Event Shapes at NLLA+NNLO

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17.11.2008

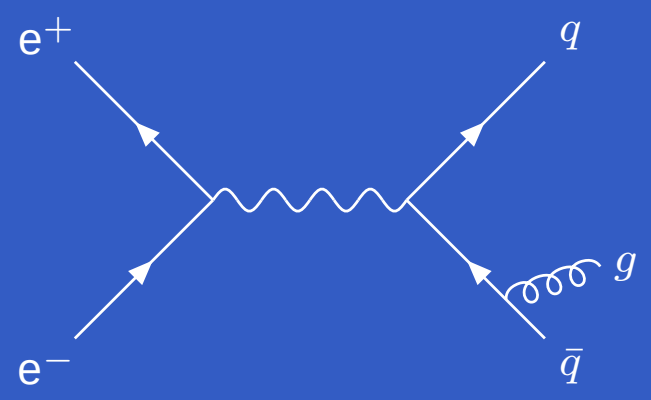
LCWS08

Outline

- Motivation
- Event shapes Observables
- Cross section calculations
  - Fixed order calculations
  - Resummed calculations
  - Matching
- Matched results
- Determination of α_S

Motivation

- $e^+e^- \to 3$ jet reaction: very prominent role for phenomenology:
  - discovery of gluon and its properties,
  - precise determination of the QCD coupling constant α_s.



- Instead of looking only at jet rates, we can study their shape in the final state: $\Longrightarrow$ EVENT SHAPE OBSERVABLES

Event shape observables

- Parametrize geometrical properties of energy-momentum flow,
- canonical example: Thrust, $T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$

- popular observables for testing QCD $\leftrightarrow$ IR & collinear safe,
- measured very precisely at LEP:
  - error in the determination of α_S mainly from theoretical uncertainty

$$\alpha_s(M_Z) = 0.1202 \pm 0.0003(\text{stat}) \pm 0.0009(\text{sys}) \pm 0.0013(\text{had}) \pm 0.0047(\text{scale})$$

[LEPQCDWG]

Event shape observables

Theoretical calculations:

- State-of-the-art one year ago:
  - NLO calculations, [Ellis, Ross, Terrano; Kunszt, Nason; Giele, Glover; Catani, Seymour.]
  - NLL resummation, [Catani, Trentadue, Turnock, Webber; Banfi, Salam, Zanderighi.]

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  - NLL resummation, [Catani, Trentadue, Turnock, Webber; Banfi, Salam, Zanderighi.]
- Very important progress in the last year
  - NNLO calculations and matching with NLLA of the LEP standard set of event shape observables,
    [Gehrmann, Gehrmann-De-Ridder, Glover, Heinrich; Gehrmann, G.L., Stenzel]
  - N^{3}LL resummation in SCET and matching with NNLO for T,
    [Schwartz; Becher, Schwartz]
  - Non-perturbative $1/Q$ corrections to NLLA+NNLO for T,
    [Davison, Webber]

Fixed Order Calculations

- For an observable y the differential cross section at NNLO is given by $\left(\bar{\alpha}_s = \frac{\alpha_s}{2\pi}\ , x_\mu = \frac{\mu}{Q}\right)$:

$$\frac{1}{\sigma_{\rm had}}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu) = \bar{\alpha}_s(\mu)\frac{\mathrm{d}\bar{A}}{\mathrm{d}y}(y) + \bar{\alpha}_s^2(\mu)\frac{\mathrm{d}\bar{B}}{\mathrm{d}y}(y,x_\mu) + \bar{\alpha}_s^3(\mu)\frac{\mathrm{d}\bar{C}}{\mathrm{d}y}(y,x_\mu) + \mathcal{O}(\bar{\alpha}_s^4)\,.$$

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LO	$\gamma^* \to q\bar{q}g$	**tree level**	**NNLO**	$\gamma^* \to q\bar{q}g$	**two loop**
				$\gamma^* \to q\bar{q}gg$	**one loop**
NLO	$\gamma^* \to q\bar{q}g$	**one loop**		$\gamma^* \to q\bar{q}q\bar{q}$	**one loop**
	$\gamma^* \to q\bar{q}gg$	**tree level**		$\gamma^* \to q\bar{q}q\bar{q}g$	**tree level**
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- Coefficient functions $\frac{d\bar{A}}{dy}$, $\frac{d\bar{B}}{dy}$, $\frac{d\bar{C}}{dy}$ are functions of $L \equiv \ln\frac{1}{y}$,
  - describe the enhancement due to soft and collinear emissions.

Fixed Order Calculations

- NLO and NNLO calculations: [Gehrmann, Gehrmann-De-Ridder, Glover, Heinrich]
  - careful subtraction of real and virtual divergencies using antenna method:

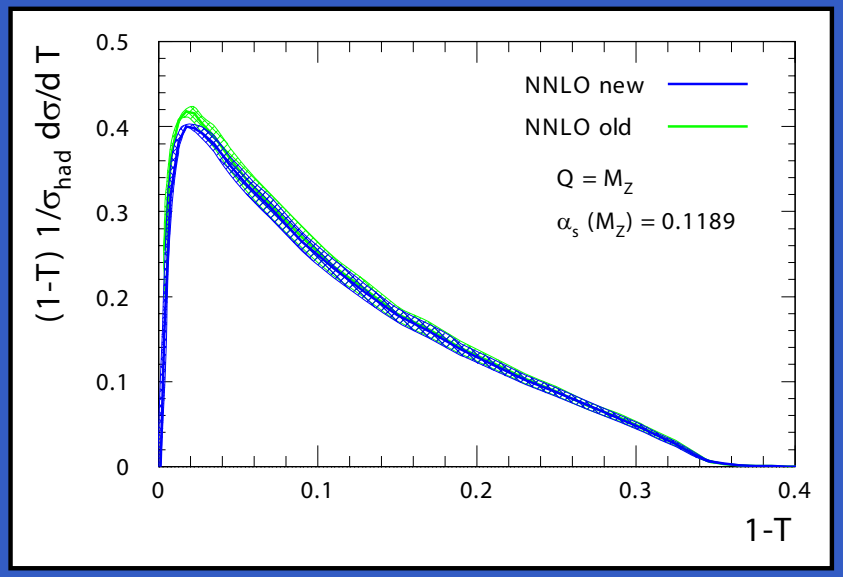
$$
\begin{aligned}
d\sigma_{\mathrm{NLO}} &= \int_{d\Phi_{m+1}} \left(d\sigma^{\mathrm{R}}_{\mathrm{NLO}} - d\sigma^{\mathrm{S}}_{\mathrm{NLO}} \right) + \left[\int_{d\Phi_{m+1}} d\sigma^{\mathrm{S}}_{\mathrm{NLO}} + \int_{d\Phi_{m}} d\sigma^{V}_{\mathrm{NLO}} \right] \\
d\sigma_{\mathrm{NNLO}} &= \int_{d\Phi_{m+2}} \left(d\sigma^{\mathrm{R}}_{\mathrm{NNLO}} - d\sigma^{\mathrm{S}}_{\mathrm{NNLO}} \right) + \int_{d\Phi_{m+2}} d\sigma^{\mathrm{S}}_{\mathrm{NNLO}} \\
&\quad + \int_{d\Phi_{m+1}} \left(d\sigma^{\mathrm{V},1}_{\mathrm{NNLO}} - d\sigma^{\mathrm{VS},1}_{\mathrm{NNLO}} \right) + \int_{d\Phi_{m+1}} d\sigma^{\mathrm{VS},1}_{\mathrm{NNLO}} \\
&\quad + \int_{d\Phi_{m}} d\sigma^{\mathrm{V},2}_{\mathrm{NNLO}} .
\end{aligned}
$$

