

Measuring the parameters of the Lagrangian

Theory Rally
Vancouver Linear Collider Workshop
Vancouver, 07/21/06

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LAL Orsay

- **Introduction**
- **SUSY measurements**
- **Reconstruction of the fundamental parameters**
- **Conclusions**

Introduction

Supersymmetry:

- solves hierarchy problem
- proper extension of Lorentz algebra
- has a light Higgs boson mass (EW data)
- promises rich collider phenomenology @TeVscale

LHC and the ILC will provide a wealth of measurements:

- masses
- mass differences
- cross sections
- branching ratios
- mixtures of all of the above and more.....

spin-0	spin-1/2	spin-1
Squarks: \tilde{q}_R, \tilde{q}_L	q	
	Gluino: \tilde{g}	g
Sleptons: $\tilde{\ell}_R, \tilde{\ell}_L$	ℓ	
h,H,A	Neutralino $\chi_{i=1-4}$	Z, γ
H[±]	Charginos: $\chi_{i=1-2}^{\pm}$	W[±]

Transform measurements of (s)particle properties into measurements of fundamental parameters

Need to specify a model (more or less constrained):

- **mSUGRA (top-down) and MSSM (bottom-up) with conservation of R-parity**

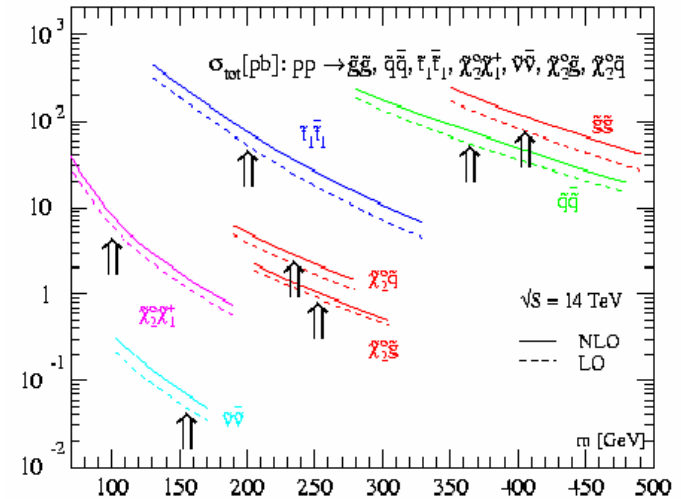
Main difficulties:

- need predictions for all observables matching theoretical and experimental precision
- observables sensitive to several parameters: correlations, error propagation

Thus studies to determine supersymmetric parameters from measurements need brain power and sophisticated tools:

- Mass spectra generated by SOFTSUSY, SUSPECT, SPHENO
 - typically 2-loop RGEs, radiative corrections to particle masses, dominant 2-loop Higgs mass contributions
- Branching ratios by MSMLIB, SPHENO, SDecay
 - 2, 3, 4 body-decays (including transition 2-3), including QCD corrections and EW corrections, 1-loop SUSY-QCD
- e+e- cross sections (polarized) by SPHENO
 - ISR and gluon-exchange corrections
- NLO proton cross sections by Prospino2.0

Beenakker et al



Putting it all together (error propagation, search for minima etc):

FITTINO: P. Bechtle, K. Desch, P. Wienemann with W. Porod

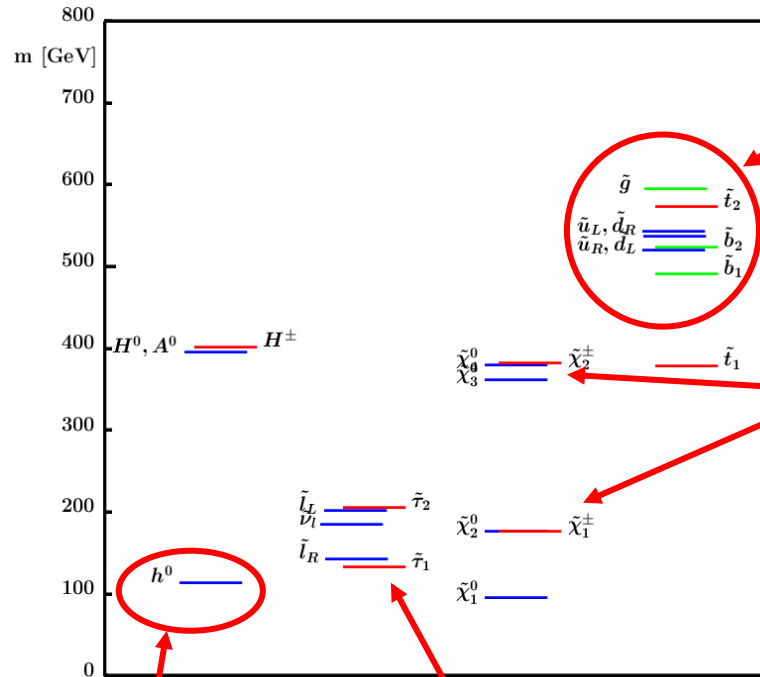
SFITTER: R. Lafaye, T. Plehn, D. Z.

P. Skands et al., SUSY Les Houches accord (SLHA), Interfacing SUSY spectrum calculators, decay packages, and event generators, JHEP 0407 (2004) 036

SPS1a and SPA1

$m_0 = 100\text{GeV}$ $m_{1/2} = 250\text{GeV}$ $A_0 = -100\text{GeV}$ $\tan\beta = 10$ $\text{sign}(\mu) = +$
 favourable for LHC and ILC (Complementarity)

$m_0 = 70\text{GeV}$ $A_0 = -300\text{GeV}$ compatible with Ω_{h^2}



Moderately heavy gluinos and squarks

“Physics Interplay of the LHC and ILC”
 Editor G. Weiglein hep-ph/0410364

Heavy and light gauginos

$\tilde{\tau}_1$ lighter than lightest χ^\pm :

- χ^\pm BR 100% $\tilde{\tau}\tilde{\nu}$
- χ_2 BR 90% $\tau\tilde{\tau}$

• cascade:

$\tilde{q}_L \rightarrow \chi_2 q \rightarrow \tilde{\ell}_R \ell q \rightarrow \ell \ell q \chi_1$
 visible

