Precise Calculations for the r/cMSSM Higgs Sector: FeynHiggs2.4

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based on collaboration with
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1. Motivation

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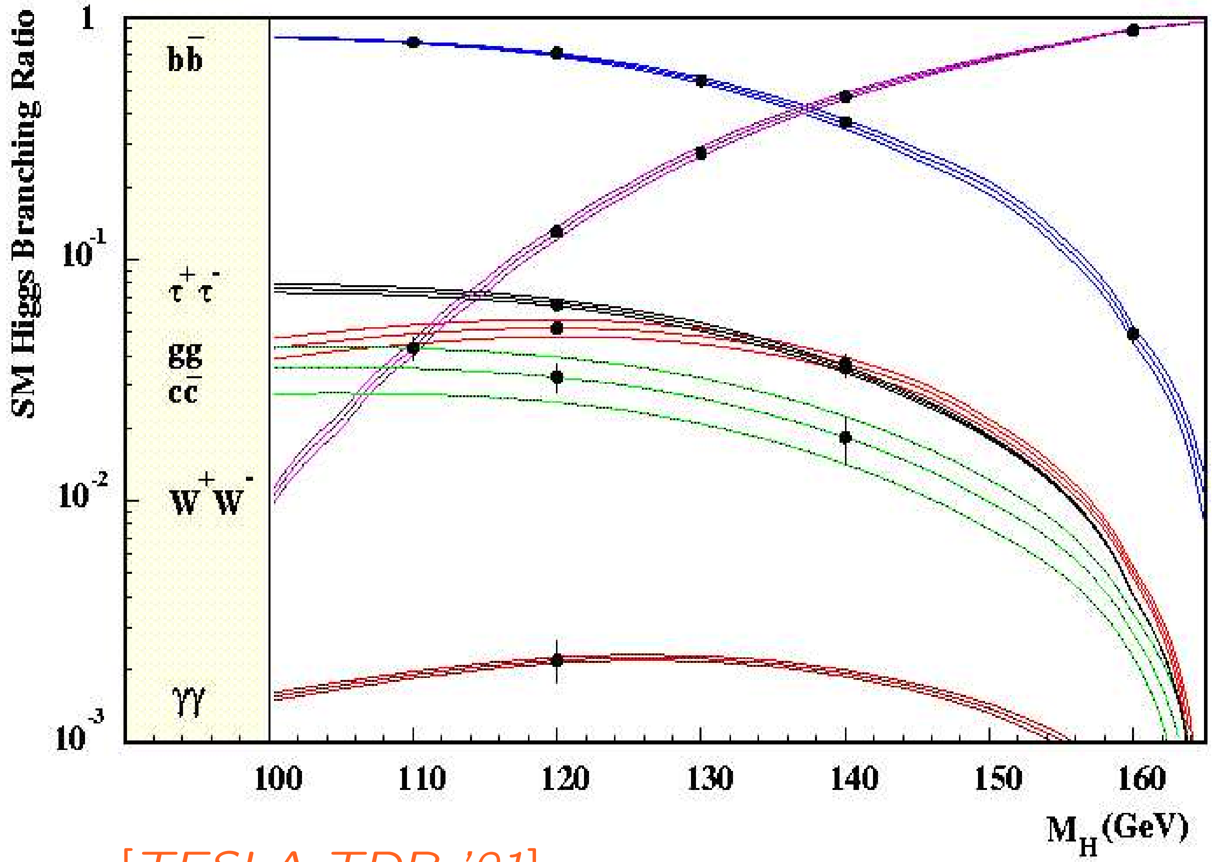
1. Motivation

SM Higgs @ ILC:
Precise measurement of:

1. Higgs boson mass,
   $\delta M_H \approx 50$ MeV
2. Higgs boson width
   (direct/indirect)
3. Higgs boson couplings,
   $\mathcal{O}$ (few%) ⇒
4. Higgs boson quantum
   numbers: spin, . . .



