Estimate of the BPM resolution for FONT at ATF2 by means of tracking simulations.

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1 The three BPM method

Let's estimate the resolution of the three BPMs used for the FONT system at ATF2, which have been allocated in a dispersion-free section of the extraction line. From the transverse beam position at two BPMs we can predict the transverse beam position at the third one. For instance, let s_1, s_2 and s_3 be the position of the three FONT BPMs or pick ups, such as it is shown in schematic Fig. 1.

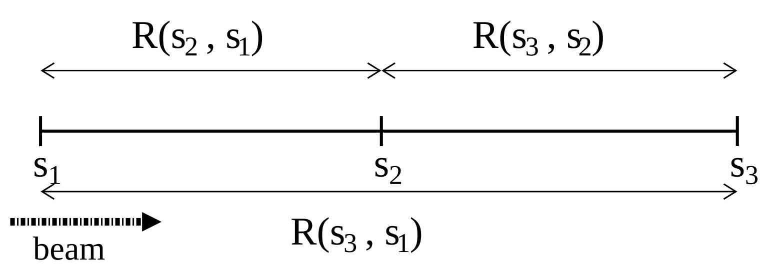

Figure 1: Schematic of three arbitrary BPM positions

If the elements of the transfer matrix in between the BPMs are known, then we can easily define the following linear system of equations:

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$$
\begin{aligned}
y_2 &= R_{33}(s_2,s_1)y_1 + R_{34}(s_2,s_1)y_1' \ , && (1)\\
y_3 &= R_{33}(s_3,s_2)y_2 + R_{34}(s_3,s_2)y_2' \ , && (2)\\
y_3 &= R_{33}(s_3,s_1)y_1 + R_{34}(s_3,s_1)y_1' \ , && (3)
\end{aligned}
$$

where $R_{33}(s_i,s_j)$ and $R_{34}(s_i,s_j)$ denote the transfer matrix elements between positions s_i and s_j. Here we have assumed linear transport.
For example, using (1) and (3) one obtains the beam coordinate y_3 in terms of y_1 and y_2:

$$
y_3 = \left(R_{33}(s_3,s_1) - \frac{R_{34}(s_3,s_1)R_{33}(s_2,s_1)}{R_{34}(s_2,s_1)}\right) y_1 + \frac{R_{34}(s_3,s_1)}{R_{34}(s_2,s_1)} y_2 \ , \tag{4}
$$

Similarly, using the Eqs. (1) and (3), we can obtain y_2 in terms of y_1 and y_3:

$$
y_2 = \left(R_{33}(s_2,s_1) - \frac{R_{34}(s_2,s_1)R_{33}(s_3,s_1)}{R_{34}(s_3,s_1)}\right) y_1 + \frac{R_{34}(s_2,s_1)}{R_{34}(s_3,s_1)} y_3 \ , \tag{5}
$$

By symmetry, in order to predict the coordinate y_1 from y_2 and y_3, we can consider the beam transport from the right to the left of the Fig. 1. In this case we can then write:

$$
y_1 = \left(R_{33}(s_1,s_3) - \frac{R_{34}(s_1,s_3)R_{33}(s_2,s_3)}{R_{34}(s_2,s_3)}\right) y_3 + \frac{R_{34}(s_1,s_3)}{R_{34}(s_2,s_3)} y_2 \ , \tag{6}
$$

where we have to take into account that $\mathbf{R}(s_1,s_3) \equiv \mathbf{R}(s_3,s_1)^{-1}$, $\mathbf{R}(s_2,s_3) \equiv \mathbf{R}(s_3,s_2)^{-1}$ and $\mathbf{R}(s_1,s_2) \equiv \mathbf{R}(s_2,s_1)^{-1}$. In this case the corresponding inverse transfer matrix is:

$$
\mathbf{R}^{-1} = \frac{1}{\det\{\mathbf{R}\}} \begin{pmatrix} R_{44} & -R_{34} \\ -R_{43} & R_{33} \end{pmatrix} \ . \tag{7}
$$

Note that $R_{33}(s_1,s_3) = R_{44}(s_3,s_1)$, $R_{34}(s_1,s_3) = -R_{34}(s_3,s_1)$, $R_{43}(s_1,s_3) = -R_{43}(s_3,s_1)$ and $R_{44}(S_1,s_3) = R_{33}(s_3,s_1)$. The determinant of the optics matrix $\mathbf{R}$ fulfils the condition $\det\{\mathbf{R}\} = 1$.
We can rewrite Eqs. (4), (5) and (6) as follows:

$$
y_i = \left(R_{33}(s_i,s_j) - \frac{R_{34}(s_i,s_j)R_{33}(s_k,s_j)}{R_{34}(s_k,s_j)}\right) y_j + \frac{R_{34}(s_i,s_j)}{R_{34}(s_k,s_j)} y_k \ , \tag{8}
$$

