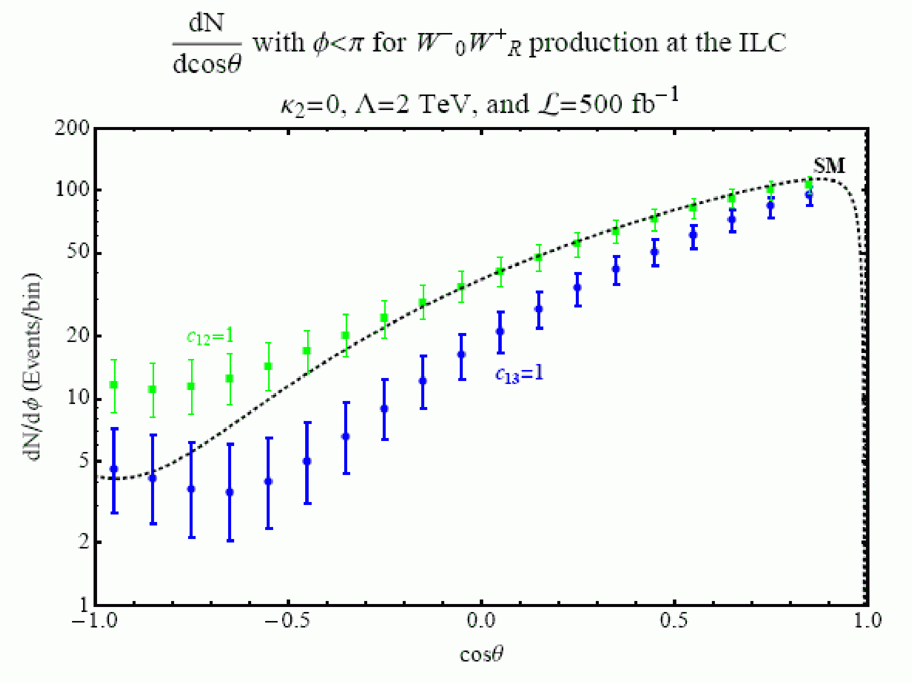
Using $\mathcal{L} = 500\ \mathrm{fb}^{-1}$ and $P_{e^-} = 0.9$, $P_{e^+} = 0.6$, $\Delta P/P = 0.25\%$. Observables summed over all final state helicities.

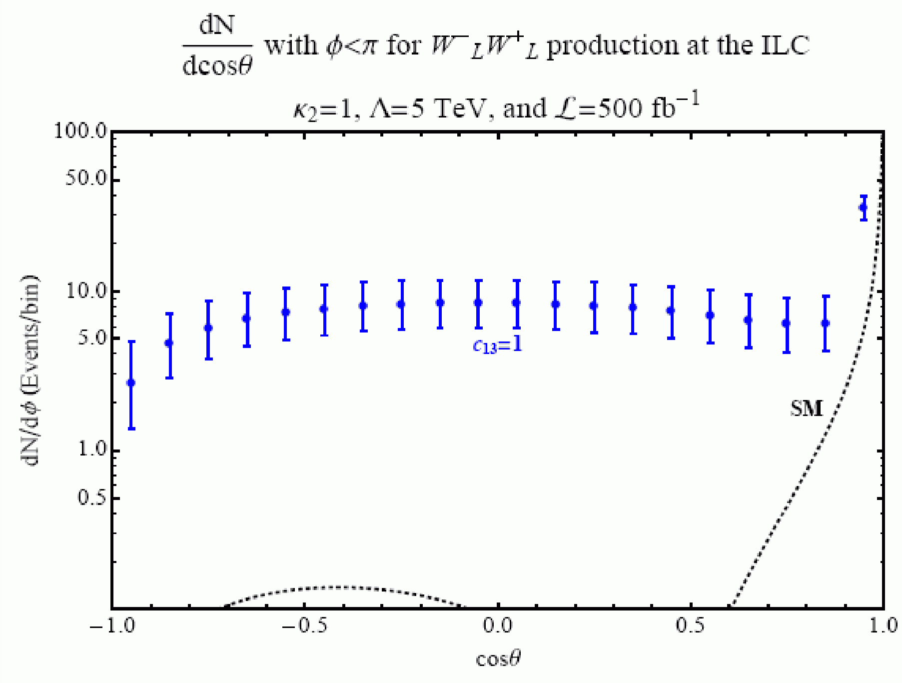
1 TeV ILC search reach.



Using $\mathcal{L} = 500\ \text{fb}^{-1}$ and $P_{e^-} = 0.9$, $P_{e^+} = 0.6$, $\Delta P/P = 0.25\%$. Observables summed over all final state helicities.

Polarized W's at 1 TeV ILC.





Comparison to Other Studies

- OPAL collaboration hep-ex/0303035: $e^+e^- \to \gamma\gamma$ gives $\Lambda > 141\ \mathrm{GeV}$.
- Alboteanu, Ohl, and Rückl, hep-ph/0608155: $pp \to Z\gamma$ gives search reach to $\Lambda \sim 1\ \mathrm{TeV}$.
- Alboteanu, Ohl, and Rückl, arXiv:0708.2359:

$(K_{Z\gamma\gamma}, K_{ZZ\gamma})$	$\lvert\vec{E}\rvert^2 = 1, \vec{B} = 0$	$\vec{E} = 0, \lvert\vec{B}\rvert^2 = 1$
$K_0 \equiv (0,0)$	$\Lambda \gtrsim 2\,\mathrm{TeV}$	$\Lambda \gtrsim 0.4\,\mathrm{TeV}$
$K_1 \equiv (-0.333, 0.035)$	$\Lambda \gtrsim 5.9\,\mathrm{TeV}$	$\Lambda \gtrsim 0.9\,\mathrm{TeV}$
$K_5 \equiv (0.095, 0.155)$	$\Lambda \gtrsim 2.6\,\mathrm{TeV}$	$\Lambda \gtrsim 0.25\,\mathrm{TeV}$
$K_3 \equiv (-0.254, -0.048)$	$\Lambda \gtrsim 5.4\,\mathrm{TeV}$	$\Lambda \gtrsim 0.9\,\mathrm{TeV}$

Table: Bounds on Λ from $e^+e^- \to Z\gamma$ at the ILC, for the minimal (first row) and nonminimal NCSM

Summary.

- The NCSM is a model of noncommutativity that could be observed in future experiments.
- Unitarity and LEP constrain but don't rule out the NCSM.
- At the LHC, the search reach is not much better than the unitarity bound.
- The ILC can probe large regions of NCSM parameter space and is sensitive to all of the parameters through W polarization measurements.