19/02/08

i is the BPM number

y_i are the BPM readings

Δ_{ij} are calculated

$\sigma^2_{\Delta_{ij}}$ are calculated

$\sigma^2_{\Delta_{12}} \neq \sigma^2_{\Delta_{13}} \neq \sigma^2_{\Delta_{23}}$ generally

$$\begin{cases} y_1 - y_2 = \Delta_{12} \\ y_1 - y_3 = \Delta_{13} \\ y_2 - y_3 = \Delta_{23} \end{cases}$$

$$\begin{cases} \sigma_1^2 + \sigma_2^2 = \sigma^2_{\Delta_{12}} \\ \sigma_1^2 + \sigma_3^2 = \sigma^2_{\Delta_{13}} \\ \sigma_2^2 + \sigma_3^2 = \sigma^2_{\Delta_{23}} \end{cases}$$

$$\begin{cases} \sigma_1^2 + \sigma_2^2 \qquad\quad = \sigma^2_{\Delta_{12}} \\ \sigma_1^2 \qquad\quad + \sigma_3^2 = \sigma^2_{\Delta_{13}} \\ \qquad\quad \sigma_2^2 + \sigma_3^2 = \sigma^2_{\Delta_{23}} \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -1 - 1 = -2$$

$$D_1 = \begin{vmatrix} \sigma^2_{\Delta_{12}} & 1 & 0 \\ \sigma^2_{\Delta_{13}} & 0 & 1 \\ \sigma^2_{\Delta_{23}} & 1 & 1 \end{vmatrix} \neq 0$$

$$D_2 = \begin{vmatrix} 1 & \sigma^2_{\Delta_{12}} & 0 \\ 1 & \sigma^2_{\Delta_{13}} & 1 \\ 0 & \sigma^2_{\Delta_{23}} & 1 \end{vmatrix} \neq 0 \qquad D_3 = \begin{vmatrix} 1 & 1 & \sigma^2_{\Delta_{12}} \\ 1 & 0 & \sigma^2_{\Delta_{13}} \\ 0 & 1 & \sigma^2_{\Delta_{23}} \end{vmatrix} \neq 0$$

```
1   %finite length gaussian distribution number array, mean=0, std=1
N=3000;
M=3;
Y=randn(N,M);
mY=mean(Y);
rmsY=std(Y);
%hist(Y,20);

2   %BPM readings for resolution 1um, 2um and 3um
for m=1:M        %BPM number
    for j=1:N    %shot number
        y(j,m)=m*Y(j,m);
    end
end
%hist(y,20);

3   %a zero offset calculated & a 'true' rms calculated (to be
    %compared with the RMS obtained below with the method)
zero_offs=mean(y);
for m=1:M
    for j=1:N
    yy(j,m)=y(j,m)-zero_offs(m);
    end
end
rmsy=std(yy);
%hist(yy,20);

4   %the residue calculated & its rms^2 as the right side of the
equation
delta12=y(:,1)-y(:,2);
delta13=y(:,1)-y(:,3);
delta23=y(:,2)-y(:,3);
md12=mean(delta12);
md13=mean(delta13);
md23=mean(delta23);
d12=delta12-md12;
d13=delta13-md13;
d23=delta23-md23;

rmsd12=std(d12);
rmsd13=std(d13);
rmsd23=std(d23);
R12=rmsd12^2;
R13=rmsd13^2;
R23=rmsd23^2;

5   %determinants
A=[1 1 0; 1 0 1; 0 1 1];
A1=[R12 1 0; R13 0 1; R23 1 1];
A2=[1 R12 0; 1 R13 1; 0 R23 1];
A3=[1 1 R12; 1 0 R13; 0 1 R23];
D=det(A);
D1=det(A1);
D2=det(A2);
D3=det(A3);

6   %the BPM's RMSs as system solutions to be compared with the true rms
above
RMSy1=sqrt(D1/D);
RMSy2=sqrt(D2/D);
RMSy3=sqrt(D3/D);

7777
```