[*TESLA TDR '01*]

MSSM: similar precision expected (possible problems from loop corrections)

Q: Can this precision be utilized in the MSSM Higgs sector?

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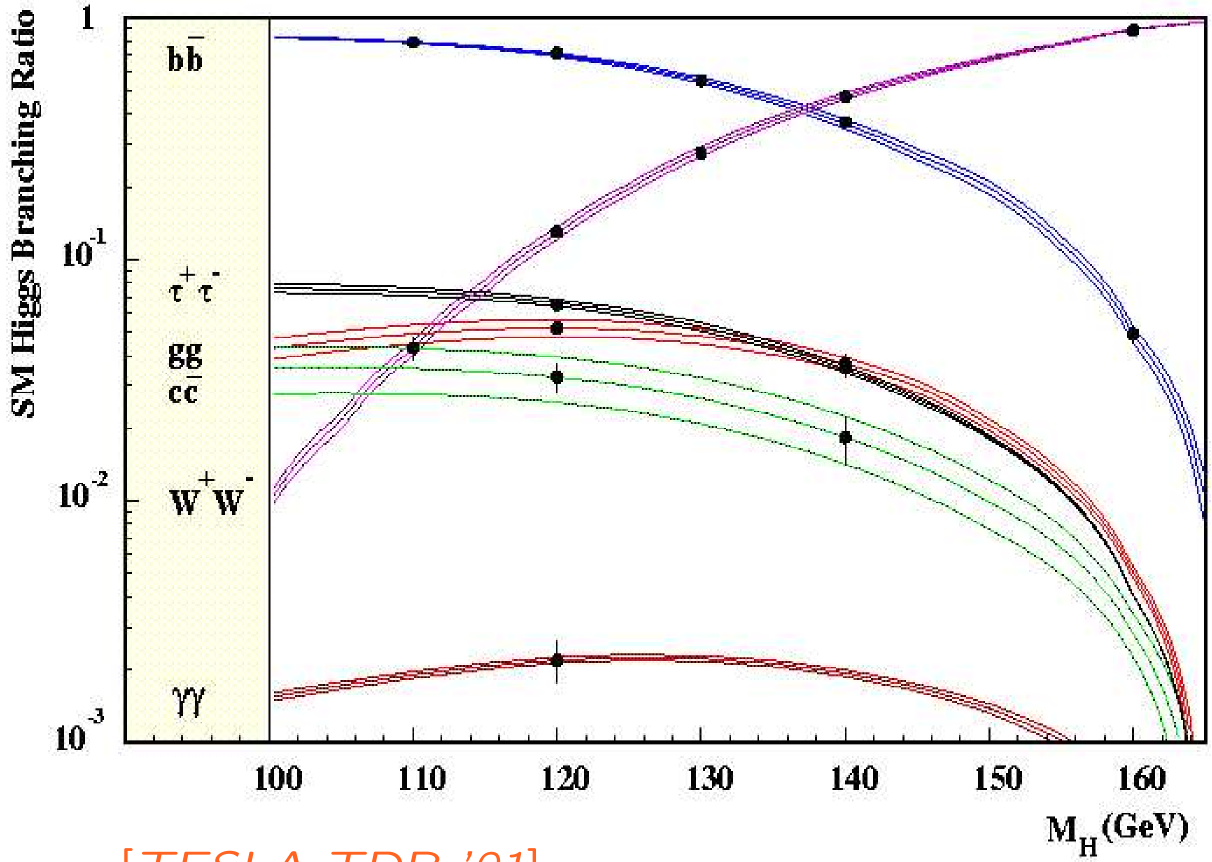

[*TESLA TDR '01*]

MSSM: similar precision expected (possible problems from loop corrections)

Q: Can this precision be utilized in the MSSM Higgs sector?