  - Implemented in the EERAD3 integration programme.

Fixed Order Calculations

- Inconsistency in the treatment of large-angle radiation, [Weinzierl]
  - inconsistency was corrected and cross-checks are in progress,
  - numerically minor changes in the kinematical region of interest for phenomenology.

PRELIMINARY



Fixed Order Calculations

- Consider cumulative cross section $R(y,Q,\mu) \equiv \frac{1}{\sigma_{\rm had}} \int_0^y \frac{d\sigma(x,Q,\mu)}{dx} dx$,

$$R(y,Q,\mu) = 1 + \mathcal{A}(y)\,\bar{\alpha}_s(\mu) + \mathcal{B}(y,x_\mu)\,\bar{\alpha}_s^2(\mu) + \mathcal{C}(y,x_\mu)\,\bar{\alpha}_s^3(\mu).$$

$\bar{\alpha}_s\mathcal{A}(y)$	$\bar{\alpha}_s L$	$\bar{\alpha}_s L^2$				
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Contribution becomes smaller

- If L is NOT large, contributions become smaller line-by-line.
- In phase space region where $y \to 0$, $L \to \infty$:

$\longrightarrow$ coefficient functions become large spoiling the convergence of the series expansion.

- Main contribution comes from highest power of the logarithms.

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Need RESUMMATION!

Resummed Calculations

- Idea: resum the highest powers of the logarithms to all orders in perturbation theory

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- Leading logarithms
- Next-to-Leading logarithms
- From trivial exponentiation

Resummed Calculations

- For suitable observables, resummation of logarithms leads to exponentiation

$$\Sigma(y) = e^{L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \ldots}$$

with

$$L\, g_1(\alpha_s L) = G_{12} L^2 \bar{\alpha}_s + G_{23} L^3 \bar{\alpha}_s^2 + G_{34} L^4 \bar{\alpha}_s^3 + \ldots \quad \text{(LL)}$$

$$g_2(\alpha_s L) = G_{11} L \bar{\alpha}_s + G_{22} L^2 \bar{\alpha}_s^2 + G_{33} L^3 \bar{\alpha}_s^3 + \ldots \quad \text{(NLL)}$$

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- Integrated cross section at NLLA to be matched with NNLO:

$$\begin{aligned} R(y) &= \left(1 + C_1 \alpha_s + C_2 \alpha_s^2 + C_3 \alpha_s^3\right) \times \\ &\quad e^{L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \bar{\alpha}_s^2 G_{21} L + \bar{\alpha}_s^3 G_{32} L^2 + \bar{\alpha}_s^3 G_{31} L} + D(y) \\ &= \underbrace{C(\alpha_s)\, \Sigma(y)}_{\text{logarithmic part}} + \underbrace{D(y)}_{\text{remainder function: } \longrightarrow 0 \text{ as } y \longrightarrow 0} \end{aligned}$$

C_1, C_2, C_3, G_{21}, G_{32}, G_{31}, $D(y)$: to be determined by matching with fixed order.

Matching

- Different matching schemes
  - R-matching scheme:
    - Two predictions for $R(y)$ are matched and double-counting terms are subtracted.
    - Unknown matching coefficients C_1, C_2, C_3, G_{21}, G_{32}, G_{31} numerically determined from fixed order result.

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  - Log(R)-matching scheme:
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Log-R matching scheme

- To NLLA + NNLO the integrated cross section in the Log-R matching scheme is given by

$$
\begin{aligned}
\ln\left(R\left(y,\alpha_S\right)\right) \quad = \quad & L\, g_1\left(\alpha_s L\right) + g_2\left(\alpha_s L\right) \\
& + \bar{\alpha}_S\left(\mathcal{A}\left(y\right) - G_{11}L - G_{12}L^2\right) + \\
& + \bar{\alpha}_S^2\left(\mathcal{B}\left(y\right) - \frac{1}{2}\mathcal{A}^2\left(y\right) - G_{22}L^2 - G_{23}L^3\right) \\
& + \bar{\alpha}_S^3\left(\mathcal{C}\left(y\right) - \mathcal{A}\left(y\right)\mathcal{B}\left(y\right) + \frac{1}{3}\mathcal{A}^3\left(y\right) - G_{33}L^3 - G_{34}L^4\right) \; .
\end{aligned}
$$

- fixed order
- resummation

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  - fixed order
  - resummation

- To ensure the vanishing of the matched expression at the kinematical boundary $y_{\max}$

$$L \longrightarrow \tilde{L} = \frac{1}{p}\ln\left(\left(\frac{y_0}{y\,x_L}\right)^p - \left(\frac{y_0}{y_{\max}\,x_L}\right)^p + 1\right),$$

with $y_0 = 6$ for $y = C$ and $y_0 = 1$ otherwise, ($x_L = p = 1$).

[Ford, Jones, Salam, Stenzel, Wicke.]