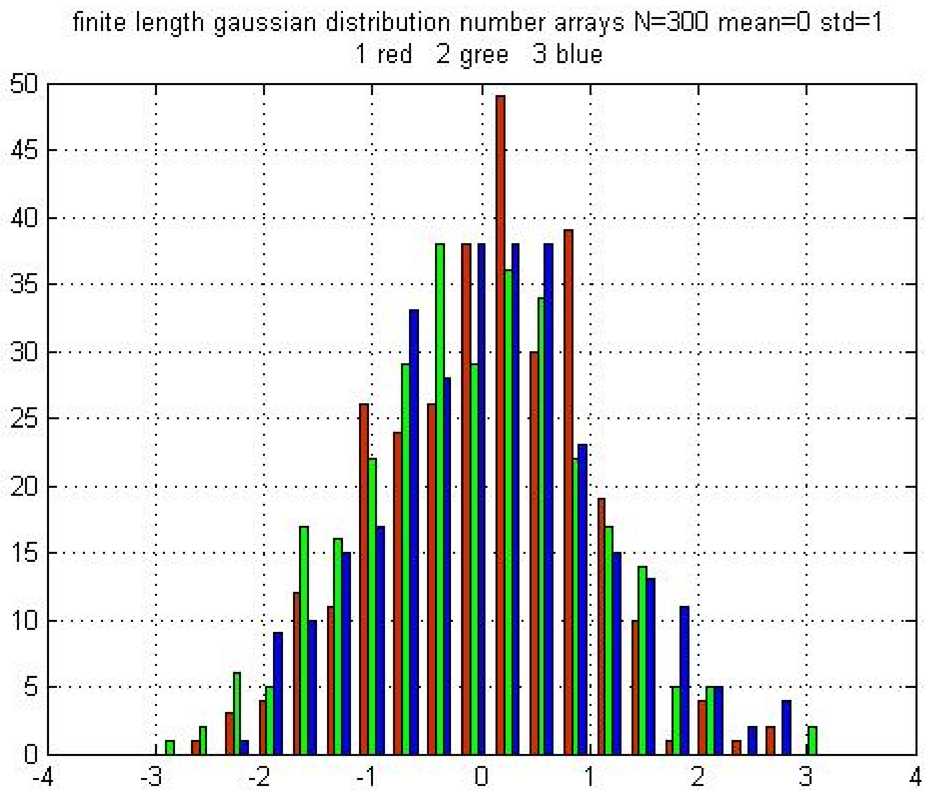
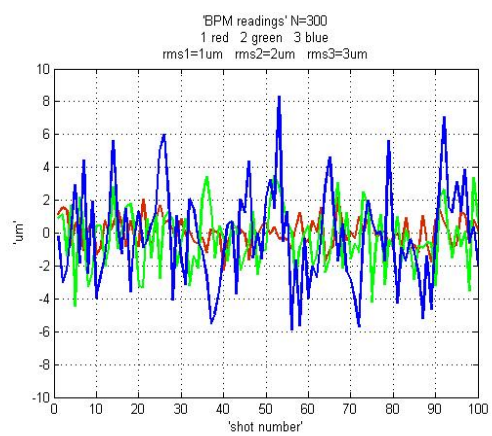


Fig.1. Random number arrays generated with Matlab.



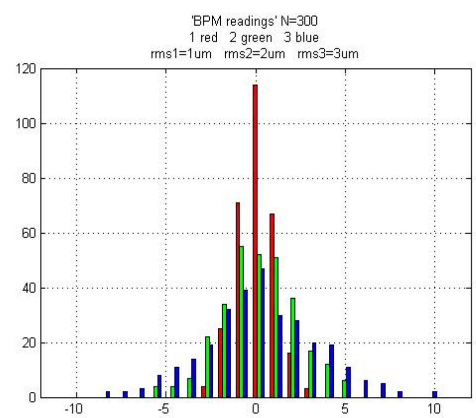


Fig.2. ‘BPM readings’ manufactured from the arrays above. Resolution of the BPMs is: 1um (red), 2um (green), 3um (blue).

