Precise Prediction of M_W in the MSSM

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based on collaboration with
W. Hollik, D. Stöckinger, A.M. Weber and *G. Weiglein*

1. Motivation

M_W is a precision observable

Experimental situation:

today: LEP2, Tevatron $\Rightarrow$ $\delta M_W^{\text{exp}} = 32$ MeV

Tevatron (8 if^{-}1) $\Rightarrow$ $\delta M_W^{\text{exp}} = 20$ MeV

LHC $\Rightarrow$ $\delta M_W^{\text{exp}} = 15$ MeV

ILC $\Rightarrow$ $\delta M_W^{\text{exp}} = 10$ MeV

GigaZ $\Rightarrow$ $\delta M_W^{\text{exp}} = 7$ MeV

Prediction in the SM:

$$\delta M_W^{\text{theory,SM,today}} \approx \pm 4 \text{ MeV}$$

$$\delta M_W^{\text{theory,SM,future}} \approx \pm 2 \text{ MeV}$$

$\delta m_t = 2.3$ GeV : $\delta M_W^{m_t,\text{para,today}} \approx \pm 14$ MeV

$\delta m_t = 0.1$ GeV : $\delta M_W^{m_t,\text{para,future}} \approx \pm 1$ MeV

$\delta(\Delta\alpha_{\text{had}})$: $\delta M_W^{\Delta\alpha,\text{para,today}} \approx \pm 6.5$ MeV

$\delta(\Delta\alpha_{\text{had}})$: $\delta M_W^{\Delta\alpha,\text{para,future}} \approx \pm 1$ MeV

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Prediction in the SM:

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SUSY?

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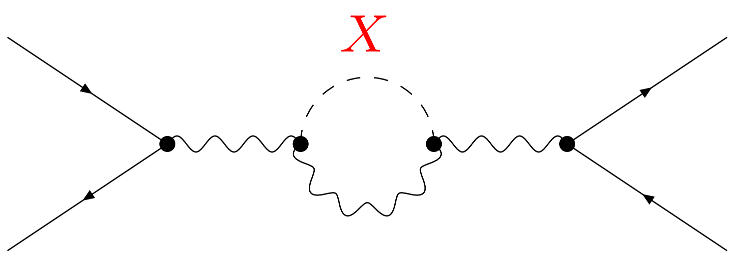
Precision Observables (POs):

Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
M_W, $\sin^2\theta_{\text{eff}}, a_\mu$, ...	$\leftrightarrow$	SM, MSSM , ...

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections



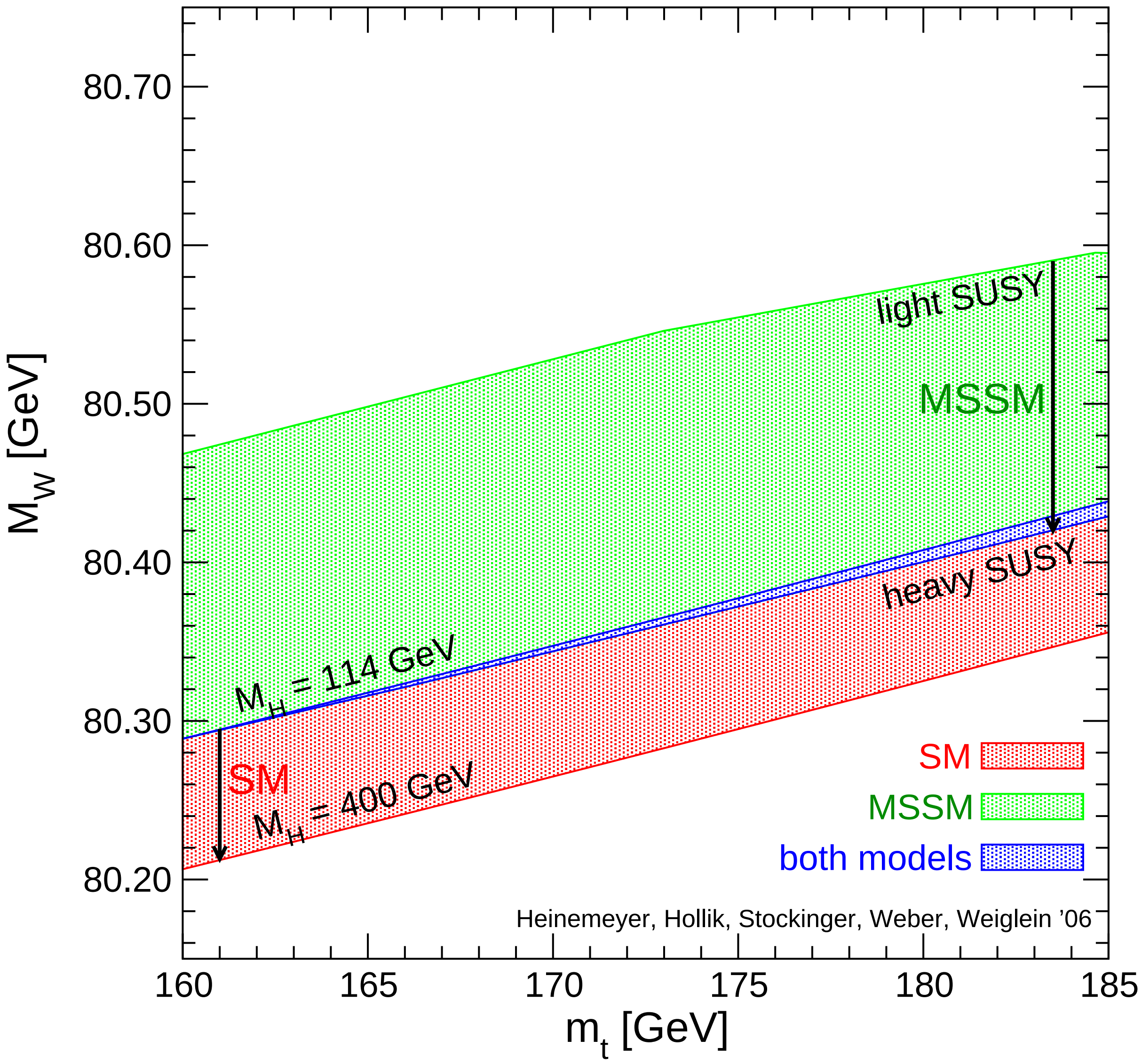
$\Downarrow$

Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

Prediction for M_W in the SM and the MSSM :

[*S.H., W. Hollik, D. Stockinger, A.M. Weber, G. Weiglein '06*]



MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of M_H^{SM}

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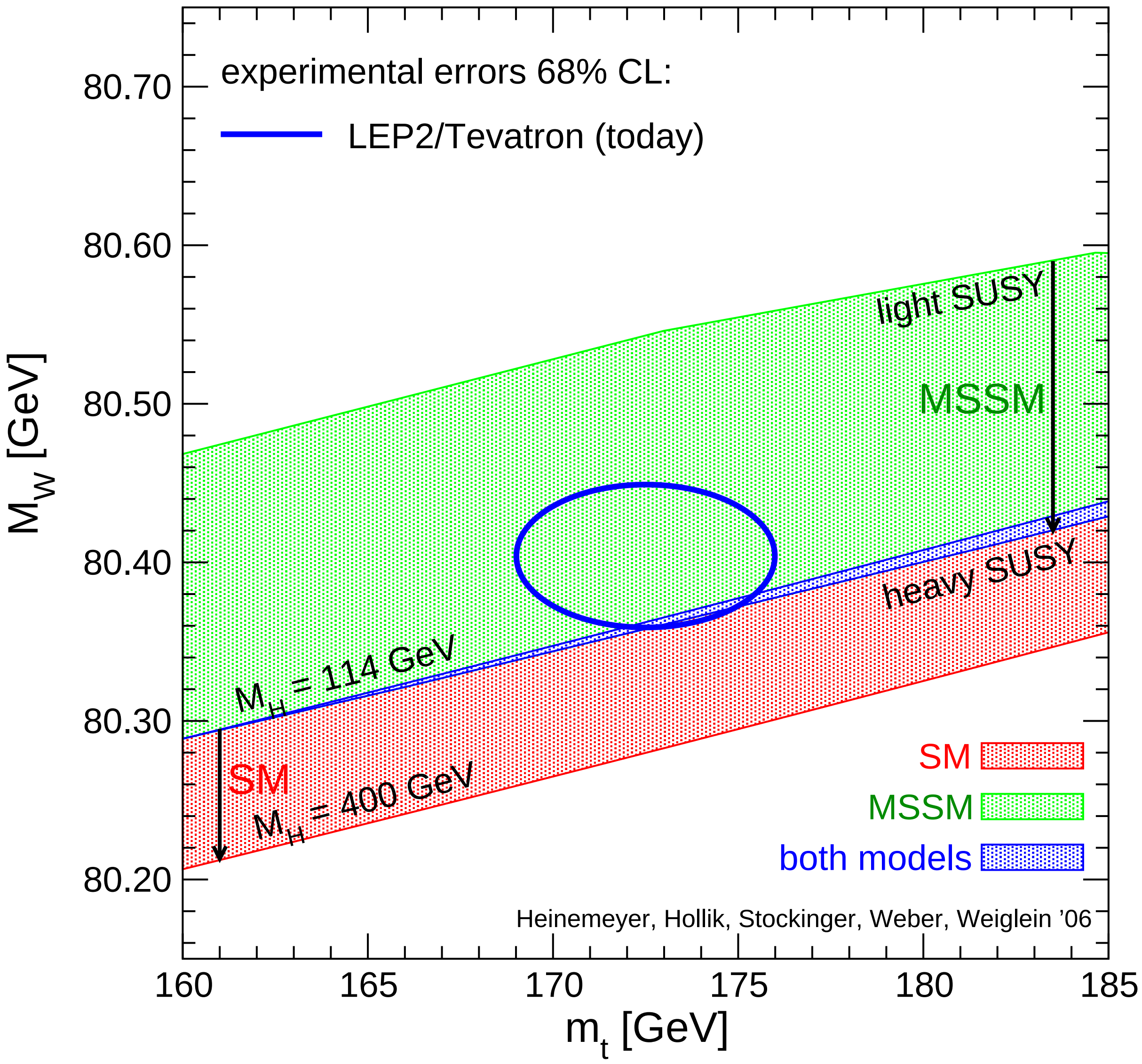

