Precision Measurements at the ILC

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Linear Collider Workshop of the Americas,
Albuquerque, Sept 2009

The ILC is being designed with precision measurements in mind.

- **Standard Model:**

  Higgs mass & couplings,
  Precision electroweak, Weak boson couplings, $\alpha_s(Q)$
  Top Width, m_t , top couplings, $\ldots$

- **Beyond the SM:**

  The most exciting precision measurements are of the mass and couplings of particles we have not yet seen. In this regard the ILC is crucial to decipher the new physics we "plan" to observed at the LHC.

This talk is not a review of all possible precision measurements.

Rather I will focus in detail on two:

$\alpha_s(Q)$ and m_t from e^+e^- Colliders

Measure $\alpha_s(Q)$ and m_t from e^+e^- colliders

Using:

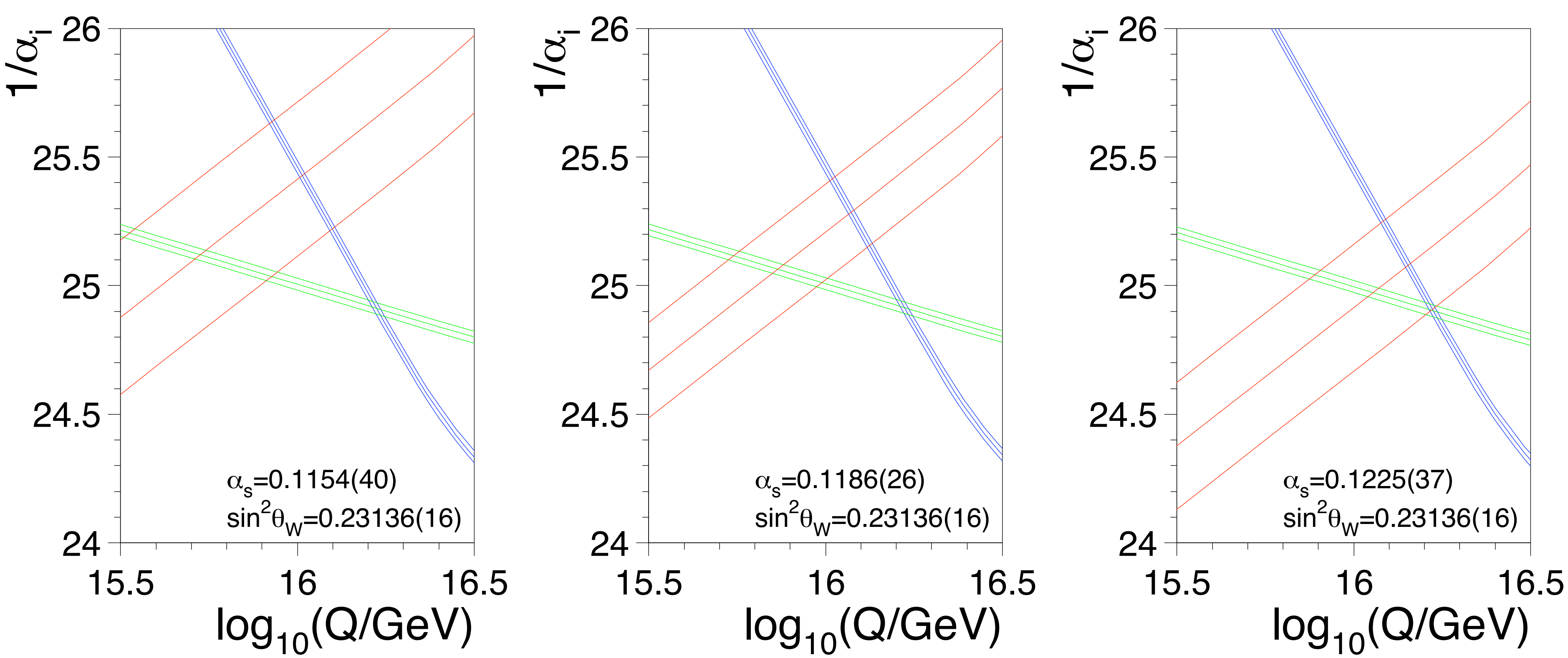
- $e^+e^- \rightarrow \text{jets}$, event shape measurements of $\alpha_s(m_Z)$
- $e^+e^- \rightarrow t\bar{t}$ at threshold $Q \simeq 2m_t$
- $e^+e^- \rightarrow t\bar{t}$ above threshold $Q > 2m_t$

- Discuss recent theoretical advances in QCD that have an impact on precision physics at the ILC:
  i) fixed order computations,
  ii) resummation,
  iii) improved theoretical framework for computations
  *Factorization & Soft-Collinear Effective Theory (SCET)

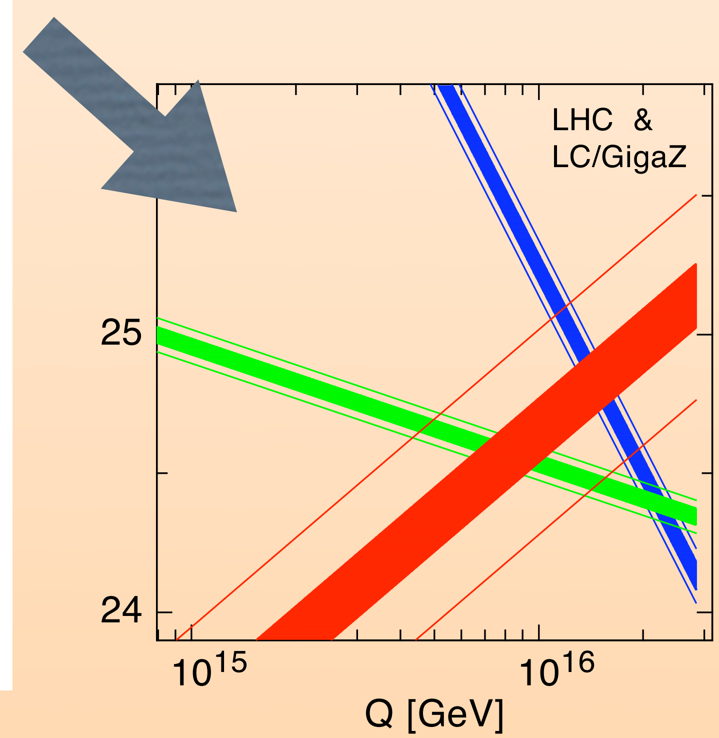
$$\alpha_s(Q)$$

Motivation

- $\alpha_s(m_Z)$ enters the analysis of all collider data (LHC, ILC, ...)
- It also plays a role in searches for new physics
  - indirectly in precision electroweak analyses, B$\rightarrow X_s\gamma$, etc.
  - directly through the unification of couplings:



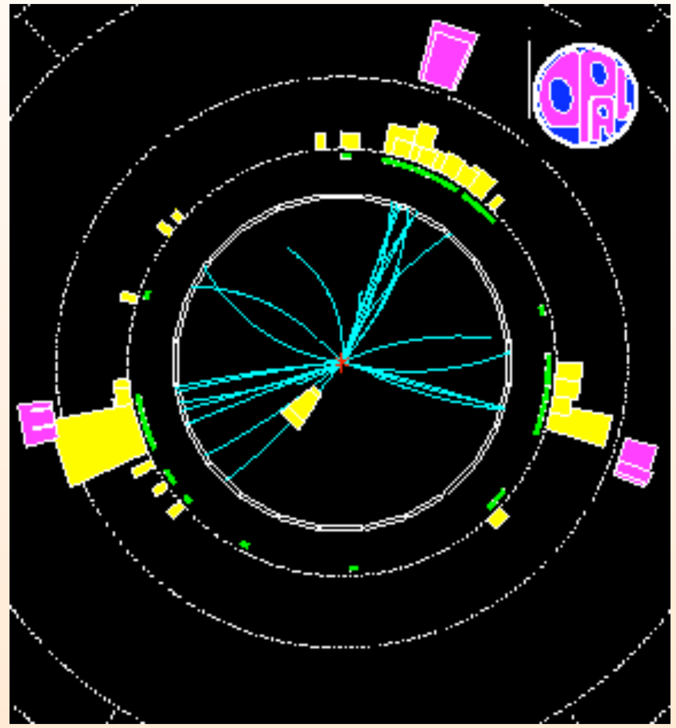
mSUGRA (from Boer & Sander '03)



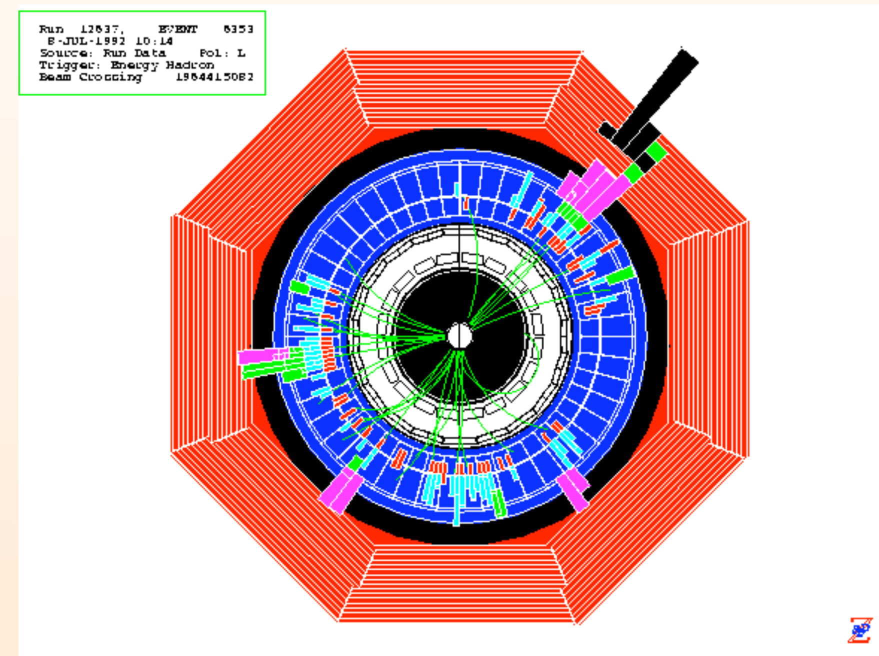
Allanach et.al. '04

Event Shapes are a classic method for determining $\alpha_s(m_Z)$

LEP 2 jet event

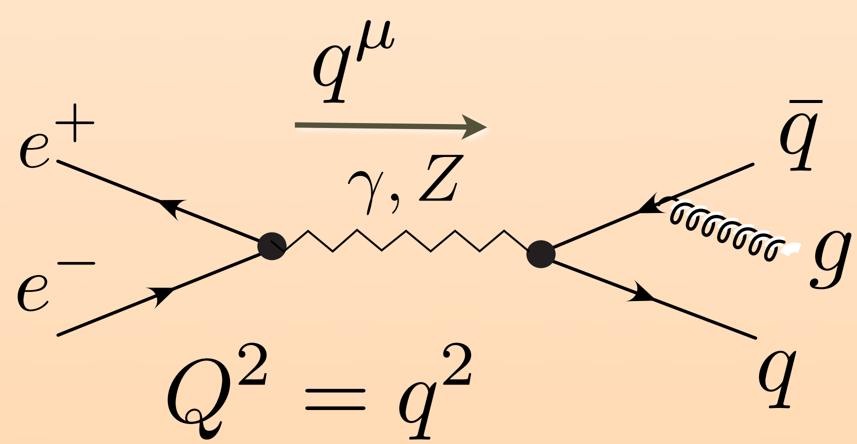


OPAL 3 jet event



SLD 3 jet event

Three jet events are proportional to α_s , good sensitivity



LEP era Results

S. Bethke's Review 2006

e^+e^- event shapes

- theory errors dominate

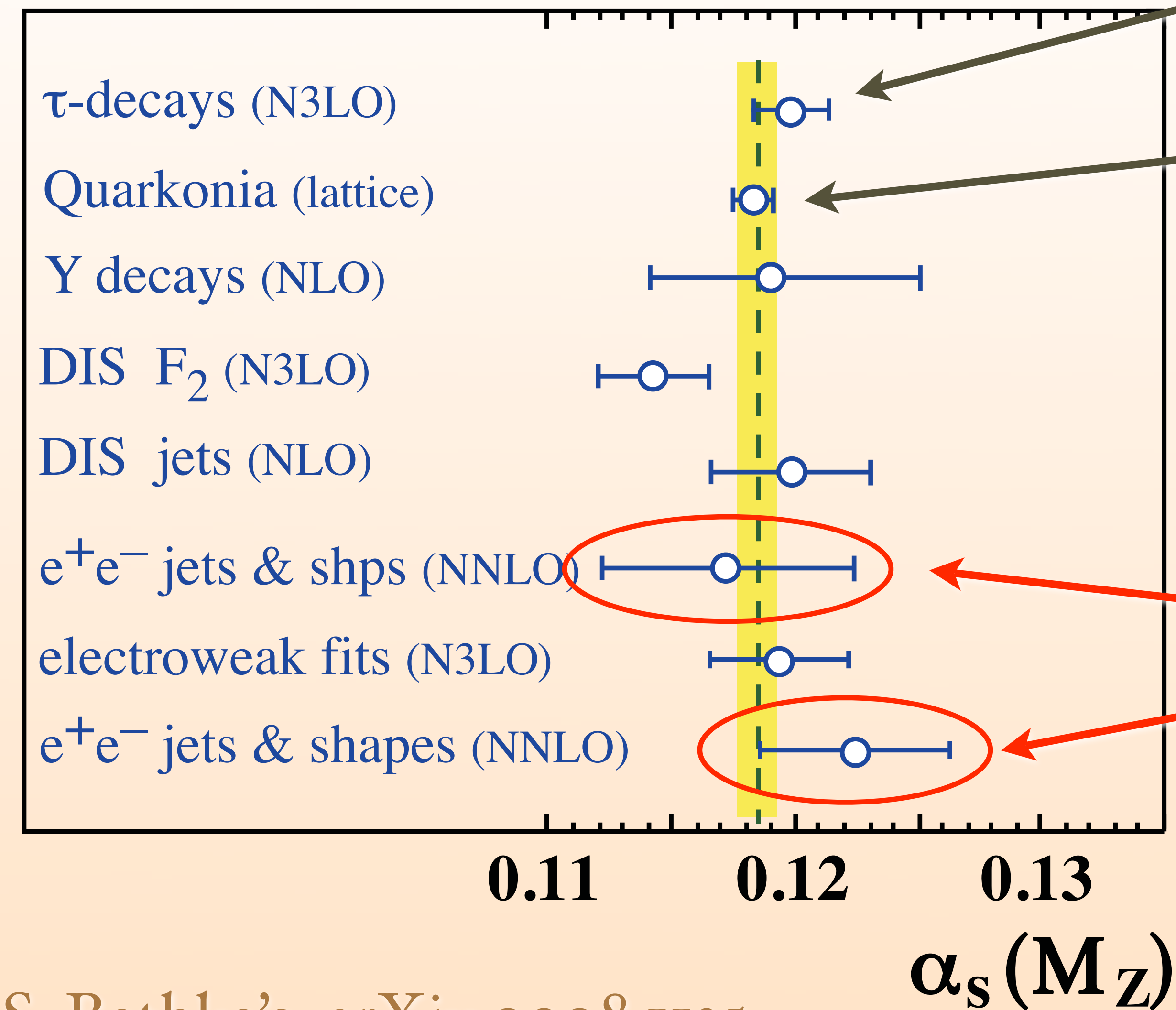
no longer true!

- fit for each Q

theoretical advances make a rigorous GLOBAL FIT possible

Process	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	Theory	refs.
DIS [pol. SF]	0.7 - 8		$0.113^{+0.010}_{-0.008}$	± 0.004	$^{+0.009}_{-0.006}$	NLO	[76]
DIS [Bj-SR]	1.58	$0.375^{+0.062}_{-0.081}$	$0.121^{+0.005}_{-0.009}$	–	–	NNLO	[77]
DIS [GLS-SR]	1.73	$0.280^{+0.070}_{-0.068}$	$0.112^{+0.009}_{-0.012}$	$^{+0.008}_{-0.010}$	0.005	NNLO	[78]
τ-decays	1.78	0.345 ± 0.010	0.1215 ± 0.0012	0.0004	0.0011	NNLO	[70]
DIS [ν; xF_3]	2.8 - 11		$0.119^{+0.007}_{-0.006}$	0.005	$^{+0.005}_{-0.003}$	NNLO	[79]
DIS [e/μ; F_2]	2 - 15		0.1166 ± 0.0022	0.0009	0.0020	NNLO	[80, 81]
DIS [e-p $\rightarrow$ jets]	6 - 100		0.1186 ± 0.0051	0.0011	0.0050	NLO	[67]
Υ decays	4.75	0.217 ± 0.021	0.118 ± 0.006	–	–	NNLO	[82]
$Q\overline{Q}$ states	7.5	0.1886 ± 0.0032	0.1170 ± 0.0012	0.0000	0.0012	LGT	[73]
e^+e^- [F_2^γ]	1.4 - 28		$0.1198^{+0.0044}_{-0.0054}$	0.0028	$^{+0.0034}_{-0.0046}$	NLO	[83]
e^+e^- [σ_{had}]	10.52	0.20 ± 0.06	$0.130^{+0.021}_{-0.029}$	$^{+0.021}_{-0.029}$	0.002	NNLO	[84]
e^+e^- [jets & shps]	14.0	$0.170^{+0.021}_{-0.017}$	$0.120^{+0.010}_{-0.008}$	0.002	$^{+0.009}_{-0.008}$	resum	[85]
e^+e^- [jets & shps]	22.0	$0.151^{+0.015}_{-0.013}$	$0.118^{+0.009}_{-0.008}$	0.003	$^{+0.009}_{-0.007}$	resum	[85]
e^+e^- [jets & shps]	35.0	$0.145^{+0.012}_{-0.007}$	$0.123^{+0.008}_{-0.006}$	0.002	$^{+0.008}_{-0.005}$	resum	[85]
e^+e^- [σ_{had}]	42.4	0.144 ± 0.029	0.126 ± 0.022	0.022	0.002	NNLO	[86, 32]
e^+e^- [jets & shps]	44.0	$0.139^{+0.011}_{-0.008}$	$0.123^{+0.008}_{-0.006}$	0.003	$^{+0.007}_{-0.005}$	resum	[85]
e^+e^- [jets & shps]	58.0	0.132 ± 0.008	0.123 ± 0.007	0.003	0.007	resum	[87]
$p\bar{p} \rightarrow b\bar{b}X$	20.0	$0.145^{+0.018}_{-0.019}$	0.113 ± 0.011	$^{+0.007}_{-0.006}$	$^{+0.008}_{-0.009}$	NLO	[88]
$p\bar{p}$, $pp \rightarrow \gamma X$	24.3	$0.135^{+0.012}_{-0.008}$	$0.110^{+0.008}_{-0.005}$	0.004	$^{+0.007}_{-0.003}$	NLO	[89]
$\sigma(p\bar{p} \rightarrow$ jets)	40 - 250		0.118 ± 0.012	$^{+0.008}_{-0.010}$	$^{+0.009}_{-0.008}$	NLO	[90]
e^+e^- $\Gamma(Z \rightarrow$ had)	91.2	$0.1226^{+0.0058}_{-0.0038}$	$0.1226^{+0.0058}_{-0.0038}$	± 0.0038	$^{+0.0043}_{-0.0005}$	NNLO	[91]
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022	0.1176 ± 0.0022	0.0010	0.0020	NLO	[92]
e^+e^- [jets & shps]	91.2	0.121 ± 0.006	0.121 ± 0.006	0.001	0.006	resum	[32]
e^+e^- [jets & shps]	133	0.113 ± 0.008	0.120 ± 0.007	0.003	0.006	resum	[32]
e^+e^- [jets & shps]	161	0.109 ± 0.007	0.118 ± 0.008	0.005	0.006	resum	[32]
e^+e^- [jets & shps]	172	0.104 ± 0.007	0.114 ± 0.008	0.005	0.006	resum	[32]
e^+e^- [jets & shps]	183	0.109 ± 0.005	0.121 ± 0.006	0.002	0.005	resum	[32]
$\mathbf{e^+e^-}$ [jets & shps]	189	0.109 ± 0.004	0.121 ± 0.005	0.001	0.005	resum	[32]
e^+e^- [jets & shps]	195	0.109 ± 0.005	0.122 ± 0.006	0.001	0.006	resum	[81]
e^+e^- [jets & shps]	201	0.110 ± 0.005	0.124 ± 0.006	0.002	0.006	resum	[81]
e^+e^- [jets & shps]	206	0.110 ± 0.005	0.124 ± 0.006	0.001	0.006	resum	[81]

Latest World Average



S. Bethke's, arXiv:0908.1135

errors inflated to account for variation in literature

fit to Υ-splittings, Wilson loops

$$\alpha_s(m_Z) = 0.1183 \pm 0.0008$$

HPQCD 0807.1687

event shape results at fixed order

I will show that by
i) improving the theory and
ii) performing a global fit,
that LEP data already gives a precision comparable to the lattice result.

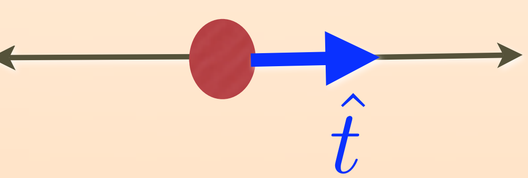
With an ILC we can do even better.

Thrust is a classic example of an “event-shape”

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \qquad \tau = 1 - T$$

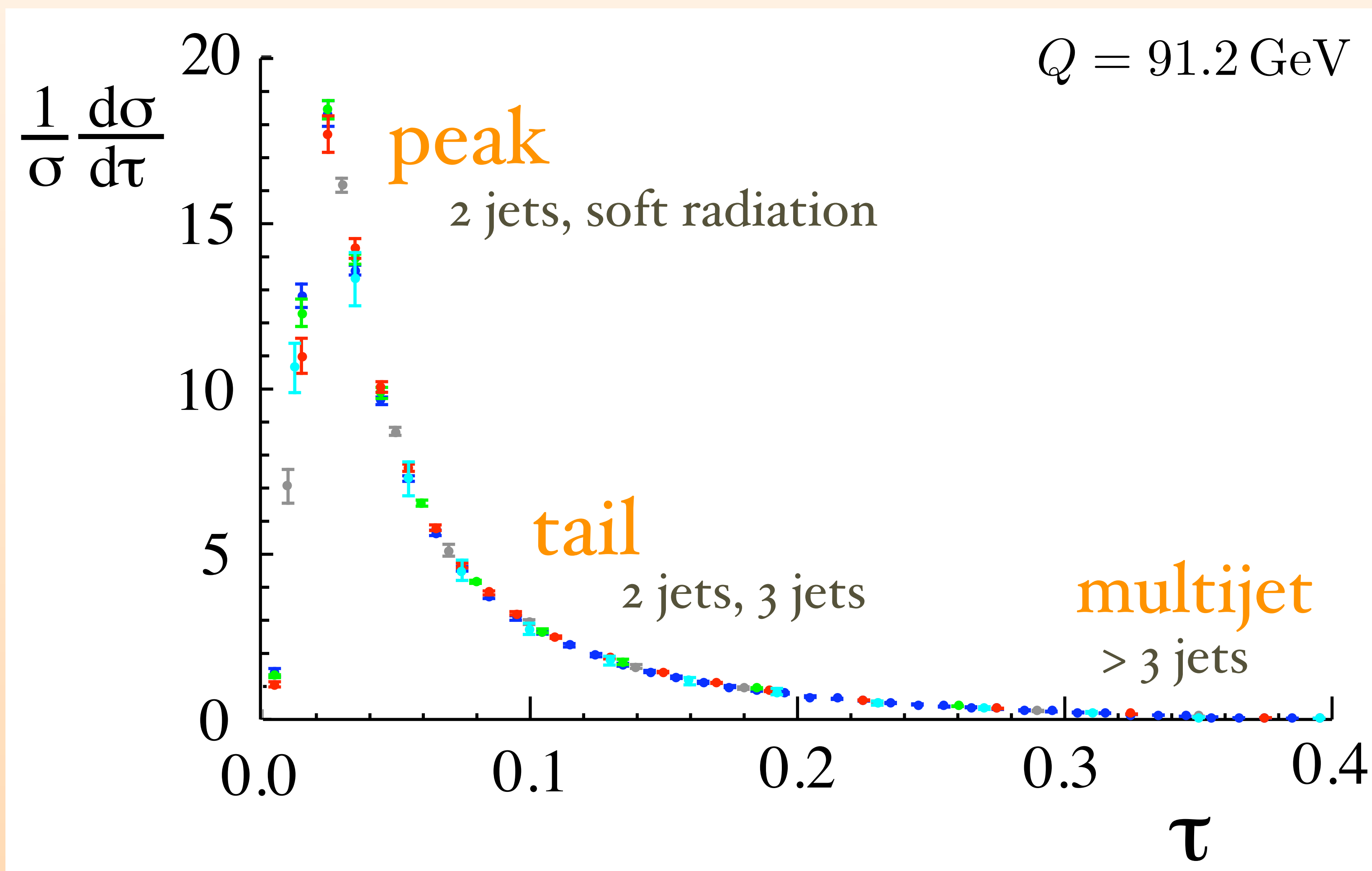
ALEPH, DELPHI, L3, OPAL, SLD

2 jets

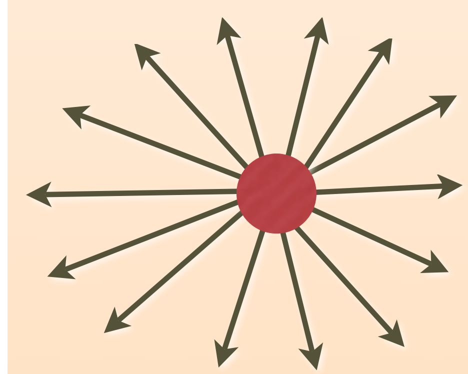


$T = 1$

$\tau = 0$



spherical event



$T = 1/2$

$\tau = 1/2$

Almost all event shape fits cut on τ, eg. keep $\tau \in \{0.09, 0.25\}$.

