Electron beam in undulators of e+ source
- Emittance and orbit angle with quad misalignment and corrections
- Effect of beam pipe wakefiled

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Part1. Emittance and orbit angle with quad misalignment and corrections

Review of old work

Report in ILC LET meeting at Daresbury

Main Concerns were:

- Radiation in undulators may increase transverse emittance if there is dispersion
- Section by section orbit angle change, then gamma-ray angle change, can make polarization of e+ impossible.

Lattice and assumption

- Undulator:
  - Field   1T
  - Period 14 mm
  - Total undulator length 119 m
    - 11.9 m between quadrupole magnets, 10 sections
  - No field errors, no misalignment
- Lattice:
  - Quadrupole magnet every 12.4 m,
  - x 70 deg. y 90 deg. per FODO cell
  - Quad-BPM-dipole corrector package
- Errors
  - Quad offset 0.3 mm, rotation 0.3 mrad
  - BPM offset 10 micron w.r.t. quad, rotation 0.3 mrad
- No wakefield

Parameters are not exactly the same as present design, but probably the difference is not too significant for rough estimations here. (or, scaling will be possible.)

Kick Minimization

Quad magnet, BPM and steering magnets should be attached.

$$\text{Minimize } r\sum_i (x_i^2 + y_i^2) + \sum_i \left[\left(\theta_{x,i} + k_i x_i\right)^2 + \left(\theta_{y,i} - k_i y_i\right)^2 \right],$$

$\theta_{x(y)i}$: Additional kick angle (additional to designed kick) of steering at i-th quad

$x(y)_i$: Offset from designed orbit at i-th quad

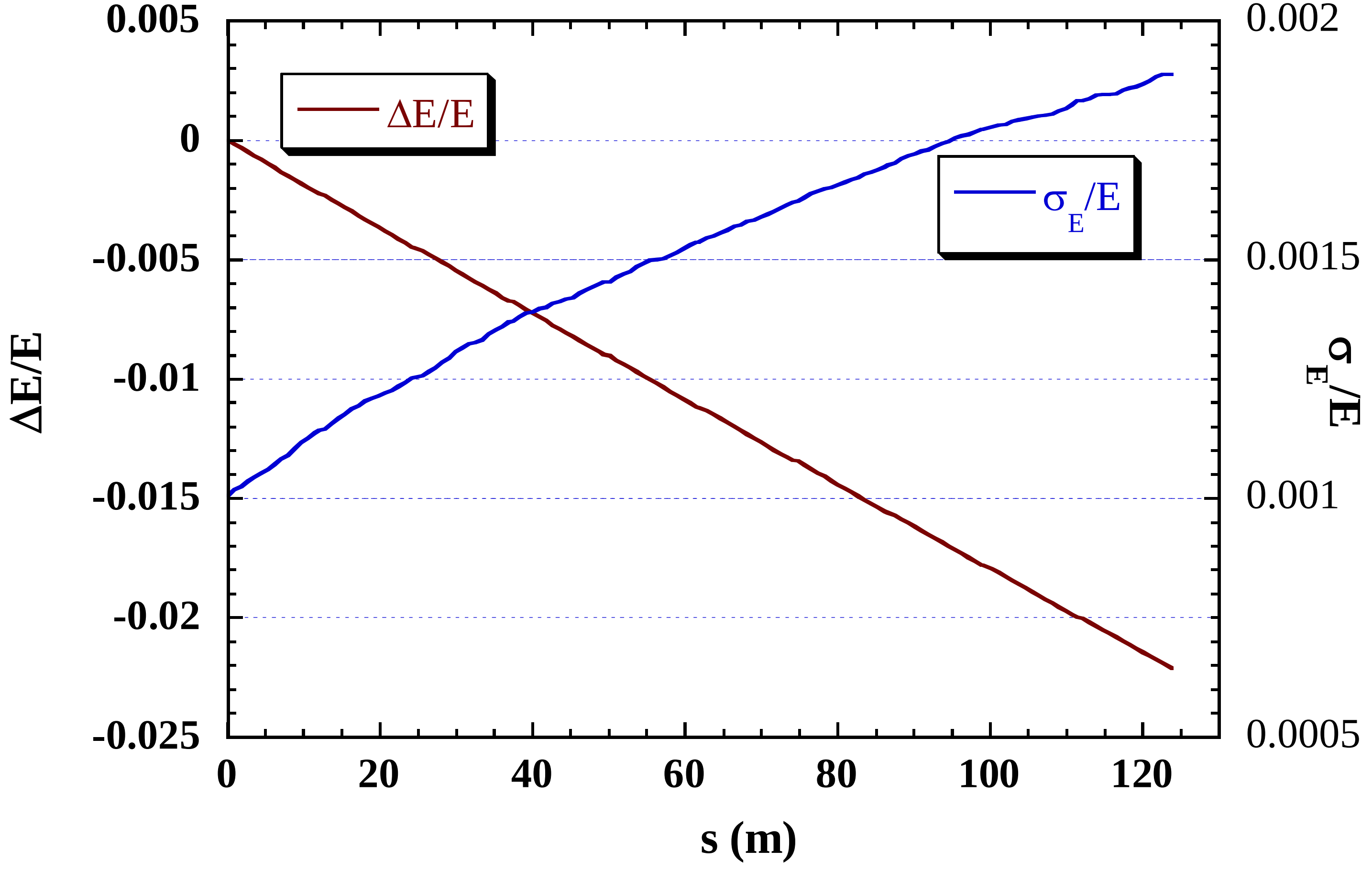
k_i : K-value (inverse of focal length) of the i-th quad