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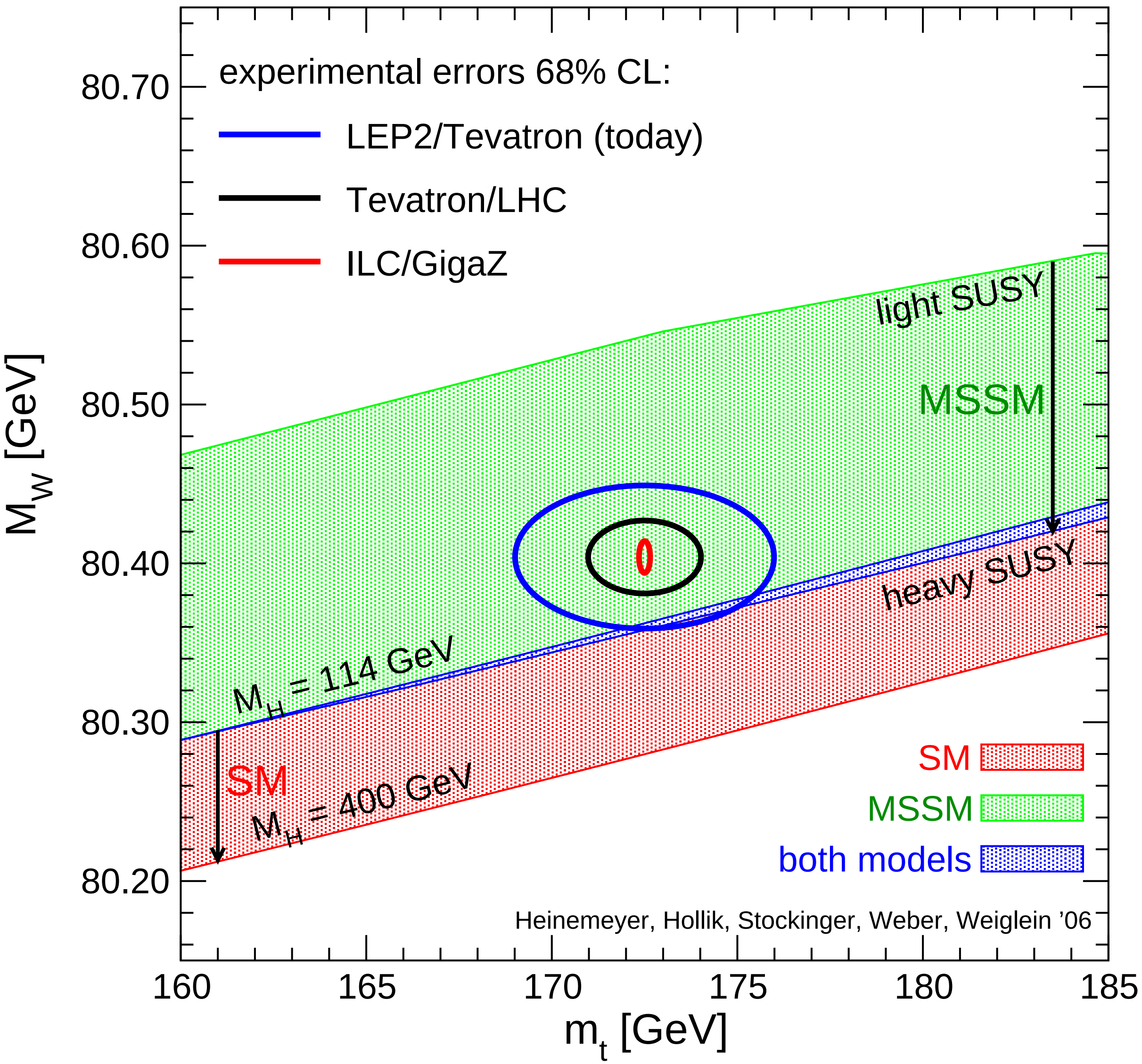

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SM band:
variation of M_H^{SM}

2. Calculation of M_W in the MSSM

Theoretical prediction for M_W in terms of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi \, \alpha}{\sqrt{2}\, G_\mu} (1 + \Delta r)$$

$\Updownarrow$

loop corrections

Evaluate Δr from μ decay $\Rightarrow M_W$

One-loop result for M_W in the SM:

[*A. Sirlin '80*] , [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{\text{1-loop}} = \quad \Delta\alpha \quad - \quad \frac{c_{\text{W}}^2}{s_{\text{W}}^2} \Delta\rho \quad + \quad \Delta r_{\text{rem}}(M_H)$$

$$\sim \log \frac{M_Z}{m_f} \qquad \sim m_t^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

Combination of SM and MSSM result

$\rightarrow$ use best available SM result

$\rightarrow$ add all available MSSM corrections

$\rightarrow$ subtract double counting

$\Rightarrow$ decoupling limit ok ($M_{\text{SUSY}} \rightarrow \infty$)

. . . but some corrections included only via their SM part

$\Rightarrow$ best solution

Status of SM calculation

- Most recently full electroweak two-loop result available
  [*Awramik, Czakon, Freitas, Hollik, Onishenko, Veretin, Walter, Weiglein '03, '04*]
- Most recent electroweak three-loop corrections via $\Delta\rho$
  [*Faisst, Kühn, Seidensticker, Veretin '04*]
- Compact parametric formula used
  [*Awramik, Czakon, Freitas, Weiglein '04*]

Status of MSSM calculations of M_W

- MSSM, Δr: full one-loop corrections
  [*P. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski, J. Rosiek '94*]
  [*D. Garcia, J. Solà '94*]
- MSSM, $\Delta\rho$: leading $\mathcal{O}(\alpha\alpha_s)$ corrections
  [*A. Djouadi, P. Gambino, S.H., W. Hollik, C. Jünger, G. Weiglein '97*]
- MSSM, Δr: leading gluonic $\mathcal{O}(\alpha\alpha_s)$ corr.
  [*S.H. '98*] [*S.H., W. Hollik, G. Weiglein '04*]
- MSSM, $\Delta\rho$: leading $\mathcal{O}\left(\alpha_t^2, \alpha_t\alpha_b, \alpha_b^2\right)$ corrections
  [*S.H., G. Weiglein '01, '03*]
  [*J. Haestier, S.H., D. Stöckinger, G. Weiglein '05*]

Most recent:

- MSSM, Δr: one-loop with complex parameters
  [*S.H., W. Hollik, D. Stöckinger, A.M. Weber, G. Weiglein '06*]

Most dominant:

- $\Delta\rho$: one- or two-loop, contribution from $\tilde{t}/\tilde{b}$ sector

More details about our calculation:

- Complex phases in the squark sector enter only via shift in squark masses (explicit dependence drops out)

$$|X_t|^2 = |A_t|^2 + |\mu \cot\beta|^2 - 2|A_t| \cdot |\mu| \cot\beta \cos(\phi_{A_t} + \phi_\mu)$$
$$|X_b|^2 = |A_b|^2 + |\mu \tan\beta|^2 - 2|A_b| \cdot |\mu| \tan\beta \cos(\phi_{A_b} + \phi_\mu)$$

  Only some phase combinations are physical,
  other phases can be rotated away.
  Examples for physical combinations:

$$\phi_{A_t} + \phi_\mu$$
$$\phi_{A_b} + \phi_\mu$$

- Higgs mass dependence of the two-loop contributions is know to be very strong
  $\Rightarrow$ we use *FeynHiggs* (www.feynhiggs.de)

- All one-loop calculations have been performed with *FeynArts* and *FormCalc*
  [*T. Hahn et al '00 - '05*]

Treatment of the phase dependence beyond one-loop order

Phase dependence at the two-loop level approximated by a simple interpolation based on:

full phase dependence at the one-loop level, $M_W^{\text{1L}}(\phi)$,

two-loop results for real parameters, $M_W^{\text{full}}(0)$, $M_W^{\text{full}}(\pi)$

⇒ Two-loop result for complex phase ϕ:

$$M_W^{\text{full}}(\phi) = M_W^{\text{1L}}(\phi) + \left[M_W^{\text{full}}(0) - M_W^{\text{1L}}(0)\right] \times \frac{1+\cos\phi}{2} + \left[M_W^{\text{full}}(\pi) - M_W^{\text{1L}}(\pi)\right] \times \frac{1-\cos\phi}{2}$$

3. Numerical results

3A) phase dependence from squark sector

Complex phases in the squark sector enter only
via shift in squark masses (explicit dependence drops out)
$\Rightarrow$ phase dependence must be reflected in the squark masses

Only some phase combinations are physical,
other phases can be rotated away.
Examples for physical combinations:

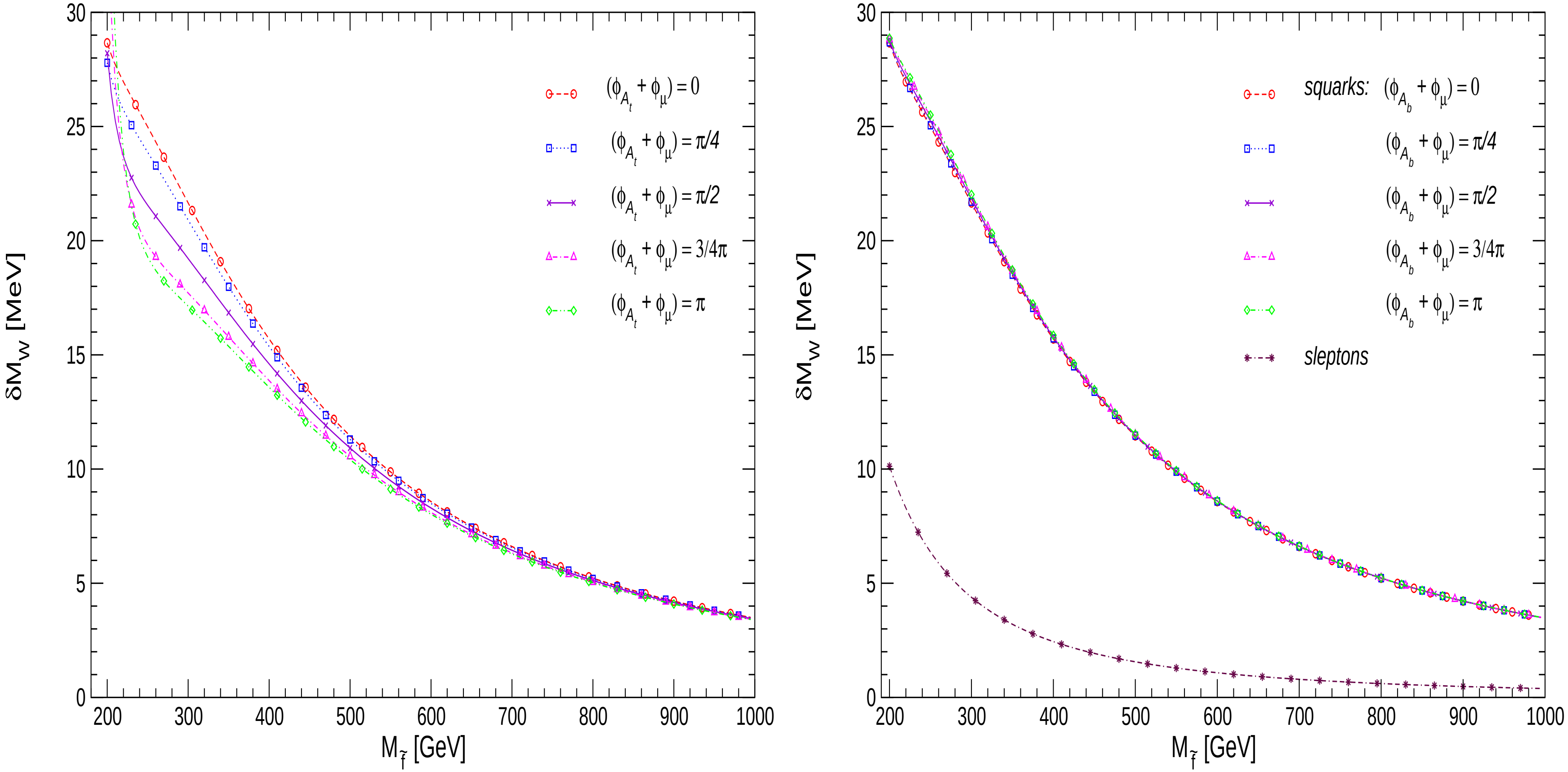
$$\phi_{A_t} + \phi_\mu$$
$$\phi_{A_b} + \phi_\mu$$

δM_W evaluated from

$$\delta M_W = -\frac{M_W}{2}\frac{s_\mathrm{W}^2}{c_\mathrm{W}^2 - s_\mathrm{W}^2}\Delta r$$

δM_W dependence on ϕ_{A_t} and ϕ_{A_b} (I):