Complete result:

For $\tau > 0$ singular non-singular

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = \sum_{n,m} \alpha_s^n \frac{\ln^m \tau}{\tau} + \sum_{n,m} \alpha_s^n \ln^m \tau + \sum_{n,m} \alpha_s^n f_m(\tau)$$

$$+ f(\tau, \Lambda_{\rm QCD}/Q)$$

nonperturbative power corrections

Factorization Theorem:

Hard Function Jet Function Soft Function

$$\boxed{\frac{d\sigma}{d\tau} = \sigma_0 H(Q, m_Z, \mu)\, Q \int d\ell\, J_T\Big(Q^2\tau - Q\ell, \mu\Big)\, S_T(\ell, \mu)} + \left(\frac{d\sigma}{d\tau}\right)_{\rm nonsingular}$$

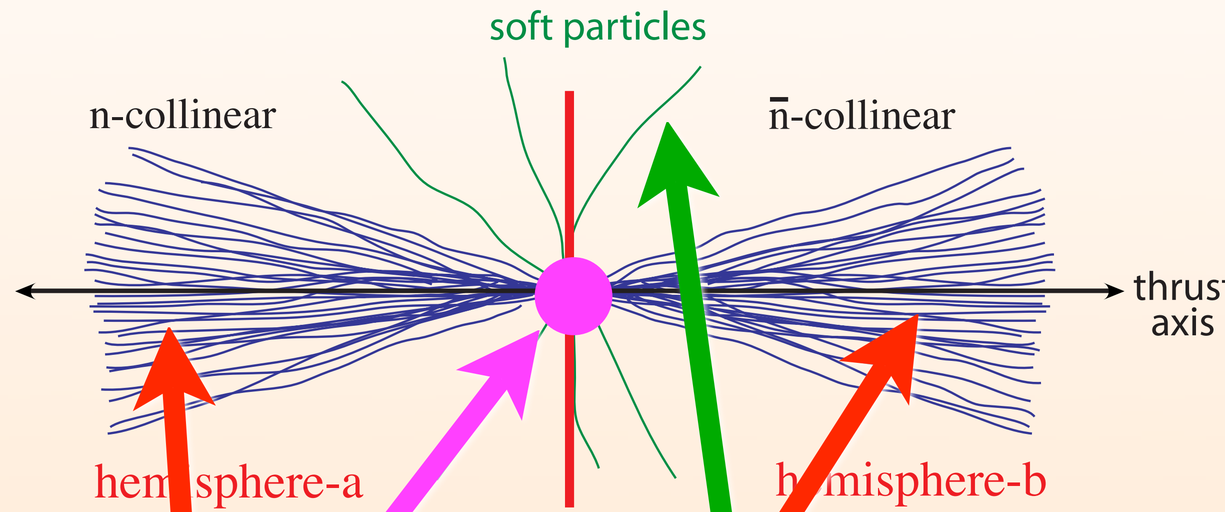
singular terms

Renormalization group evolution sums logs of τ

encodes dominant power corrections by a universal function

Subleading SCET factorization theorems tells us how power corrections enter here too eg. $e^+e^- \to Z \to 2\,\text{jets} + X_{\text{soft}}$

$$m_Z^2 \gg M_{\text{jet}}^2 \gg E_{\text{soft}}^2$$



$$\mu_Q \simeq m_Z = 91.2\,\text{GeV}$$

$$\mu_J \simeq M_{\text{jet}} \simeq 20\,\text{GeV}$$

$$\mu_S \simeq E_{\text{soft}} \simeq 5\,\text{GeV or smaller,}$$

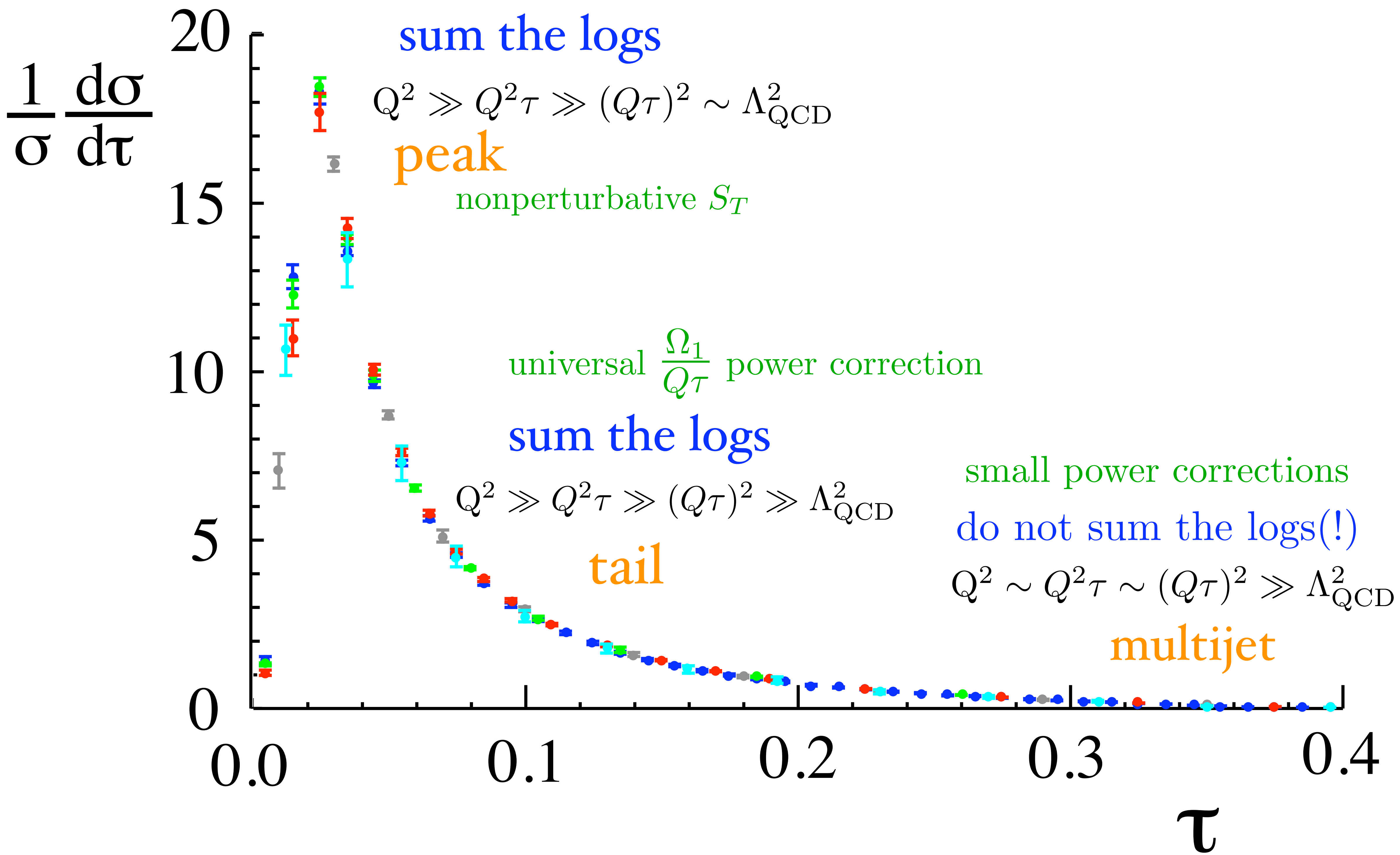
down to Λ_{QCD}

$$\mathrm{Q}^2 \gg Q^2\tau \gg (Q\tau)^2$$

hard jet soft

$$\boxed{\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu)\, Q \int d\ell\, J_T\Big(Q^2\tau - Q\ell, \mu\Big) S_T(\ell,\mu)}$$

Our Three Regions:



Recent Literature

i) $\mathcal{O}(\alpha_s^3)$ fixed order results (numerical) — Gehrmann, Gehrmann-De Ridder, Glover, Heinrich; S.Weinzierl

ii) summation of large logs to $\mathrm{N}^3\mathrm{LL}$ (analytic with Soft-Collinear EFT) — Becher and Schwartz

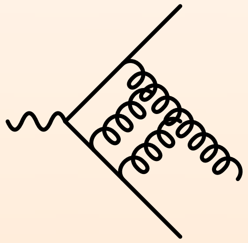
iii) power corrections — Davison & Webber; Lee & Sterman; Hoang & I.S.; Ligeti, I.S., Tackmann.

iv) All together, a Global Thrust Fit for alphas — Abbate, Fickinger, Hoang, Mateu, I.S.

• $\mathcal{O}(\alpha_s^3)$ fixed order results

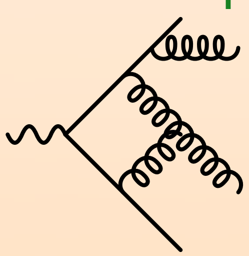
- Two-loop matrix elements

  $|\mathcal{M}|^2_{\text{2-loop},3 \text{ partons}}$



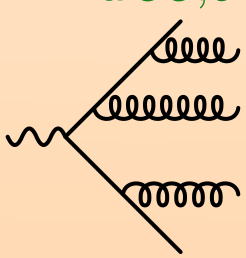
- One-loop matrix elements

  $|\mathcal{M}|^2_{\text{1-loop},4 \text{ partons}}$



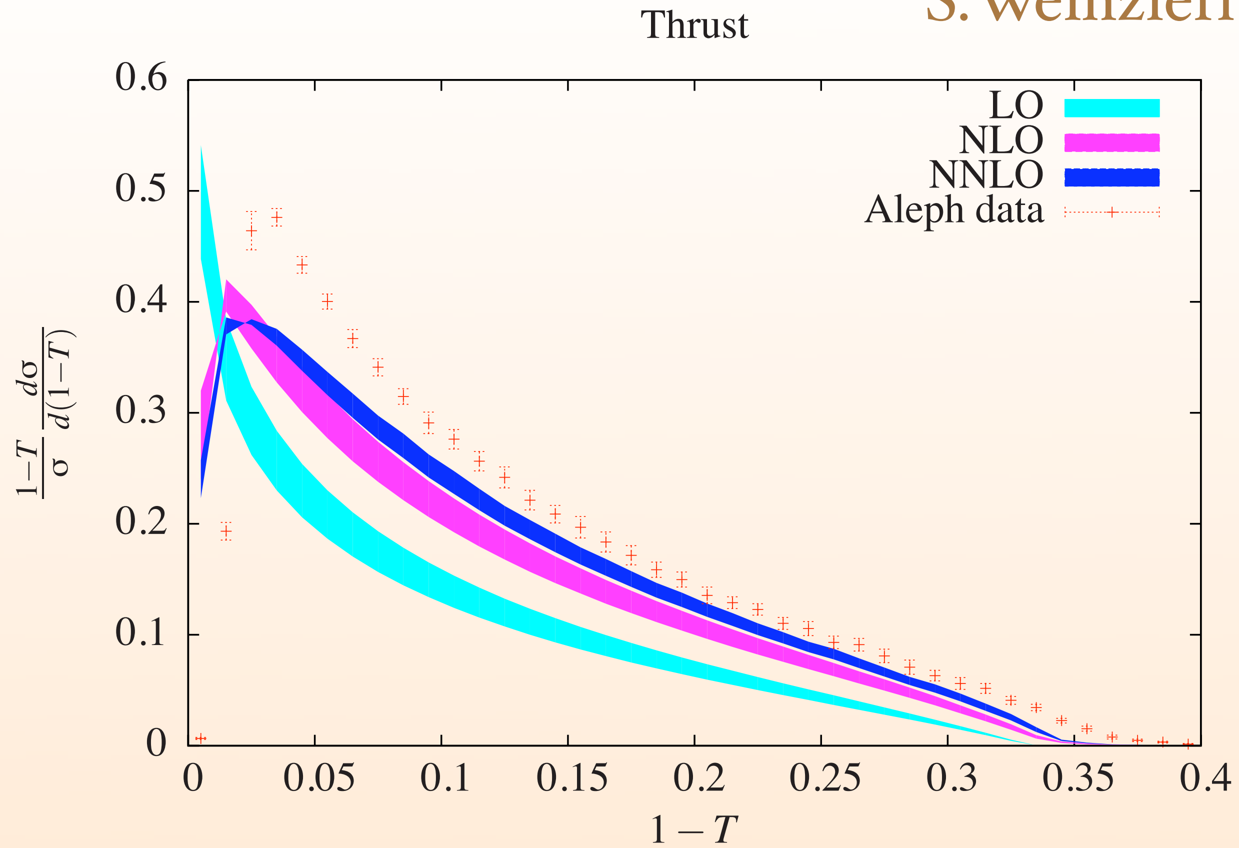
- Tree level matrix elements

  $|\mathcal{M}|^2_{\text{tree},5 \text{ partons}}$

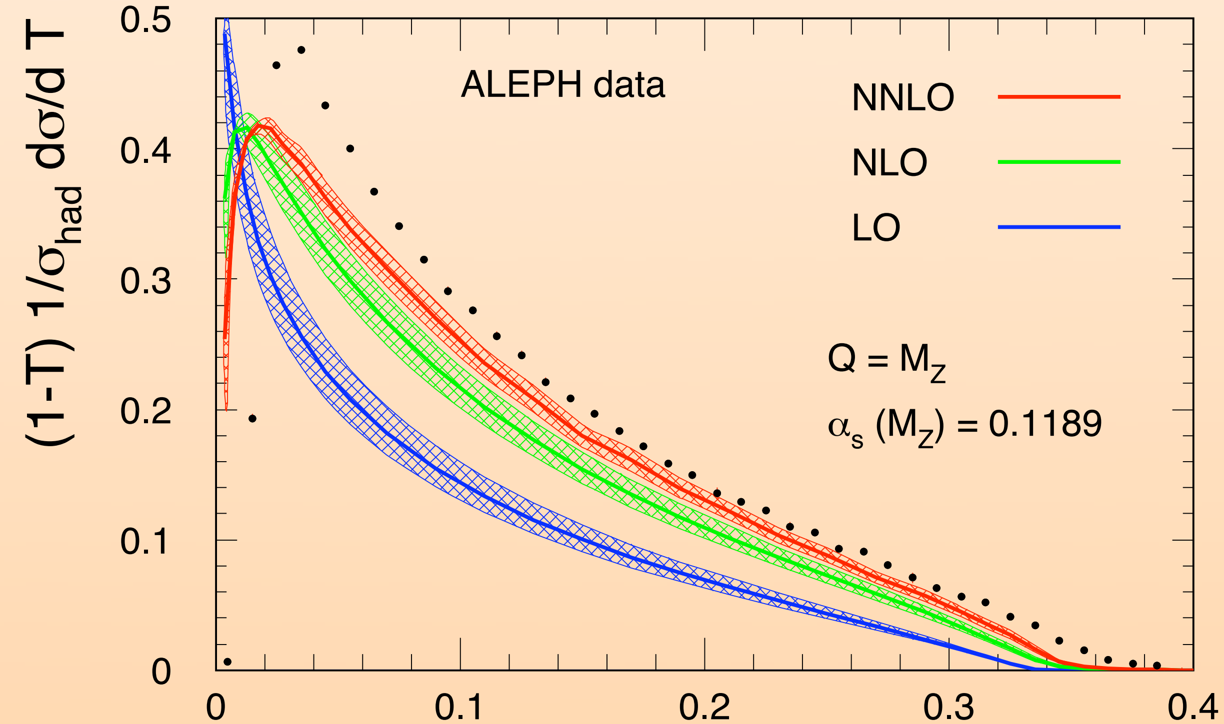


Infrared Poles cancel in the sum

S.Weinzierl



Gehrmann, Gehrmann-De Ridder, Glover, Heinrich



convergence? μ dependence?

- summation of large logs to N^3LL (analytic with SCET)

Catani et.al.

Becher and Schwartz

LL, NLL, $NNLL$, N^3LL

$$\ln\frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s(\alpha_s \ln)^k + \alpha_s^2(\alpha_s \ln)^k + \ldots$$

LL NLL NNLL N^3LL

$y =$ Fourier transform of τ

		cusp	non-cusp	matching	alphas
standard counting	LL	1	–	tree	1
	NLL	2	1	tree	2
	NNLL	3	2	1	3
	N^3LL	4^{pade}	3	2	4
primed counting	LL'	1	–	tree	1
	NLL'	2	1	1	2
	$NNLL'$	3	2	2	3
	N^3LL'	4^{pade}	3	3	4

when fixed order results are important primed counting is better

- summation of large logs to $\mathrm{N^3LL}$ (analytic with SCET)

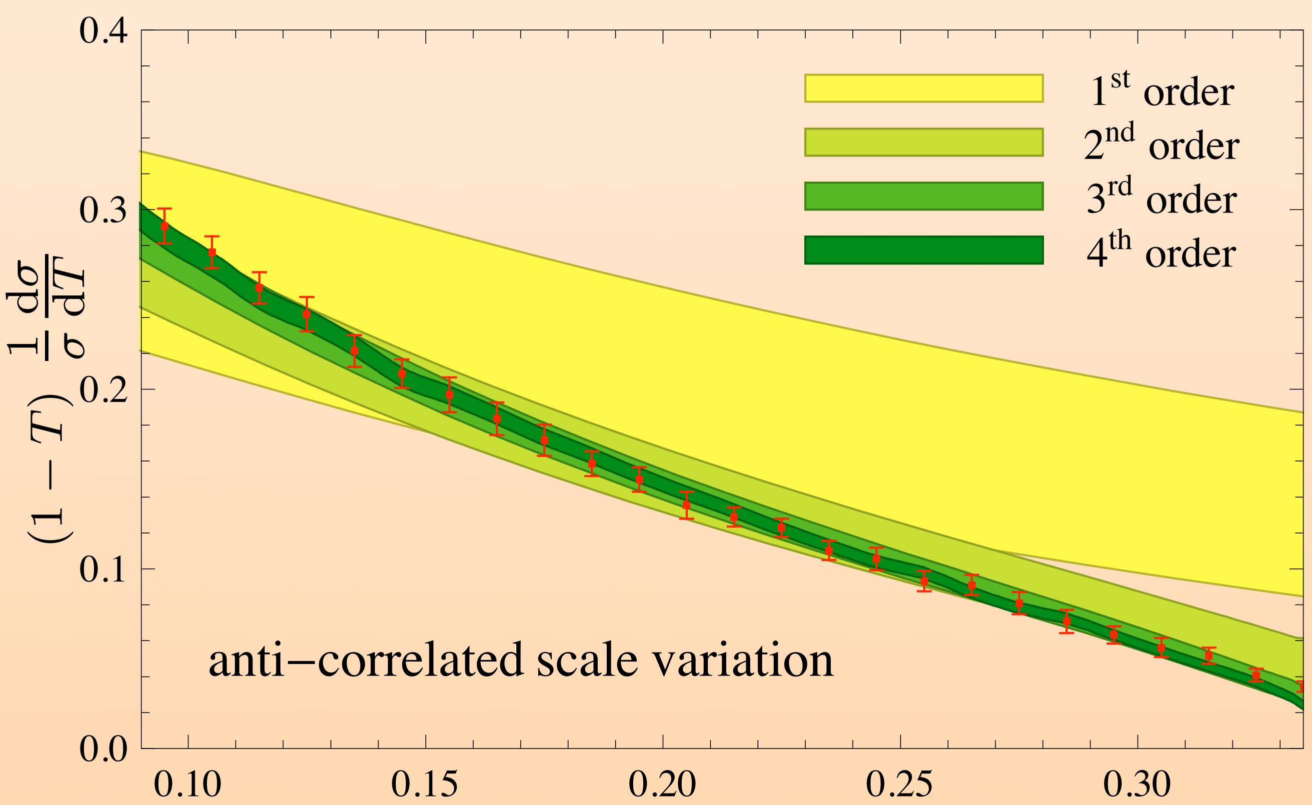
Catani et.al.

Becher and Schwartz

$\mathrm{LL},\ \mathrm{NLL},\ \mathrm{NNLL},\ \mathrm{N^3LL}$

$$\ln\frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s(\alpha_s \ln)^k + \alpha_s^2(\alpha_s \ln)^k + \ldots$$

LL NLL NNLL $\mathrm{N^3LL}$



better convergence

nice μ dependence

- summation of large logs to $\mathrm{N^3LL}$ (analytic with SCET)

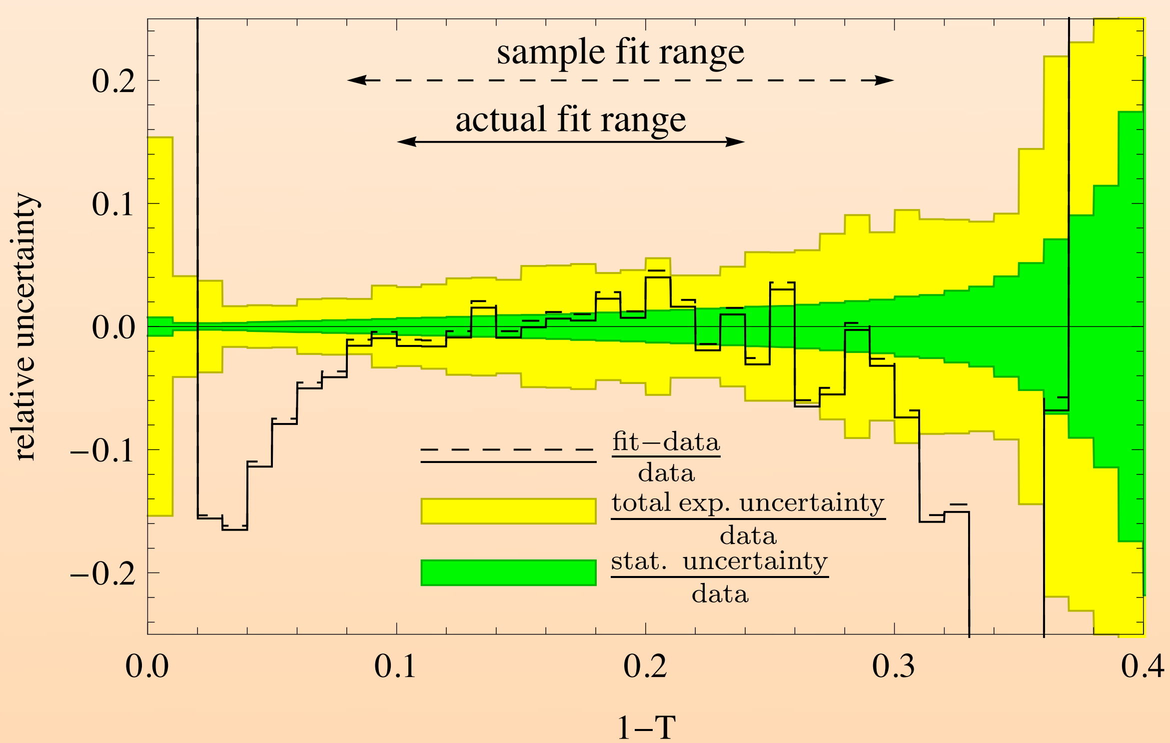
Catani et.al.