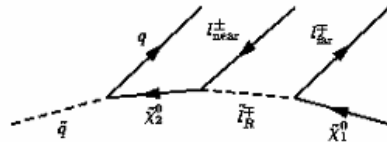
Higgs at the limit
 of LEP reach

light sleptons

LHC:

Abundant production of gluinos and squarks decaying through cascade decays via neutralinos and sleptons

Leptons at the LHC: electrons and muons



$$(m_{ll}^2)^{\text{edge}} = \frac{(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_R}^2}$$

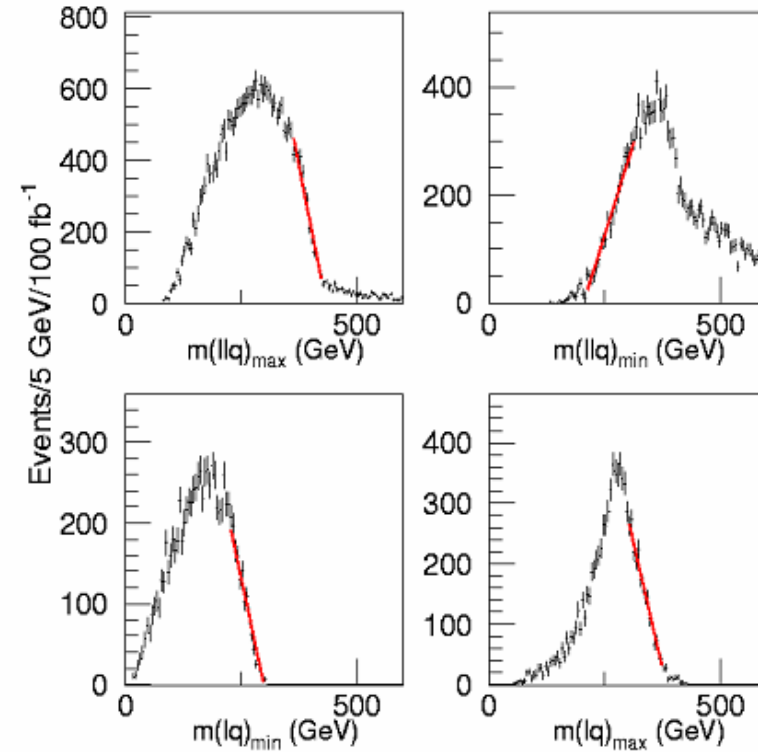
$$(m_{qll}^2)^{\text{edge}} = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{\chi}_2^0}^2}$$

$$(m_{ql}^2)_{\text{min}}^{\text{edge}} = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)}{m_{\tilde{\chi}_2^0}^2}$$

$$(m_{ql}^2)_{\text{max}}^{\text{edge}} = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_R}^2}$$

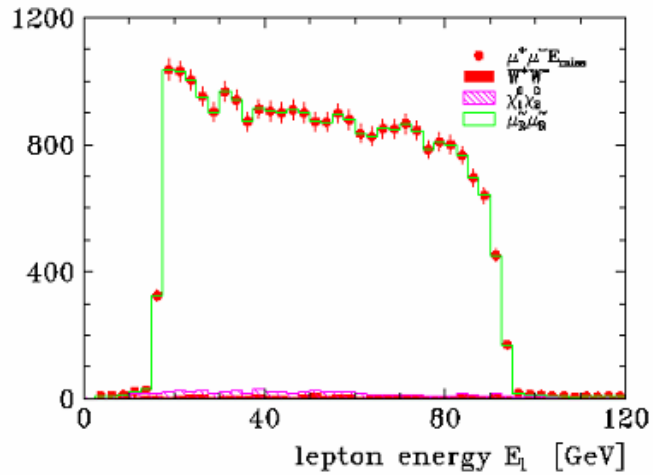
$$(m_{qll}^2)^{\text{thres}} = \frac{[(m_{\tilde{q}_L}^2 + m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2) - (m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)\sqrt{(m_{\tilde{\chi}_2^0}^2 + m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 + m_{\tilde{\chi}_1^0}^2)^2 - 16m_{\tilde{\chi}_2^0}^2 m_{\tilde{l}_R}^4 m_{\tilde{\chi}_1^0}^2} + 2m_{\tilde{l}_R}^2(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)]/(4m_{\tilde{l}_R}^2 m_{\tilde{\chi}_2^0}^2)}$$

Mass determination for 300fb⁻¹ (thus 2014) LHC:
Toy MC from edges, thresholds to masses

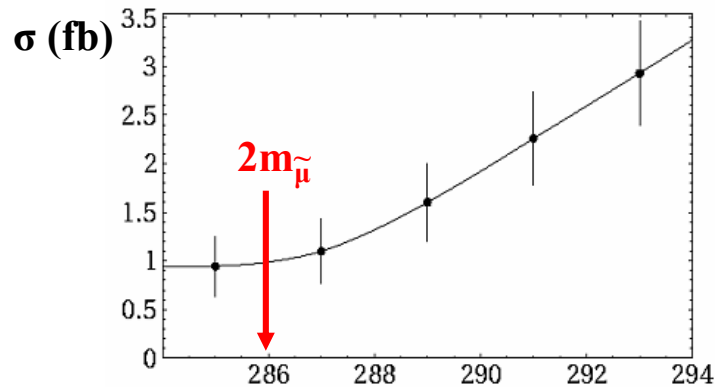


Variable	Value (GeV)	Errors		
		Stat. (GeV)	Scale (GeV)	Total
$m_{\ell\ell}^{\text{max}}$	77.07	0.03	0.08	0.08
$m_{\ell\ell q}^{\text{max}}$	428.5	1.4	4.3	4.5
$m_{\ell\ell}^{\text{low}}$	300.3	0.9	3.0	3.1
$m_{\ell\ell q}^{\text{high}}$	378.0	1.0	3.8	3.9
$m_{\ell\ell q}^{\text{min}}$	201.9	1.6	2.0	2.6
$m_{\ell\ell b}^{\text{min}}$	183.1	3.6	1.8	4.1
$m(\ell_L) - m(\tilde{\chi}_1^0)$	106.1	1.6	0.1	1.6
$m_{\ell\ell}^{\text{max}}(\tilde{\chi}_4^0)$	280.9	2.3	0.3	2.3
$m_{\tau\tau}^{\text{max}}$	80.6	5.0	0.8	5.1
$m(\tilde{g}) - 0.99 \times m(\tilde{\chi}_1^0)$	500.0	2.3	6.0	6.4
$m(\tilde{q}_R) - m(\tilde{\chi}_1^0)$	424.2	10.0	4.2	10.9
$m(\tilde{g}) - m(b_1)$	103.3	1.5	1.0	1.8
$m(\tilde{g}) - m(b_2)$	70.6	2.5	0.7	2.6