$(|A_{t,b}| = 350$ GeV, $\mu = 300$ GeV, $\tan\beta = 10)$

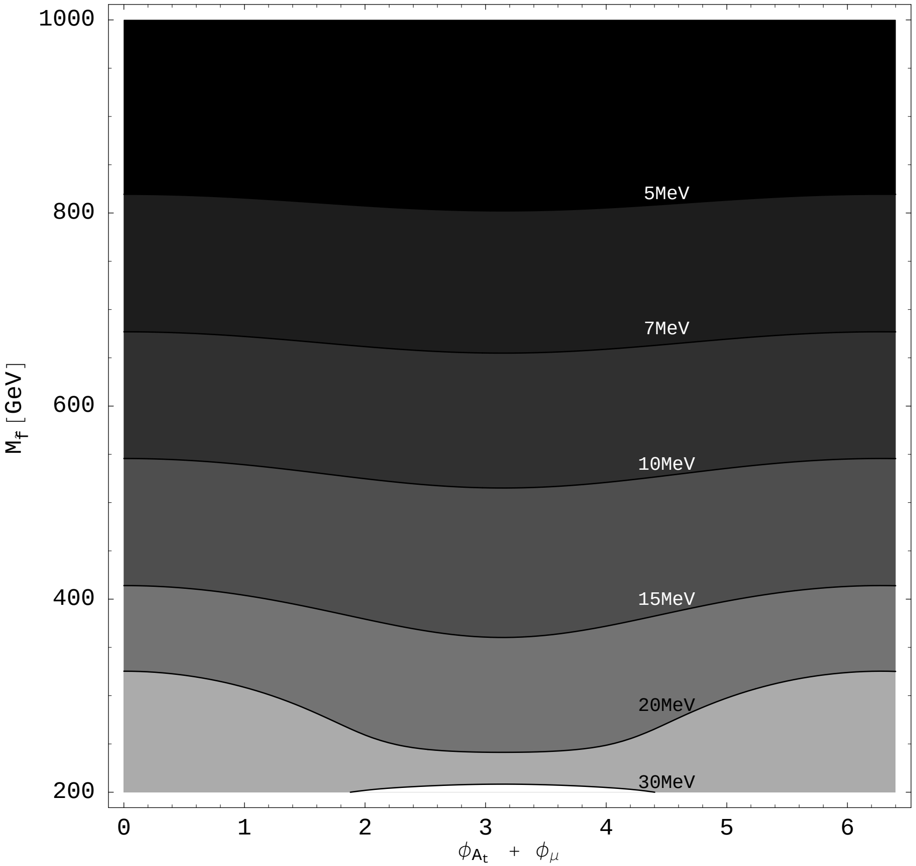


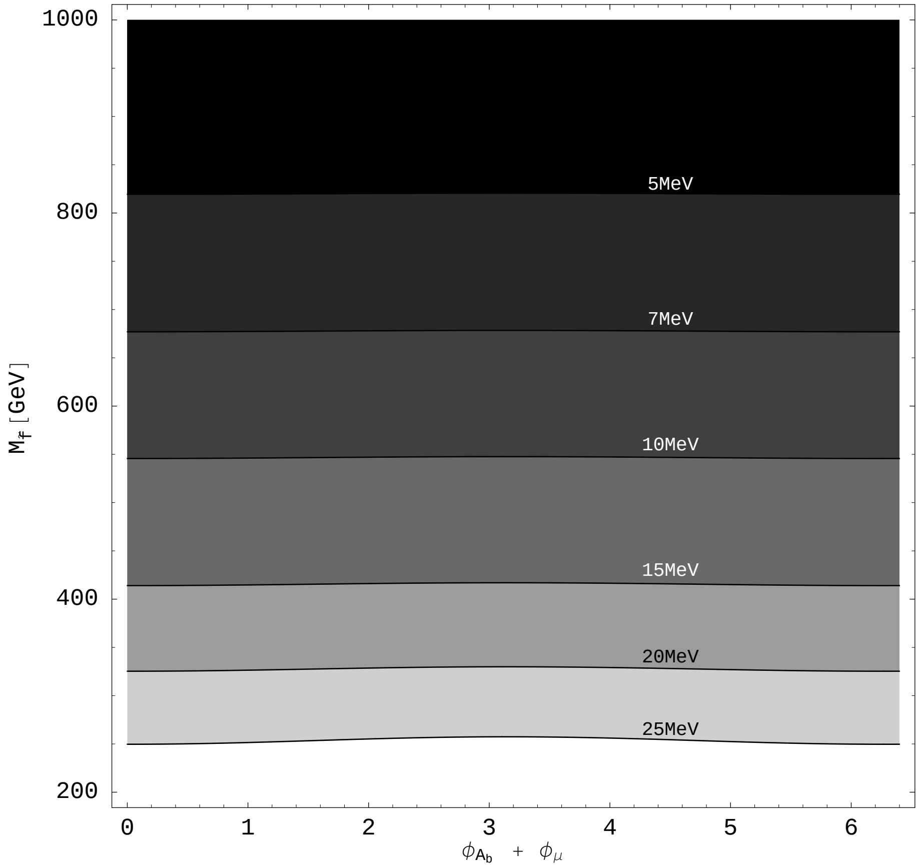
$\Rightarrow$ large squark contribution, decoupling with M_{SUSY}

$\Rightarrow$ ϕ_{A_t} dependence, but no ϕ_{A_b} effects

δM_W dependence on ϕ_{A_t} and ϕ_{A_b} (II):