Becher and Schwartz

LL, NLL, NNLL, $\mathrm{N^3LL}$

$$\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \dots$$

LL NLL NNLL $\mathrm{N^3LL}$



$$\alpha_s(m_Z) = 0.1172 \pm 0.0022$$

error competitive with WA

- Nonperturbative corrections not included in central value

tuning of programs like Pythia does not properly separate nonpert. & pert. corrections

Nonperturbative Corrections

Universal Soft Function

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\underbrace{\overline{Y}_{\bar n}\, Y_n}(0)|X_s\rangle\langle X_s|\underbrace{Y_n^\dagger\,\overline{Y}_{\bar n}^\dagger}(0)|0\rangle$$

soft Wilson lines

$S_T(\tau)$ is symmetric projection

OPE:

Korchemsky, Sterman, Lee & Sterman

$$S_T(\tau) = S_{\rm pert}(\tau) - S'_{\rm pert}(\tau)\frac{2\Omega_1}{Q} + \ldots$$

$$= S_{\rm pert}(\tau - 2\Omega_1/Q) + \ldots$$

shifts distributions to the right

Dokshitzer & Webber;

$\Omega_1 \sim \Lambda_{\rm QCD}$ a universal parameter

Nonperturbative Corrections

Universal Soft Function

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\underbrace{\overline{Y}_{\bar n}\, Y_n}(0)|X_s\rangle\langle X_s|\underbrace{Y_n^\dagger\,\overline{Y}_{\bar n}^\dagger}(0)|0\rangle$$

soft Wilson lines

$S_T(\tau)$ is symmetric projection

Perturbative & Nonperturbative parts:

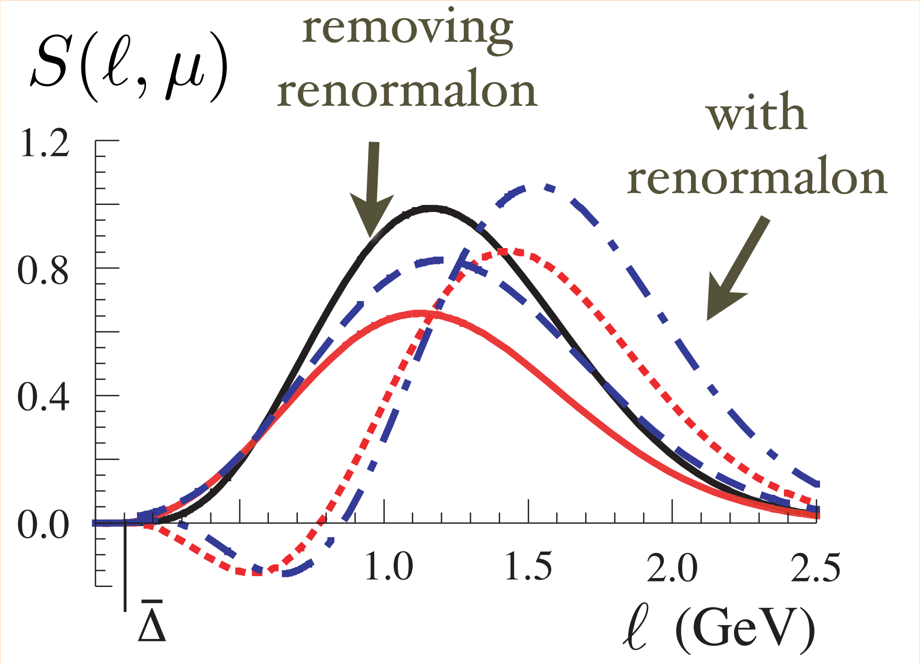
Hoang & I.S.;

Ligeti, I.S., Tackmann

$$S(\ell,\mu) = \int d\ell'\ \underbrace{S_{\rm part}(\ell-\ell',\mu)}_{\text{partonic soft function at fixed order}}\ \underbrace{F(\ell')}_{\text{normalized model function, complete basis (must have exponential fall off!)}}$$

In general, Pert. and Nonpert. parts are hard to separate (renormalons).

Use renormalon free scheme for parameters in F, such as Ω_1

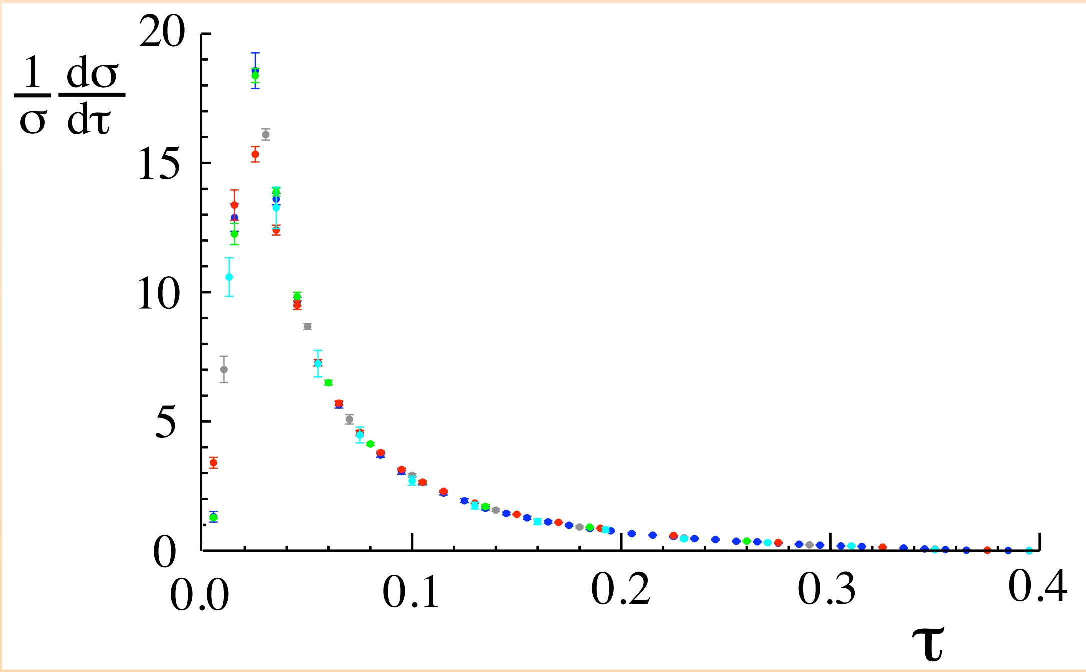


Thrust Data Sets

Experiment:	Values of Q :
ALEPH	{91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0}
DELPHI	{45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0}
OPAL	{91.0, 133.0, 177.0, 197.0}
L3	{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2}
SLD	{91.2}
TASSO	{14.0, 22.0, 35.0, 44.0}
JADE	{35.0, 44.0}
AMY	{55.2}

At each Q there is a distribution in τ

Lots of Data: 807 bins



Ingredients for Global Analysis

- SCET Factorization Theorems, Sum Large Logs: $\sum_k (\alpha_s \ln^2)^k$

  LL, NLL, NNLL, $\mathrm{N^3LL}$ and/or $\mathrm{LL'}$, $\mathrm{NLL'}$, $\mathrm{NNLL'}$, $\mathrm{N^3LL'}$

- Power Corrections: $\dfrac{\Lambda_{\rm QCD}}{\mu_S}, \ \dfrac{\Lambda_{\rm QCD}}{\mu_J}, \ \dfrac{\Lambda_{\rm QCD}}{\mu_h}, \ \dfrac{\mu_S}{\mu_J}$

- Multiple Regions:

  need smooth transitions

  $i)$ peak: $\mu_h \gg \mu_J \gg \mu_S \sim \Lambda_{\rm QCD}$

  $ii)$ tail: $\mu_h \gg \mu_J \gg \mu_S \gg \Lambda_{\rm QCD}$

  $iii)$ far tail: (multi jet) $\mu_h \sim \mu_J \sim \mu_S \gg \Lambda_{\rm QCD}$

- Renormalon Subtractions (Mass, Gap), R-RGE

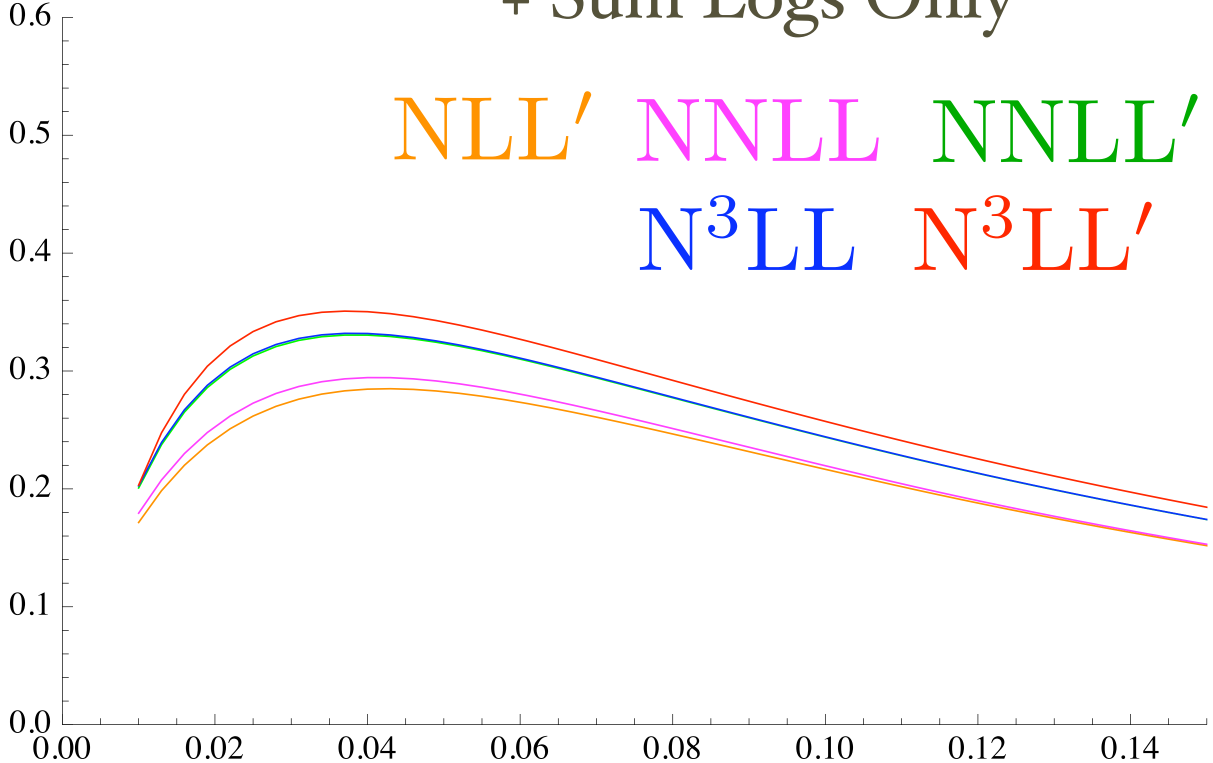
- Complete Basis for modeling Hadronic functions

- Final State QED radiation, with resummation of Sudakov

- Rigorous treatment of b-quark mass effects (factorization)

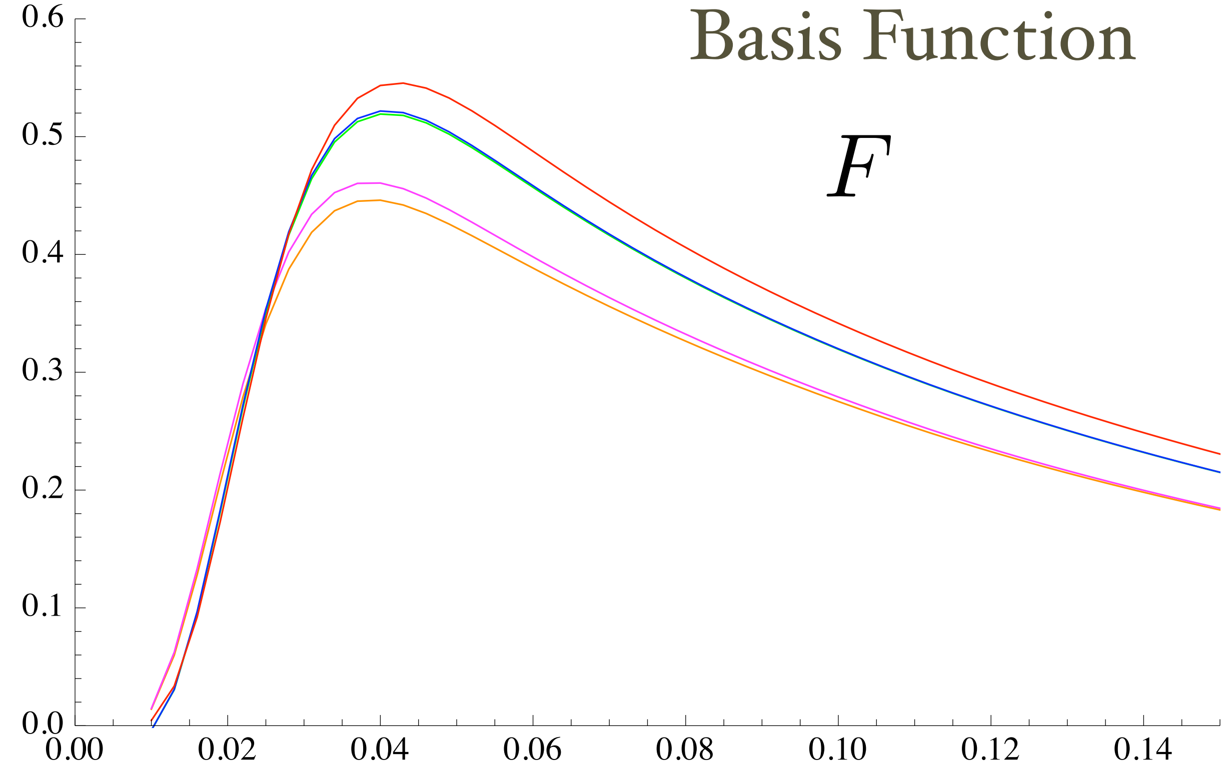
$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$

Perturbation Theory
+ Sum Logs Only



τ

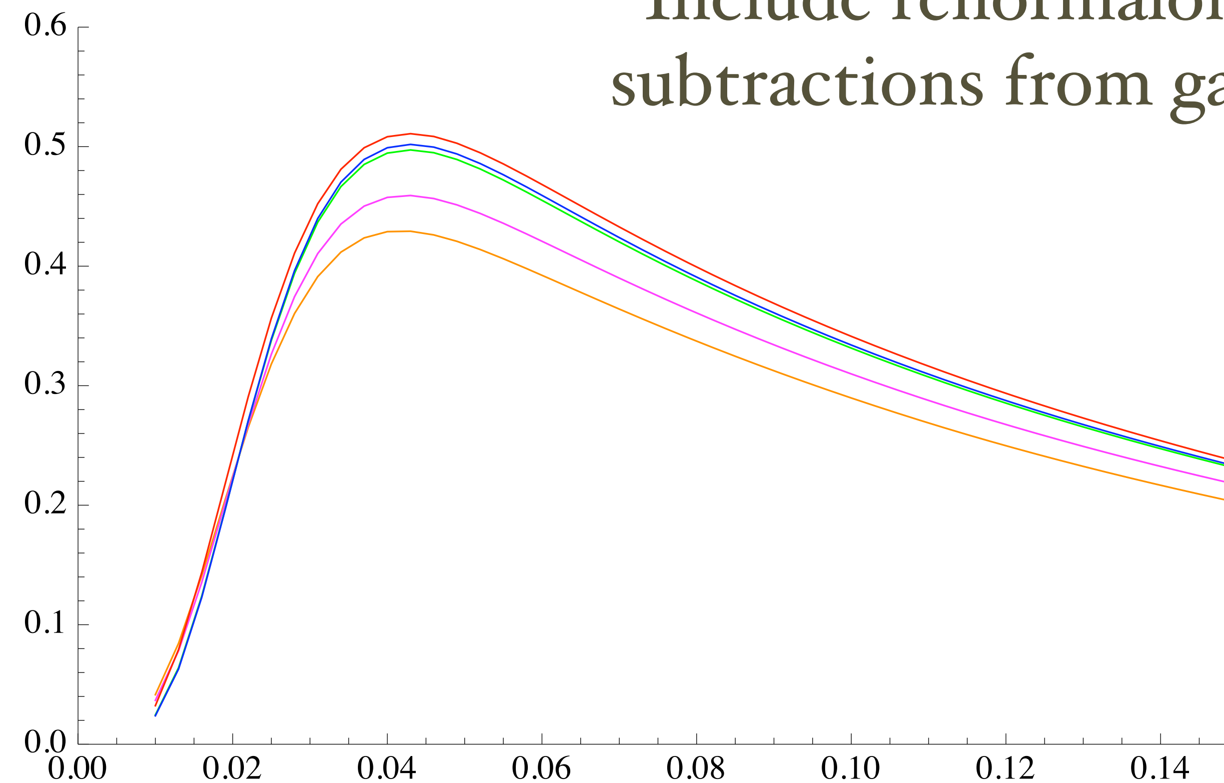
Add Soft Radiation
Basis Function



AFHMS

stable peaks
predictions

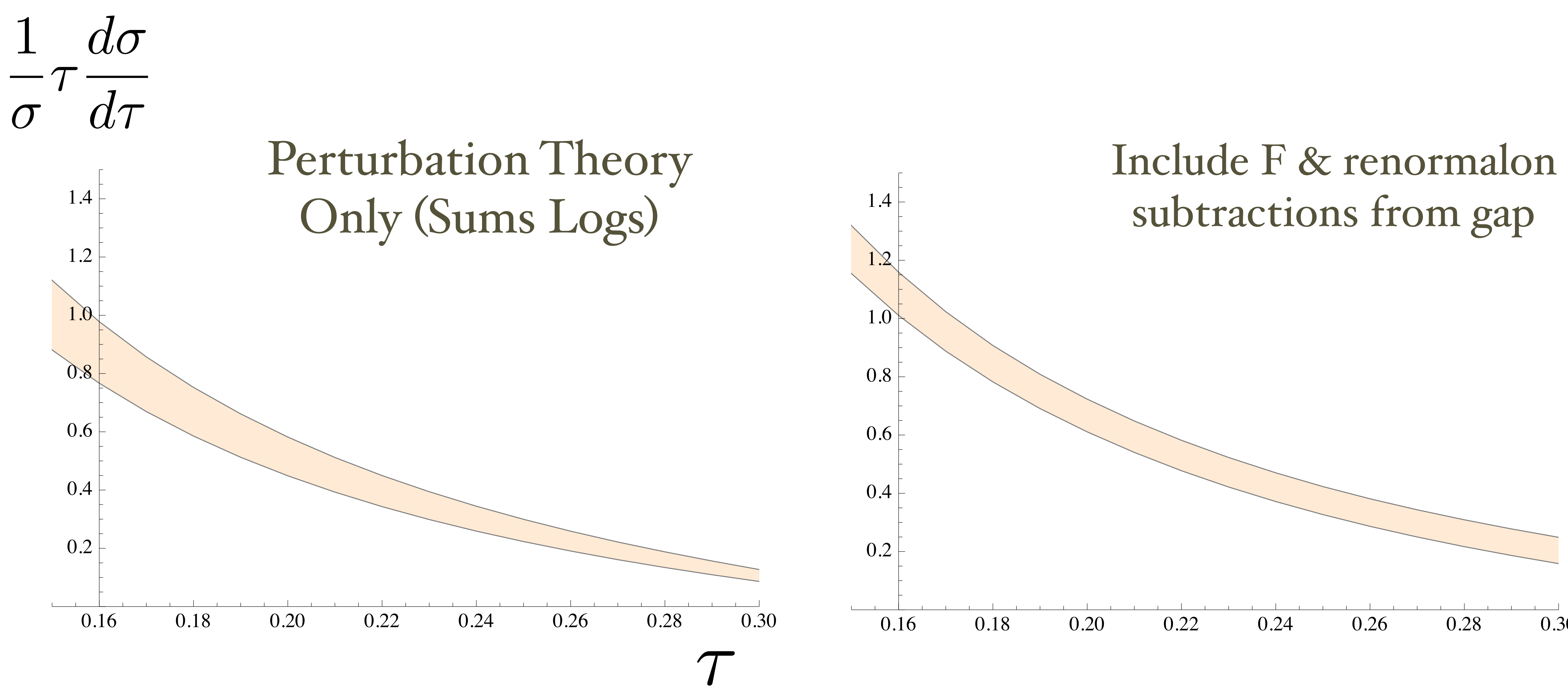
Include renormalon
subtractions from gap