Renormalization scale dependence

- The full renormalization scale dependence is given by making the following replacements,

$$\alpha_s \quad \rightarrow \quad \alpha_s(\mu) \ ,$$

$$\mathcal{B}(y) \quad \rightarrow \quad \mathcal{B}(y,\mu) = 2\,\beta_0\,\ln x_\mu\,\mathcal{A}(y) + \mathcal{B}(y) \ ,$$

$$\mathcal{C}(y) \quad \rightarrow \quad \mathcal{C}(y,\mu) = (2\,\beta_0\,\ln x_\mu)^2\,\mathcal{A}(y) + 2\,\ln x_\mu\,\left[2\,\beta_0\mathcal{B}(y) + 2\,\beta_1\,\mathcal{A}(y)\right] + \mathcal{C}(y) \ ,$$

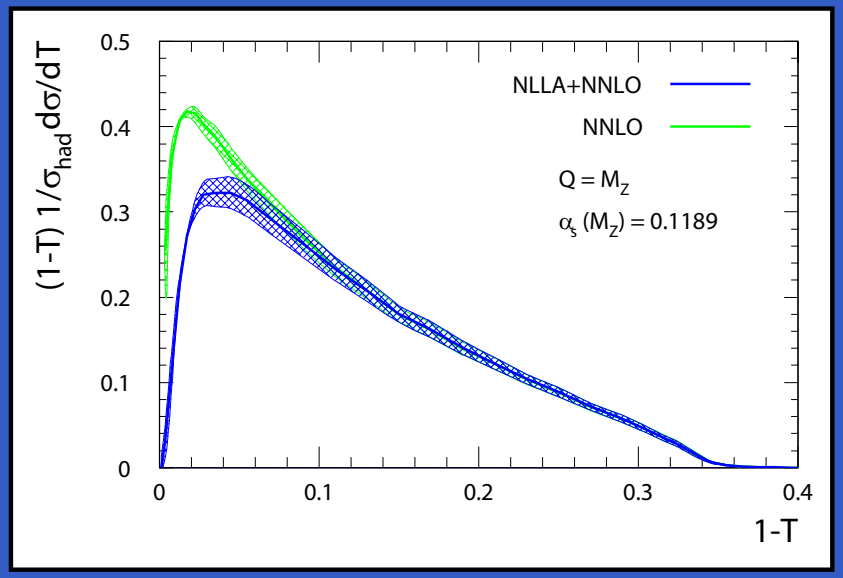
$$g_2(\alpha_S L) \quad \rightarrow \quad g_2\left(\alpha_S L, \mu^2\right) = g_2(\alpha_S L) + \frac{\beta_0}{\pi}(\alpha_S L)^2\,g_1'(\alpha_S L)\,\ln x_\mu \ ,$$

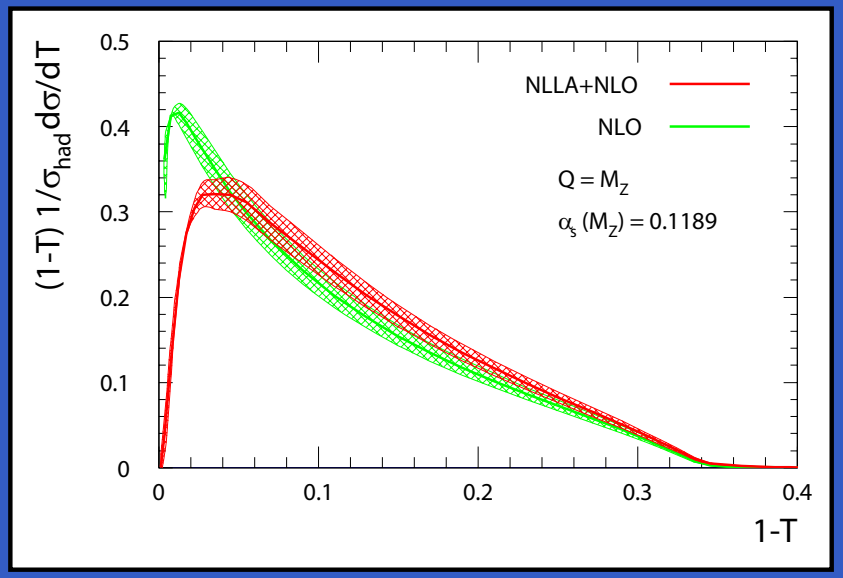
$$G_{22} \quad \rightarrow \quad G_{22}(\mu) = G_{22} + 2\beta_0 G_{12}\ln x_\mu \ ,$$

$$G_{33} \quad \rightarrow \quad G_{33}(\mu) = G_{33} + 4\beta_0 G_{23}\ln x_\mu \ .$$