r : Weight ratio : (Quad-BPM offset)^2/(Quad offset)^2

Helical Undulator in SAD

- Simulation used computer code SAD
  - Helical undulator is not supported as a standard magnet.
- Represented by series of bend magnets
  - 16 bend magnets/period (0.875 mm/magnet)
  - Rotate i-th magnet by (360/16)*i deg.

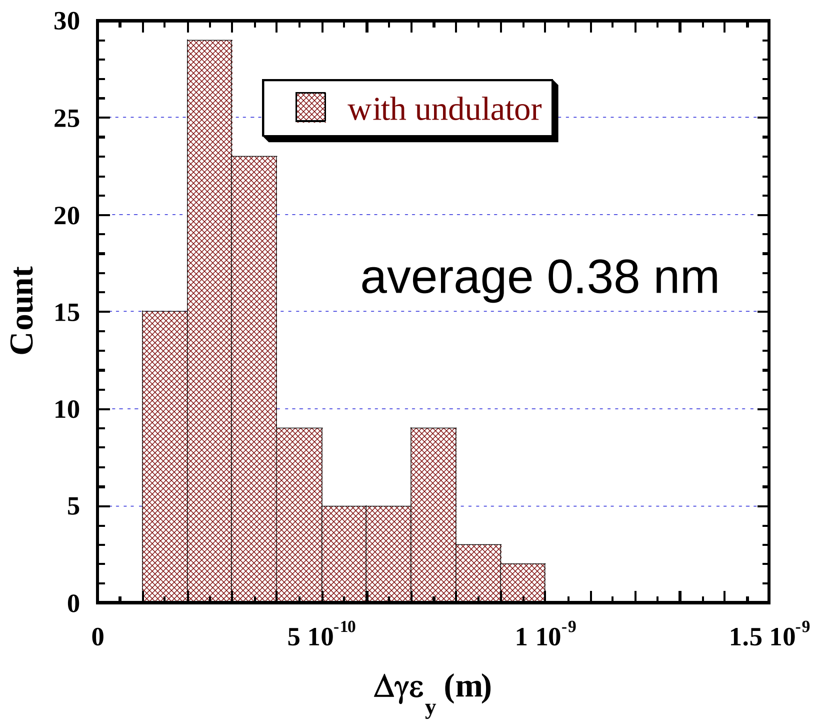