A: Yes! . . . if the theory predictions are as precise

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 - i\chi_1)/\sqrt{2} \\ -\phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$
$$+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters:

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

Contrary to the SM:

M_h is not a free parameter

MSSM tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:

$$\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Measurement of M_h, Higgs couplings $\Rightarrow$ test of the theory

LHC: $\Delta M_h \approx 0.2$ GeV, ILC: $\Delta M_h \approx 0.05$ GeV

$\Rightarrow$ aim for theoretical precision!

($\Rightarrow M_h$ will be (the best?) electroweak precision observable)

The complex case:

Higgs potential of the cMSSM contains two Higgs doublets:

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 - i\chi_1)/\sqrt{2} \\ -\phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

Five physical states: $h^0, H^0, A^0, H^\pm$ (no $\mathcal{CP}$V at tree-level)

2 $\mathcal{CP}$-violating phases: ξ, $\arg(m_{12})$ $\Rightarrow$ can be set/rotated to zero

Input parameters: $\tan\beta = \frac{v_2}{v_1}$ and $M_{H^\pm}$

Effects of complex parameters in the Higgs sector:

Complex parameters enter via loop corrections:

– μ : Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings $\Rightarrow$ $X_{t,b,\tau} = A_{t,b} - \mu^* \{\cot\beta, \tan\beta\}$ complex

– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– $m_{\tilde{g}}$: gluino mass

$\Rightarrow$ can induce $\mathcal{CP}$-violating effects

Result:

$$(A, H, h) \rightarrow (h_3, h_2, h_1)$$

with

$$M_{h_3} > M_{h_2} > M_{h_1}$$

How to include higher-order corrections: ($\to$ Feynman-diagrammatic approach)

Propagator / mass matrix with higher-order corrections:

$$M^2_{hHA}(q^2) = \begin{pmatrix} q^2 - M_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i,j = h, H, A)$: renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0$ $\Rightarrow$ $\mathcal{CP}$V, $\mathcal{CP}$-even and $\mathcal{CP}$-odd fields can mix

Our result for $\hat{\Sigma}_{ij}$:

– full 1-loop: complex phases, q^2-dep., imaginary parts

– currently implemented: cMSSM $\mathcal{O}(\alpha_t\alpha_s)$ corrections in the FD approach
rMSSM: difference between FD and RGiEP approach $\mathcal{O}$ (few GeV)

$\Rightarrow$ numerical search for the complex roots of det$(M^2_{hHA}(q^2))$

Result for $q^2 \neq 0$:

$$(A, H, h) \to (h_3, h_2, h_1)$$

Limit of $\hat{\Sigma}(q^2) \to \hat{\Sigma}(0)$:

$$\begin{pmatrix} M^2_{h_3}(0) & 0 & 0 \\ 0 & M^2_{h_2}(0) & 0 \\ 0 & 0 & M^2_{h_1}(0) \end{pmatrix} = U \; M^2_{hHA}(0) \; U^+$$

$$\begin{pmatrix} h_3 \\ h_2 \\ h_1 \end{pmatrix} = U \cdot \begin{pmatrix} A \\ H \\ h \end{pmatrix} , \qquad U = \begin{pmatrix} U_{33} & U_{32} & U_{31} \\ U_{23} & U_{22} & U_{21} \\ U_{13} & U_{12} & U_{11} \end{pmatrix}$$

$q^2 = 0 \quad \Rightarrow \quad \hat{\Sigma}(0)$ real $\quad \Rightarrow \quad U$ unitary

Treatment of "higher-order" corrected Higgs bosons:

1. external/on-shell Higgs bosons

   amplitude with on-shell Higgs boson i:

$$A_{h_i xy} \sim \sqrt{\hat{Z}_i} \left(\hat{Z}_{ih} C_{hxy} + \hat{Z}_{iH} C_{Hxy} + \hat{Z}_{iA} C_{Axy} \right)$$