Results: renormalization scale dependence

- Thrust T: consider $\tau = 1 - T$

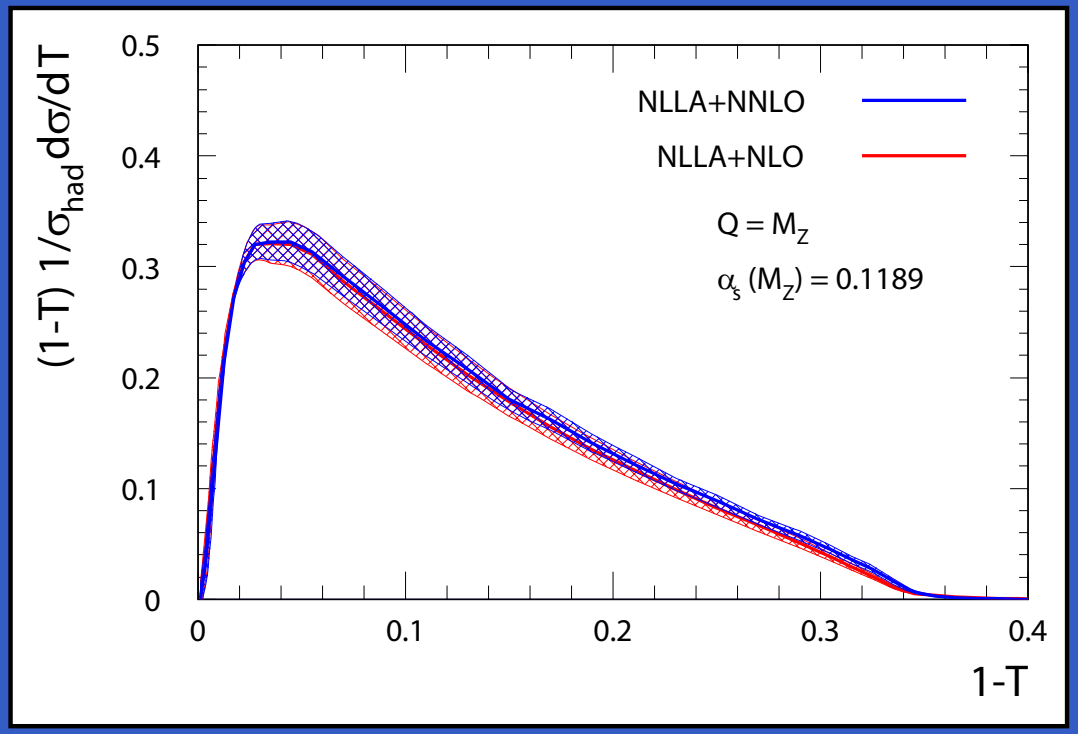




- Difference between NLLA+NNLO and NNLO restricted to the two-jet region, whereas NLLA+NLO differ in normalisation throughout the full kinematical range.

Results: renormalization scale dependence

- Thrust T: consider $\tau = 1 - T$



- Difference between NLLA+NNLO and NLLA+NLO moderate in the three-jet region.
- Renormalization scale dependence reduced in three-jet region.

Results: renormalization scale dependence

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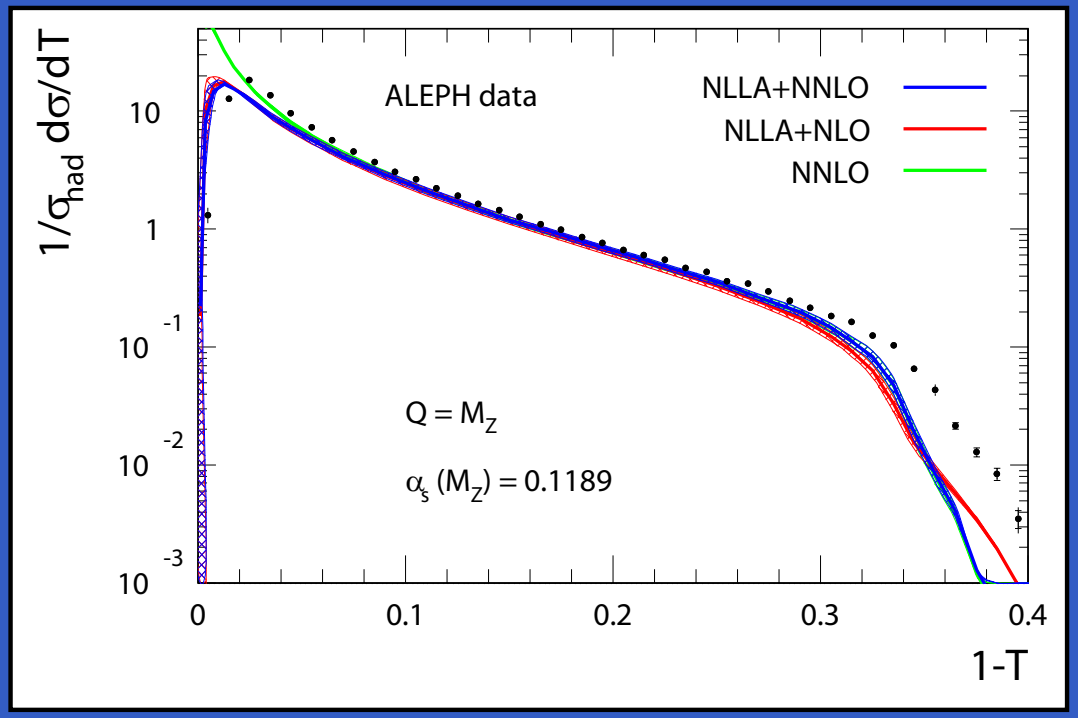

- Description of the hadron-level data improves between parton-level NLLA+NLO and parton-level NLLA+NNLO, especially in the three-jet region.

Results: renormalization scale dependence

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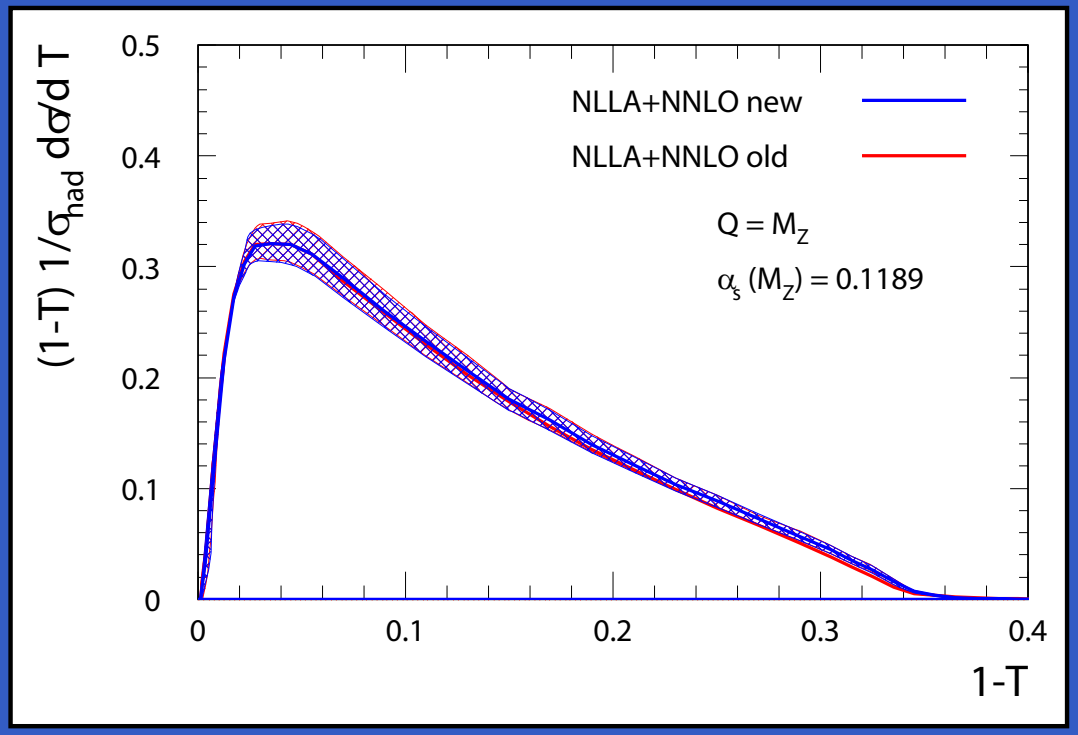
PRELIMINARY



- Comparison between matched results using old and corrected new histograms:

  the small difference in the IR region disappears, resummation becomes dominant.