which is an expression valid for 3 BPMs at arbitrary different positions s_i, s_j and s_k.
For the particular case of a drift in between the BPMs, the transfer matrix elements are: $R_{33} = 1$, $R_{34} = L$, $R_{43} = 0$ and $R_{44} = 1$, where L is the length of the drift. Therefore, in this case the Eq. (8) can be rewritten as:

$$
y_i = \frac{L(s_i,s_j)}{L(s_k,s_j)} y_k - \frac{L(s_i,s_k)}{L_(s_k,s_j)} y_j \ , \tag{9}
$$

where $L(s_i,s_j)$ denotes the length of the drift in between the positions s_j and s_i.

The code Placet-octave allow us to obtain the first order transfer matrix elements in a transport line. Using Eq. (8) for the three BPM method we can estimate the resolution of the FONT BPMs at ATF2. The definition of the position resolution is the rms of the residual between the measured and predicted beam position at BPM Pi,

$$\sigma_{\text{reso},i} = \sqrt{\langle (y_{i,\text{measured}} - y_{i,\text{predicted}})^2 \rangle} \ . \qquad (10)$$

We have obtained $\sigma_{\text{reso},i}$ from a Gaussian fit to the residual distribution. Simulation results are shown in Fig. 2 for P1, Fig. 3 for P2 and Fig. 4 for P3. At this point it is necessary to mention that these results come from computer simulations, where the "measured" variable y_i is simply the centroid of the beam at position s_i from macro-particle tracking studies, and the "predicted" variable y_i is calculated using the expression Eq. (8) with the first order transport matrices and the known variables y_j and y_k (obtained from beam tracking simulations). Table 1 summarises the results for each BPM. From these results we have estimated a BPM resolution of approximately 2 μm.

Table 1: Estimate of the BPM resolution for FONT at ATF2 by means of tracking simulations.

BPM Pi	$\sigma_{\text{reso},i}$
P1	1.4094 μm
P2	2.4002 μm
P3	1.5542 μm

1.1 Transfer matrix reconstruction

The transfer matrix between two positions in a line can be constructed using two BPMs. Considering only the linear optics approximation, the point transfer map between two BPMs is given by

$$\begin{pmatrix} y \\ y' \end{pmatrix}_2 = \begin{pmatrix} R_{33} & R_{34} \\ R_{43} & R_{44} \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}_1 \ . \qquad (11)$$

Let the point 1 (BPM P1) be adjacent to a corrector or kicker (K1), as it is shown in the schematic Fig. for the FONT layout. Then two measurements are required to determine R_{34}. For example, we can use $y_2^{\{1\}}$ at the BPM P2 obtained with the nominal trajectory (the index $\{1\}$ indicates first measurement) and the initial values $(y, y')_1$ at BPM P1 position. Then, in a second measure we can obtain $y_2^{\{2\}}$ and use the initial vertical position and slope $(y, y' + \Delta\theta_1)_1$, where $\Delta\theta_1$ is an arbitrary kick angle introduced by the corrector K1. The difference in position $(y_2^{\{2\}} - y_1^{\{1\}}) \equiv \Delta y_2$ is related to R_{34} by

$$R_{34}(s_2, s_1) = \Delta y_2 / \Delta\theta_1 \ . \qquad (12)$$

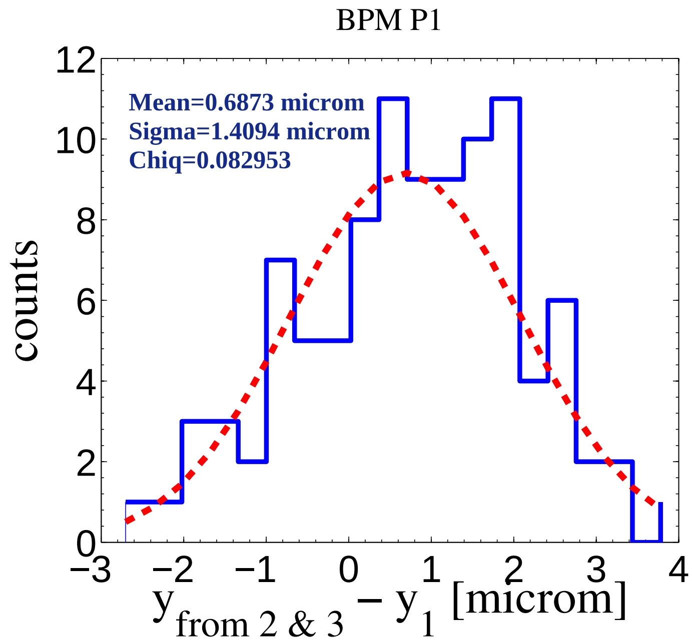

Figure 2: Left:residual distribution from the difference between the predicted beam position y (using the BPMs P2 and P3) and the simulated measurement of the position y with BPM P1.