$(|A_{t,b}| = 350$ GeV, $\mu = 300$ GeV, $\tan\beta = 10)$

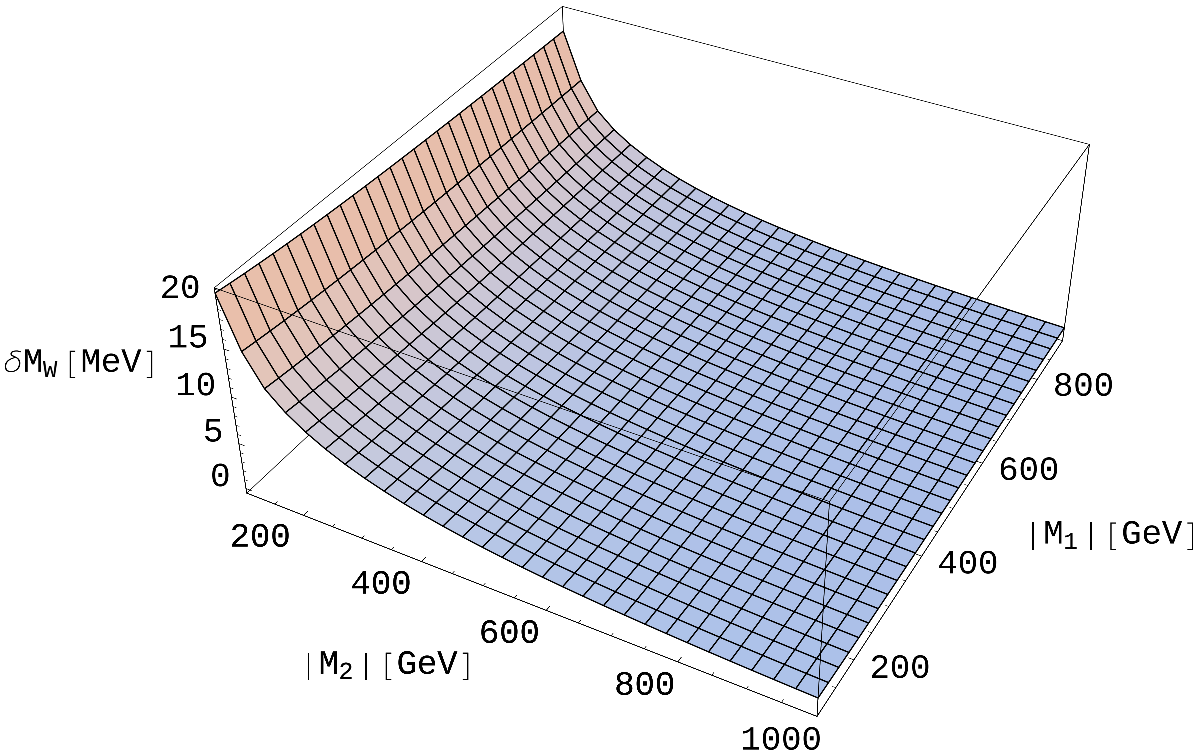




$\Rightarrow$ large squark contribution, decoupling with $M_{\rm SUSY}$

$\Rightarrow$ ϕ_{A_t} dependence, but no ϕ_{A_b} effects

3B) dependence on chargino/neutralino sector



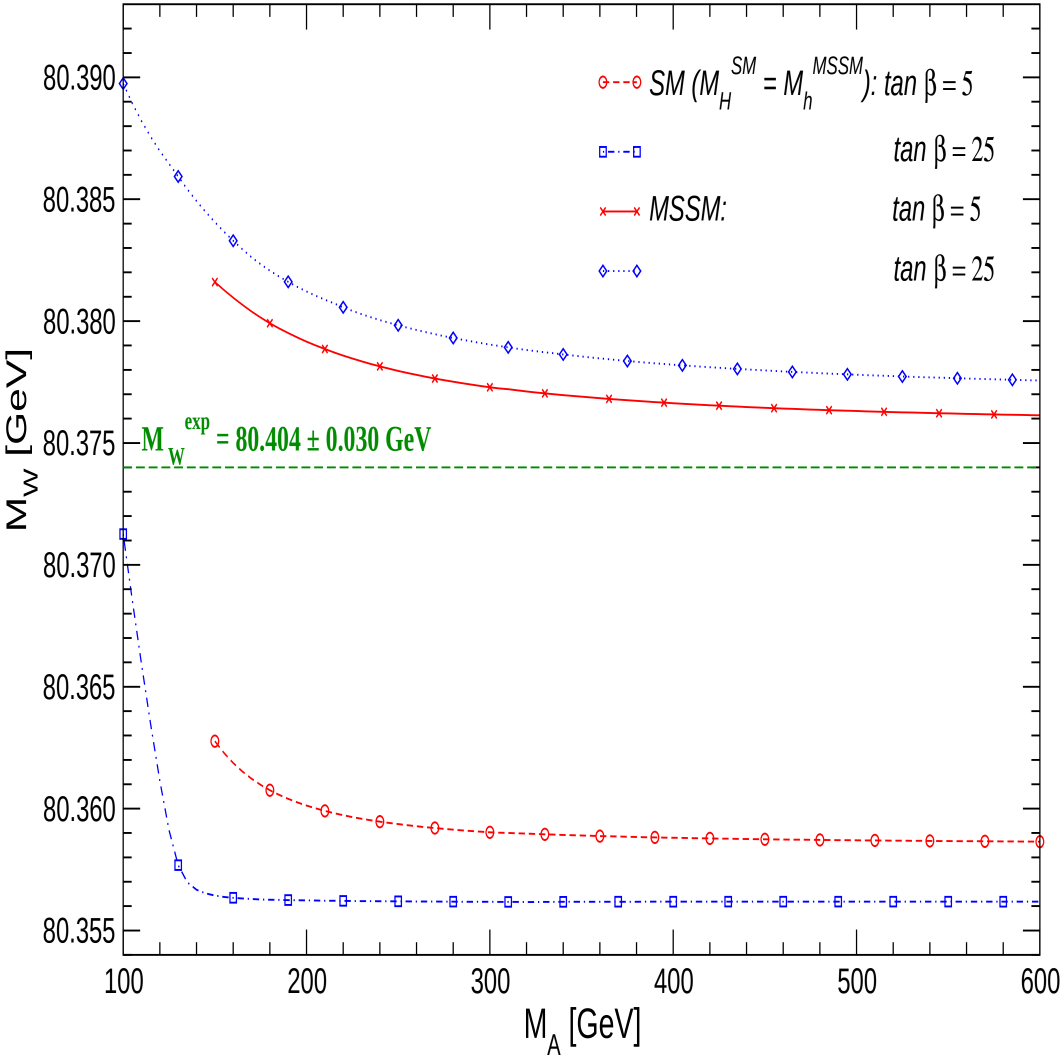
– hardly any M_1 dependence

– up to 20 MeV contribution via M_2

other parameters:
$M_{SUSY} = 250$ GeV
$\mu = 300$ GeV,
$\phi_{M_{1,2}} = 0$
$\tan\beta = 10$

3C) Prediction of M_W

Comparison with SM:



other parameters:

$M_{\text{SUSY}} = 600$ GeV

$A_{t,b} = 1200$ GeV

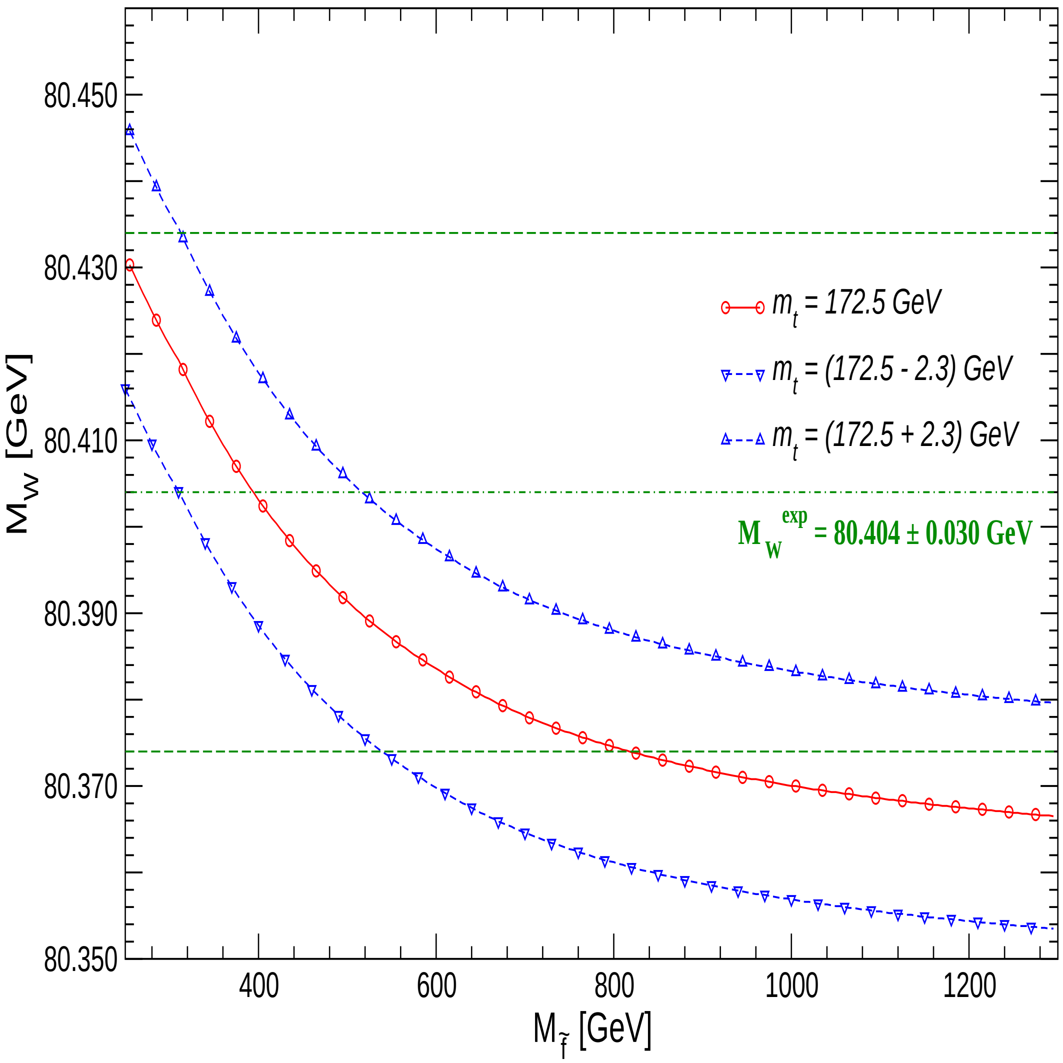
$\mu = M_2 = m_{\tilde{g}} = 300$ GeV,

$\phi_x = 0$

$M_W^{\text{SUSY}} - M_W^{\text{SM}} \gtrsim 20$ MeV
(typical difference)

Comparison with M_W^{exp}:
MSSM agrees at 1σ
SM agrees at 2σ

Prediction of M_W: variation of m_t



other parameters:

$A_{t,b} = 2\, M_{\text{SUSY}}$

$\mu = M_2 = m_{\tilde{g}} = 300$ GeV,

$\phi_x = 0$

$\tan\beta = 10$

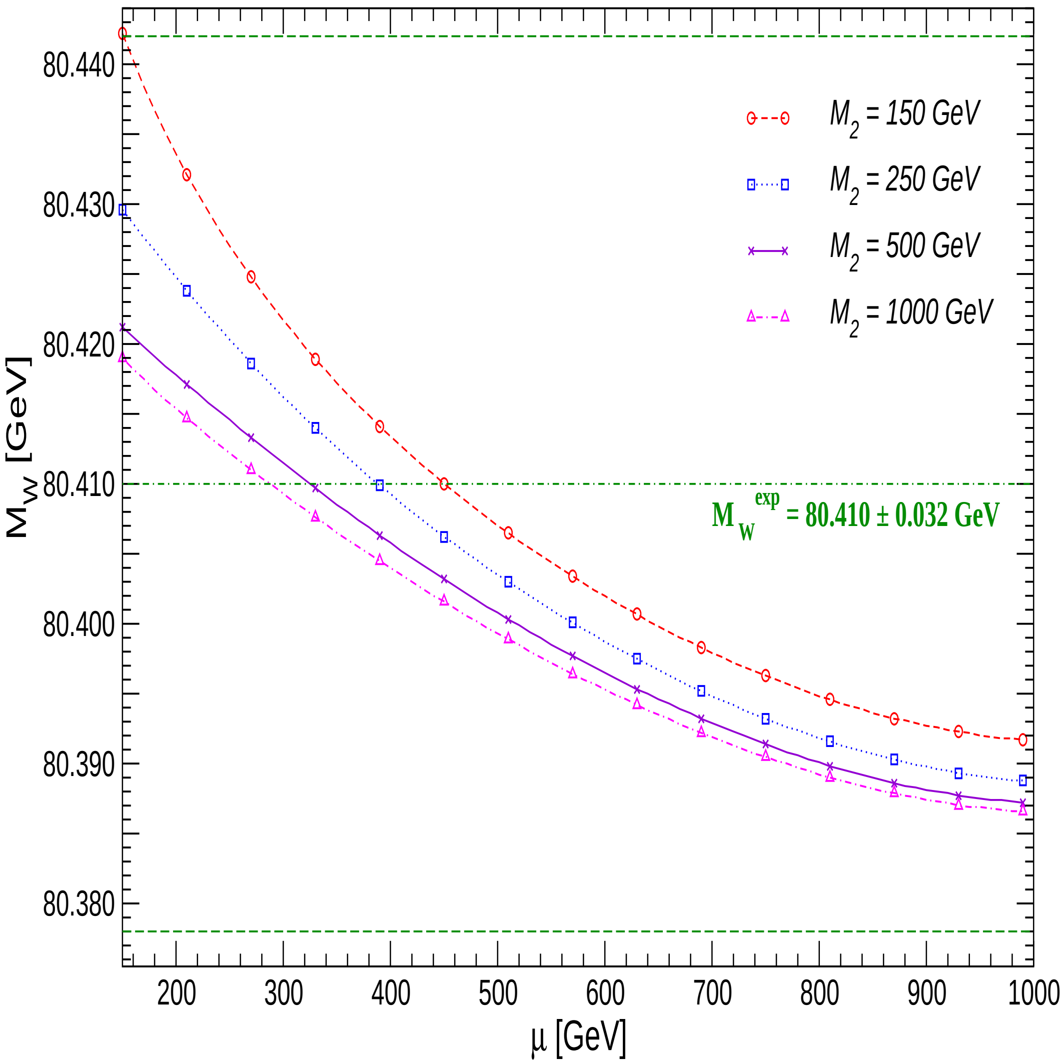
$\delta m_t = \pm 2.3$ GeV

$\Rightarrow \delta M_W \approx \pm 14$ MeV

lower m_t disfavored

$\Rightarrow$ largest parametric uncertainty

Prediction of M_W: variation of μ and M_2



other parameters:

$M_{\text{SUSY}} \quad = \quad 300$ GeV

$A_{t,b} = 2\,M_{\text{SUSY}}$

$m_{\tilde{g}} = 300$ GeV,

$\phi_x = 0$

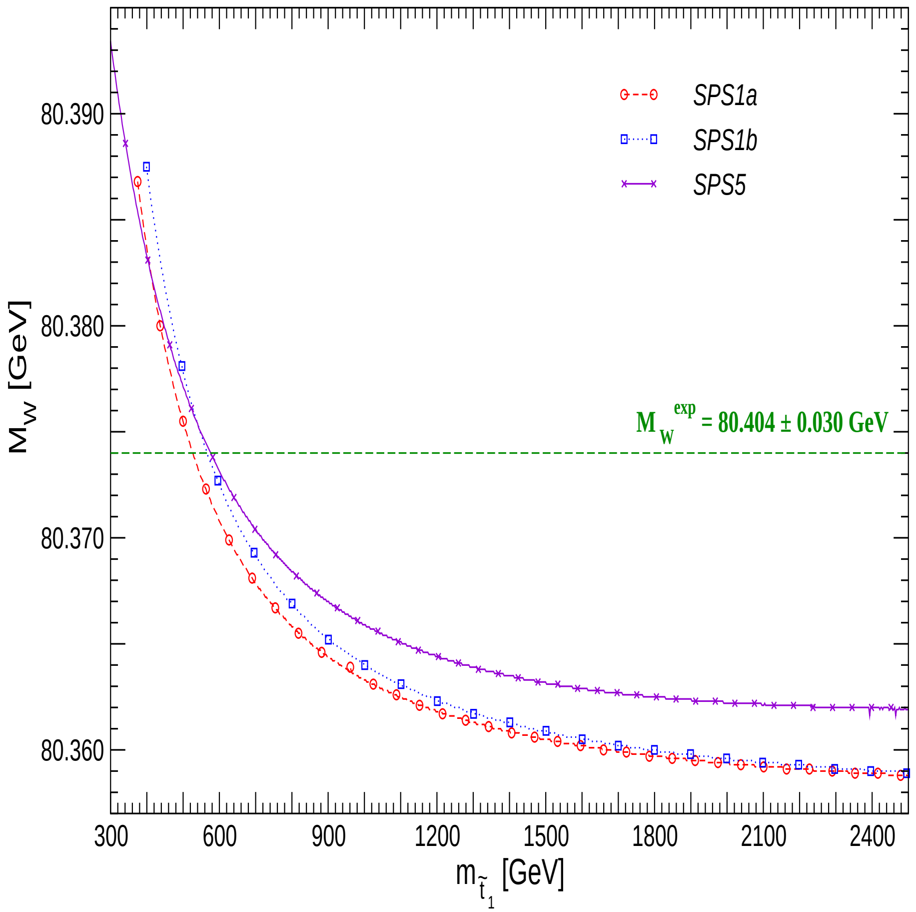
$\tan\beta = 10$

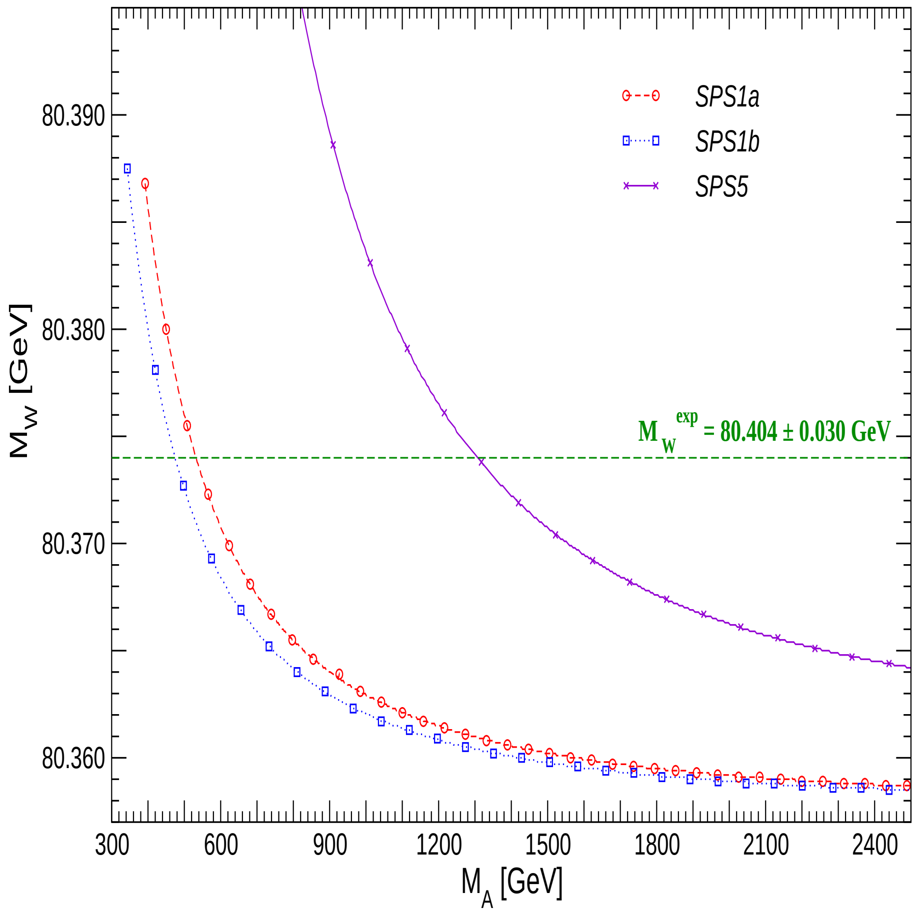
$M_A = 1000$ GeV

$\Rightarrow$ large variation with μ

$\Rightarrow$ smaller variation with M_2

Prediction of M_W: typical scenarios: SPS 1a, 1b, 5

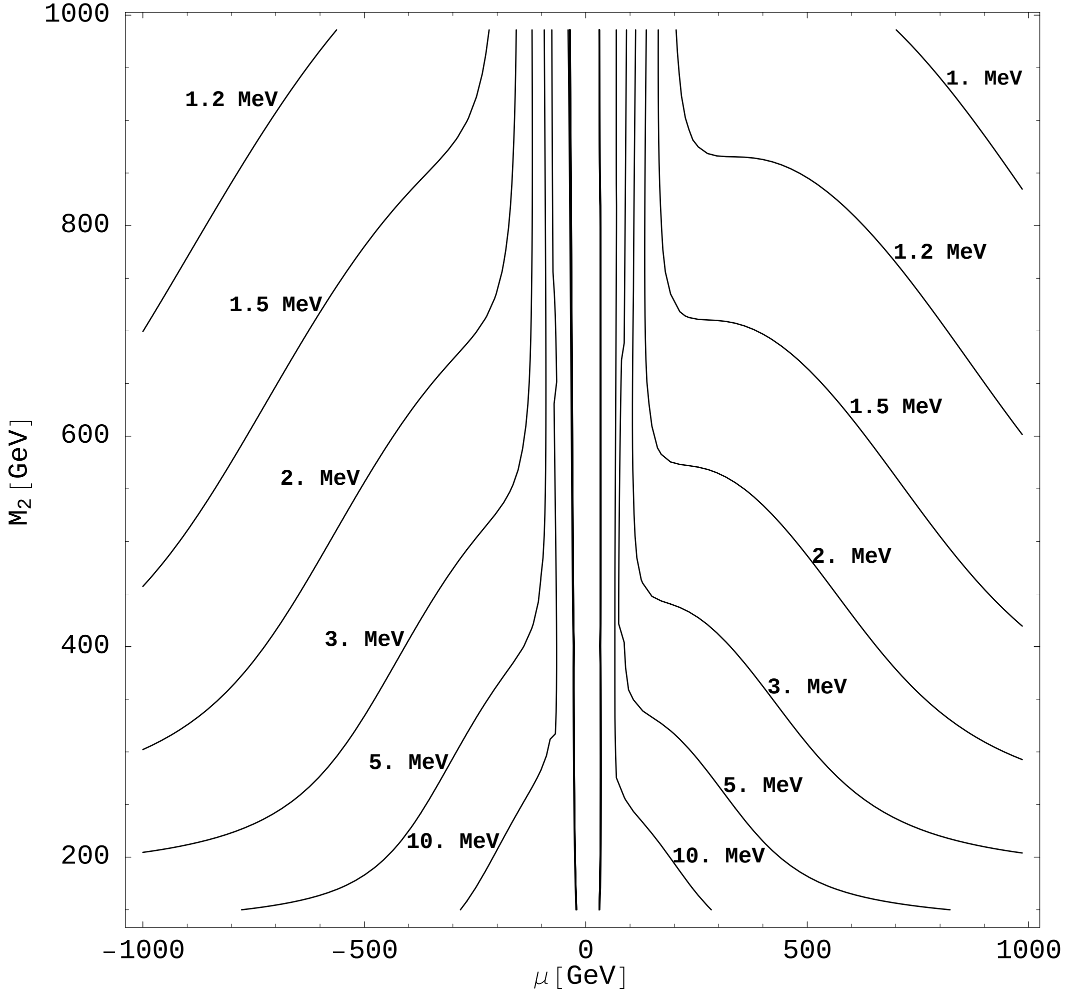




$\Rightarrow$ only "lower" masses show agreement at 1σ

Prediction of M_W: Split SUSY