Beam Energy and Energy Spread vs. s



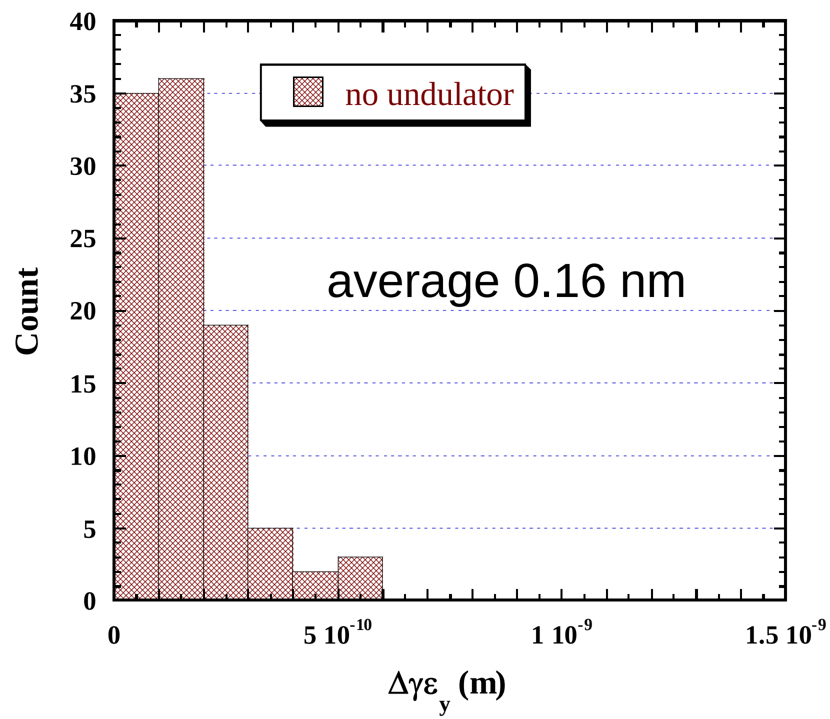
Emittance increase in the undulator section

Quad offset 0.3 mm, rotation 0.3 mrad
BPM offset 10 micron w.r.t. quad, rotation 0.3 mrad
KM steering, 100 random seeds.

With undulator (radiation)

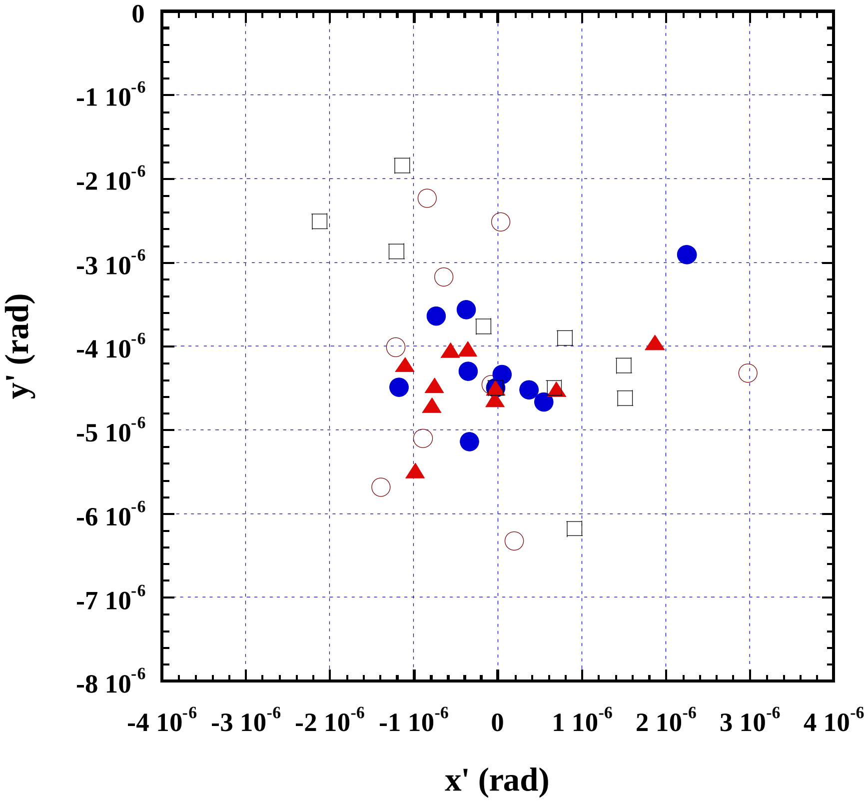


No undulator (replaced by drift space)