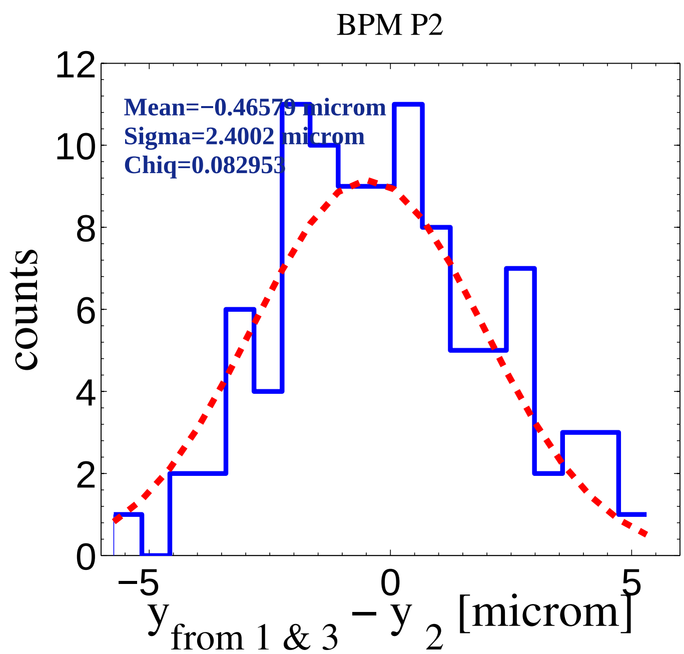

Figure 3: Residual distribution from the difference between the predicted beam position y (using the BPMs P1 and P3) and the simulated measurement of the position y with BPM P2.

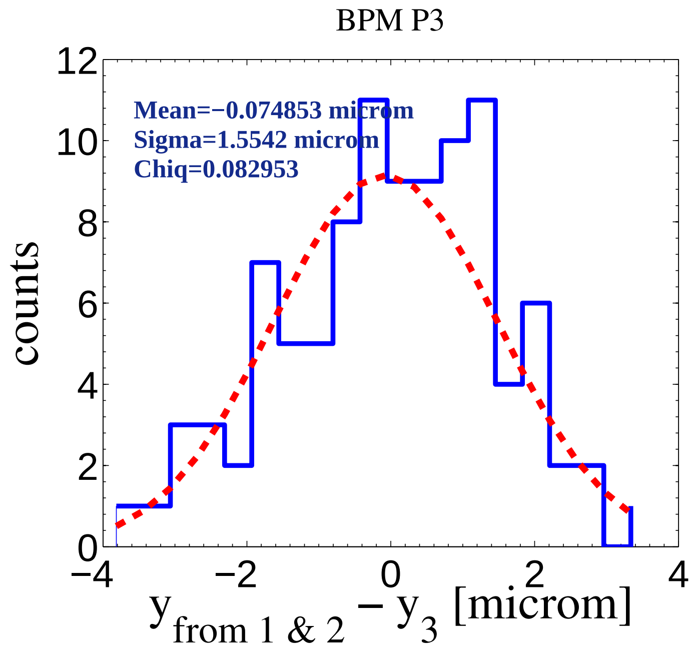

Figure 4: Residual distribution from the difference between the predicted beam position y (using the BPMs P1 and P2) and the simulated measurement of the position y with BPM P3.

Using a similar procedure we can obtain $R_{34}(s_3, s_2) = \Delta y_3/\Delta\theta_2$, with a kick angle $\Delta\theta_2$ produced by the kicker K2, and $R_{34}(s_3, s_1) = \Delta y_3/\Delta\theta_1$.

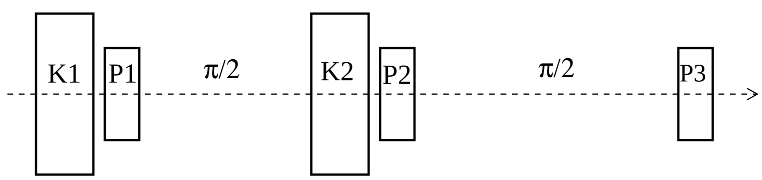

Figure 5: Simple scheme of the FONT elements.

A detailed explanation of a general method for reconstruction of 4x4 linear matrix can be found in Ref. [2], where the technique is also extended to a non-linear optics case including second (or even higher) order terms.

References

[1] D. Schulte *et al.*, https://savannah.cern.ch/projects/placet/

[2] T. Barklow, *et al.*, "Review of Lattice Measurement Techniques at the SLC", SLAC-PUB-5695, November 1991.