Wednesday, September 30, 2009

Tail Predictions with Scan over Theory Uncertainties

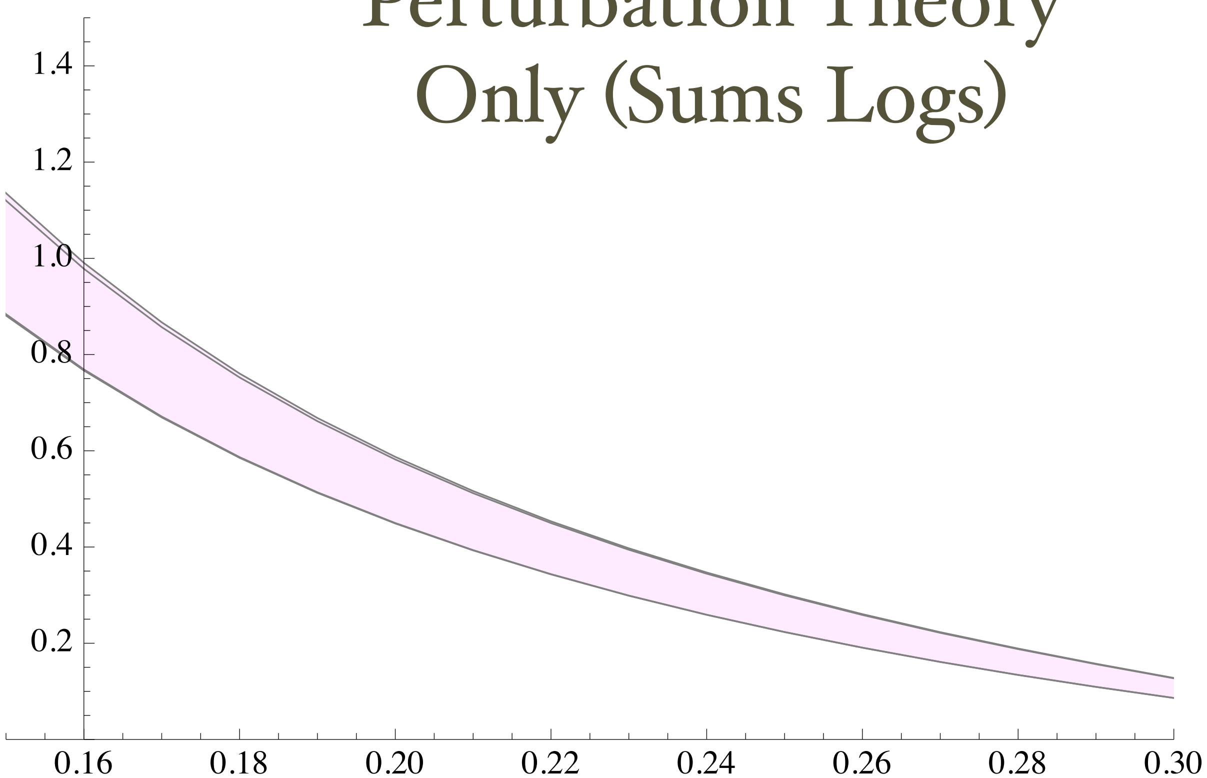
NLL′

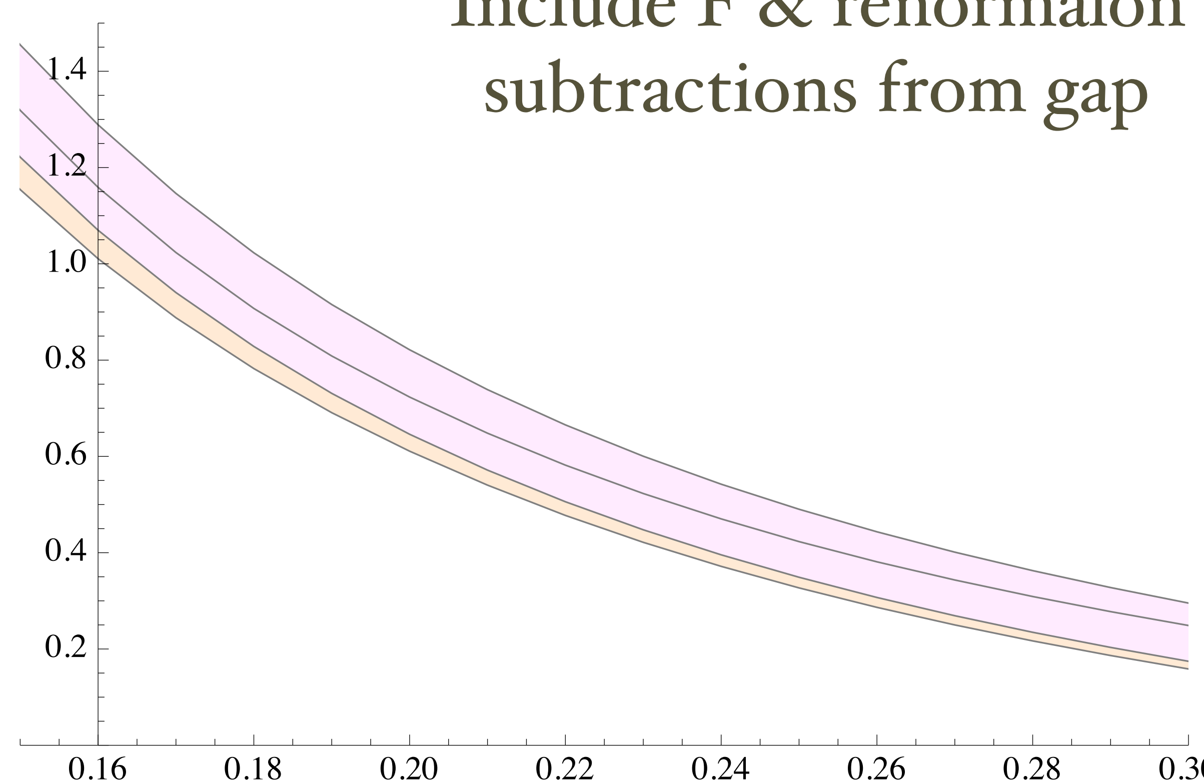


Tail Predictions with Scan over Theory Uncertainties

NLL' NNLL

$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$





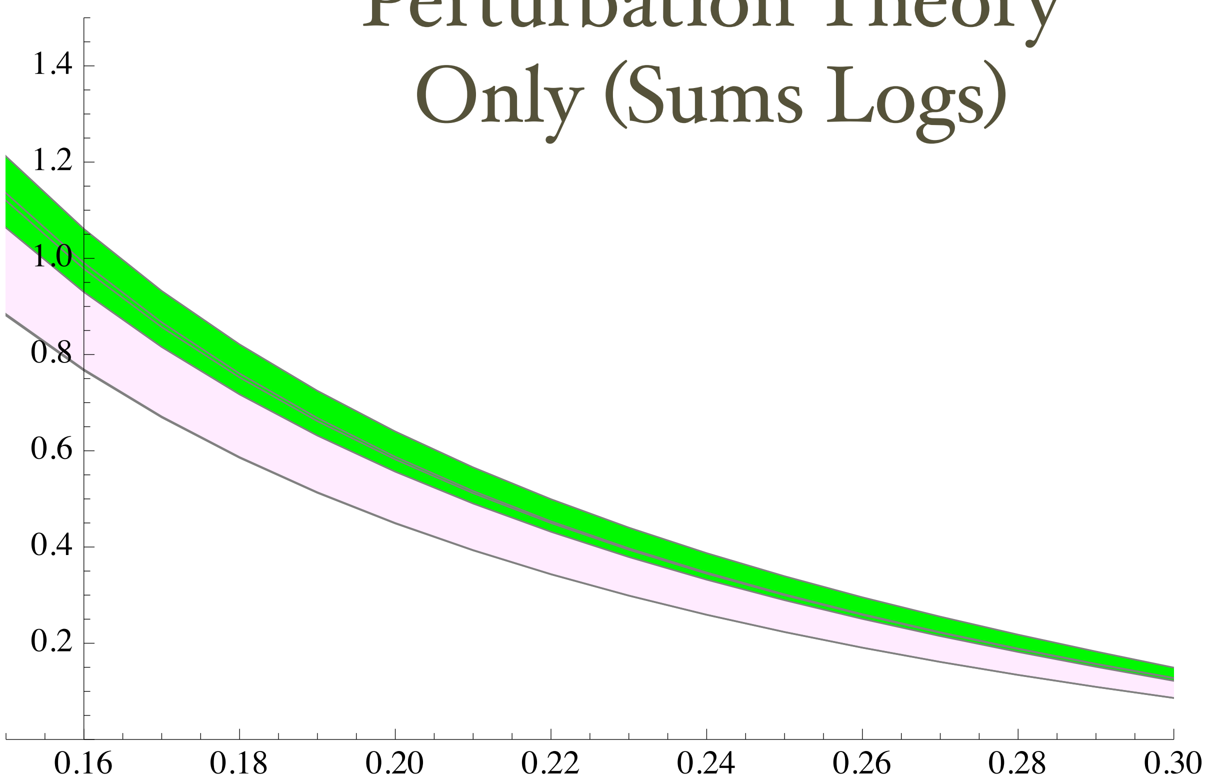
τ

Tail Predictions with Scan over Theory Uncertainties

NLL′ NNLL NNLL′

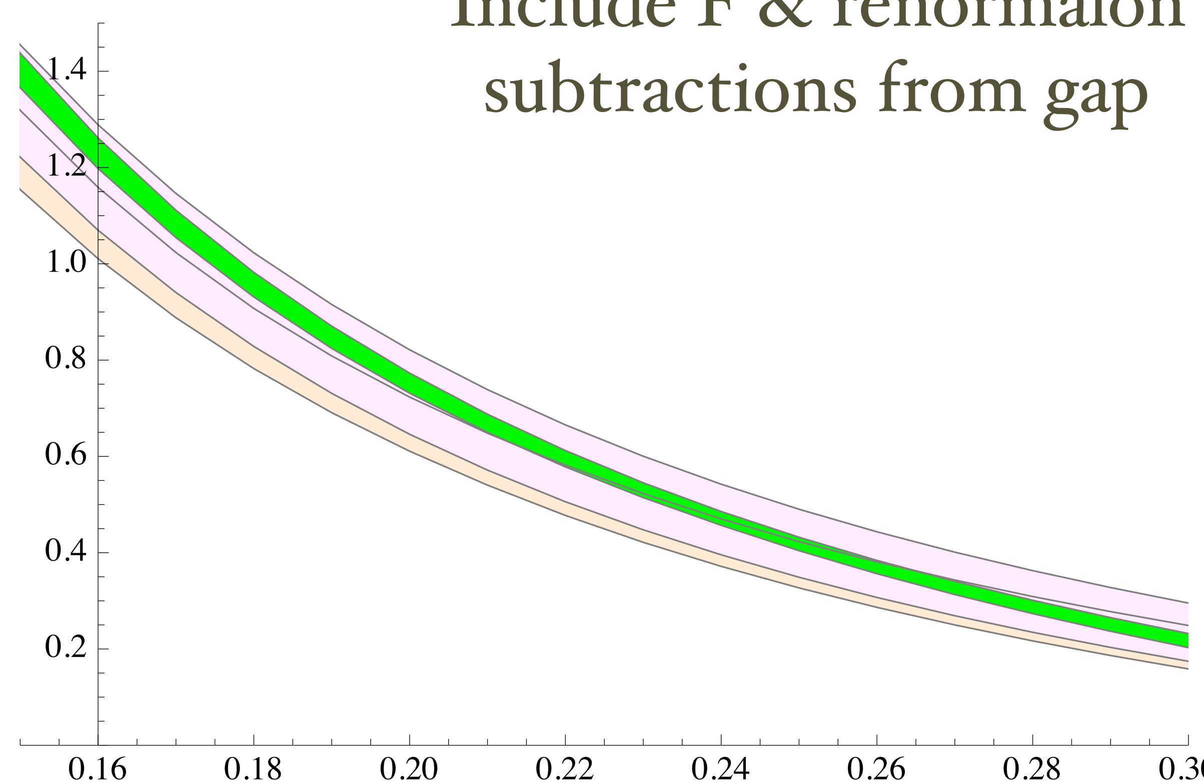
$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$

Perturbation Theory Only (Sums Logs)



τ

Include F & renormalon subtractions from gap

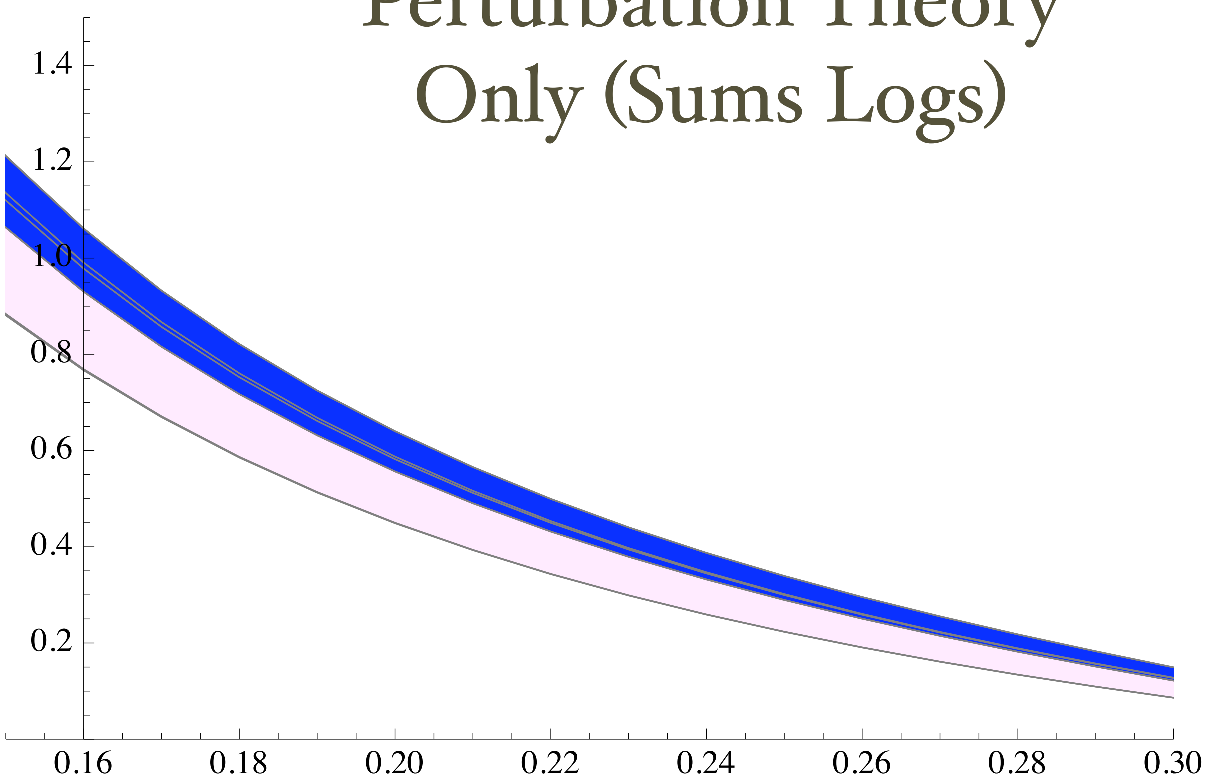


Tail Predictions with Scan over Theory Uncertainties

NLL' NNLL NNLL' N^3LL

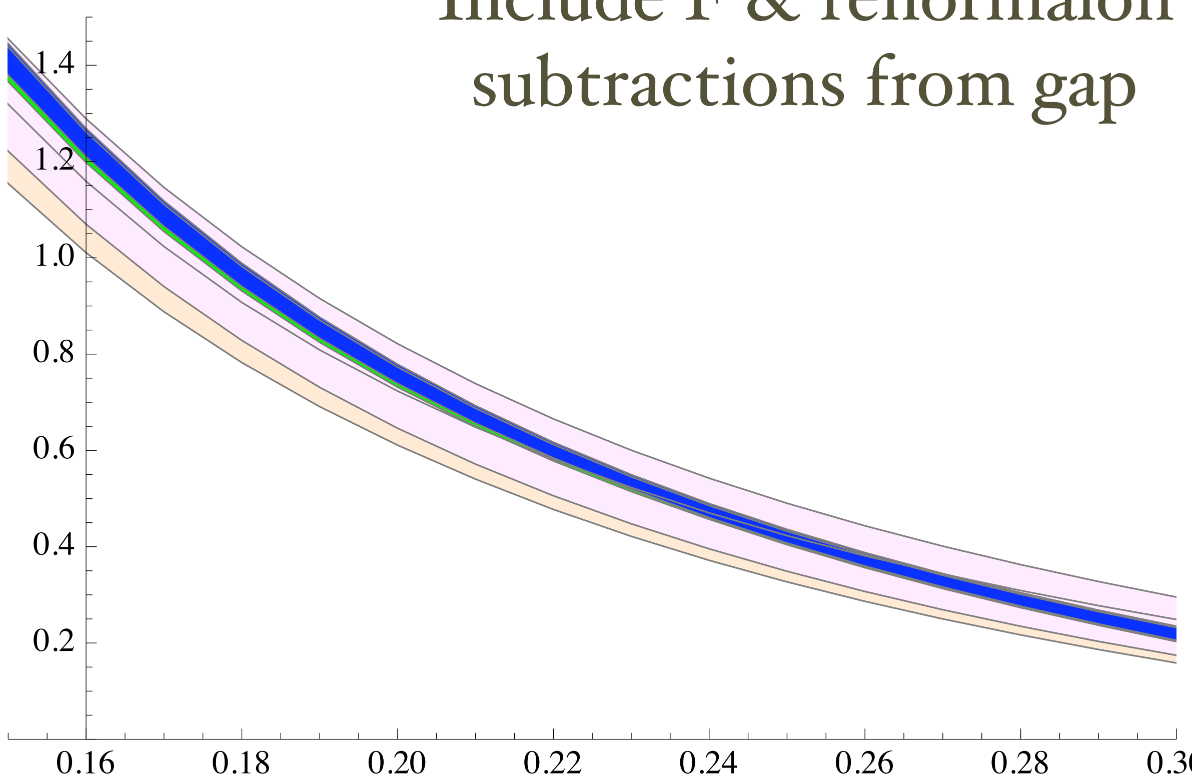
$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$

Perturbation Theory Only (Sums Logs)



τ

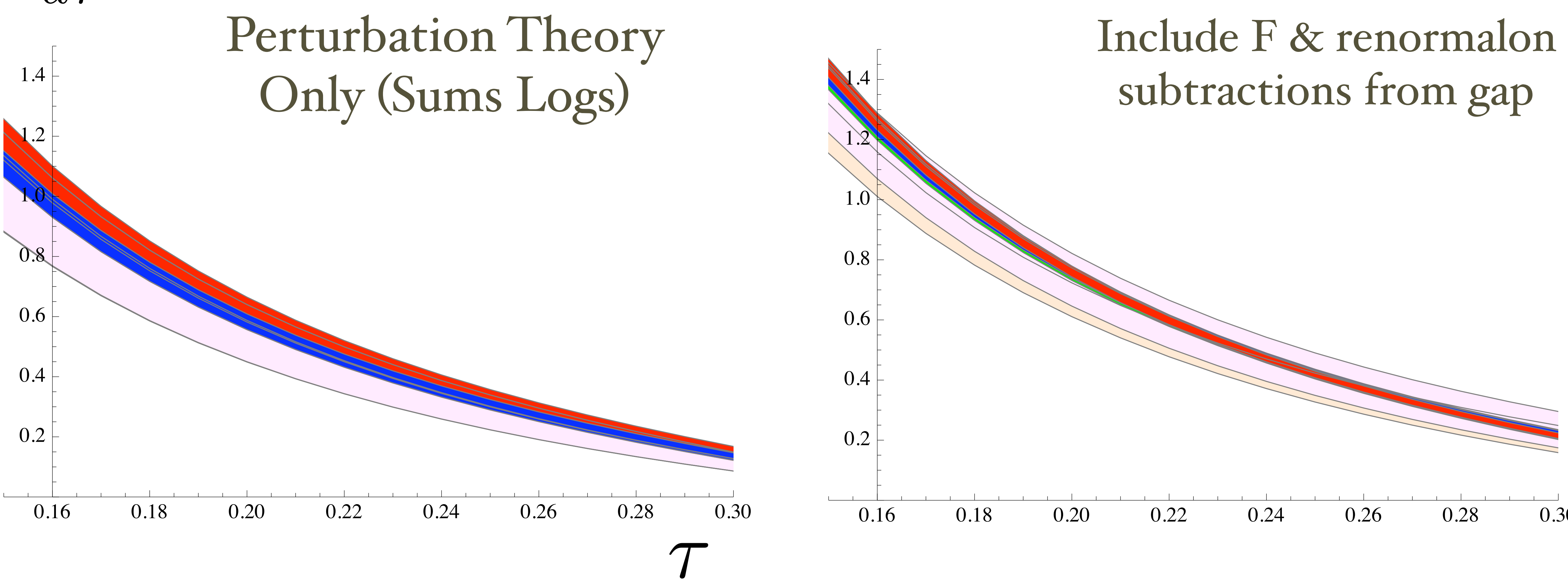
Include F & renormalon subtractions from gap



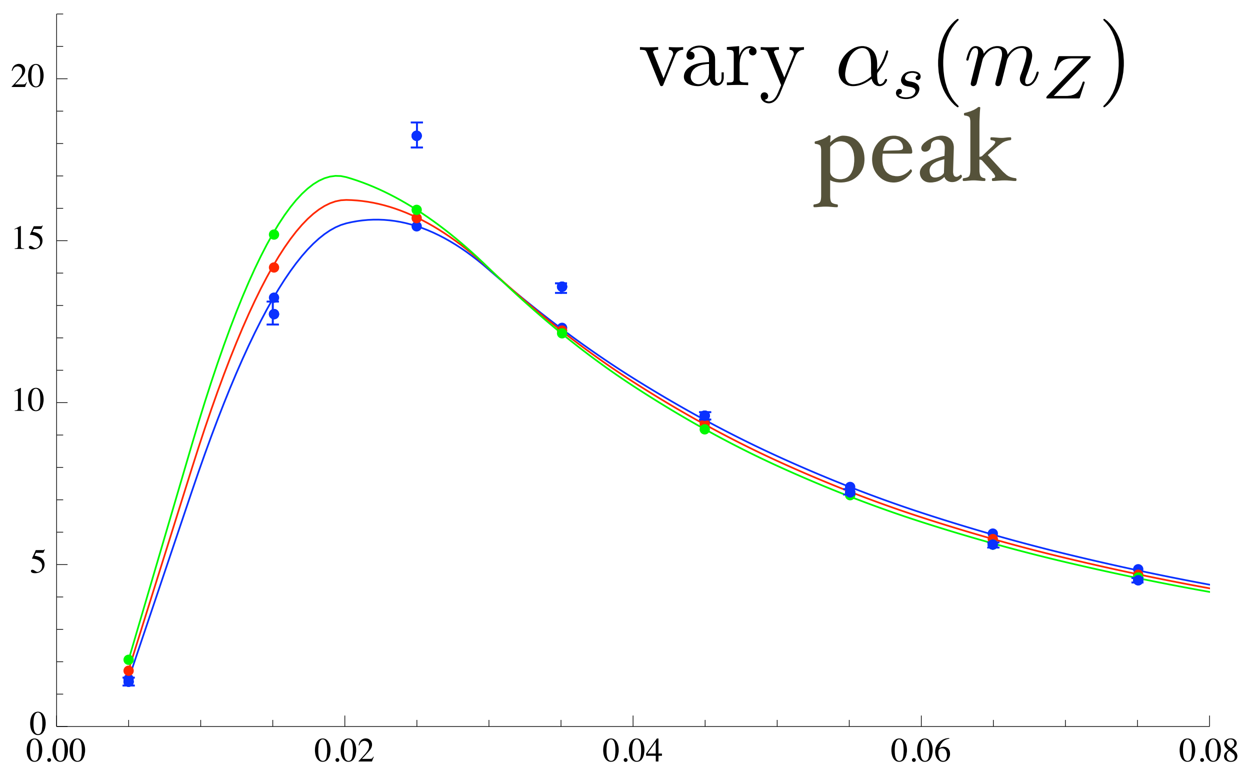
Tail Predictions with Scan over Theory Uncertainties

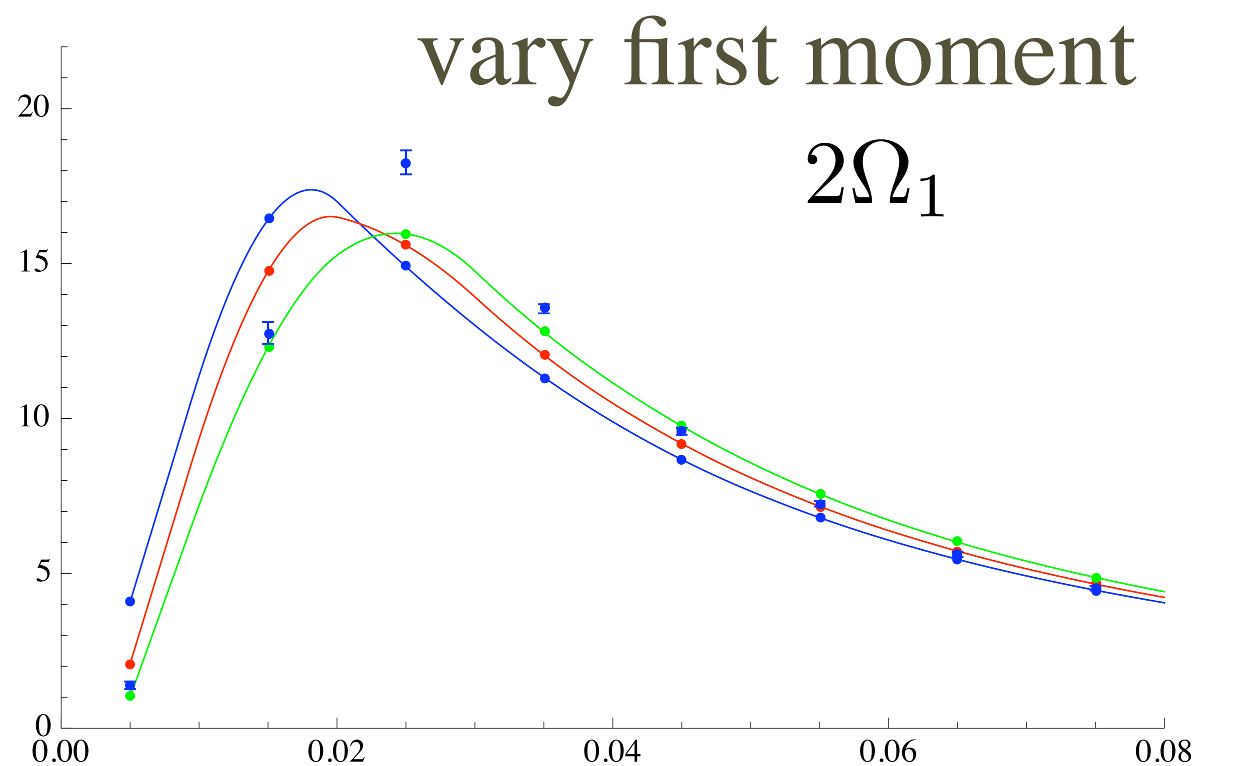
NLL′ NNLL NNLL′ N^{3}LL N^{3}LL′

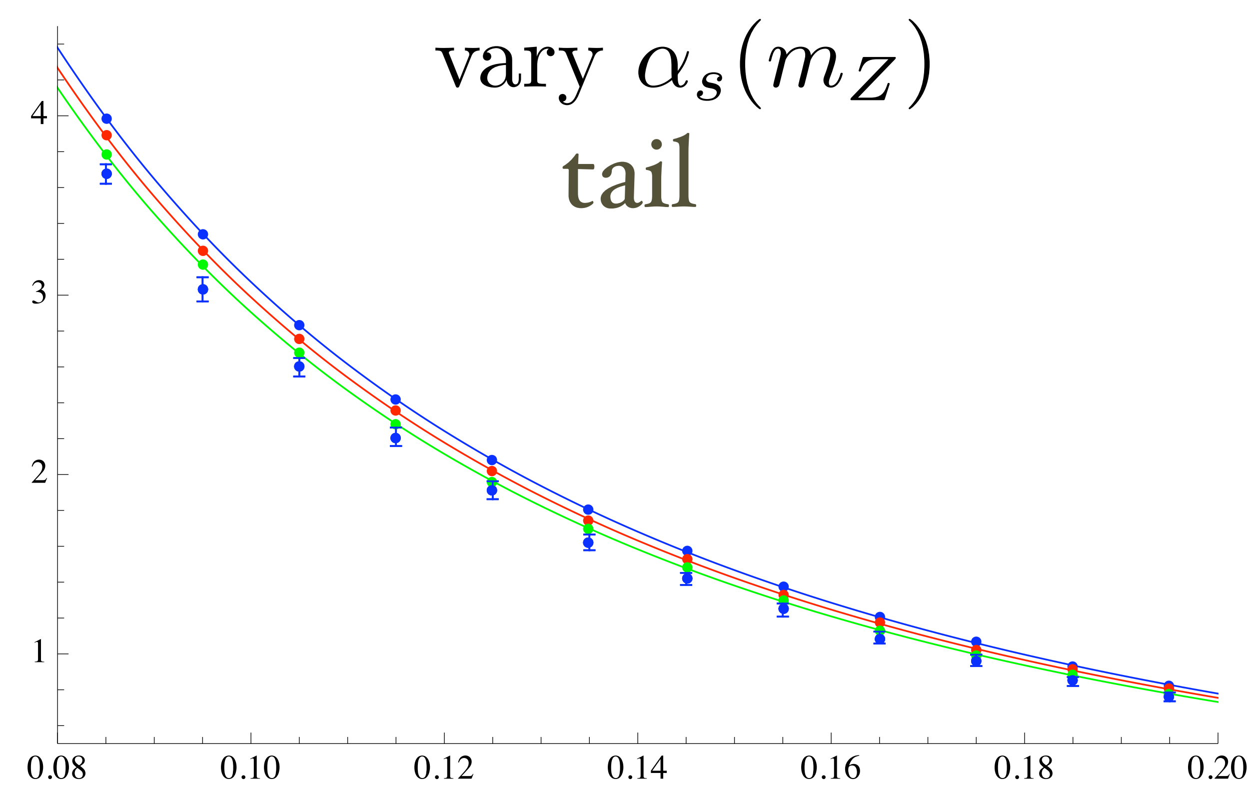
$$\frac{1}{\sigma}\tau\frac{d\sigma}{d\tau}$$