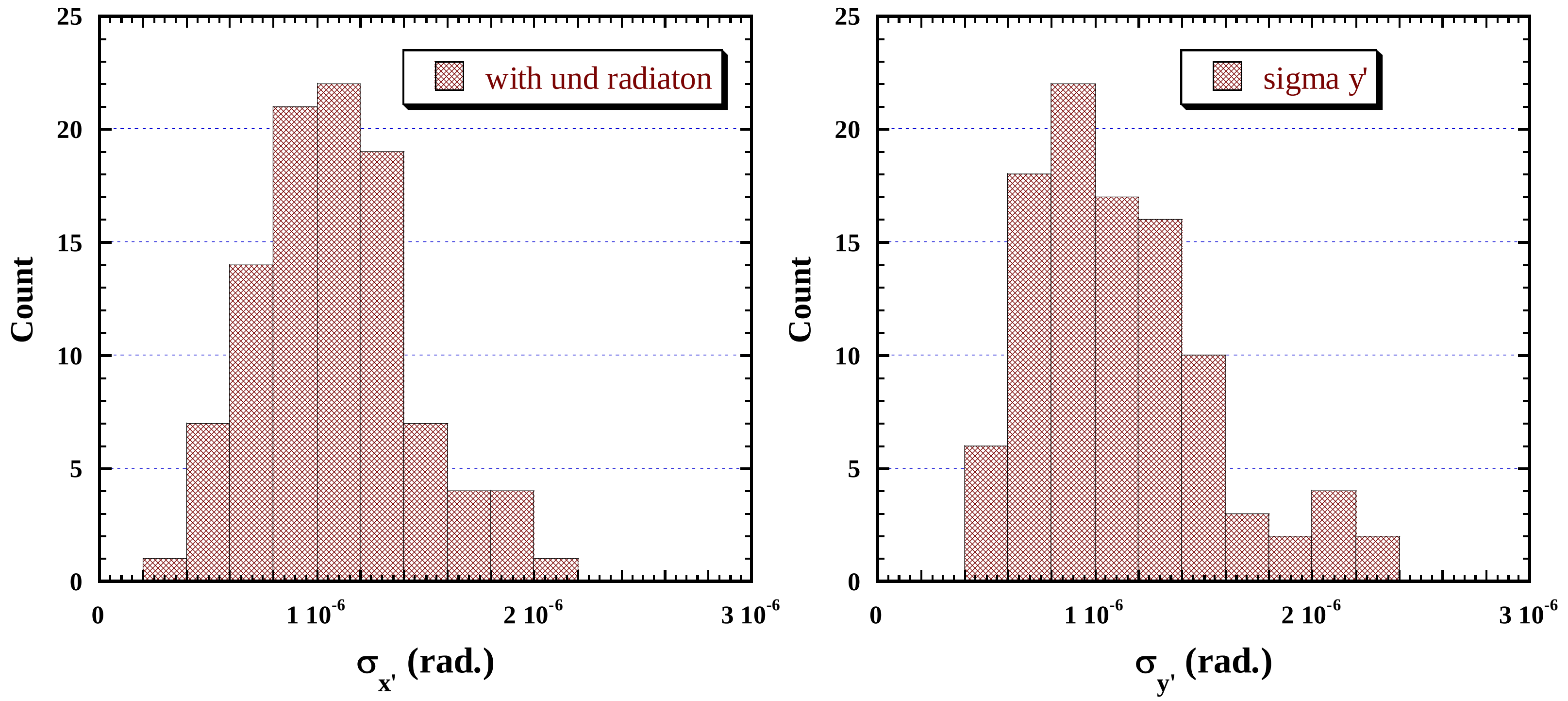
Orbit angle distribution

Each set consists of 10 points (There are 10 undulator sections, each of which is between two quadrupole magnets.).
Each point represents orbit angle at the entrance of a quadrupole magnet after each undulator section.
Note: 1/gamma = 3.3e-6. σx' ~ 2-3e-6, σy' << σx'



Note:
Design y' is not zero, to make overall orbit straight.
We assumed ends of each undulator have horizontal field.

Distribution of r.m.s. of orbit angle



Below 1/gamma

Summary of part 1

Misalignment of quadrupole magnets 0.3 mm.