   $\hat{Z}_i$, $\hat{Z}_{ij}$: finite wave function renormalizations

   Written more compact with the Z matrix:

$$(Z)_{ij} = \sqrt{\hat{Z}_i}\, \hat{Z}_{ij}$$

   resulting in

$$A_{h_i xy} \sim Z_{ih}\, C_{hxy} + Z_{iH}\, C_{Hxy} + Z_{iA}\, C_{Axy}$$

2. internal Higgs bosons

   rotate tree-level couplings with U:

$$C_{h_i xy} = U_{ih}\, C_{hxy} + U_{iH}\, C_{Hxy} + U_{iA}\, C_{Axy}$$

Limit of $\hat{\Sigma}(q^2) \to \hat{\Sigma}(0)$: $Z_{ij} \to U_{ij}$

2. The code FeynHiggs2.4

Latest version: FeynHiggs2.4.1 (06/06)

version FeynHiggs2.4.2 to be released within two weeks . . .

real MSSM:

contains all available higher-order corrections

to Higgs boson masses and couplings

FeynHiggs contains

– full 1 loop calculations

– all available 2 loop calculations (leading and subleading)

– very leading 3 loop contributions

complex MSSM:

contains nearly all available results

(we are (even currently) working on the rest)

www.feynhiggs.de

*FeynHiggs*2.2 → *FeynHiggs*2.4: main new features

- Complex contributions to Higgs mass matrix taken into account (from $\text{Im}\, B_0(\ldots) \neq 0$)
- Higgs masses are now the real part of the complex pole
- $\Rightarrow$ complex 3×3 mixing matrix Z $\Rightarrow$ on-shell Higgs bosons

  unitary 3×3 mixing matrix U $\Rightarrow$ internal Higgs bosons
- $\Rightarrow$ included in all Higgs production and decay
- inclusion of full one-loop NMFV effects
- Preliminary implementation of LEP Higgs exclusion bounds (to be refined)
- extended implementation of $(g-2)_\mu$: leading SM fermion two-loop contributions

  [*S.H., D. Stöckinger, G. Weiglein '04*]

Included in *FeynHiggs*2.4 (I):

Evaluation of all Higgs boson masses and mixing angles (rMSSM/cMSSM)

- $M_{h_1}, M_{h_2}, M_{h_3}, M_{H^\pm}$, $\alpha_{\rm eff}$, Z_{ij}, U_{ij}, . . .

Evaluation of all neutral Higgs boson decay channels (rMSSM/cMSSM)

- total decay width $\Gamma_{\rm tot}$
- $\mathrm{BR}(h_i \to f\bar{f})$: decay to SM fermions
- $\mathrm{BR}(h_i \to \gamma\gamma, ZZ^{(*)}, WW^{(*)}, gg)$: decay to SM gauge bosons
- $\mathrm{BR}(h_i \to h_1 Z^{(*)}, h_1 h_1)$: decay to gauge and Higgs bosons
- $\mathrm{BR}(h_i \to \tilde{f}_i \tilde{f}_j)$: decay to sfermions
- $\mathrm{BR}(h_i \to \tilde{\chi}_i^\pm \tilde{\chi}_j^\pm, \tilde{\chi}_i^0 \tilde{\chi}_j^0)$: decay to charginos, neutralinos

Evaluation for the SM Higgs (same masses as the three MSSM Higgses)

- total decay width $\Gamma_{\rm tot}^{\rm SM}$
- $\mathrm{BR}(h_i^{\rm SM} \to f\bar{f})$: decay to SM fermions
- $\mathrm{BR}(h_i^{\rm SM} \to \gamma\gamma, ZZ^{(*)}, WW^{(*)}, gg)$: decay to SM gauge bosons

Included in *FeynHiggs*2.4 (II):

Evaluation of all neutral Higgs boson production cross sections at Tevatron/LHC (rMSSM/cMSSM)