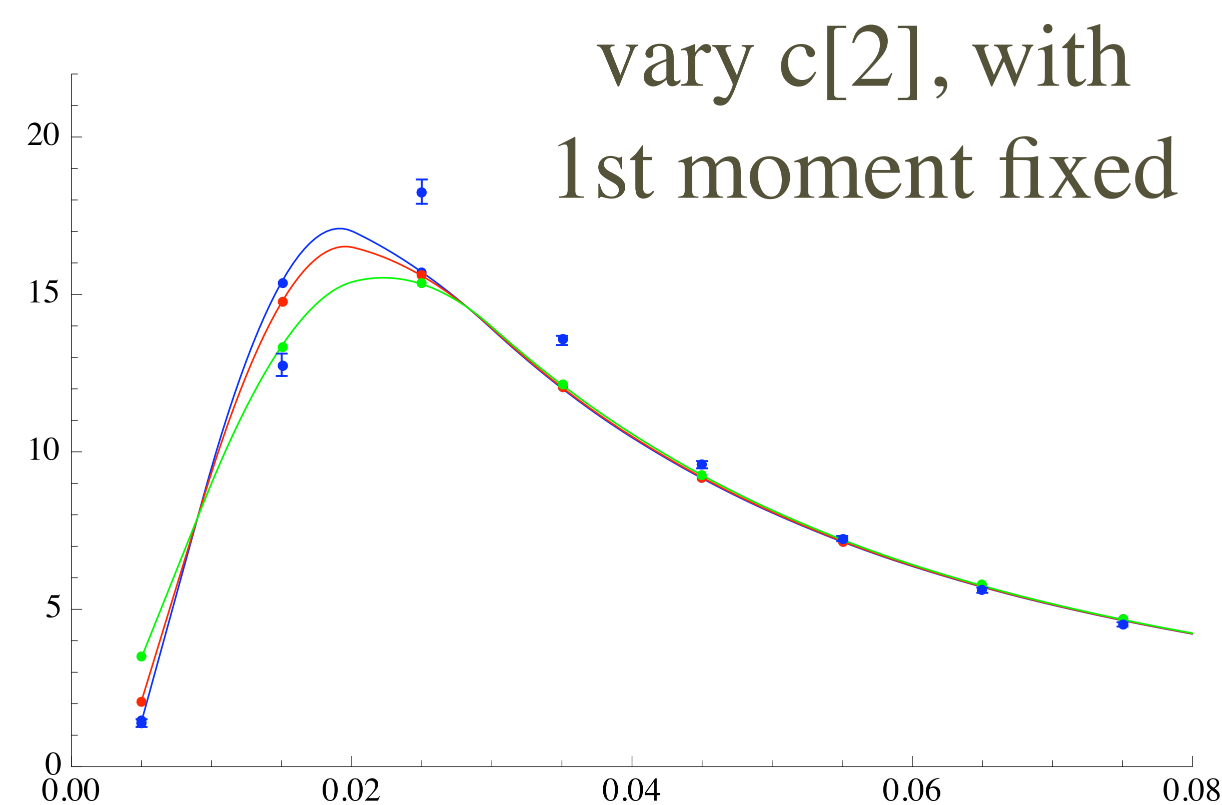


What Parameters to fit?





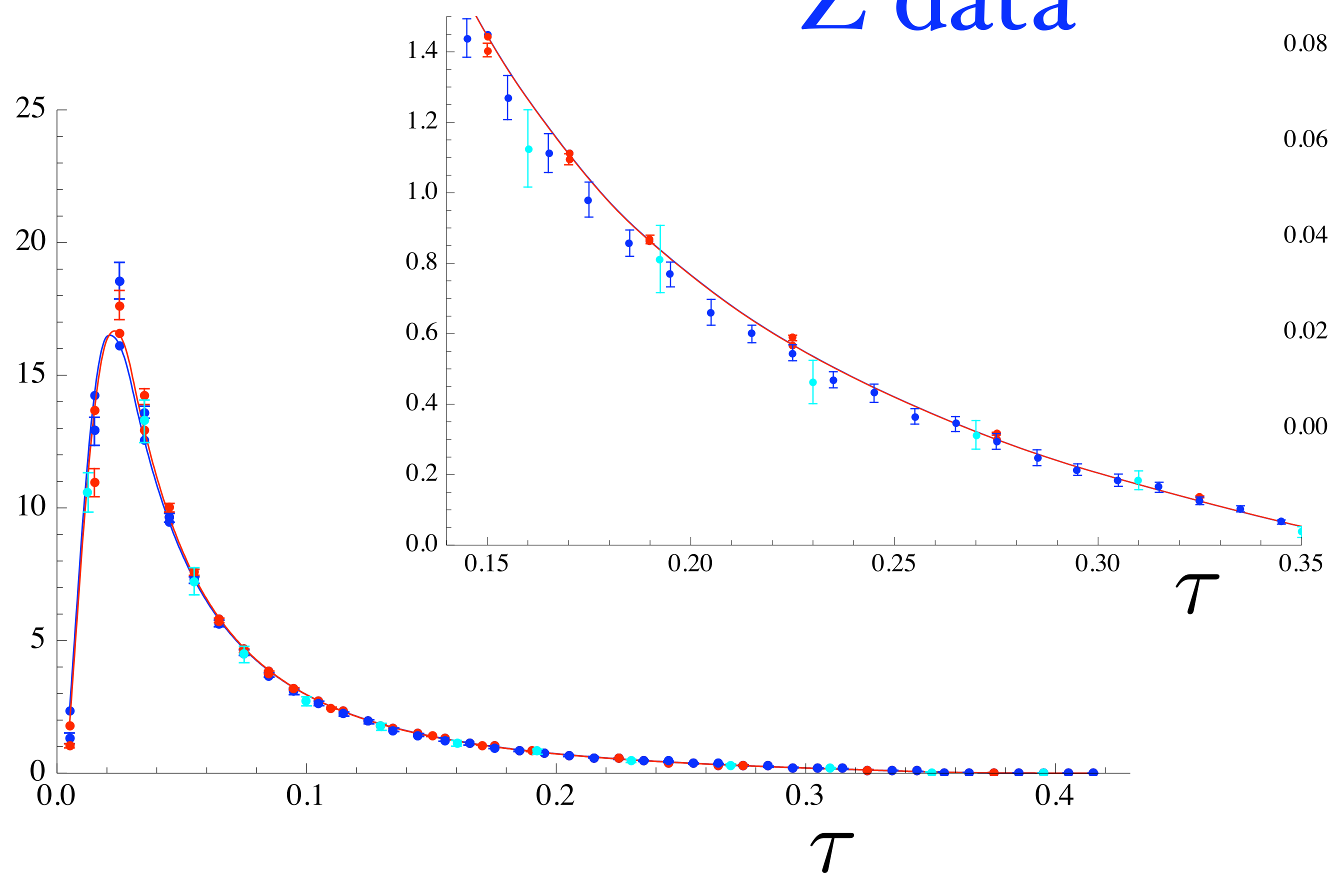




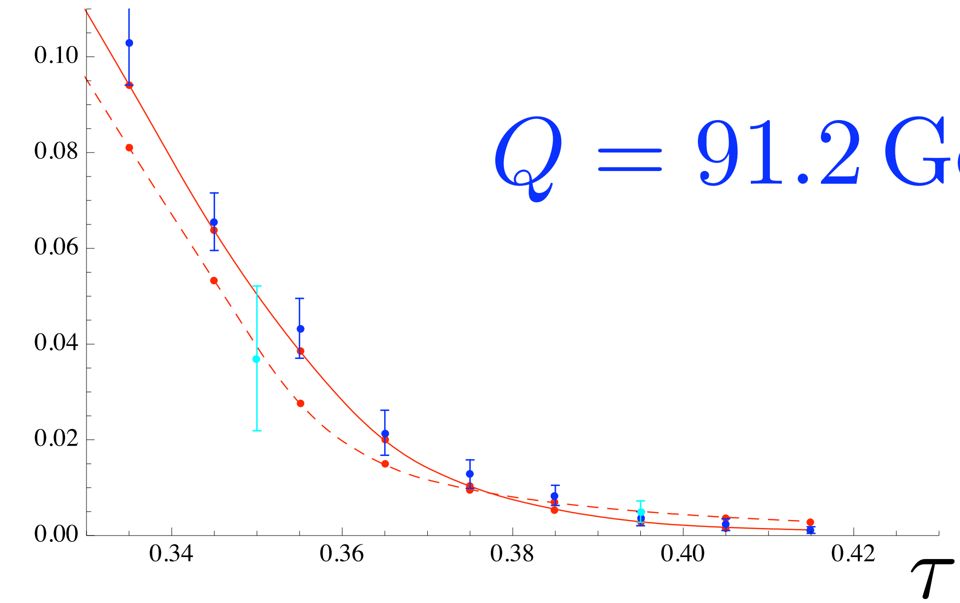
Sample Fit results:

Z data

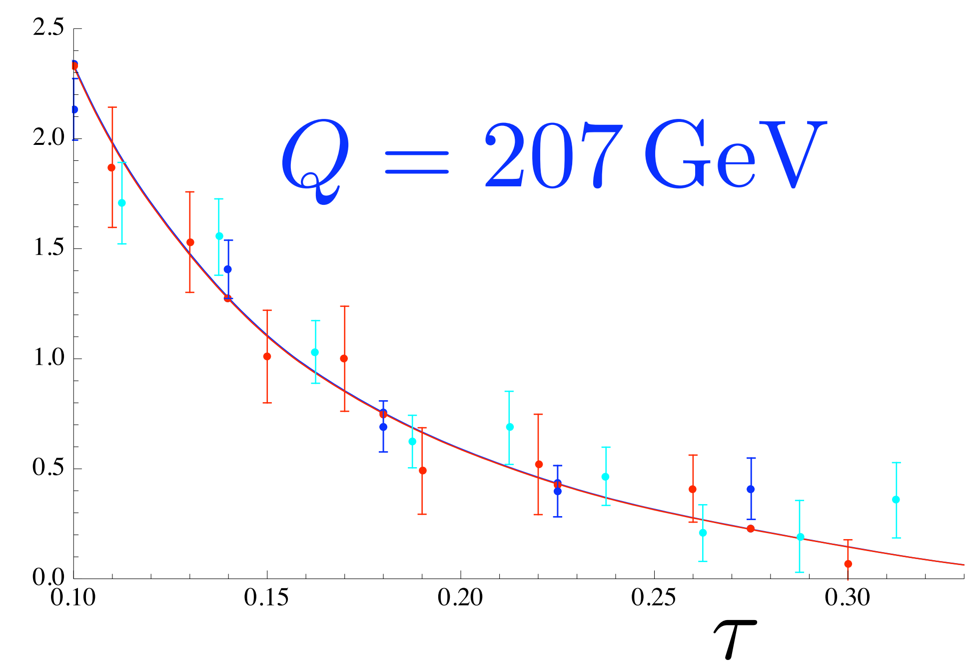
$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$$



$Q = 91.2\,\mathrm{GeV}$

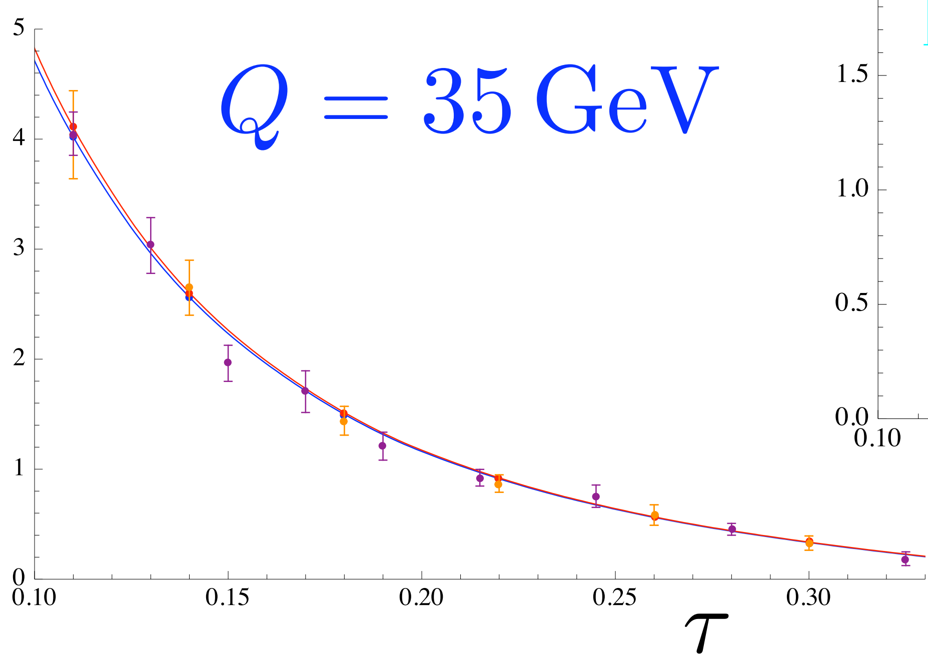


$Q = 207\,\mathrm{GeV}$

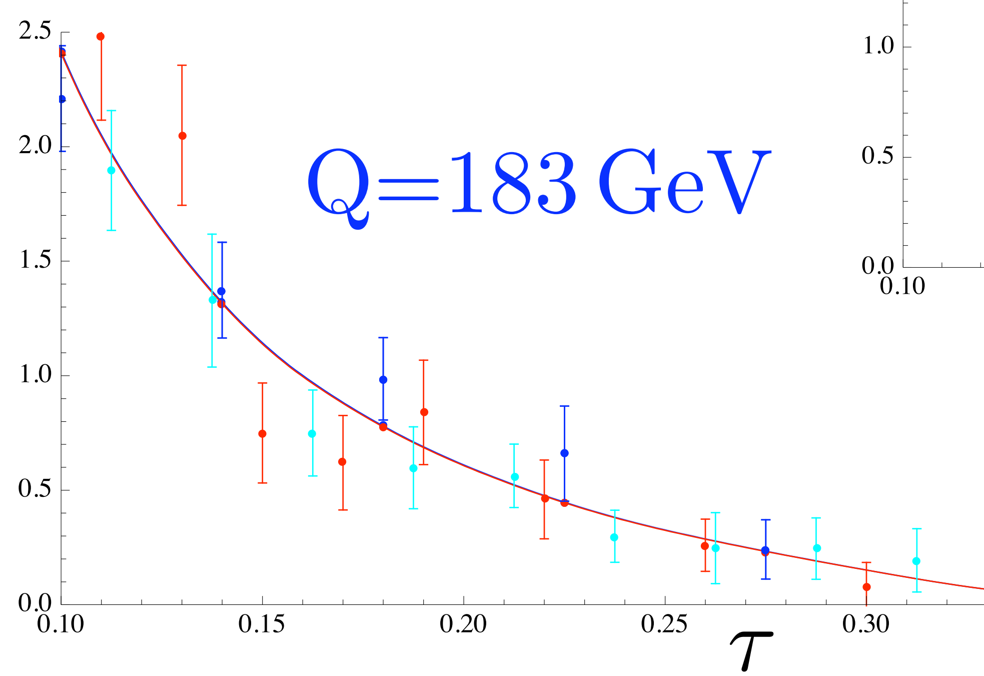


$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$$

$Q = 35\,\mathrm{GeV}$



Q=183 GeV



Here

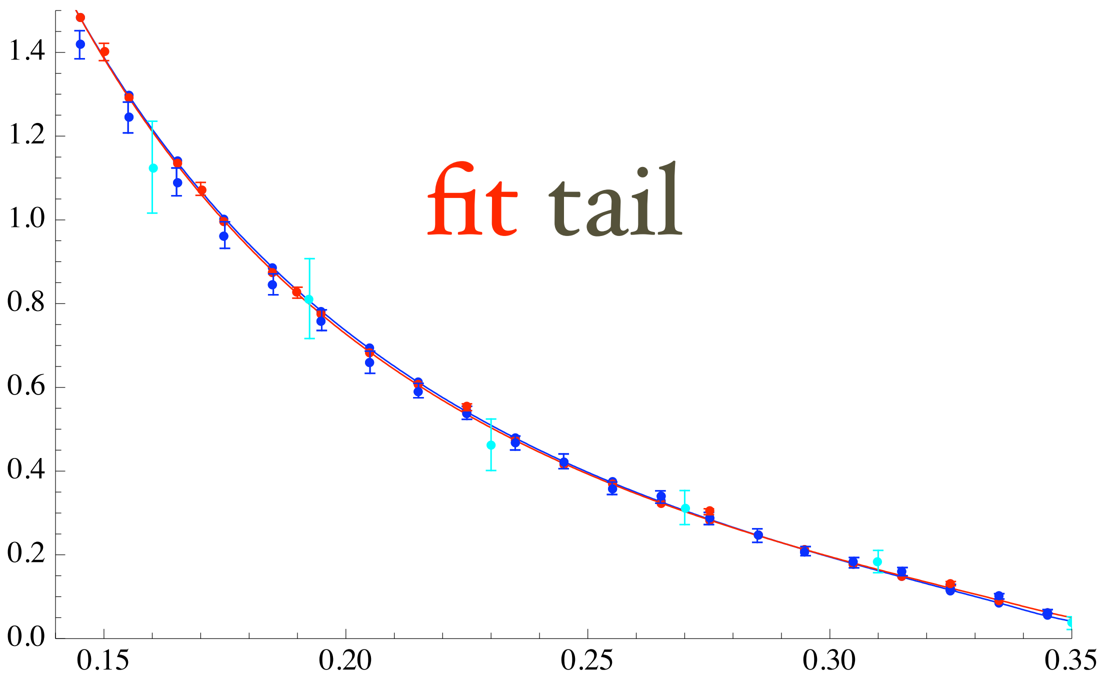
3 parameters:

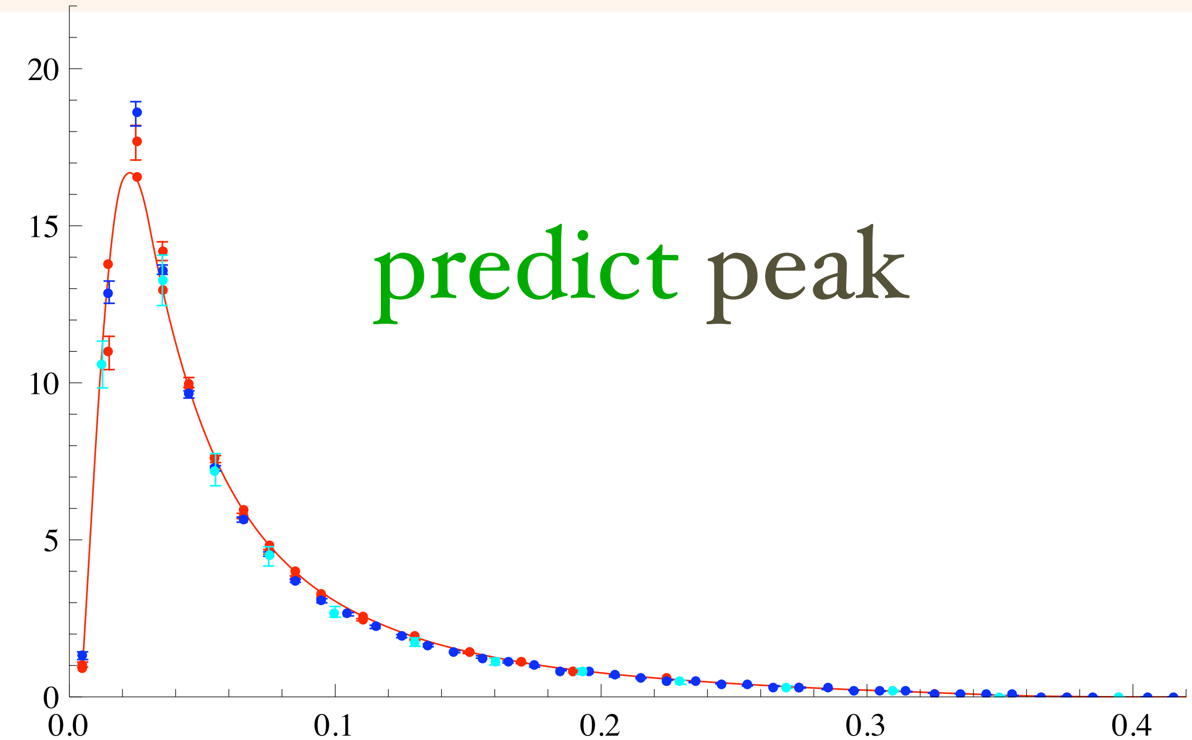
$\alpha_s(m_Z)$, Ω_1, c_2, $[\Delta_0]$

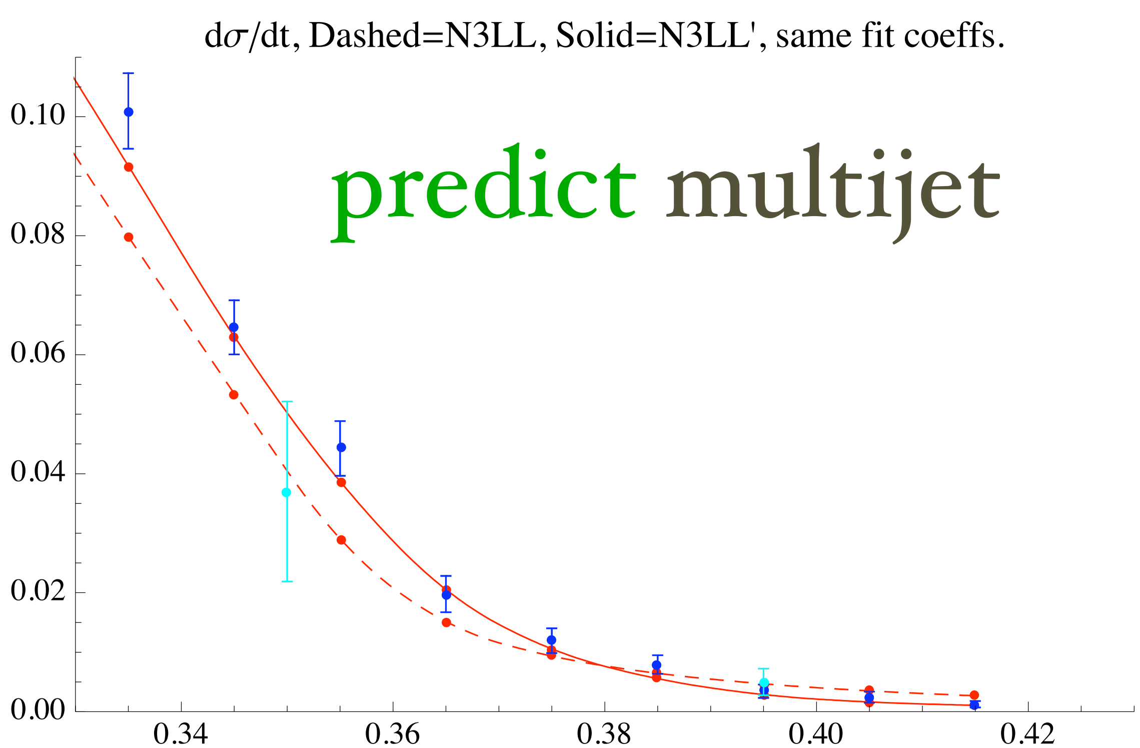
A Tail Fit

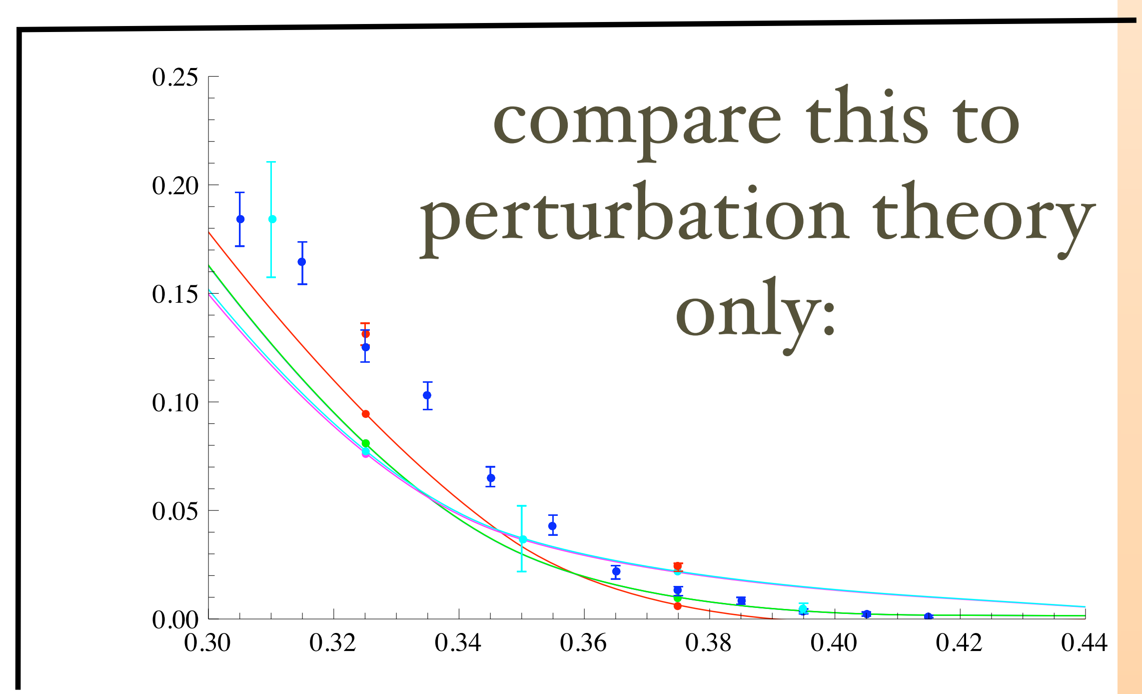
$\{\alpha_s(m_Z), \Omega_1\}$

For τ in the tail region ($Q = 91$, $\tau \in [0.09, 0.33]$, etc.) we can safely do a two parameter fit









Fit Uncertainties:

Statistical Error + Systematic Error
+ Hadronization ($2\Omega_1$)

→ Error Ellipse from Fit

Theory Uncertainties

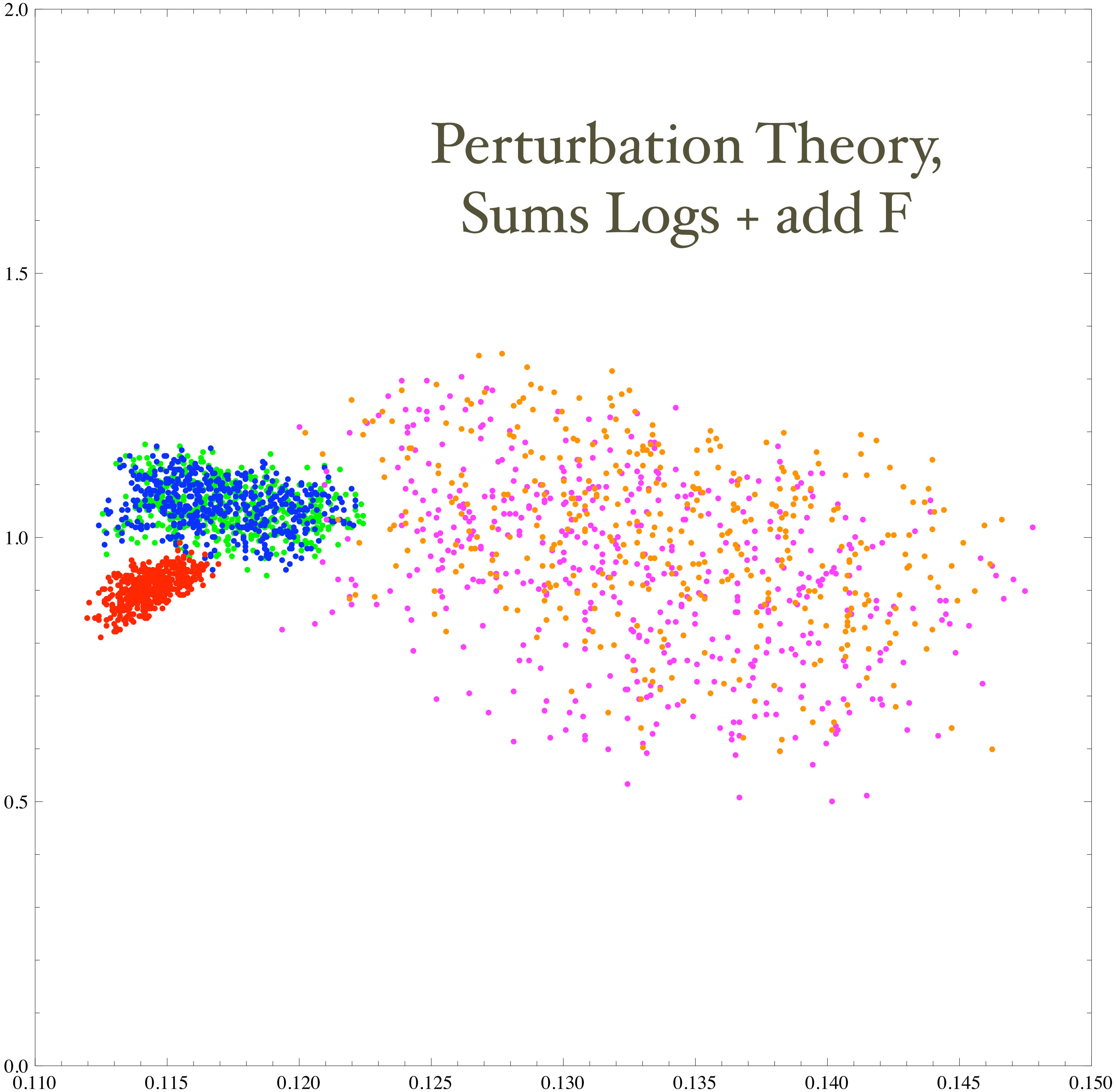
We do a flat scan over unknown theory parameters, fitting each time and take the range of central values

mu dependence: μ_0 n_1 τ_2 ϵ_J $r_h = \mu_h/Q$ n_s

2, 3 loop uncertainties: s_1 ϵ_2 ϵ_3 $\Gamma_3^{\rm cusp}$ H_3 J_3 S_3

theory MC statistics

$2\Omega_1$ (GeV)



$\alpha_s(m_Z)$

χ^2/dof

2.108

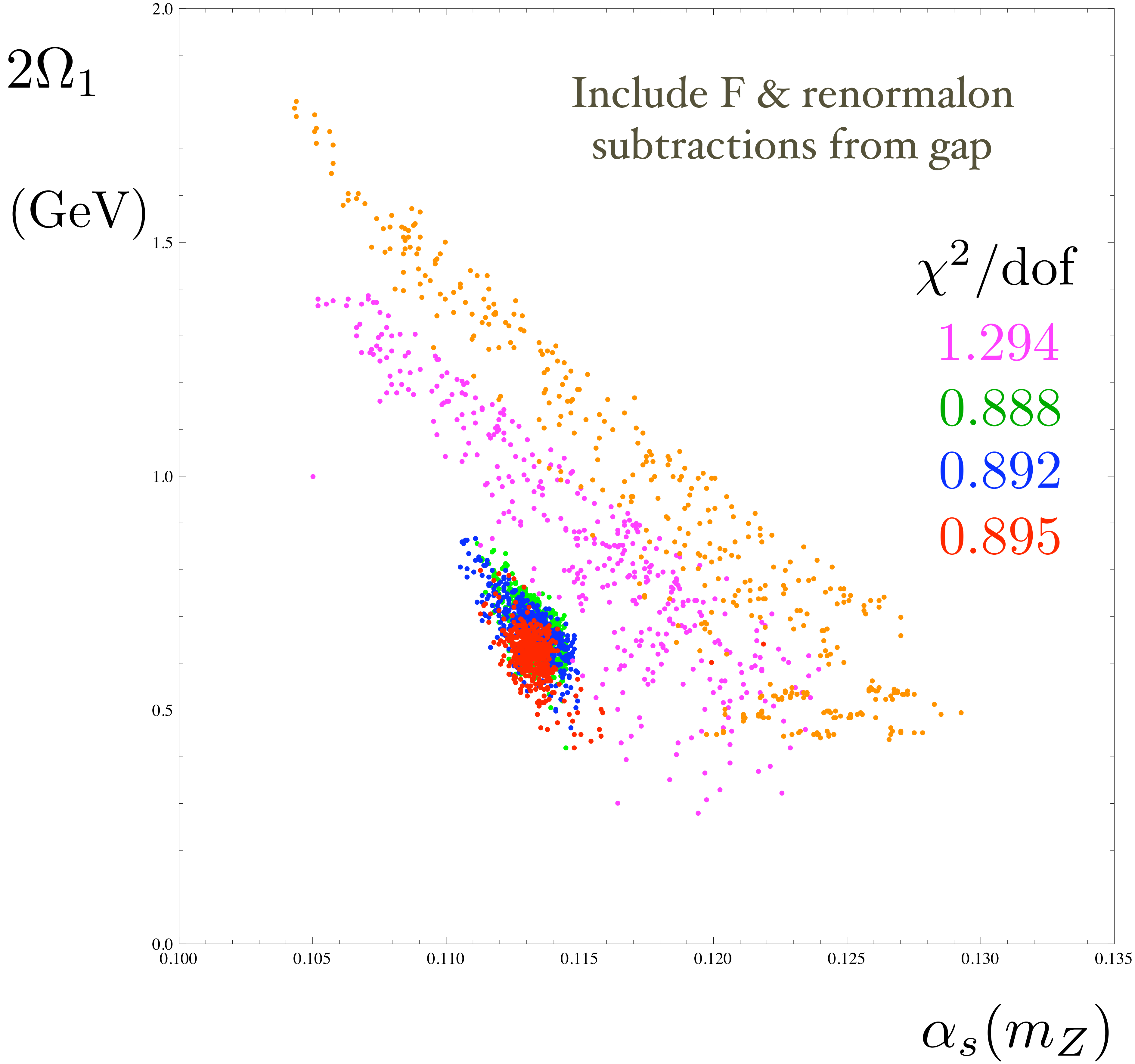
1.561

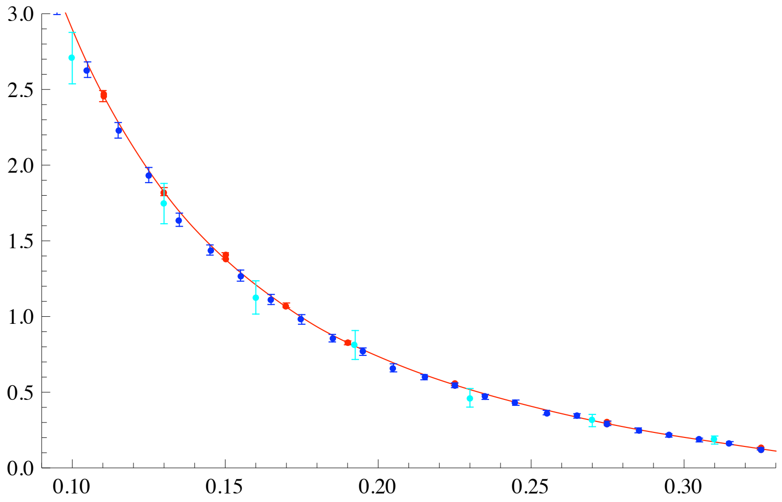
1.570

1.228

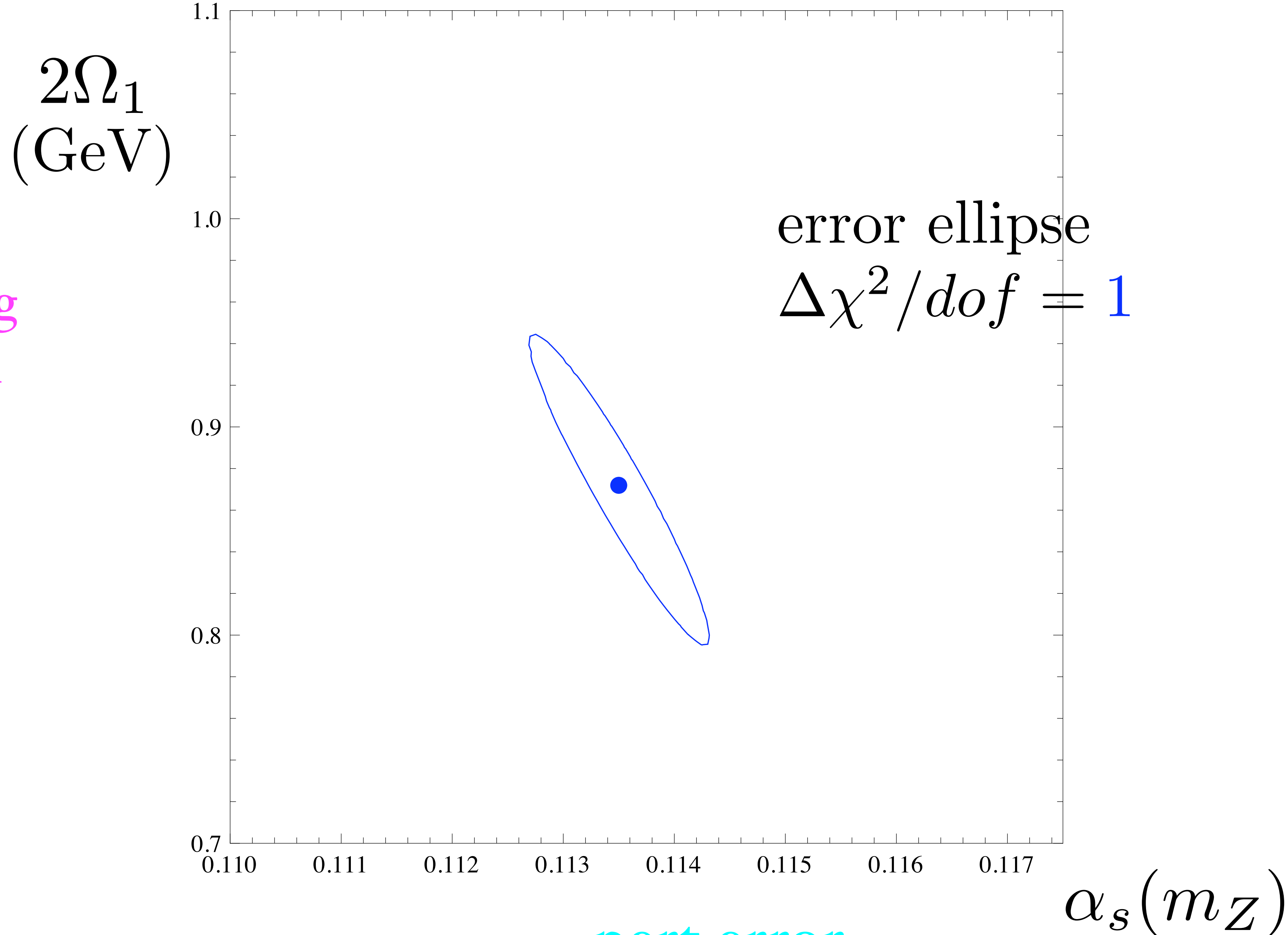
without $2\Omega_1$

$\chi^2/\mathrm{dof} \gtrsim 2$


$2\Omega_1$
(GeV)
Include F & renormalon subtractions from gap
χ^2/dof
1.294
0.888
0.892
0.895
$\alpha_s(m_Z)$
0.0
0.5
1.0
1.5
2.0
0.100
0.105
0.110
0.115
0.120
0.125
0.130
0.135


0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.10
0.15
0.20
0.25
0.30

Tail Fit Result



we use LEP working group's corr. model for syst.errors:

$$\frac{\chi^2}{dof} = \frac{385.9}{433-2} = 0.895$$

pert.error

$$\alpha_s(m_Z) = 0.1135 \pm 0.0008 \begin{array}{l} +0.0007 \\ -0.0013 \end{array}$$

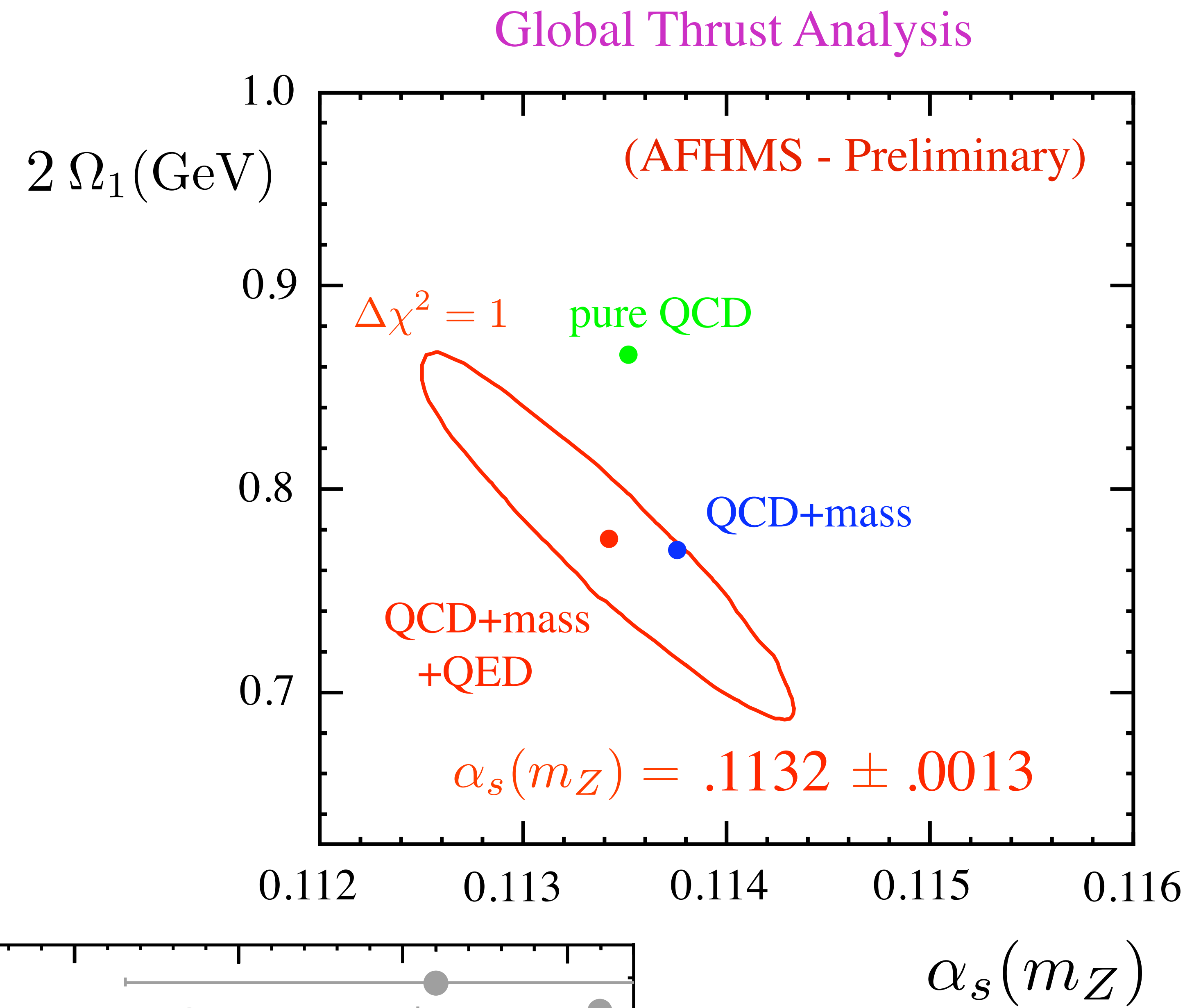
hadronization + expt. error reduced by a factor of 2-3

comparison to

$$\alpha_s(m_Z) = 0.1172 \pm 0.0010(stat) \pm 0.0008(sys) \pm 0.0012(had) \pm 0.0012(pert)$$ Becher & Schwartz fit — resum

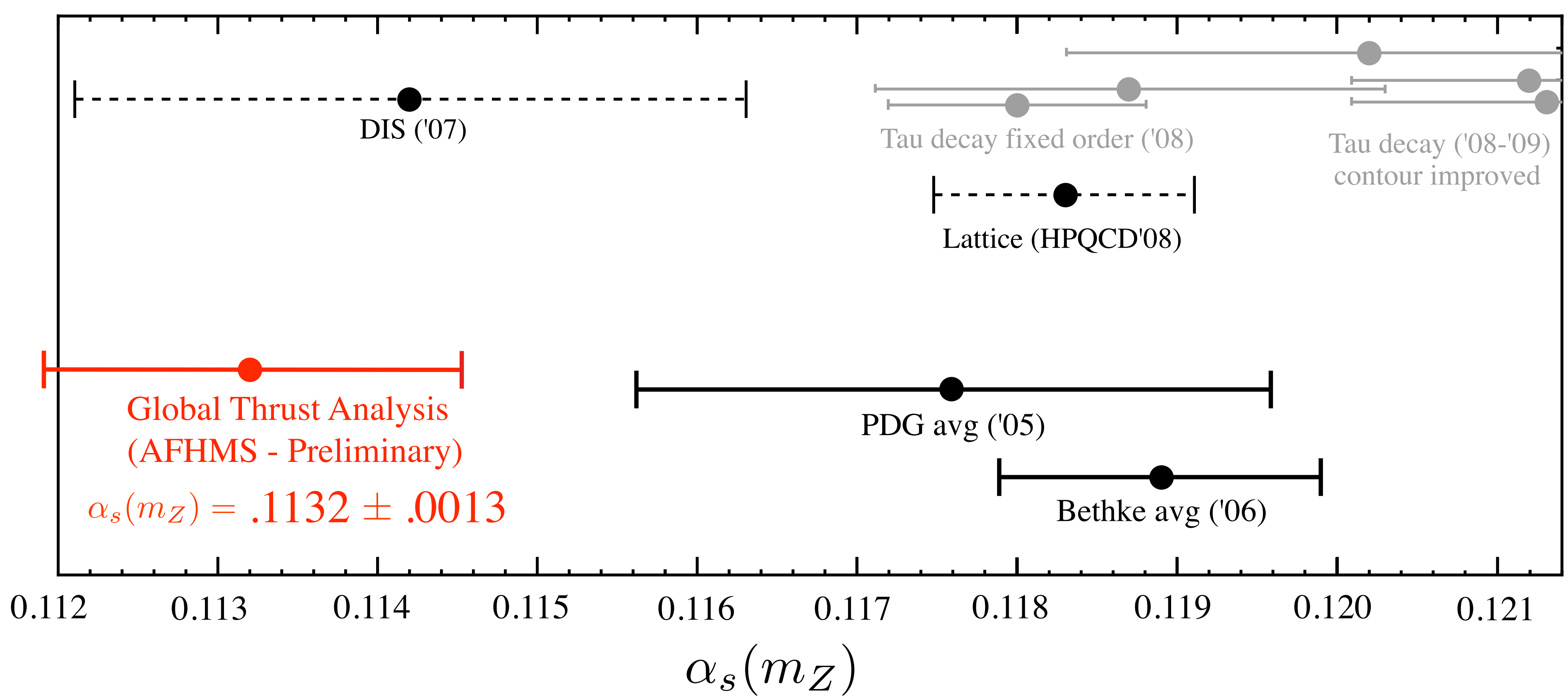
$$\alpha_s(m_Z) = 0.1224 \pm 0.0009(stat) \pm 0.0009(sys) \pm 0.0012(had) \pm 0.0035(theo)$$ Gehrmann, et al. — fixed order

Tail Fit with QED & b-mass



$$\frac{\chi^2}{dof} = \frac{377.4}{433-2} = 0.876$$

(became a bit smaller)



Implications for ILC:

- Further improvements can be made by extending the fit to other event shapes without(!) requiring additional fit parameters.