Results

- Full set of event shape variables:
  - Heavy jet mass: $\rho = \frac{M_H^2}{s} = \max_i \frac{1}{E_{\text{vis}}} \left(\sum_{k \in H_i} p_k \right)^2$
  - C-parameter: $\Theta^{\alpha\beta} = \frac{1}{\sum_k |\vec{p}_k|} \frac{\sum_k p_k^\alpha p_k^\beta}{\sum_k |\vec{p}_k|}$,

$$C = 3 \left(\Theta^{11}\Theta^{22} + \Theta^{22}\Theta^{33} + \Theta^{33}\Theta^{22} - \Theta^{12}\Theta^{12} - \Theta^{23}\Theta^{23} - \Theta^{31}\Theta^{31} \right)$$

  - Total jet broadening: $B_T = B_1 + B_2$
  - Wide jet broadening: $B_W = \max(B_1, B_2)$,

$$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_k |\vec{p}_k|}$$

  - Two-to-three jet parameter for Durham jet algorithm:

$$y_{ij,D} = \frac{2 \min \left(E_i^2, E_j^2 \right) (1 - \cos\theta_{ij})}{E_{\text{vis}}^2}$$

Determination of α_S

- Recent works:
  - α_S fit using only theoretical NNLO predictions and ALEPH data,

    [Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Stenzel.]
  - α_S fit using theoretical NNLO and NLLA+NNLO predictions and JADE data,

    [Bethke, Kluth, Pahl, Schieck and JADE Collaboration.]

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- Work still in progress:
  - α_S fit using the the matched NLLA+NNLO predictions and ALEPH data.

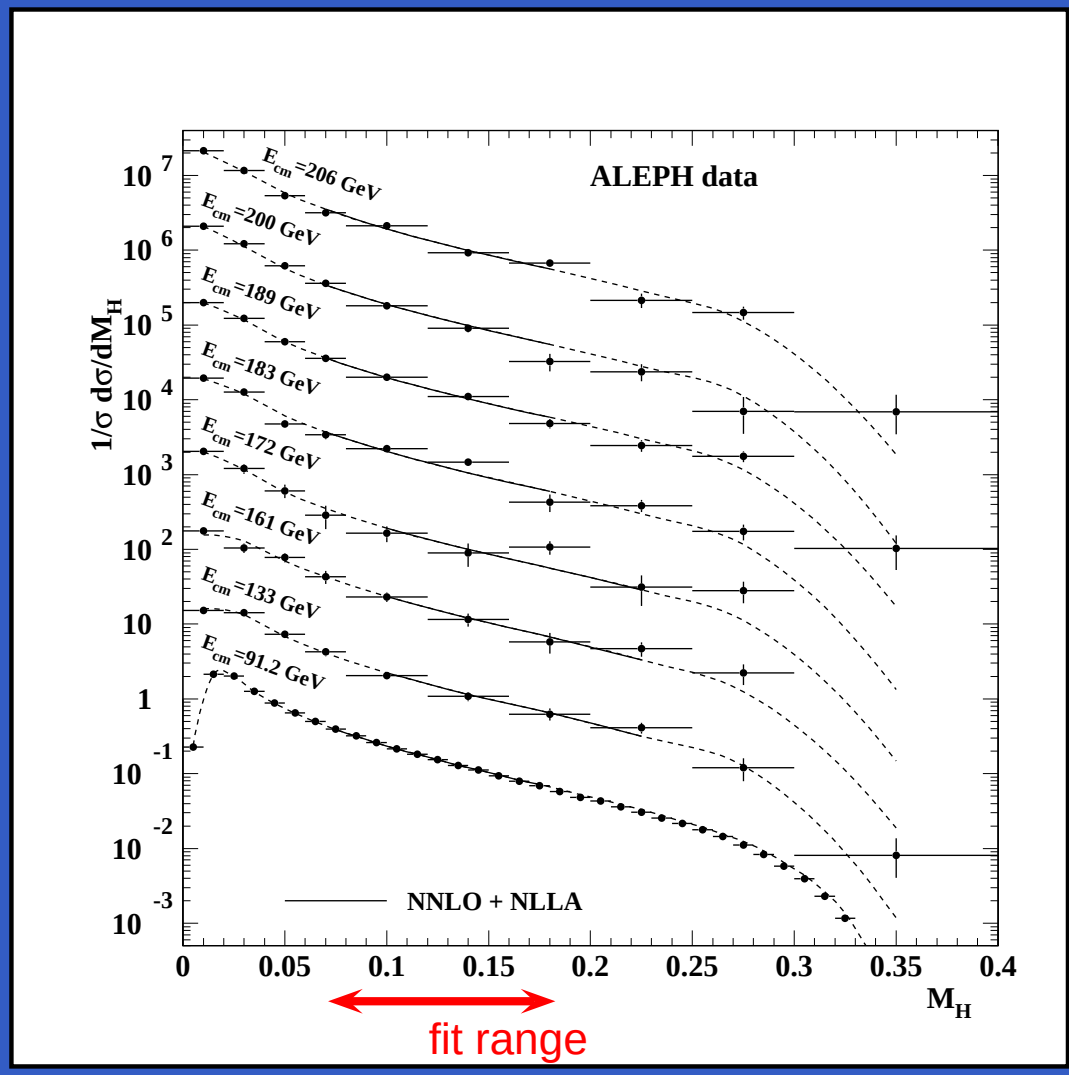
Determination of α_S

- Analysis outline
  - use public ALEPH data on event shapes, [Heister et al.]
  - data are corrected to hadron level using MC corrections accounting for ISR/FSR and background,
  - data are fitted by NNLO or NLLA+NNLO predictions, including NLO quark mass correction, folded to hadron level by MC generators,
  - combine 6 variables and 8 data sets (LEPI + LEPII)

Determination of α_S: NLLA+NNLO fits to data

PRELIMINARY

- data are fit in the central part of the event shape distribution,
- only statistical uncertainties are included in the χ^2.