SM: most up-to-date, MSSM: additional effective coupling approximation

- $gg \to h_i$: gluon fusion
- $WW \to h_i$, $ZZ \to h_i$: gauge boson fusion
- $W \to Wh_i$, $Z \to Zh_i$: Higgs strahlung
- $b\bar{b} \to b\bar{b}h_i$: Yukawa process
- $b\bar{b} \to b\bar{b}h_i$, $h_i \to b\bar{b}$, one b tagged
- $t\bar{t} \to t\bar{t}h_i$: Yukawa process

Evaluation for the SM Higgs (same masses as the three MSSM Higgses)

- all channels as above

Included in *FeynHiggs*2.4 (III):

Evaluation of all charged Higgs boson decay channels (rMSSM/cMSSM)

- total decay width $\Gamma_{\rm tot}$
- $\mathrm{BR}(H^+ \to f \bar{f}')$: decay to SM fermions
- $\mathrm{BR}(H^+ \to h_i W^+)$: decay to gauge and Higgs bosons
- $\mathrm{BR}(H^+ \to \tilde{f}_i \tilde{f}'_j)$: decay to sfermions
- $\mathrm{BR}(H^+ \to \tilde{\chi}^0_i \tilde{\chi}^+_j)$: decay to charginos and neutralinos

Evaluation of additional couplings:

- $g(V \to V h_i, h_i h_j)$: coupling of gauge and Higgs bosons
- $g(h_i h_j h_k)$: all Higgs self couplings (including charged Higgs)
- $\sigma(\gamma\gamma \to h_i)$: Higgs production XS at a γC

Included in *FeynHiggs*2.4 (IV):

Evaluation of theory error on masses and mixing

→ estimate of uncertainty in M_{h_i}, U_{ij}, Z_{ij} from unknown higher-order corr.

Evaluation of masses, mixing and decay in the NMFV MSSM

NMFV: Non Minimal Flavor Violation [*Hahn, S.H., Hollik, Merz, Peñaranda '04-'06*]
$\Rightarrow$ Connection to Flavor physics

Evaluation of additional constraints (rMSSM/cMSSM)

- ρ-parameter: $\Delta\rho^{\text{SUSY}}$ at $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha\alpha_s)$, . . . , including NMFV effects
  $\Rightarrow M_W$, $\sin^2\theta_{\text{eff}}$ via SM formula + $\Delta\rho^{\text{SUSY}}$, including NMFV effects
- anomalous magnetic moment of the μ: $(g-2)_\mu$
- BR$(b \to s\gamma)$, including NMFV effects [*T. Hahn, W. Hollik, J. Illana, S. Peñaranda '06*]
- LEP Higgs constraints [*LEP Higgs WG '06*]

Planned:

- ILC production cross sections
- EDMs of electron, neutron, Hg, . . .

3. How to install FeynHiggs2.4

1. Go to www.feynhiggs.de

2. Download the latest version

3. type `./configure`, `make`, `make install`
   $\Rightarrow$ library `libFH.a` is created

4. 4 possible ways to use *FeynHiggs*:

   A) Command-line mode

   B) called from a Fortran/C++ code

   C) called within Mathematica

   D) WWW mode

   processing of Les Houches Accord data possible

5. Detailed instructions and help are provided in the man pages

4. How to run FeynHiggs2.4

A) Command-line mode

Input File

```
MT          172.7
MB          4.7
MW          80.4
MZ          91.1
MSusy       975
MA0         200
Abs(M_2)    332
Abs(MUE)    980
TB          50
Abs(At)     -300
Abs(Ab)     1500
Abs(M_3)    975
```

Command

FeynHiggs *file flags* →

Screen Output

```
------------------- HIGGS MASSES -------------------
| Mh0     =   116.022817
| MHH     =   199.943497
| MA0     =   200.000000
| MHp     =   216.973920
| SAeff   =  -0.02685112
| UHiggs  =   0.99999346  -0.00361740   0.00000000 \
|             0.00361740   0.99999346   0.00000000 \
|             0.00000000   0.00000000   1.00000000

-------------- ESTIMATED UNCERTAINTIES -------------
| DeltaMh0  =  1.591957
| DeltaMHH  =  0.004428
| DeltaMA0  =  0.000000
| DeltaMHp  =  0.152519
  ...
```