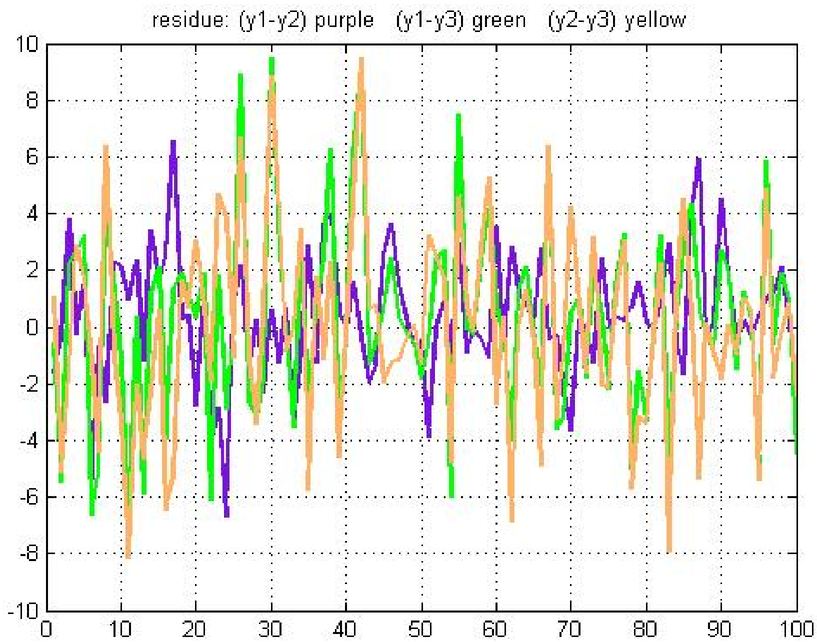


Fig.3. Residue (um) calculated with the readings above.

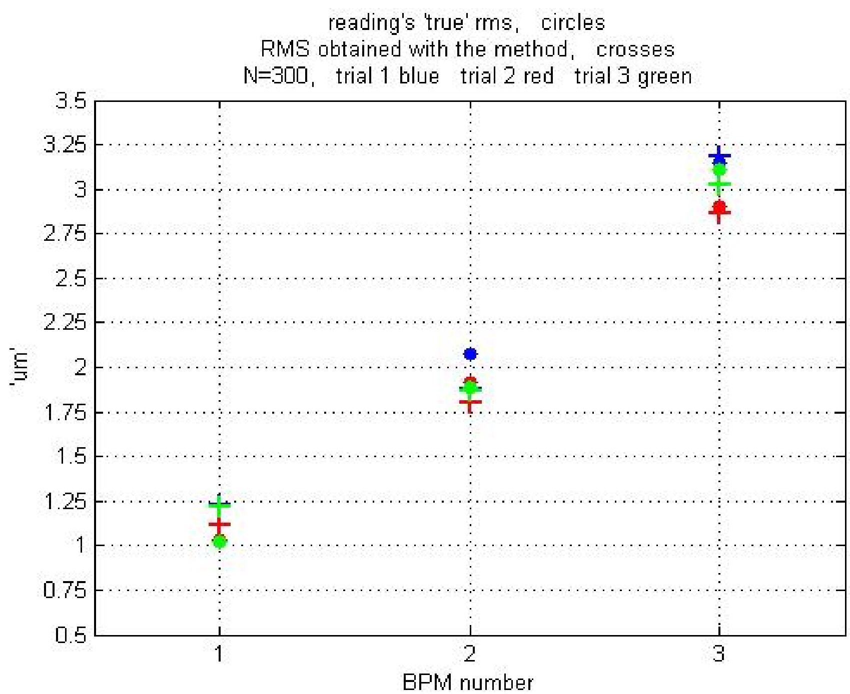


Fig.4. 'True' rms (circles) calculated directly from the residues above, and RMS obtained with the Method (crosses).
N=300. Three trials are shown blue, red and green.