Orbit correction (KM steering)

We assumed a good BPM and a dipole corrector attached to every quadrupole magnet.

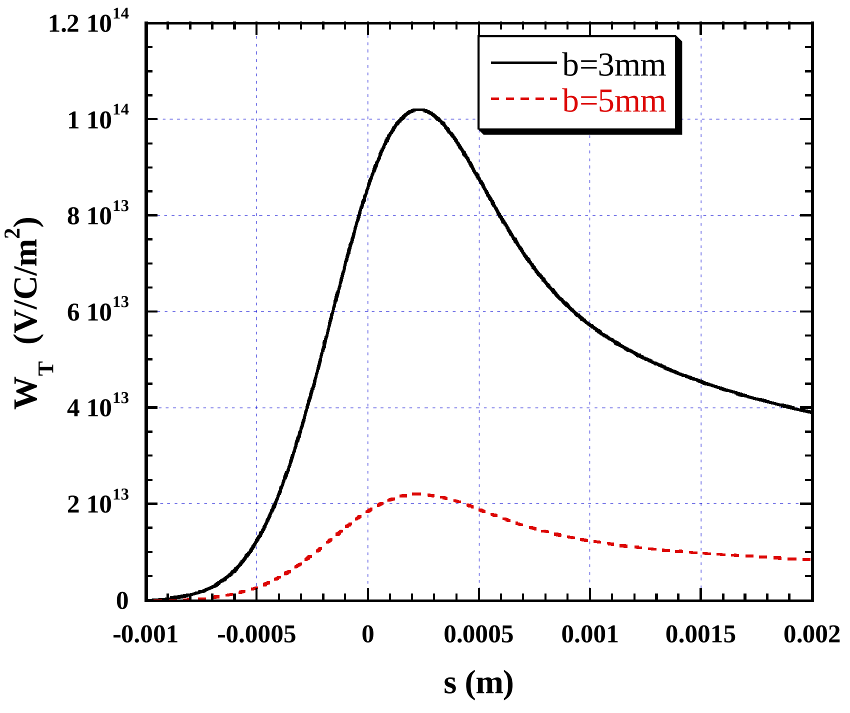
- Radiation in undulators with dispersion
  - Vertical emittance increase ~ 1 nm or less. OK.
- Section by section orbit angle change,
  - Angle distribution ~ 2 μrad or less ($<1/\gamma$). Probably OK.

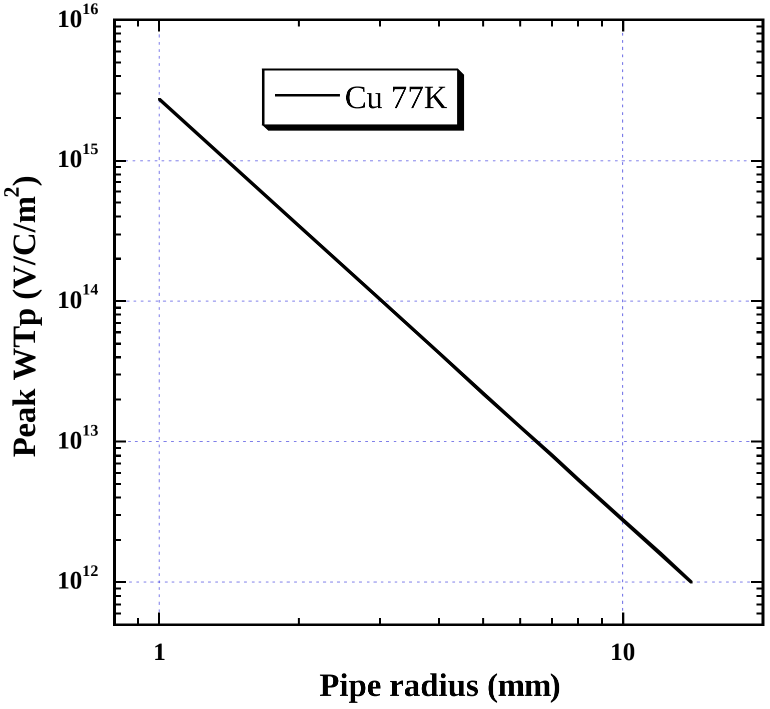
Part 2. Effect of beam pipe resistive wake

- Rough estimation of effects of resistive wall wake to transverse (vertical) motion. Dependence on Wake-potential (W_T)
- Wakefield itself was studied by:
  - Dr. Thesis of Duncan J. Scott
  - But, it assumed
    - bunch length 0.15 mm (0.3 mm is nominal. The longer the bunch, the larger the W_T)
    - beam pipe radius 1mm or 2 mm. (Looks too small.)
  - So, I did rough estimation of W_T too.