ILC:



Masses: from endpoint measurements



Masses: threshold cross section measurement

“hidden essential measurements”: particle spin
 “hidden calculations”: Xsection for mass determination

Polesello et al: use of χ_1 from ILC (high precision) in LHC analyses improves the mass determination

	Mass, ideal	“LHC”	“LC”	“LHC+LC”
$\tilde{\chi}_1^\pm$	179.7		0.55	0.55
$\tilde{\chi}_2^\pm$	382.3	–	3.0	3.0
$\tilde{\chi}_1^0$	97.2	4.8	0.05	0.05
$\tilde{\chi}_2^0$	180.7	4.7	1.2	0.08
$\tilde{\chi}_3^0$	364.7		3-5	3-5
$\tilde{\chi}_4^0$	381.9	5.1	3-5	2.23
\tilde{e}_R	143.9	4.8	0.05	0.05
\tilde{e}_L	207.1	5.0	0.2	0.2
$\tilde{\nu}_e$	191.3	–	1.2	1.2
$\tilde{\mu}_R$	143.9	4.8	0.2	0.2
$\tilde{\mu}_L$	207.1	5.0	0.5	0.5
$\tilde{\nu}_\mu$	191.3	–		
$\tilde{\tau}_1$	134.8	5-8	0.3	0.3
$\tilde{\tau}_2$	210.7	–	1.1	1.1
$\tilde{\nu}_\tau$	190.4	–	–	–
\tilde{q}_R	547.6	7-12	–	5-11
\tilde{q}_L	570.6	8.7	–	4.9
\tilde{t}_1	399.5		2.0	2.0
\tilde{t}_2	586.3		–	
\tilde{b}_1	515.1	7.5	–	5.7
\tilde{b}_2	547.1	7.9	–	6.2
\tilde{g}	604.0	8.0	–	6.5
h^0	110.8	0.25	0.05	0.05
H^0	399.8		1.5	1.5
A^0	399.4		1.5	1.5
H^\pm	407.7	–	1.5	1.5

Lagrangian@GUT scale: mSUGRA

mSUGRA advantage: few parameters, testing ground for studies of principles

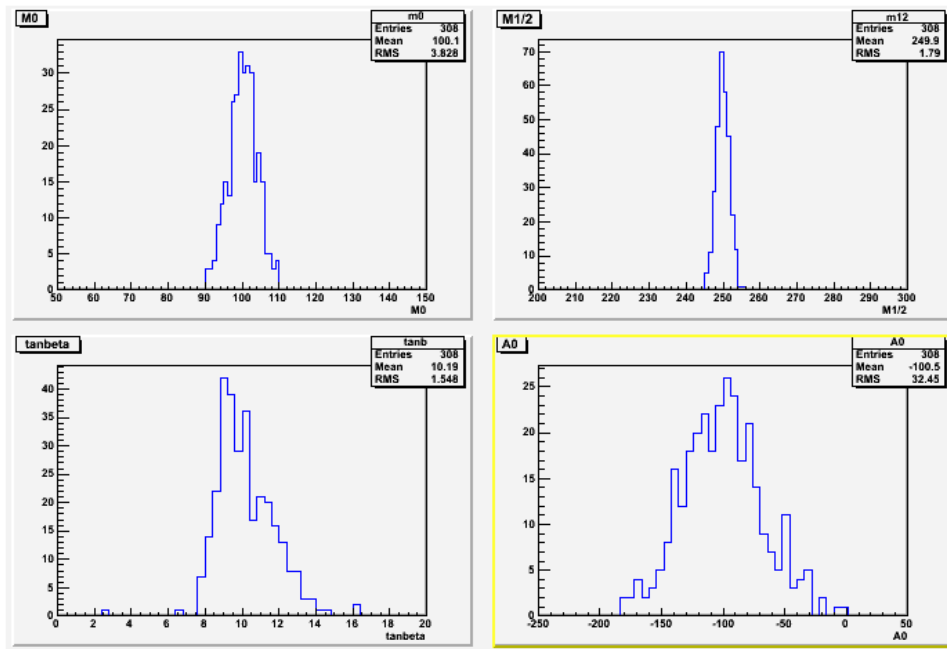
disadvantage: starts at GUT scale and adds RGE extrapolation, not the most general Lagrangian

Two separate questions:

- do we find the right point?
 - need and unbiased starting point
- what are the errors?

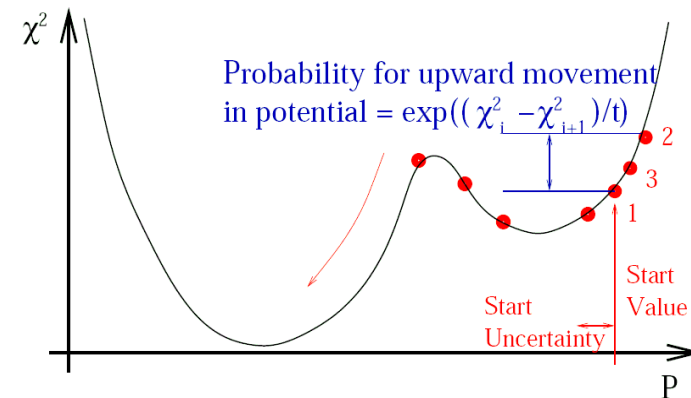
	SPS1a	Start
m_0	100	1TeV
$m_{1/2}$	250	1TeV
$\tan\beta$	10	50
A_0	-100	0GeV

Sign(μ) fixed



Fittino:

- start from tree level formula
- MINUIT
- Simulated Annealing



~300 toy experiments: convergence OK with MINUIT alone for LHC (largest errors)!