- At the ILC we will have better statistical errors. And presumably improvements in the systematics. eg. Get full correlation matrix across bins which will lead to better control (perhaps less conservative). Also better data will pin down higher moments, Ω_n, of soft function, which in turn allows more data to be used (a feedback effect).

- Event shapes are complementary and competitive with other ILC methods, like the total Z-decay rate (at Giga-Z).

➡ Together this will yield a systematic program to improve the determination of $\alpha_s(m_Z)$, at an ILC.

$$m_t$$

Motivation

- The top mass is a fundamental parameter of the Standard Model

$$m_t = 173.1 \pm 0.6_{\text{stat}} \pm 1.1_{\text{syst}}\,\text{GeV}$$

(a 0.8% error)
(theory error?
what mass is it?)

- Important for precision e.w. constraints

eg. $m_H = 76^{+33}_{-24}\,\text{GeV} \to 87$ $\qquad m_H < 182\,\text{GeV} \to 209$ $\qquad$ (95% CL)

A 2 GeV shift in m_t changes the central values by 15%

- Top Yukawa coupling is large. Top parameters are important for analyzing many new physics models. (eg. Higgs masses in MSSM)

- Top is very unstable, it decays before it has a chance to hadronize. This provides an intrinsic smearing for jet observables.

Top provides playground for future analysis of new short lived strongly interacting particles.

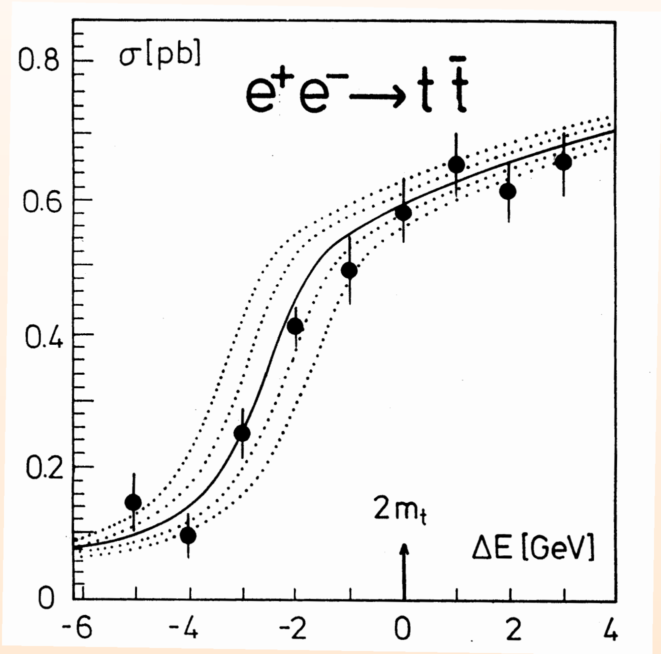
$\Gamma_t = 1.4\,\text{GeV}$

from $\quad t \to bW$

Threshold Scan $e^+e^- \to t\bar{t}$ the classic ILC method

$$\sqrt{s} \simeq 350\,\mathrm{GeV}$$

- ▷ count number of $t\bar{t}$ events
- ▷ color singlet state
- ▷ background is non-resonant
- ▷ physics well understood
  (renormalons, summations)



Precision Theory meets precision experiment:

$\to \delta m_t^{\mathrm{exp}} \simeq 50$ MeV

$\to \delta m_t^{\mathrm{th}} \simeq 100$ MeV

("peak" position)

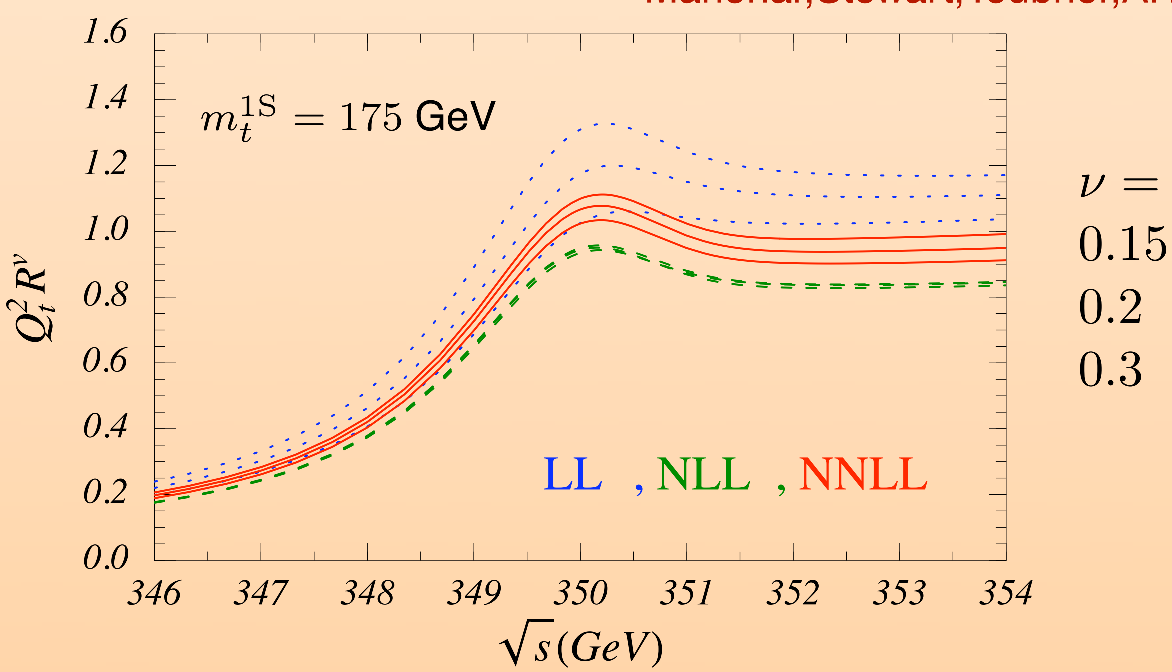
Teubner,AH; Melnikov, Yelkovski;Yakovlev; Beneke,Signer,Smirnov; Sumino, Kiyo

- Measure a short-distance top-quark mass, like m_t^{1S} NOT the top pole mass.

- Have smearing by ISR and beamstrahlung, which must be controlled precisely

1S mass - RG-improved, with NNLL non-mixing terms

Manohar,Stewart,Teubner,AH



Threshold Theory Status

goal is $\sim 3\%$ for $\delta\sigma/\sigma$

NRQCD with computable power and radiative corrections.

potential $V(r)$: full NNLL

Manohar, IS, Hoang; '99-'03
Pineda, Soto '00-'01
Peter '94, Schroeder '98

short-distance coefficints $C(\nu)$: almost NNLL

NLL: Luke etal '99
NNLL(matching): Beneke etal; Czarnecki etal '99
NNLL(non-mixing) Hoang '03
NNLL (mixing) mostly known
spin-dependent soft Penin etal. '04
usoft nf Stahlhofen, Hoang '05

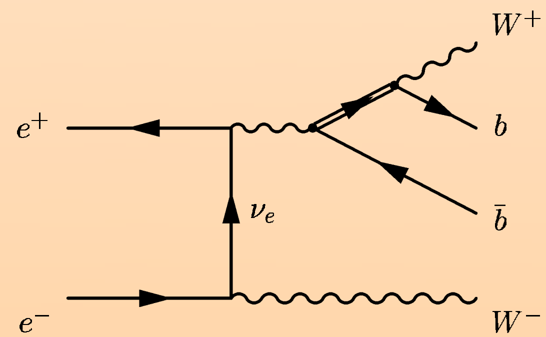
unstable top

- complex matching conditions & anomalous dimensions
- effective Lagrangian non-hermitian
- total rates through the optical theorem
- phase space matching

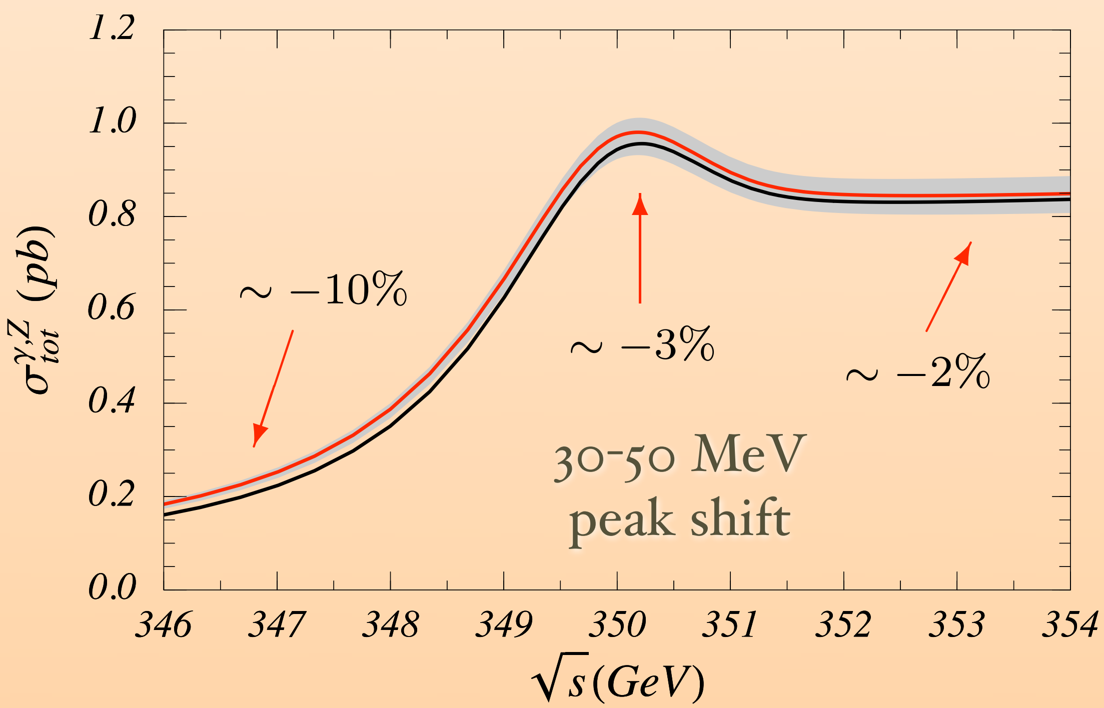
Beneke etal. '03, '04
Grzadkowski, Kuhn '87
Guth, Kuhn '92

Reisser, Hoang;'05
Reisser, Hoang '06

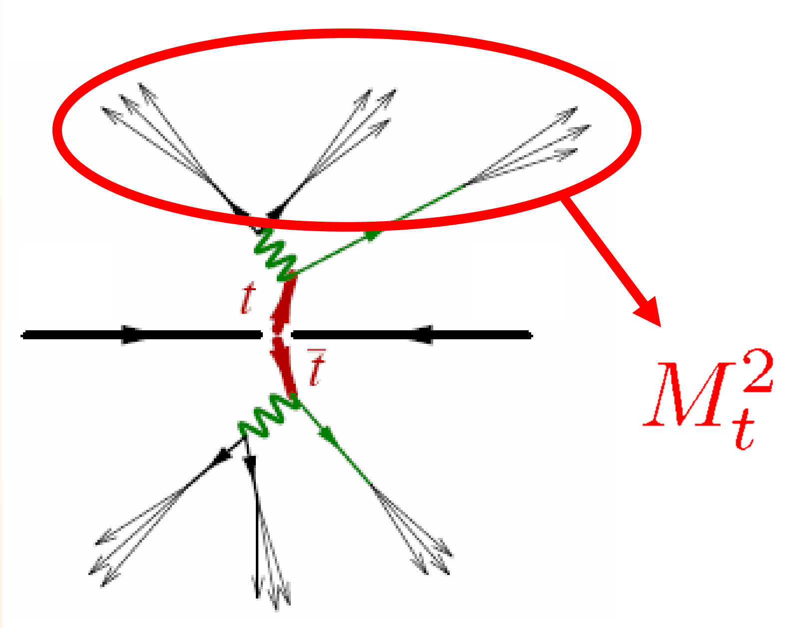
compute electroweak effects
compute non-resonant irreducible bkgnd



(e)



Above Threshold $e^+e^- \to t\bar{t}$



- jet observable ★★
- suitable top mass for jets ★
- initial state radiation
- final state radiation ★
- color reconnection ★
- sum large logs ★

$$M_t^{\rm peak} = m_t + \text{(nonperturbative effects)} + \text{(perturbative effects)}$$

To simplify things we'll work far above threshold:

$$Q \gg m_t \gg \Gamma_t$$

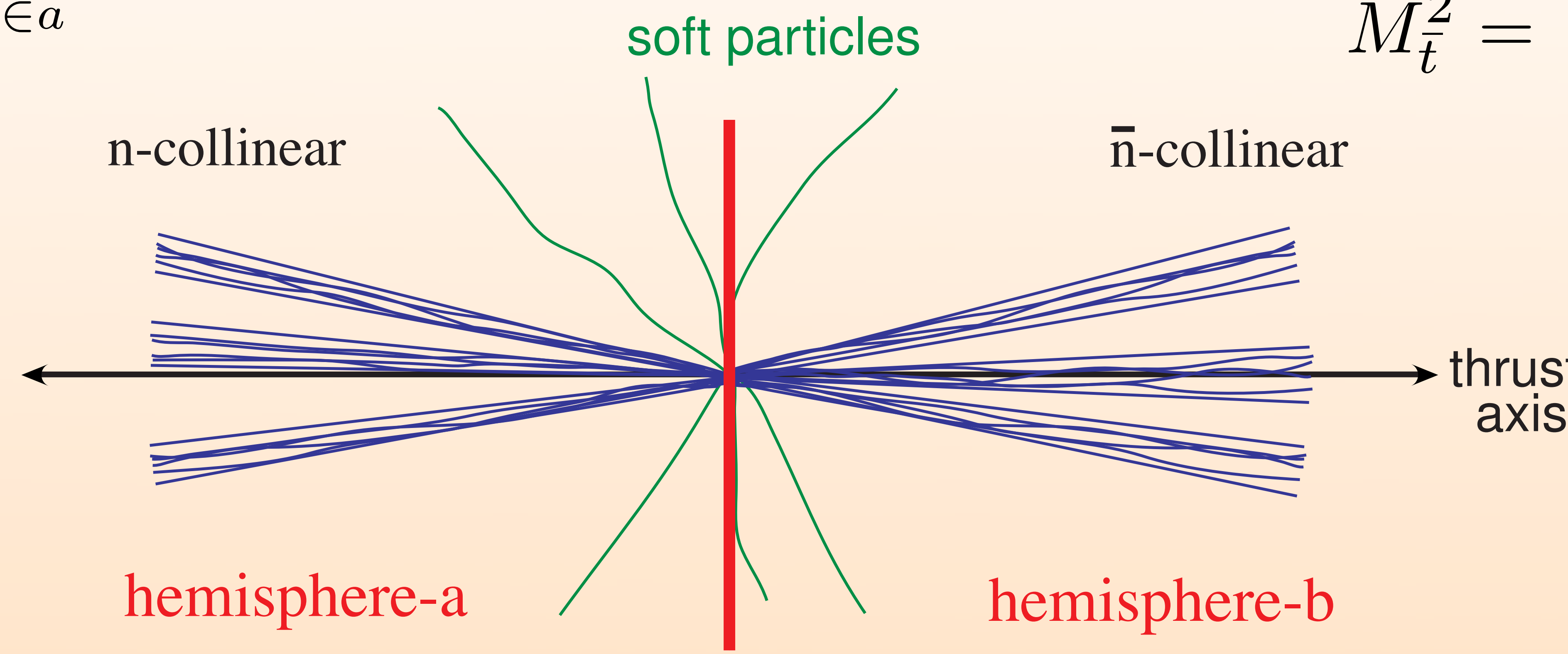
$(\frac{m_t^2}{Q^2}$ dependence can be computed)

Hemisphere Invariant Masses

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}$$

$$M_t^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2$$

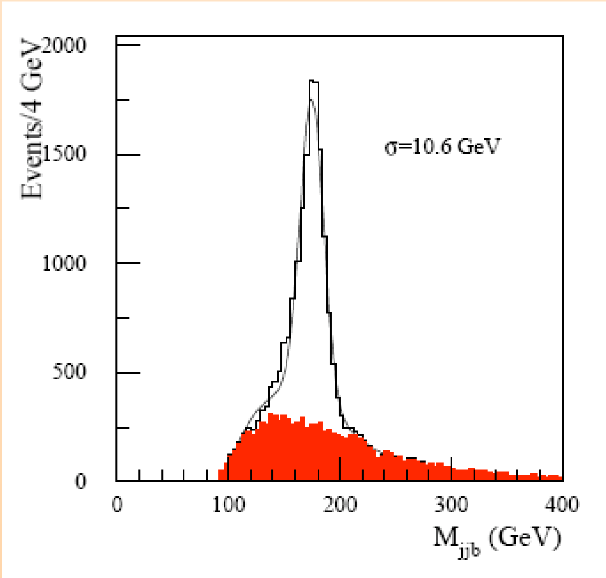
$$M_{\bar t}^2 = \Big(\sum_{i\in b} p_i^\mu\Big)^2$$



Peak region:

$$s_t \equiv M_t^2 - m^2 \sim m\Gamma \ll m^2$$

$$\boxed{\hat{s}_t \equiv \frac{M_t^2 - m^2}{m} \sim \Gamma \ll m}$$



Breit Wigner: $$\frac{m\Gamma}{s_t^2 + (m\Gamma)^2} = \Big(\frac{\Gamma}{m}\Big)\frac{1}{\hat{s}_t^2 + \Gamma^2}$$

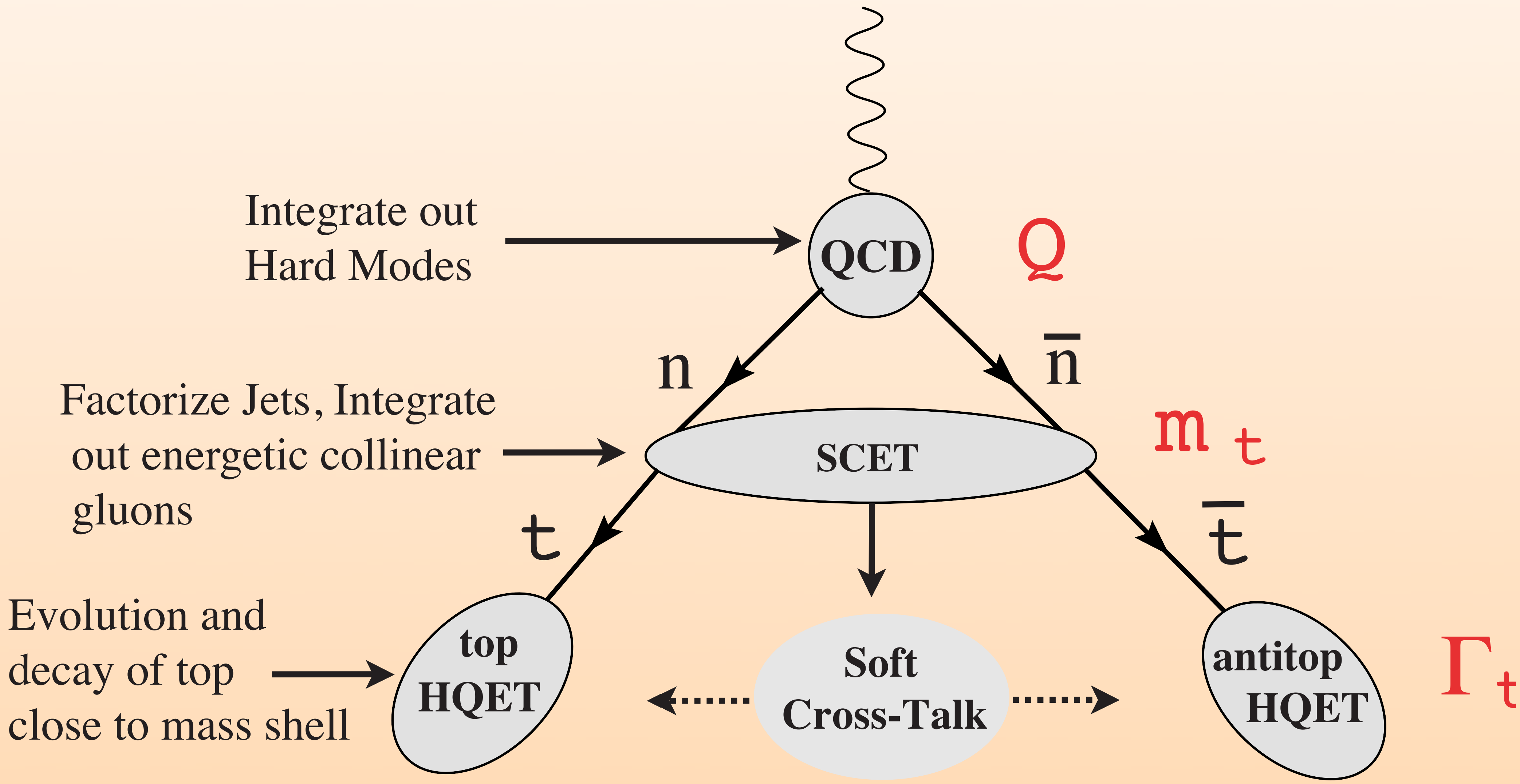
$$Q \gg m \gg \Gamma \sim \hat{s}_{t,\bar{t}}$$

Disparate Scales ➡ Effective Field Theory

QCD

SCET

HQET



Factorization Theorem:

Fleming, Hoang, Mantry, I.S.

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu).$$

Answer

$$+\,\mathcal{O}\Big(\frac{m\alpha_s(m)}{Q}\Big) + \mathcal{O}\Big(\frac{m^2}{Q^2}\Big) + \mathcal{O}\Big(\frac{\Gamma_t}{m}\Big) + \mathcal{O}\Big(\frac{s_t, s_{\bar t}}{m^2}\Big)$$

Valid to all orders in α_s

& includes leading nonperturbative effects

Factorization Theorem:

Fleming, Hoang, Mantry, I.S.