[*N. Arkani-Hamed, S. Dimopoulos '04*] [*G. Giudice, A. Romanino '04*]



Difference to M_W^{SM} with $M_H^{\text{SM}} = M_h$

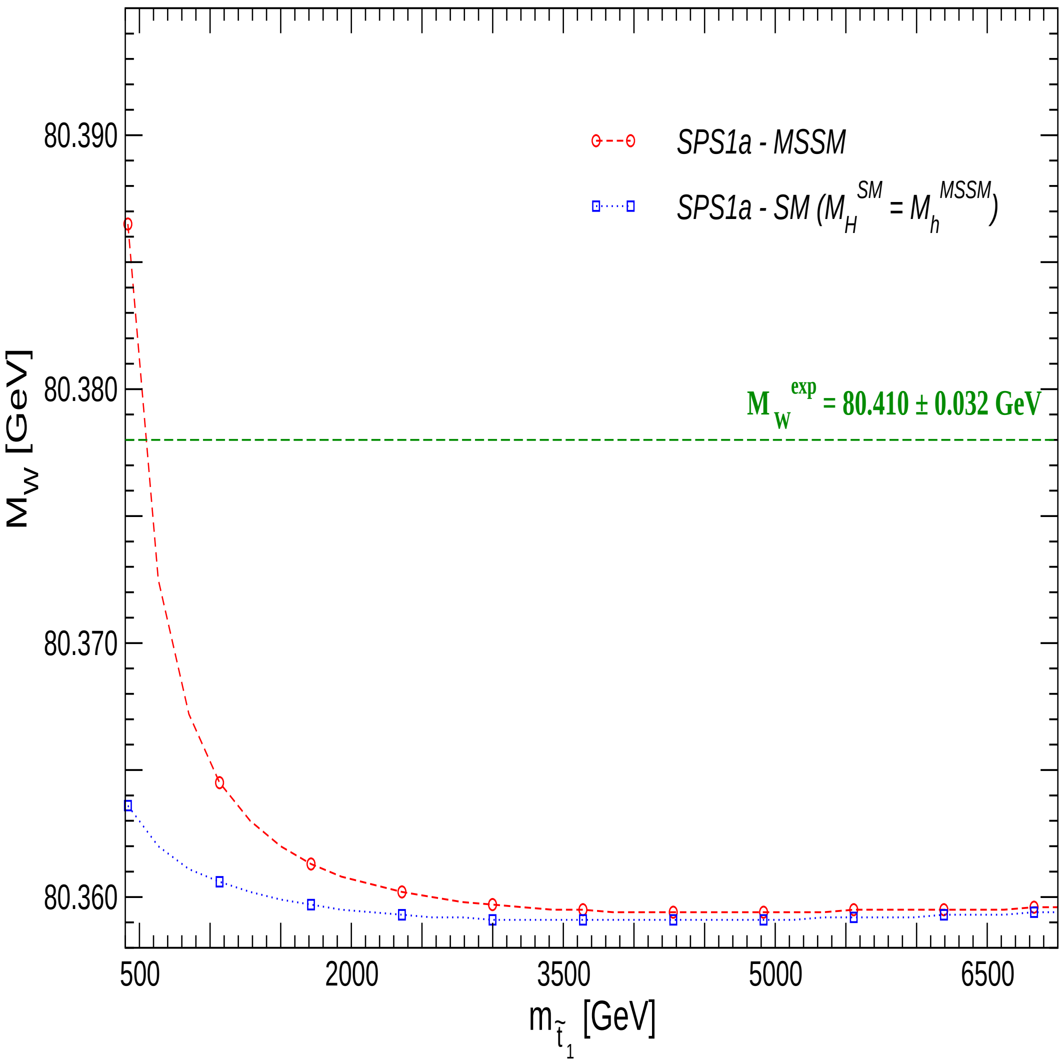
$\Rightarrow$ no deviation for Tevatron, LHC, ILC precision

$\Rightarrow$ GigaZ can confirm only very light masses

(as expected . . .)

Split SUSY disagrees with experiment at the 2σ level

Prediction of M_W: Decoupling limit



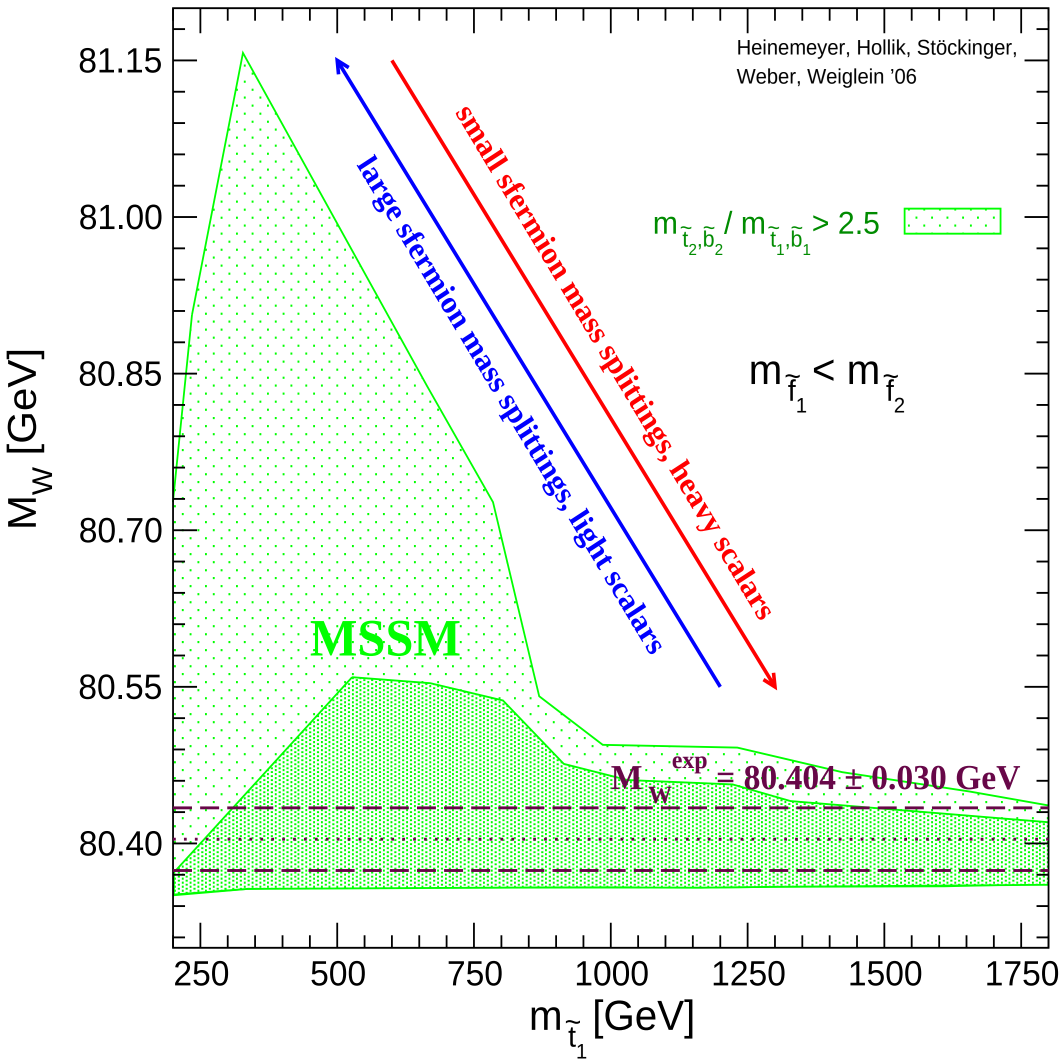
Compared to SM with
$M_H^{\text{SM}} = M_h$