Lagrangian@GUT scale: the precision for mSUGRA

	SPS1a	Δ LHC	Δ ILC	Δ CLIC	Δ LHC+ILC
m_0	100	3.9	0.09	0.08	0.08
$m_{1/2}$	250	1.7	0.13	0.13	0.11
$\tan\beta$	10	1.1	0.12	0.12	0.12
A_0	-100	33	4.8	4.6	4.3

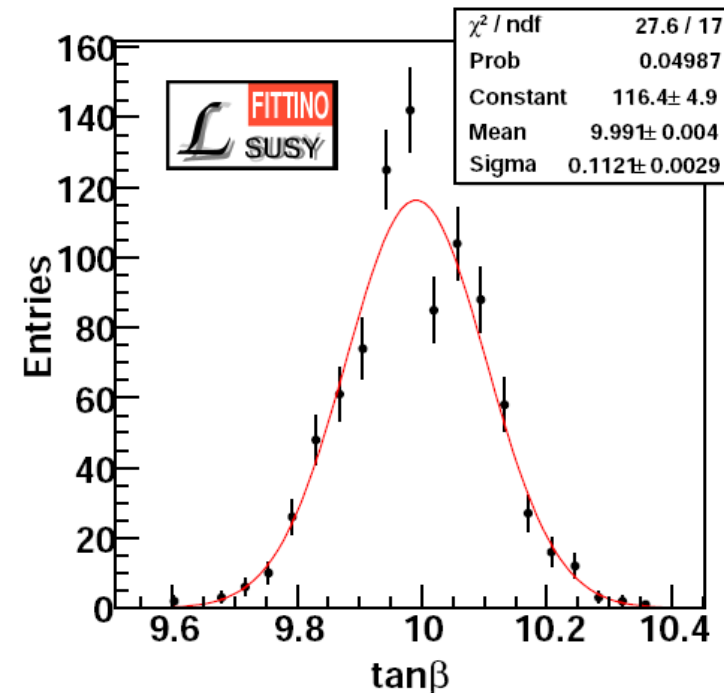
Sign(μ) fixed

errors on errors typically $\sim 10\%$

- Toy datasets (Gaussian Smearing)
- perform ~ 1000 experiments

- errors from LHC %
- errors from ILC 0.1%
- LHC+ILC: slight improvement
- low mass scalars dominate m_0

- “CLIC”: add squark measurement at 0.5%
(2-4 times better than LHC) to ILC measurements
- improves slightly m_0 and A_0 wrt ILC



Masses versus Edges (LHC)

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$m(\tilde{g}) - m(\tilde{b}_2)$	70.6	2.5	0.7	2.6

	SPS1a	Δ LHC masses	Δ LHC edges	Δ LHC top1GeV
m_0	100	3.9	1.2	1.28
$m_{1/2}$	250	1.7	1.0	1.0
$\tan\beta$	10	1.1	0.9	1.1
A0	-100	33	20	24
top	175	-	-	0.8

Sign(μ) fixed

- use of edges improves parameter determination!
- edges to masses is not a simple “coordinate” transformation:

Δm_0	Effect on $m\ell_R$	Effect on $m\ell\ell$
1GeV	0.7/5=0.14	0.4/0.08=5

Similar effect for $m_{1/2}$

- need correlations to obtain the ultimate precision from masses
- the standard model is important: top quark mass precision LHC has a non-negligible impact on SUSY parameter determination (ILC needs order of magnitude: $m_{top} \sim 0.12\text{GeV}$ affects A0....)

Negative μ mirror solution?

- μ discrete variable, therefore not suitable for fit
- fix μ to opposite sign, start from “nominal” values

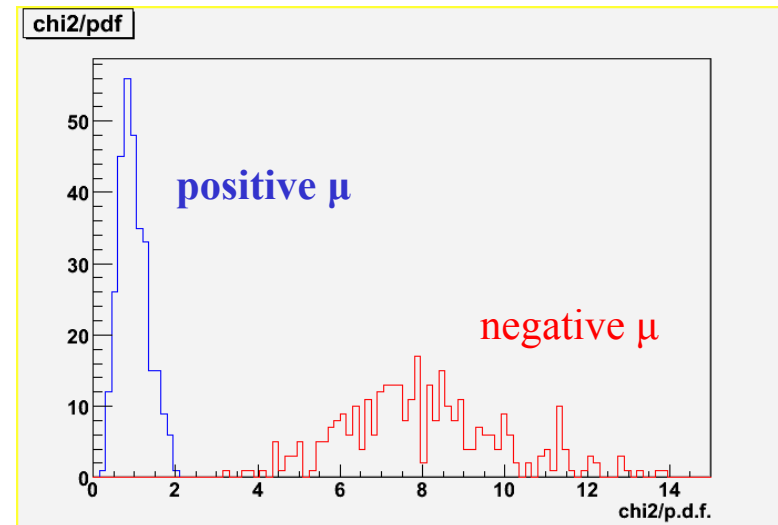
	SPS1a	LHC masses	Δ LHC
m_0	100	101.4	1.8
$m_{1/2}$	250	249.8	0.01
$\tan\beta$	10	13.8	0.002
A0	-100	-150.2	1.7
μ	+	-	

	$\Delta\chi^2$ increase (central values)
LHC masses	4
LHC edges	63
ILC	412
LHC+ILC	1400

LHC: χ^2 /pdf well separated (edges) 300 Experiments

Wrong solution might exist, but

- vary measurements with errors, no overlap in χ^2 /pdf
- very thin region (all errors “atypical”)
- “atypical” errors in more than 50% of the cases
- correlation matrix untypical
- LHC: χ_2 dominates $\Delta\chi^2$ increase
- strongest discriminating power for LHC+ILC



Total Error and down/up effect

Theoretical errors (mixture of c2c and educated guess):

Higgs	sleptons	Squarks, gluinos	Neutralinos, charginos
3GeV	1%	3%	1%

Higgs error: Sven Heinemeyer et al.