- Loops over parameter values possible (parameter scans).
- Mask off details with `FeynHiggs` *file flags* `| grep -v %`
- `table` utility converts to machine-readable format, e.g.
  `FeynHiggs` *file flags* `| table TB Mh0 > outfile`

SUSY Les Houches Accord Format

Input File

```
BLOCK MODSEL
    1     1
BLOCK MINPAR
    1    0.10000E+03   # m0
    2    0.25000E+03   # m12
    3    0.10000E+02   # tanb
    4    0.10000E+01   # sgn mu
    5   -0.10000E+03   # A
BLOCK SMINPUTS
    4    0.91187E+02   # MZ
    5    0.42500E+01   # mb(mb)
    6    0.17500E+03   # t
  ...
```

Command

`FeynHiggs` *file flags* →

file.`fh`

```
BLOCK MASS
   25    1.12697840E+02    # Mh0
   35    4.00145460E+02    # MHH
   36    3.99769788E+02    # MA0
   37    4.08050556E+02    # MHp
  ...
BLOCK ALPHA
         -1.10658125E-01    # Alpha
  ...
```

- { Uses / was developed into } the SLHA I/O Library. [*T. Hahn '04*]
- SLHA can also be used in Library Mode with `FHSetSLHA`.
- *FeynHiggs* tries to read each file in SLHA format first.
  If that fails, fallback to native format.

B) Called from a Fortran/C++ code

Link *FeynHiggs* as a subroutine ⇒ link `libFH.a`

`call FHSetFlags(...)` :
→ specification of accuracy etc.

`call FHSetPara(...)` :
→ specify input parameters

`call FHGetPara(...)` :
→ obtain derived parameters

`call FHHiggsCorr(...)` :
→ obtain Higgs boson masses and mixings

`call FHUncertainties(...)` :
→ obtain theory error on Higgs boson masses and mixings from unknown higher-order corrections

`call FHCouplings(...)` :
→ obtain decay widths, BRs, XSs, etc.

C) Called within Mathematica

- install the math link to *MFeynHiggs* , e.g.:

  `Install[''MFeynHiggs'']`

- `FHSetFlags[...]` :
  → specification of accuracy etc.

  `FHSetPara[...]` :
  → specify input parameters

  `FHGetPara[]` :
  → obtain derived parameters

  `FHHiggsCorr[]` :
  → obtain Higgs boson masses and mixings

  `FHUncertainties[]` :
  → obtain theory error on Higgs boson masses and mixings from unknown higher-order corrections

  `FHCouplings[]` :
  → obtain decay widths, BRs etc.

D) WWW mode

1. The FeynHiggs User Control Center is available at
   www.feynhiggs.de/fhucc

2. Enter you parameters on-line in the web page

3. Obtain your results with a mouse click

⇒ for single points and checks of your downloaded version of FeynHiggs
⇒ always the latest version

⇒ online presentation

Also man pages are available on-line

D) WWW mode

1. The FeynHiggs User Control Center is available at

Also man pages are available on-line

5. Conclusinos

- Very precise MSSM Higgs sector evaluation necessary to
  - exploit anticipated ILC precision
  - be sensitive to small deviations

- *FeynHiggs*2.4 provides Higgs boson masses, mixing angles, couplings, branching ratios, Tev/LHC XS, etc.
  in the MSSM with/without complex parameters (and for NMFV)

- *FeynHiggs*2.4 is available at `www.feynhiggs.de`

- On-line version is available at `www.feynhiggs.de/fhucc`

- Possible:
  Stand alone vers. - call within Fortran/C++ - call within Mathematica

- Processing of Les Houches Accord data