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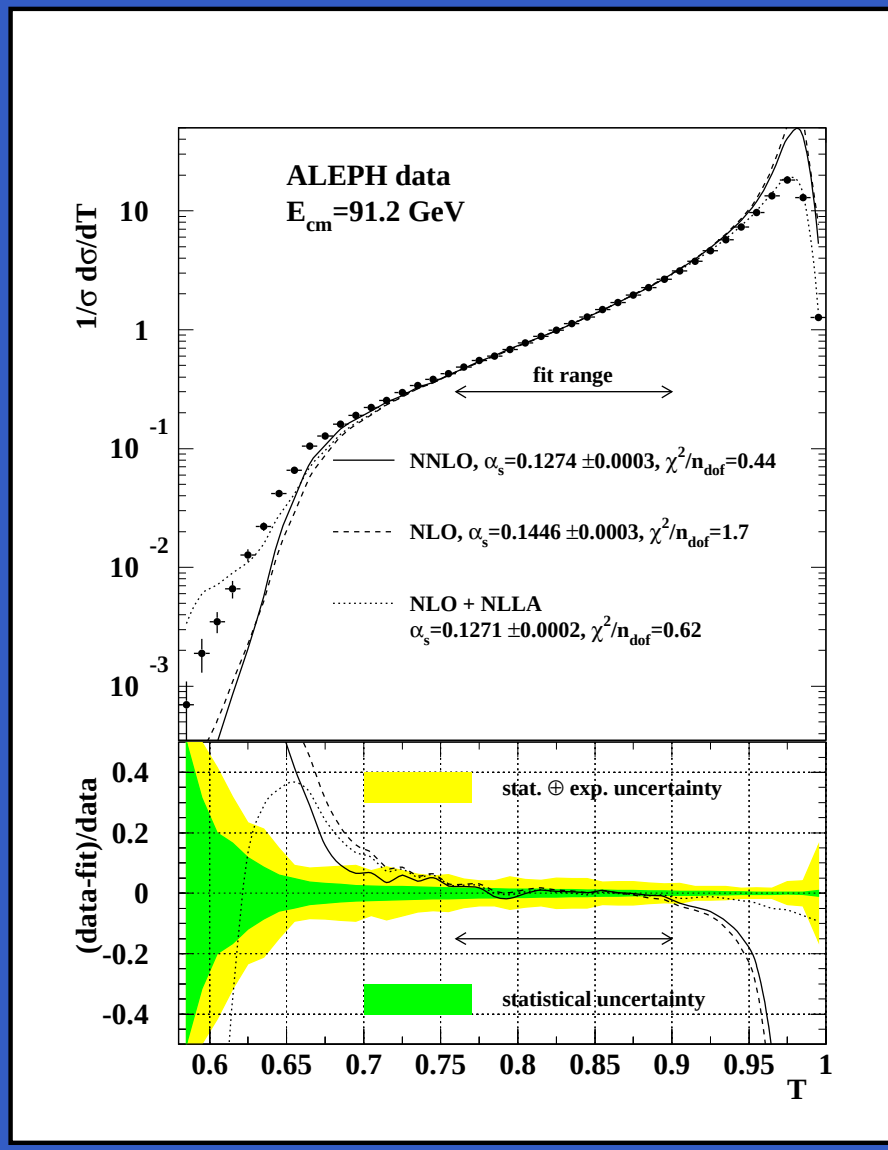
- NNLO vs NLLA+NLO
  - clear improvement of NNLO over NLO,
  - good fit quality (but includes still large statistical uncertainties of C-coefficient),
  - matched NLLA+NLO still yields a better prediction in the 2-jet region,



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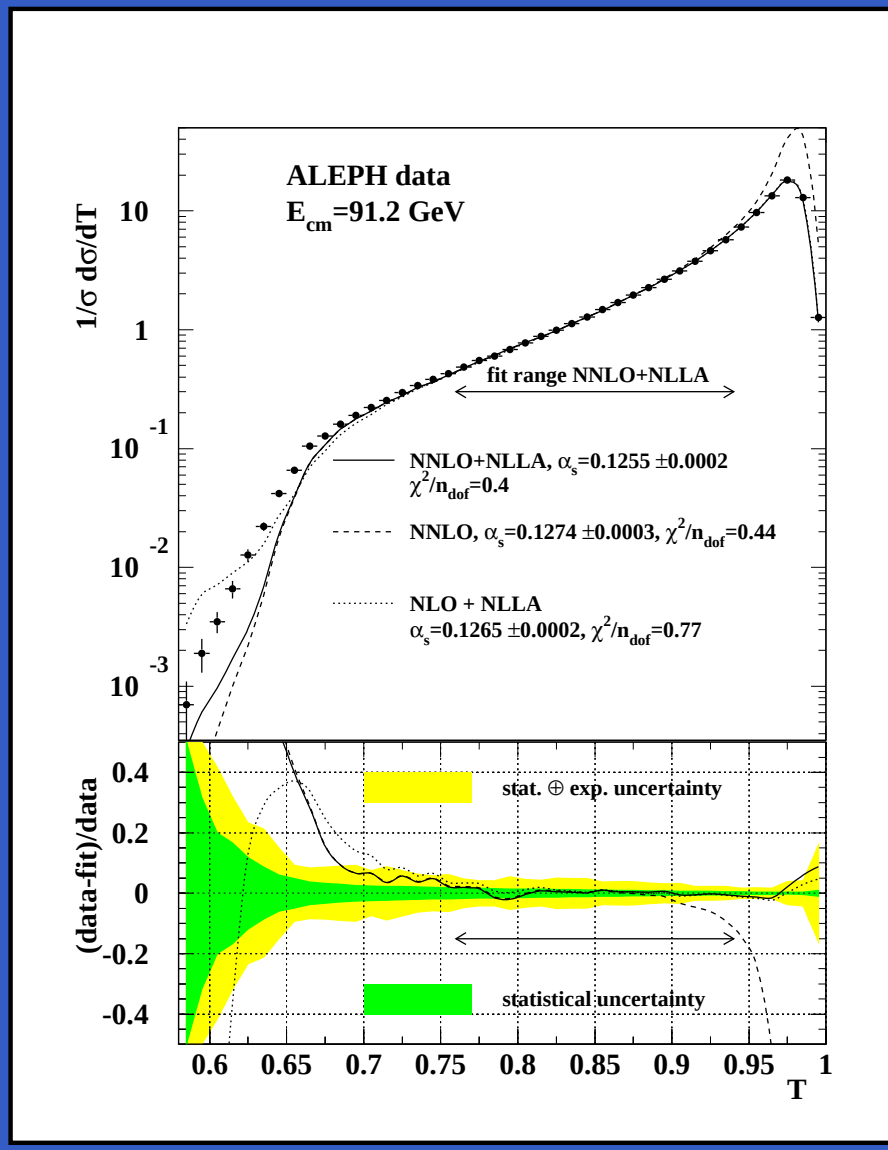
- NLLA+NNLO vs NNLO
  - better predictions in two jet region,
  - extended fit range,
- fit to fixed order calculations gives higher values for α_S,
- tendency to decrease from NLO to NNLO.