Including theory errors reduces sensitivity by an order of magnitude

	SPS1a	SoftSUSYup (Snowmass)	SoftSUSYup (Vancouver)	Δ LHC+LC (2006)
m_0	100	95.2	96.8	1.2
$m_{1/2}$	250	249.8	250.7	0.6
$\tan\beta$	10	9.82	8.4	0.5
A0	-100	-97	-109	15

	SPS1a	Δ LHC+ILCexp	Δ LHC+ILCth
m_0	100	0.08	1.2
$m_{1/2}$	250	0.11	0.7
$\tan\beta$	10	0.12	0.7
A0	-100	4.3	17

Running down/up

- spectrum generated by SUSPECT
- fit with SOFTSUSY (B. Allanach)
- central values shifted (natural)
- m_0 improved (RGE)
- overall barely compatible 1-3 σ
- theoretical errors are important

Important task SPA project: precision of theoretical calculations

The LHC neutralino enigma

χ_1	97.2	4.8
χ_2	180.8	4.7
χ_3	--	--
χ_4	381.9	5.1

Declaration of bias: 2/3 of Sfitter are in ATLAS, but:

- LHC measures the neutralino index????
- permute: χ_4 with χ_3

	SPS1a	LHC _{masses}	Δ LHC _{masses}	LHC _{edges}	Δ LHC _{edges}
m_0	100	99.6	4	100.3	2.6
$m_{1/2}$	250	250.1	1.7	248.8	2.1
$\tan\beta$	10	8.1	0.8	7.7	0.73
A0	-100	-196	30	-186	39
top	175	175	1	175.5	0.75
χ^2 /p.d.f	0	-	0.2/16	-	2/11

Exchanging χ_3 and χ_4 leads to a secondary minimum

- M_0 and $M_{1/2}$ ok, but $\tan\beta$ and A0 more than 2-3 σ from nominal values
- so in principle need ILC to see which neutralino are present....
- the predicted mass of χ_4 is about 400GeV
- the predicted branching ratios would lead us to expect more χ_4 than χ_3 in the measurement channel
- general rule: beware of the “hidden” measurements.....

SLHC+ILC

A likely scenario is concurrent running ILC plus luminosity upgrade of LHC

SPS1a results LHC 300fb⁻¹

- SLHC 3000fb⁻¹
- Some improvement
- limitation: energy scale

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SLHC	
0.08	
4.3	
3	
3.8	
2.1	
2.1	
0.5	
0.8	
1.8	
6	
5.3	
1.1	
1.1	

	SPS1a	Δ LHC before	Δ SLHC	Δ LHC+ILC	Δ SLHC+ILC
m_0	100	1.2	0.7	0.08	0.07
$m_{1/2}$	250	1.0	0.6	0.11	0.11
$\tan\beta$	10	0.9	0.7	0.12	0.12
A0	-100	20	10	4.4	3.8

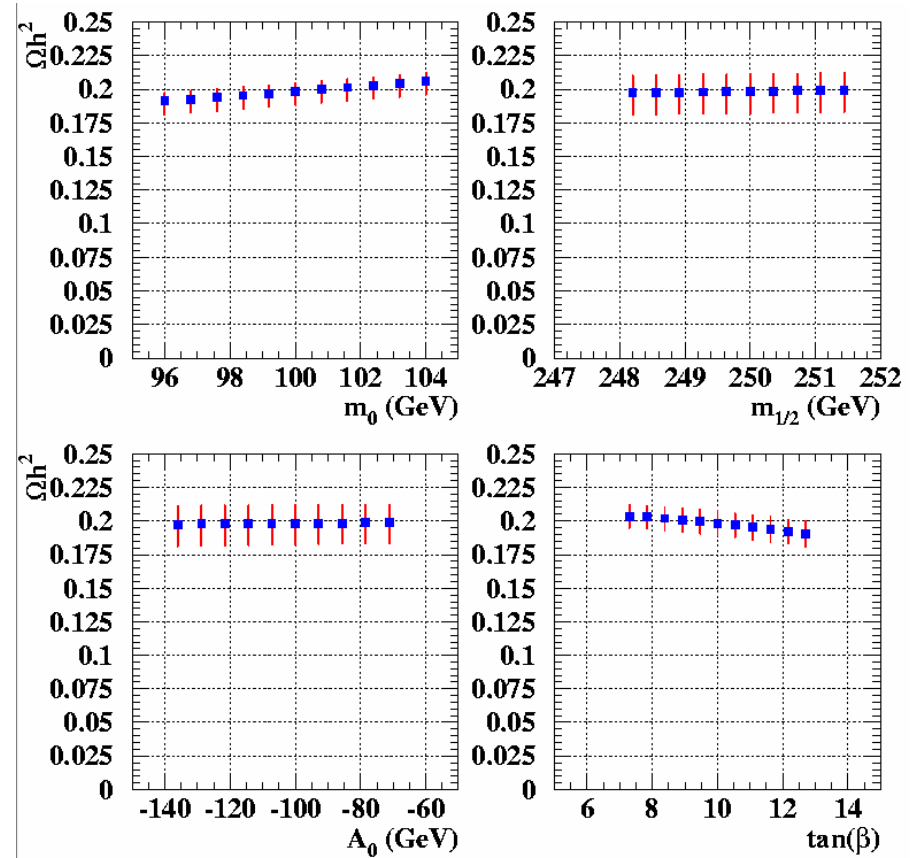
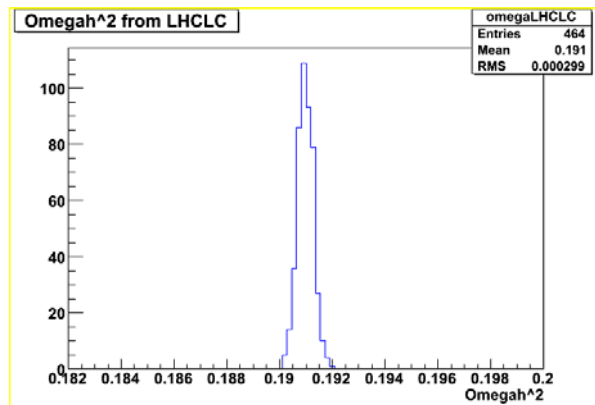
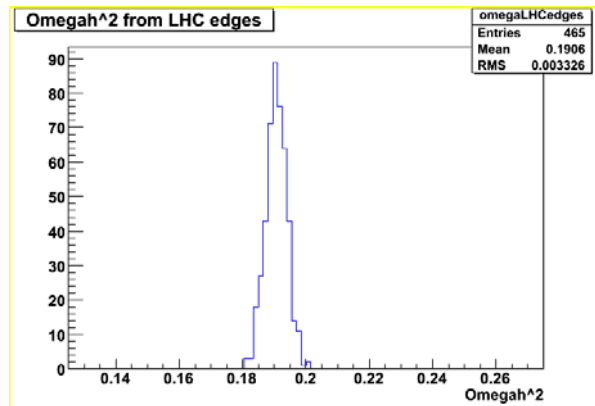
SLHC:

- factor 2 improvement
- SLHC+ILC marginal wrt LHC+ILC

Prediction of Ωh^2 at LHC+ILC

Translate determination of SUSY parameters into “Dark matter” with micrOMEGAs (Bélanger et al.):

- dependence on $m_{1/2}$, A_0 small
- dependence on $\tan\beta$, m_0 larger



- LHC : $\Omega h^2 = 0.1906 \pm 0.0033$
- LHC+ILC: $\Omega h^2 = 0.1910 \pm 0.0003$
- win order of magnitude (if theory errors are under control)

Between MSUGRA and the MSSM @LHC

Start with MSUGRA, then loosen the unification criteria,
less restricted models defined at the GUT (!) scale:

SFitter-team and Sabine Kraml
in Les Houches BSM hep-ph/0602198

	SPS1a	LHC	Δ LHC
m_0^{sleptons}	100	100	4.6
m_0^{squarks}	100	100	50
m_H^2	10000	9932	42000
$m_{1/2}$	250	250	3.5
$\tan\beta$	10	9.82	4.3
A0	-100	-100	181

- Higgs sector undetermined
 - only h (m_Z) seen
- slepton sector the same as MSUGRA
 - light scalars dominate determination of m_0
- smaller degradation in other parameters, but still % precision

	SPS1a	LHC	Δ LHC
$M_0^{1,2\text{gen}}$	100	100	4.4
$M_0^{t,b}$	100	100	59
M_0^{stau}	100	100	14
m_H^2	10000	10082	80000
$m_{1/2}$	250	250	2.6
$\tan\beta$	10	10	7.8
A0	-100	-100	323

The highest mass states do not contain the maximum information in the scalar sector, but they do in the Higgs sector!

MSSM

Parameter	"True" value	Fit value	Uncertainty (exp.)	Uncertainty (exp.+theor.)
$\tan\beta$	10.00	10.00	0.11	0.15
μ	400.4 GeV	400.4 GeV	1.2 GeV	1.3 GeV
X_τ	-4449. GeV	-4449. GeV	20. GeV	29. GeV
$M_{\tilde{t}_R}$	115.60 GeV	115.60 GeV	0.13 GeV	0.43 GeV
$M_{\tilde{t}_L}$	109.89 GeV	109.89 GeV	0.32 GeV	0.56 GeV
$M_{\tilde{b}_L}$	181.30 GeV	181.30 GeV	0.06 GeV	0.09 GeV
$M_{\tilde{b}_R}$	179.54 GeV	179.54 GeV	0.12 GeV	0.17 GeV
X_t	-565.7 GeV	-565.7 GeV	6.3 GeV	15.8 GeV
X_b	-4935. GeV	-4935. GeV	1207. GeV	1713. GeV
$M_{\tilde{\tau}_R}$	503. GeV	504. GeV	12. GeV	16. GeV
$M_{\tilde{\tau}_L}$	497. GeV	497. GeV	8. GeV	16. GeV
$M_{\tilde{d}_R}$	380.9 GeV	380.9 GeV	2.5 GeV	3.7 GeV
$M_{\tilde{d}_L}$	523. GeV	523. GeV	3.2 GeV	4.3 GeV
$M_{\tilde{e}_L}$	467.7 GeV	467.7 GeV	3.1 GeV	5.1 GeV
M_1	103.27 GeV	103.27 GeV	0.06 GeV	0.14 GeV
M_2	193.45 GeV	193.45 GeV	0.08 GeV	0.13 GeV
M_3	569. GeV	569. GeV	7. GeV	7.4 GeV
m_{Arun}	312.0 GeV	311.9 GeV	4.3 GeV	6.5 GeV
m_t	178.00 GeV	178.00 GeV	0.05 GeV	0.12 GeV

Corresponding values for the trilinear couplings:

A_τ	-445. GeV	-445. GeV	40. GeV	52. GeV
A_t	-526. GeV	-526. GeV	6. GeV	16. GeV
A_b	-931. GeV	-931. GeV	1184. GeV	1676. GeV

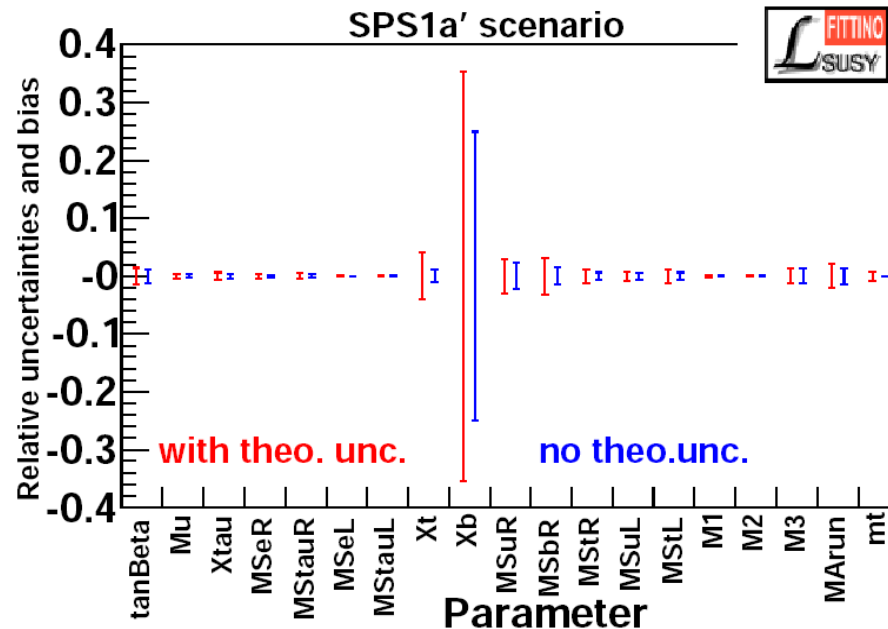
χ^2 for unsmeared observables: 2.1×10^{-5}