Transverse wake for short bunch

K.L.F.Bane, Int. J. Mod. Phys. A. vol. 22, p 3736, 2007 and thesis of Duncan J. Scott ,
For 0.3 mm bunch length,





Peak of wake potential ~ $\sigma^{-1/2}\, b^{-3}$

~ 200 V/pC/m^2 for Cu 77K (σ=500 MΩ^{-1}), radius= 2.38 mm

Rough estimation of effect to beam motion

- Wake potential: W_T [V/pC/m^2]
- Relevant beam pipe length: L [m]
- Beam - beam pipe center offset: y [micron]
- Bunch charge = 3.2 nC
- Beam energy = 150 GeV

→ Kick angle of bunch tail:

$$\Delta\theta \text{ [nrad]} \sim 2 \times 10^{-5} * W_T * L * y$$

The kick angle should be compared with angular divergence:

$$\sigma_{y'} = sqrt(\varepsilon_y / \beta_y) \sim 60 \text{ nrad}$$

- $\varepsilon_y \sim 2\ 10^{-8} / \gamma = 7 \times 10^{-14}$ m
- $\beta_y \sim 20$ m

Effect of Beam Orbit Jitter (Faster than corrections)

- Wake potential: W_T [V/pC/m^2]
- Relevant beam pipe length: L [m] ~ 200
- Vertical beam jitter: y [micron] ~ σ_y ~ 1

→ $\Delta\theta$ [nrad] ~ 4 x 10^{-3} * W_T

Requiring

$\Delta\theta \ll \sigma_{y'}$ ~ 60 [nrad]

→ $W_T \ll 1.5 \times 10^4$ [V/pC/m^2]

This limit is 75 times bigger than the estimated W_T for the case of Cu 77K (σ=500 $M\Omega^{-1}$) and radius= 2.38 mm.

OK.

Effect of Beam Pipe misalignment
(Pipe center - beam offset)

(Relevant beam pipe length can be set as beta-function.)

- Wake potential: W_T [V/pC/m^2]
- Relevant beam pipe length: L [m] ~ β ~ 20
- Total length/relevant length: n ~ 200/20 = 10
- Vertical movement: y [micron]
- → $\Delta\theta$ [nrad] ~ $4 \times 10^{-4} * W_T * y$*sqrt($n$)
- (Assuming each 20 m is randomly misaligned)

Requiring

$$\Delta\theta \ll \sigma_{y'} \sim 60 \text{ [nrad]}$$

→ W_T [V/pC/m^2] * y [micron] << 4.7×10^4

Assuming W_T for the case of Cu 77K (σ = 500 $M\Omega^{-1}$) and radius= 2.38 mm, this means

y [micron] << 240

Assume corrections (minimize down stream emittance, changing either orbit or pipe position), the tolerance for static misalignment will be larger.

→ Probably OK. But we will need diagnostics (emittance measurement) section after the undulators.

Summary

Misalignment of quadrupole magnets 0.3 mm.

With a good BPM and a dipole corrector attached to every quadrupole magnet.

- Radiation in undulators with dispersion
  - Vertical emittance increase ~ 1 nm or less. OK.
- Section by section orbit angle change,
  - Angle distribution ~ 2 μrad or less ($<1/\gamma$). Probably OK.

Effect of resistive wall wakes

Assuming 2.38 mm radius, Cold Cu beam pipe

- Effect of beam orbit jitter will be negligible.
- Alignment (pipe center to beam) should be much better than 240 micron, assuming no corrections.
  - Will be loosened with corrections.
    - Need diagnostics section (emittance measurement) after the undulators.

Question: Whether all important issues have been studied or not.