$M_{\text{SUSY}} = 500$ GeV
$\Rightarrow \delta M_W^{\text{MSSM}-\text{SM}} > 20$ MeV

$M_{\text{SUSY}} = 1$ TeV
$\Rightarrow \delta M_W^{\text{MSSM}-\text{SM}} < 10$ MeV

$M_{\text{SUSY}} > 3$ TeV
$\Rightarrow \delta M_W^{\text{MSSM}-\text{SM}} < 1$ MeV

Prediction of M_W: Parameter scan



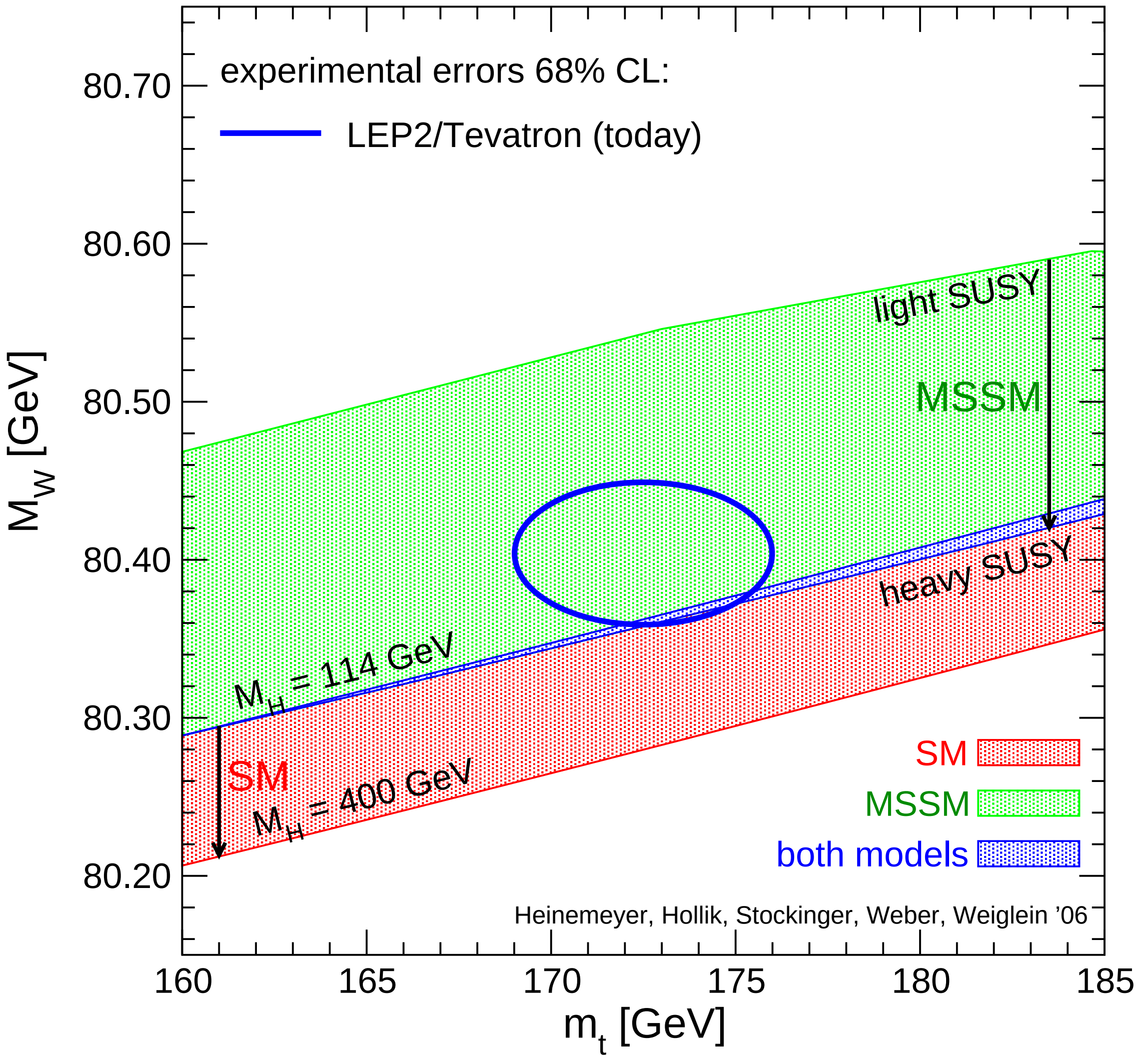
Scatter all relevant parameters independently over wide ranges:
masses $\lesssim 2$ TeV
$\tan\beta = 1.1 \ldots 60$

Experimental constraints from LEP, Tevatron included

huge splitting
$\Rightarrow$ huge correction to M_W
(experimentally excluded)

Impact of M_W on preferred SUSY masses?

Prediction for M_W in the SM and the MSSM :



MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of M_H^{SM}

$\Rightarrow$ M_W is not enough

$\Rightarrow$ use more precision obs.

$\rightarrow$ talk by S.H. later today :-)

4. Remaining theoretical (intrinsic) uncertainties

[J. Haestier, S.H., D. Stöckinger, G. Weiglein '05]

[S.H., W. Hollik, D. Stöckinger, A.M. Weber, G. Weiglein '06]

Estimate missing SUSY corrections order by order:

- $\mathcal{O}\left(\alpha_t^2, \alpha_t\alpha_b, \alpha_b^2\right)$: beyond existing leading contributions
- $\mathcal{O}\left(\alpha\alpha_s\right)$: beyond $\Delta\rho$ approx.
- $\mathcal{O}\left(\alpha\alpha_s^2\right)$
- $\mathcal{O}\left(\alpha^2\alpha_s\right)$
- $\mathcal{O}\left(\alpha^3\right)$
- missing phase dependence at two-loop

$\Rightarrow$ evaluate for $M_{\text{SUSY}} = 300, 500, 1000$ GeV

Combine with SM uncertainty: $\delta M_W^{\text{SM,intr.}} = 4$ MeV

$$\delta M_W^{\text{SUSY,intr.}} = 5 - 11 \text{ MeV}$$

(depending on M_{SUSY})

5. Conclusinos

- Precision observables
  - can give valuable information about the "true" Lagrangian
  - can provide bounds on SUSY parameter space

  $\Rightarrow M_W$ prominent example

- Combination of
  - full SM result
  - all available MSSM corr. $\Rightarrow$ best prediction of M_W in the MSSM

  $\Rightarrow$ remaining uncertainty: $\delta M_W^{\text{intr.}} = 5 - 11$ MeV (depending on M_{SUSY})

- Effects of certain sectors:
  - $\tilde{t}/\tilde{b}$ sector: $\delta M_W \gtrsim 20$ MeV , slepton sector: much smaller
  - complex phases in $\tilde{t}/\tilde{b}$ sector: $\delta M_W \gtrsim 5$ MeV
  - chargino/neutralino sector: $\delta M_W \gtrsim 20$ MeV
  - $\delta m_t = \pm 2.3$ GeV $\Rightarrow \delta M_W \approx \pm 14$ MeV $\Rightarrow$ largest parametric unc.

- Prediction of M_W in special scenarios:
  - SPS: only "lower" masses show agreement at 1σ
  - Split SUSY: basically no visible effect
  - Decoupling limit: $M_{\text{SUSY}} > 3$ TeV $\Rightarrow \delta M_W^{\text{MSSM}-\text{SM}} < 1$ MeV
  - Parameter scan: very large M_W only for very large splitting in $\tilde{t}/\tilde{b}$