Determination of α_S: perturbative uncertainty

- Uncertainty band method to estimate missing higher orders

  [Ford, Jones, Salam, Stenzel, Wicke.]

  - evaluate distribution of event shape y for a given value of α_S with a reference theory,
  - calculate theoretical uncertainties for $y \longrightarrow$ uncertainty band,
  - fill the uncertainty band with the nominal prediction by varying α_s,
  - corresponding variation range for α_S is assigned as systematic uncertainty.
- Parameter taken into account
  - for NNLO fit: only x_μ variation,
  - for NLLA+NNLO fit: variation of x_μ, x_L, y_{max}, p and matching scheme.

Determi… uncertainty

PRELIMINARY

- Uncertaint… rders

  [Ford, Jones, Salam,…

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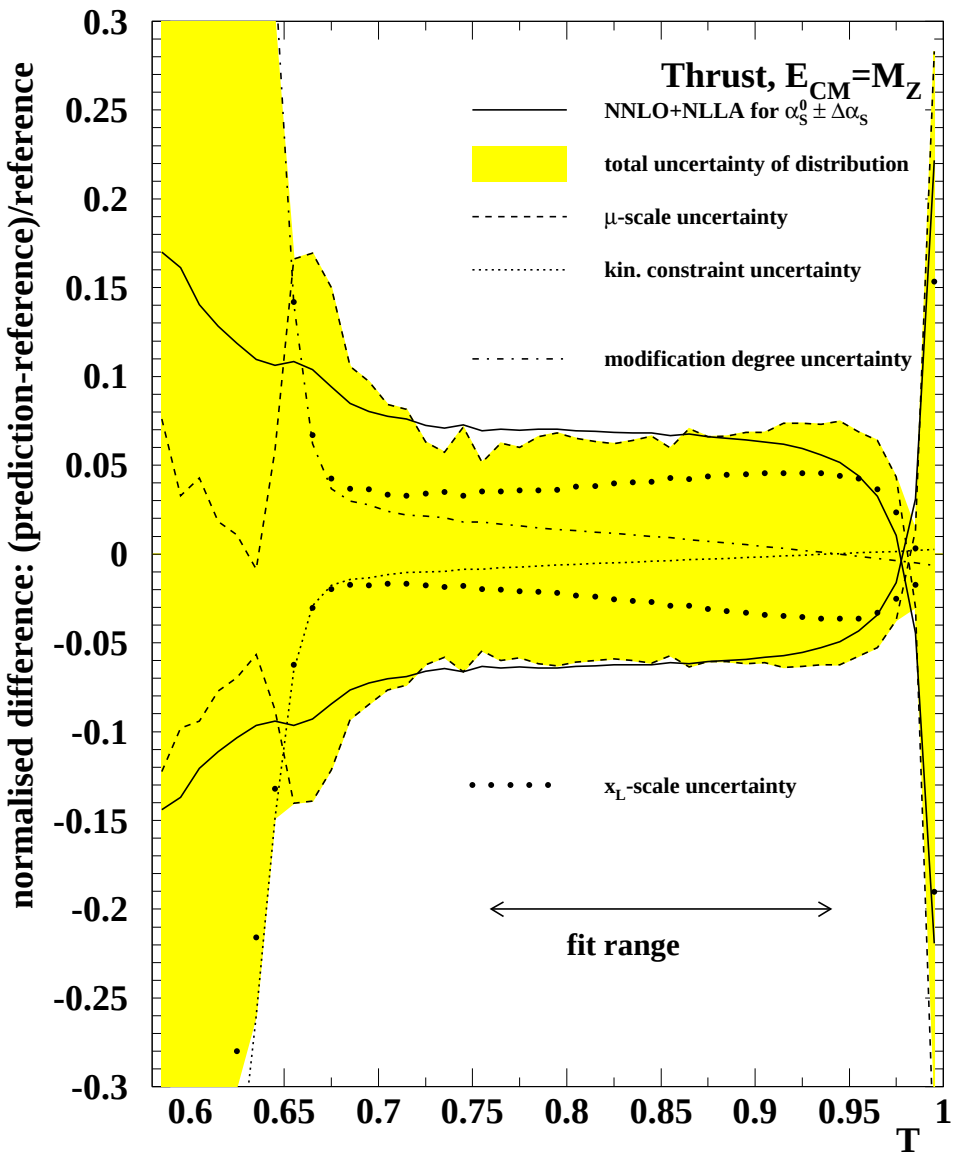
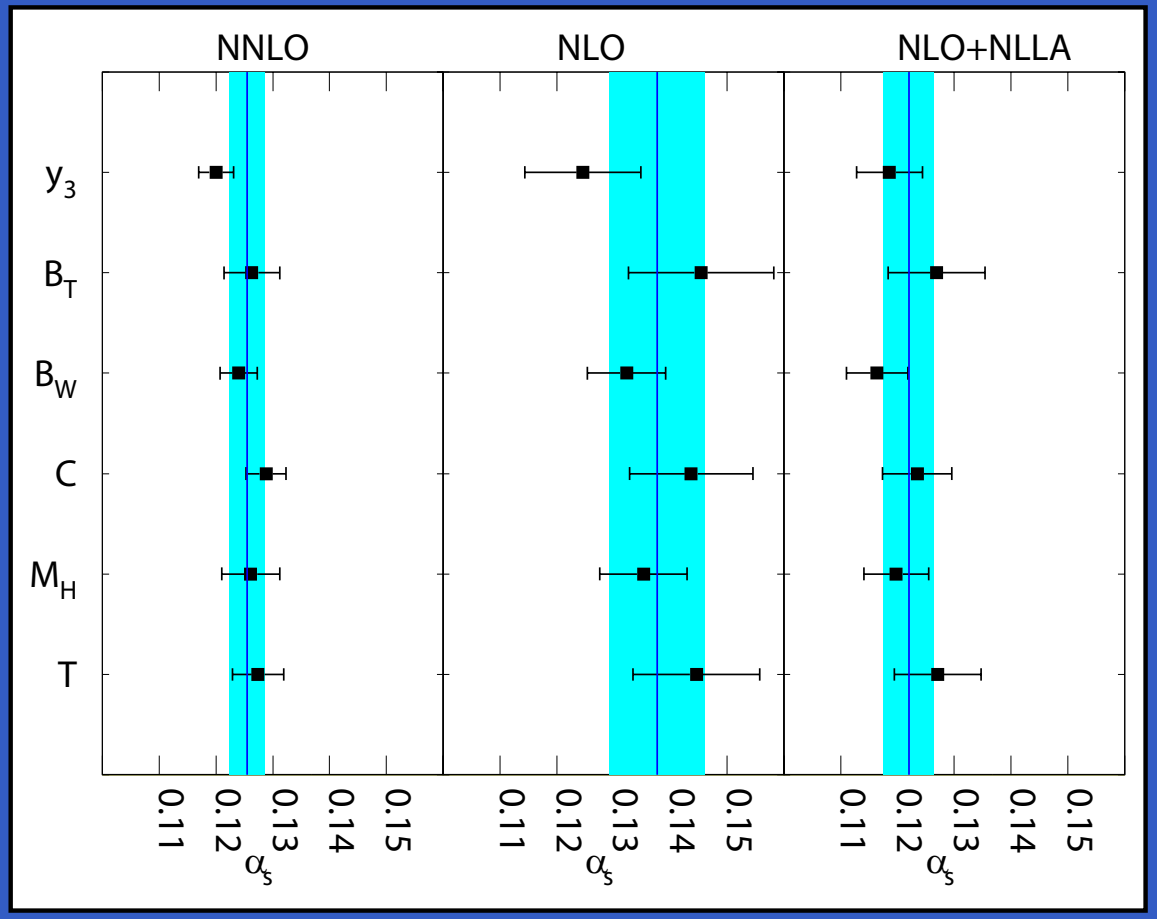