- good precision for LHC+ILC
- theoretical errors impact strongly the precision

- MSSM with 18 parameters:
- no intergenerational mixing
- no mixing between first 2 generations
- universality of same type sfermion parameters of 1st and 2nd generation

Additional measurements

- branching ratio ratios (Higgs LHC)
- cross sections ILC
- branching ratios ILC + mixtures



MSSM

		LHC	ILC	CLIC	ILC LHC	CLIC LHC
		LHC	ILC	CLIC	ILC+LHC	CLIC+LHC
TAN β	9.9999	79	1.4	1.8	1.1	1.3
M1	101.4	16	0.2	0.26	0.2	0.21
M2	191.6	47	0.9	1.1	0.6	0.6
M3	586.7	33	FIXED	408	8	7.9
MSTAUL	195.89	FIXED	2.7	2.5	2.4	3
MSTAUR	133.25	9	3.5	3.2	3	4
MSMUONL	195.5	5.7	0.5	0.5	0.5	0.5
MSMUONR	136	6.0	0.2	0.2	0.2	0.2
MSELECL	195.5	5.7	0.2	0.2	0.2	0.2
MSELECR	136	6.0	0.07	0.07	0.06	0.07
MSQL3GEN	497.1	32	39	12	39	11
MSTOPR	421.62	FIXED	24	13	21	11
MSBOTTOMR	522.5	43	FIXED	14	38	12
MSQL2GEN	545.9	13.6	FIXED	5.2	7.1	3.6
MSCHARMR	527.78	20	FIXED	5.6	16	5.1
MSSTRNGR	525.93	20	FIXED	5.6	16	5.1
MSQL1GEN	545.9	13.6	FIXED	5.2	7.1	3.6
MSUPR	527.76	20	FIXED	5.6	16	5.1
MSDOWNR	525.96	20	FIXED	5.6	16	5.1
ASTAU	-229.12	FIXED	939	787	641	833
ASTOP	-494.63	1547	17	24	12	17
ASBOTTOM	-795.29	FIXED	FIXED	5907	19290	5685
MA	398.86	FIXED	0.9	0.9	0.9	0.9
MU	357	45	2.3	2.3	1.9	1.8

MSSM which MSSM??

24 parameters at the EW scale

SFitter choice: do not unify 1st and 2nd generation: data should tell us ...

LHC or ILC alone:

- certain parameters must be fixed
- E.Turley and SFitter: study of syst. Error due to fix (bias)

LHC+ILC:

- all parameters fitted
- several parameters improved

CLIC wrt ILC:

- no fixed parameters with good precision

LHC+CLIC

- improvement essentially in the squark sector with factor 2-3 on errors as expected from the mass measurement improvement wrt LHC

Note: if at LHC mSUGRA has a secondary minimum, MSSM will have even more.....

Can the LHC do more in the MSSM?

- from edges cinematically to masses
- Cross section prediction (Prospino2.0) for squarkR accurate to about 10%
- Deviation of cross section measurement ratio is 400% wrt to single squark
- thus 4 pseudo measurements for every measurement involving squarks (equivalent to unification of breaking terms)
- LHC separates well electrons and muons: **enough measurements to do full fit**

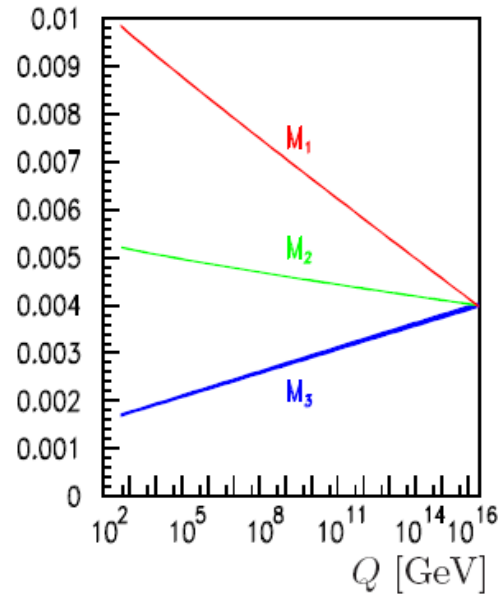
tanb	10	10	1.6	MselecL	195.5	195.5	2.5	MSQL1	545.9	545.9	7.9
M1	101.4	101.4	0.7	MselecR	136	136	0.9	MsuR	527.9	527.8	23
M2	191.6	191.6	0.6	MSQL3	497.1	497.1	13	MsdR	525.8	526	23
M3	586.7	586.7	8.7	MstopR	421.6	421.6	217	Astau	-251.7	-229	27000
MstaL	194.7	195.9	5600	MsbR	522.5	522.5	14	Astop	-494.6	-494.6	83
MstaR	133.5	133.3	1400	MSQL2	545.9	545.9	7.9	Asb	-795.3	-795.3	5733
MsmL	195.5	195.5	2.5	MscR	527.9	527.8	23	MA	398.9	398.9	913
MsmR	136	136	0.9	MssR	525.8	526	23	mu	357	357	4

Caveats:

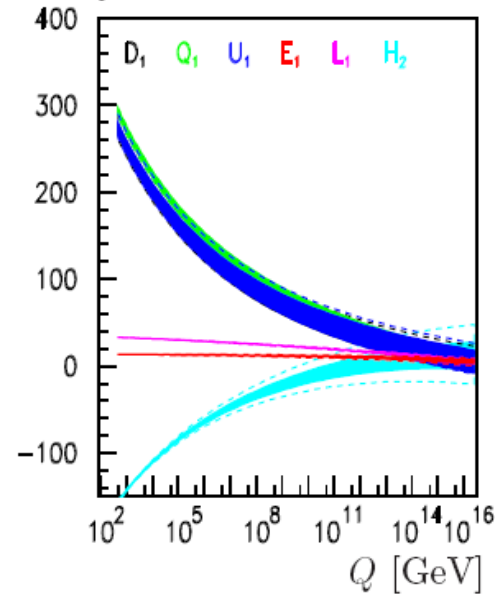
- serious experimental estimate of Xsection uncertainty needed (especially SquarkL)
- net effect wrt stat+syst error of fixing should be small (to be investigated)

Extrapolation to the High Scale

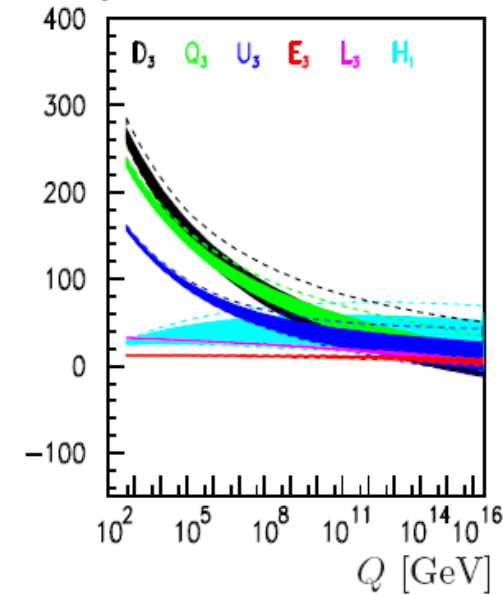
a) M_i^{-1} [GeV⁻¹]



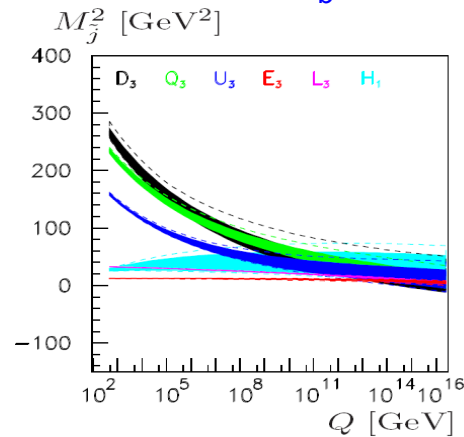
b) M_j^2 [GeV²]



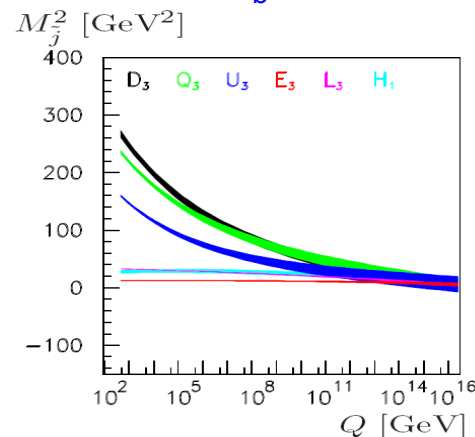
c) M_j^2 [GeV²]



130 % / 180 % A_b precision:



50 % A_b precision:



- Blair et al.,
Fittino with W. Porod:**
- extrapolation shows unification of soft breaking params
 - A_b difficult to measure, search for new observables

Beyond SPS1a @LHC and ILC

No restriction to SPS1a

$m_0 = 1400 \text{ GeV}$ $m_{1/2} = 180 \text{ GeV}$

$A_0 = 700 \text{ GeV}$ $\tan\beta = 51$ $\mu > 0$

LHC Measurements:

- Higgs masses h, H, A
- mass difference $\chi_2 - \chi_1$
- mass difference $\tilde{g} - \chi_2$

Sufficient for MSUGRA

ILC:

- Higgs mass h, H, A
- gauginos

SFitter with P. Gris, L. Serin, L. Tompkins
in Les Houches BSM hep-ph/0602198

Dominant Processes at the LHC:

- $g + g \rightarrow \tilde{g} + \tilde{g}$ (50%)
- $q + \bar{q} \rightarrow \tilde{\chi}_2^0 + \tilde{\chi}_1^\pm$ (20%)
- $f + \bar{f} \rightarrow \tilde{\chi}_1^- + \tilde{\chi}_1^+$ (10%)

Uncertainties:

- b quark mass
- t quark mass
- Higgs mass prediction
- h mass determines m_0
- H, A $\tan\beta$
- essential to take t, b mass and theory errors into account

LHCLC: don't forget $M_{top@LC}$

Similar point studied in MSSM:
Desch et al. hep-ph/0607104

	Egret	$\Delta\text{ILC} + \text{LHC}^{\text{exp}}$	$\Delta\text{ILC}^{\text{exp}}$	$\Delta\text{LHC}^{\text{exp}}$	ΔLHC	$\Delta\text{LHC}^{\text{all}}$
					$\Delta m_t = 1$	
m_0	1400	6.8	7.3	50	95	480
$m_{1/2}$	180	0.1	0.1	2.3	2.3	11
$\tan\beta$	51	0.04	0.03	0.3	2.8	3
A_0	700	13	14	181	300	656

Conclusions

- **Sophisticated tools such as Fittino and SFitter will be essential to determine the fundamental parameters of Supersymmetry:**
 - mass differences, edges and thresholds are more sensitive than masses
 - the LHC will be able to measure the parameters at the level %
 - ILC will improve errors by a factor 10
 - LHC+ILC reduces the model dependence
 - intermediate models (beyond MSUGRA before MSSM) can be studied
 - SLHC reduces LHC errors by factor 2
 - MSSM can be probed at both colliders with sensitivities to different regions of the parameter space
- **Future Studies (esp MSSM) with M.Rauch: MCMC and Markov Chains for characterization of secondary minima (a la Allanach hep-ph/0601089)**
- **The SPA project can help to understand differences between predictions of observables**
- **Above all that: it's difficult, exciting, therefore hope for an early discovery of SUSY at the LHC soon.....**