Determination of α_S: NNLO results

- $\alpha_S\,(M_Z)$

- consistent results at NNLO,
- scattering between variables much reduced.
- calculate weighted average for $\alpha_S\,(Q)$ from 6 variables

$$\bar{\alpha}_S = \sum_{i=1}^{6} w_i\,\alpha_S^i, \qquad w_i \propto \frac{1}{\sigma_i^2}$$

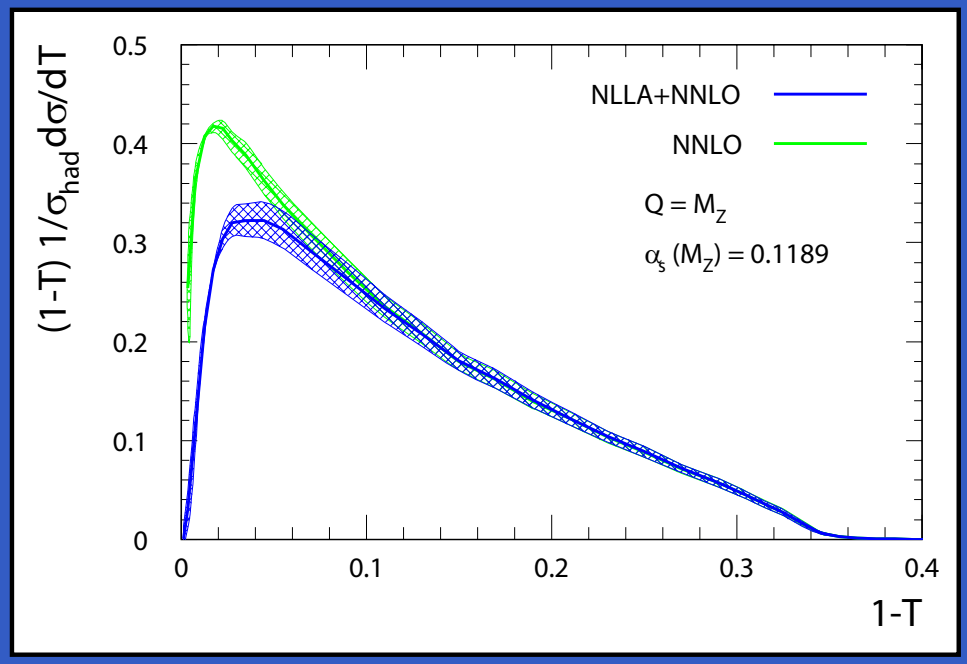


$$\Rightarrow \quad \bar{\alpha}_S\,(M_Z) \;=\; 0.1240 \pm 0.0033$$

Determination of α_S: NLLA+NNLO results

PRELIMINARY

- Not yet, but still some hints:
  - improvement from NNLO to NLLA+NNLO smaller than from NLO to NLLA+NLO: two loop running terms not compensated in NLLA.



- Tendency confirmed by JADE Collaboration analysis.

Conclusions and outlook

- Matched NLLA+NNLO distributions in the log-R matching scheme,
  - NLLA+NNLO results improved wrt. NLLA+NLO especially in 3-jet region,
  - in 2-jet region better prediction than NNLO, but far from perfect.

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- Further steps and improvements:
  - combinations of α_S measurements using NLLA+NNLO, [work in progress]
  - resummation of subleading logarithms (for all observables),
  - inclusion of EW-corrections and hadronization corrections from modern NLO+PS MC.