Hard Production modes integrated out

"Hard" collinear gluons integrated out

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu).$$

Answer

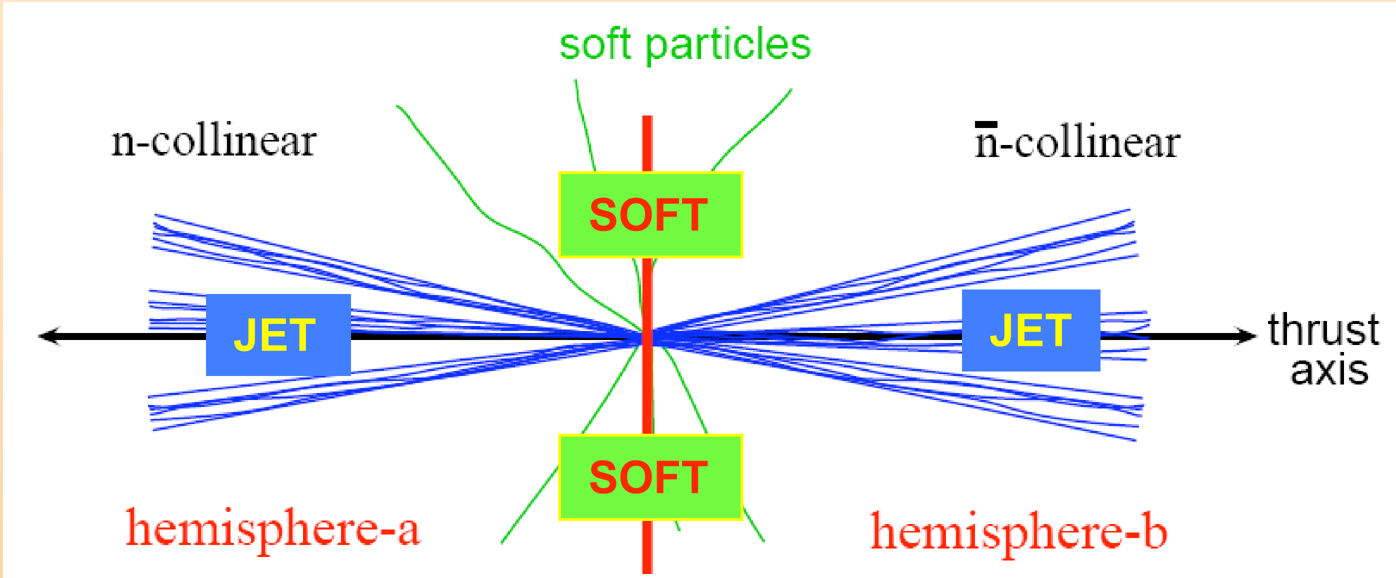
A useful event shape for massive unstable particles

Jet Functions

Evolution and decay of top quark close to mass shell

Soft Function

Non-perturbative Cross talk



Implications

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu).$$

Answer

$$M^{\rm peak} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{Q\Lambda_{\rm QCD}}{m_t}$$

measure this

extract this

compute this

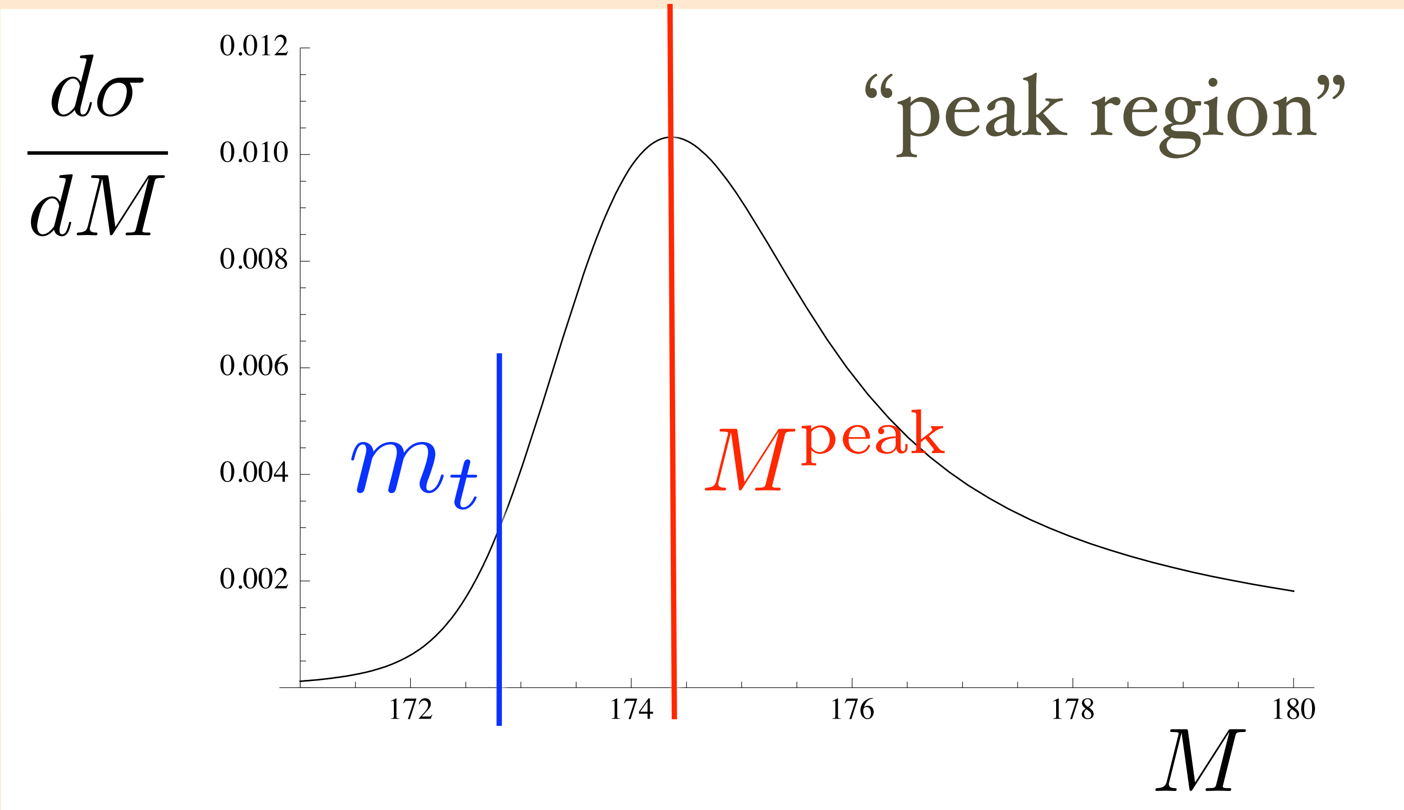
soft radiation shifts the measured mass

$\frac{Q\Lambda_{\rm QCD}}{m_t}$ is predominantly $\frac{Q\Omega_1}{m_t}$!

known to 10% from fit in part I

$$(\delta m_t)^{\Omega_1} \simeq 200\,\text{MeV}$$

will be known even better with ILC



Short Distance Mass Scheme for Jets

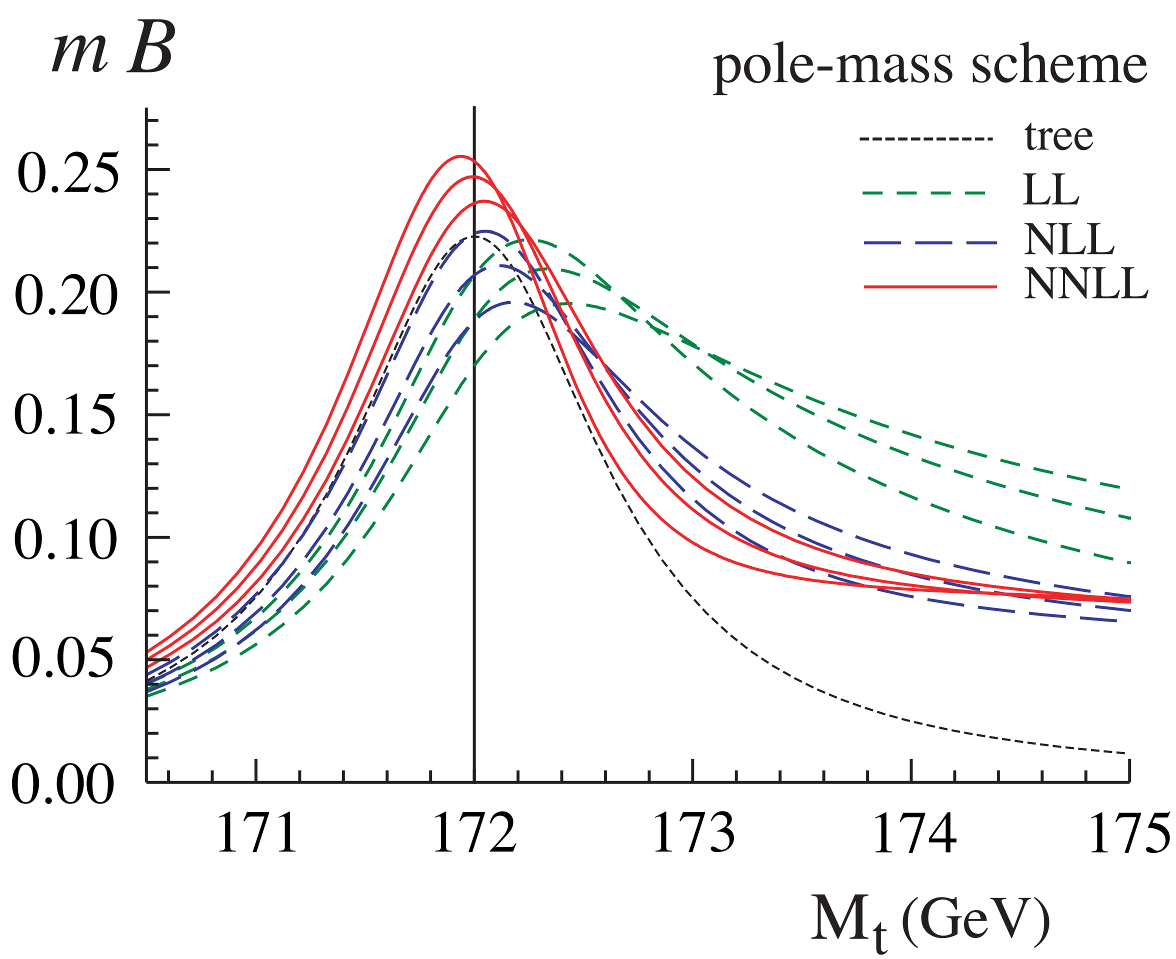
- top $\overline{\mathrm{MS}}$ mass? — Can not be treated consistently with Breit-Wigner for decay products
- pole mass? — Breit-Wigner is fine, but has renormalon problem (instability)
- $1S$ mass? — Also couples scales in an ugly fashion.
- top jet mass — Breit-Wigner is fine & no renormalon

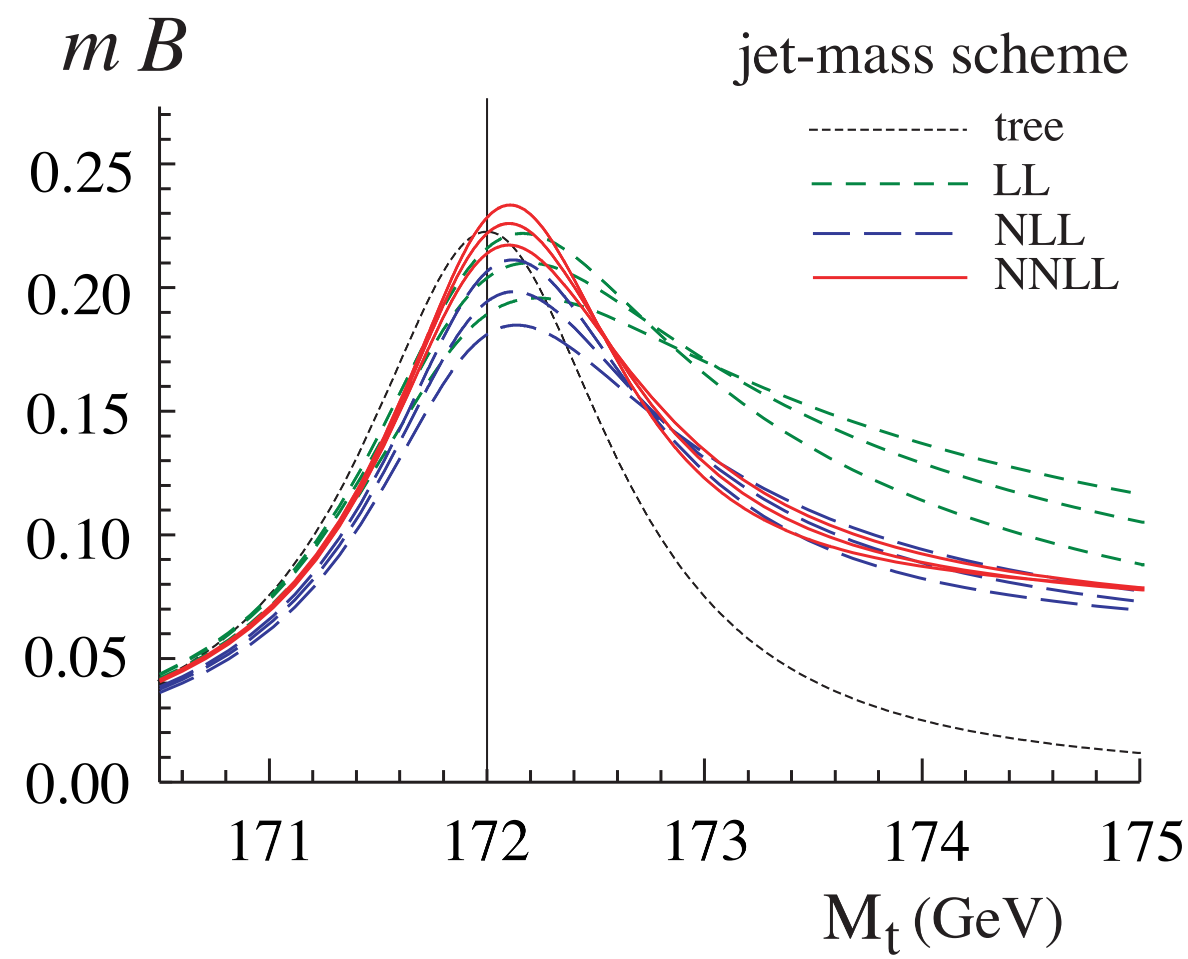
Good!

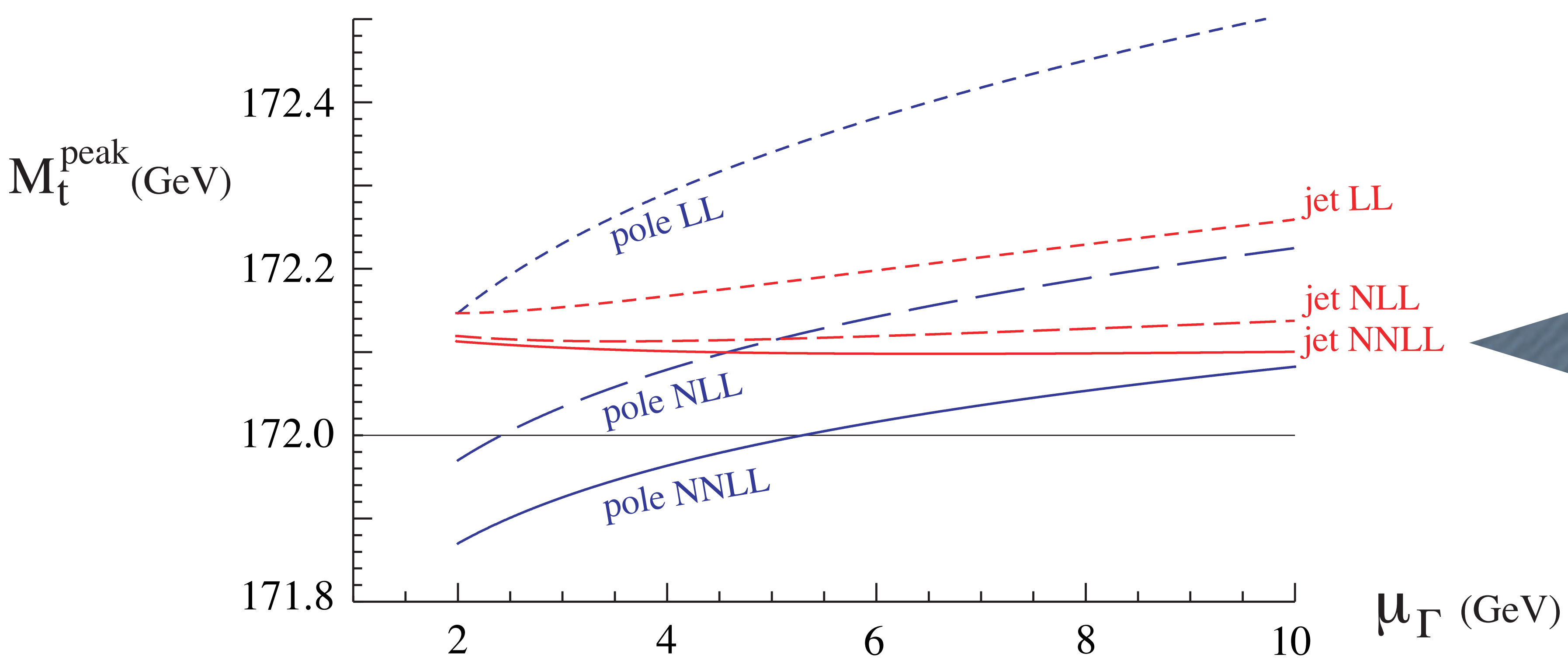
$$m^{\mathrm{pole}} - m_t^{\mathrm{jet}} \sim \alpha_s \Gamma$$

Use heavy quark jet function B to define the series

Jet Function Results up to NNLL: (3 curves vary μ_Γ)



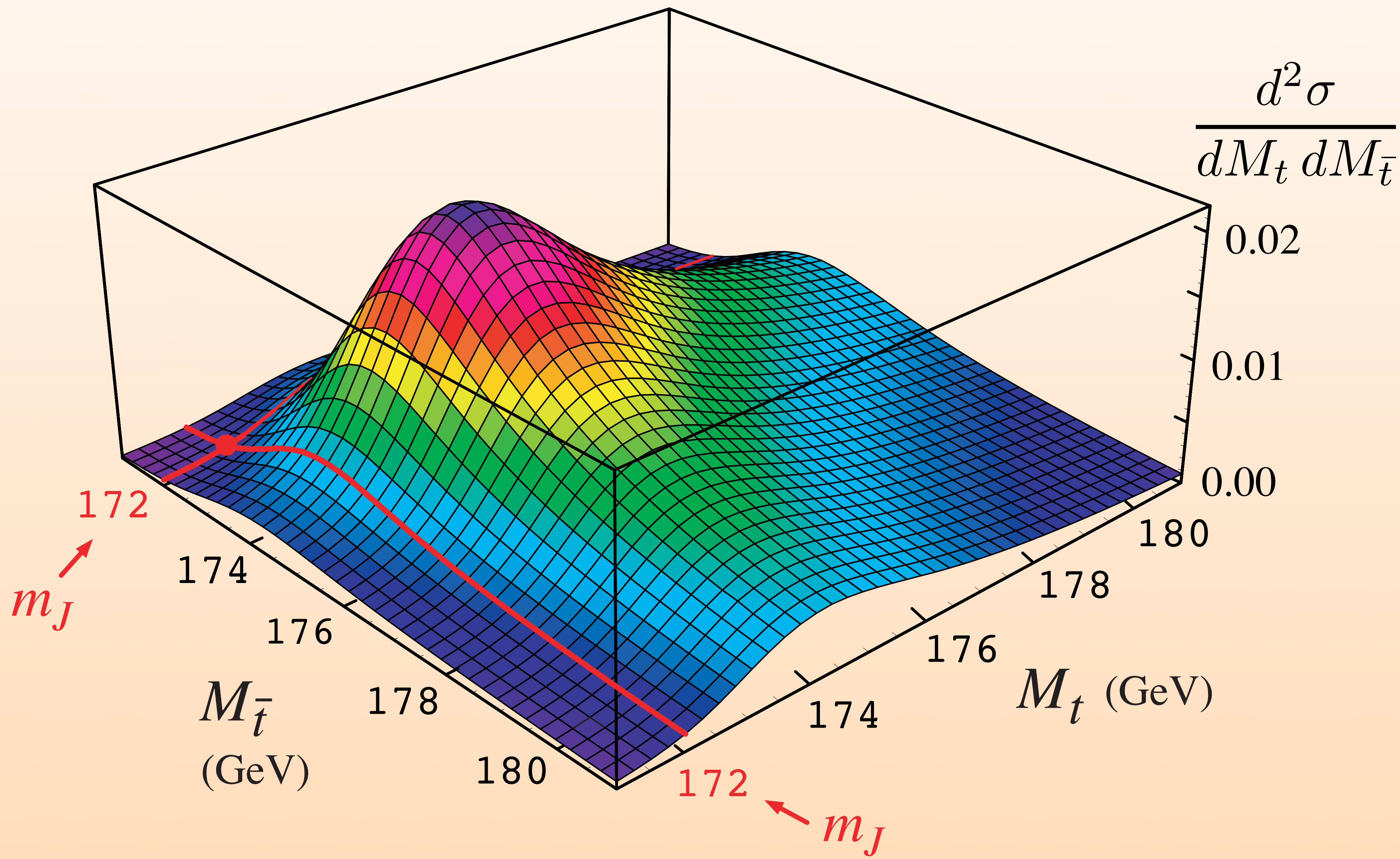




Jain, Scimemi, I.S.

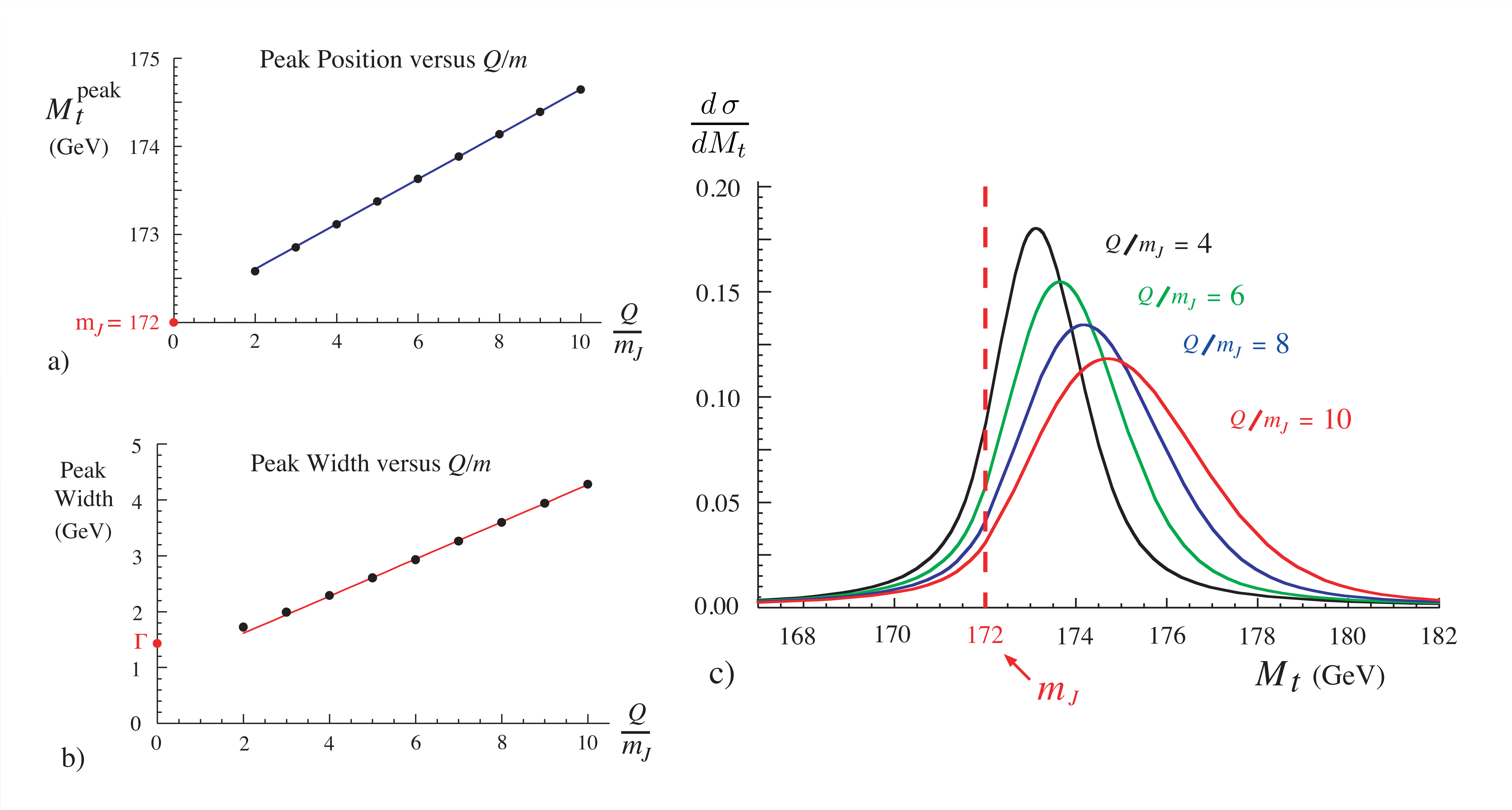
very stable perturbative peak!

NLL Cross-Section Results



soft non-perturbative radiation shifts the peak $\sim 2\,\mathrm{GeV}$, and broadens the distribution

Symmetric mass projection:



soft non-perturbative radiation shifts the peak, and broadens the distribution

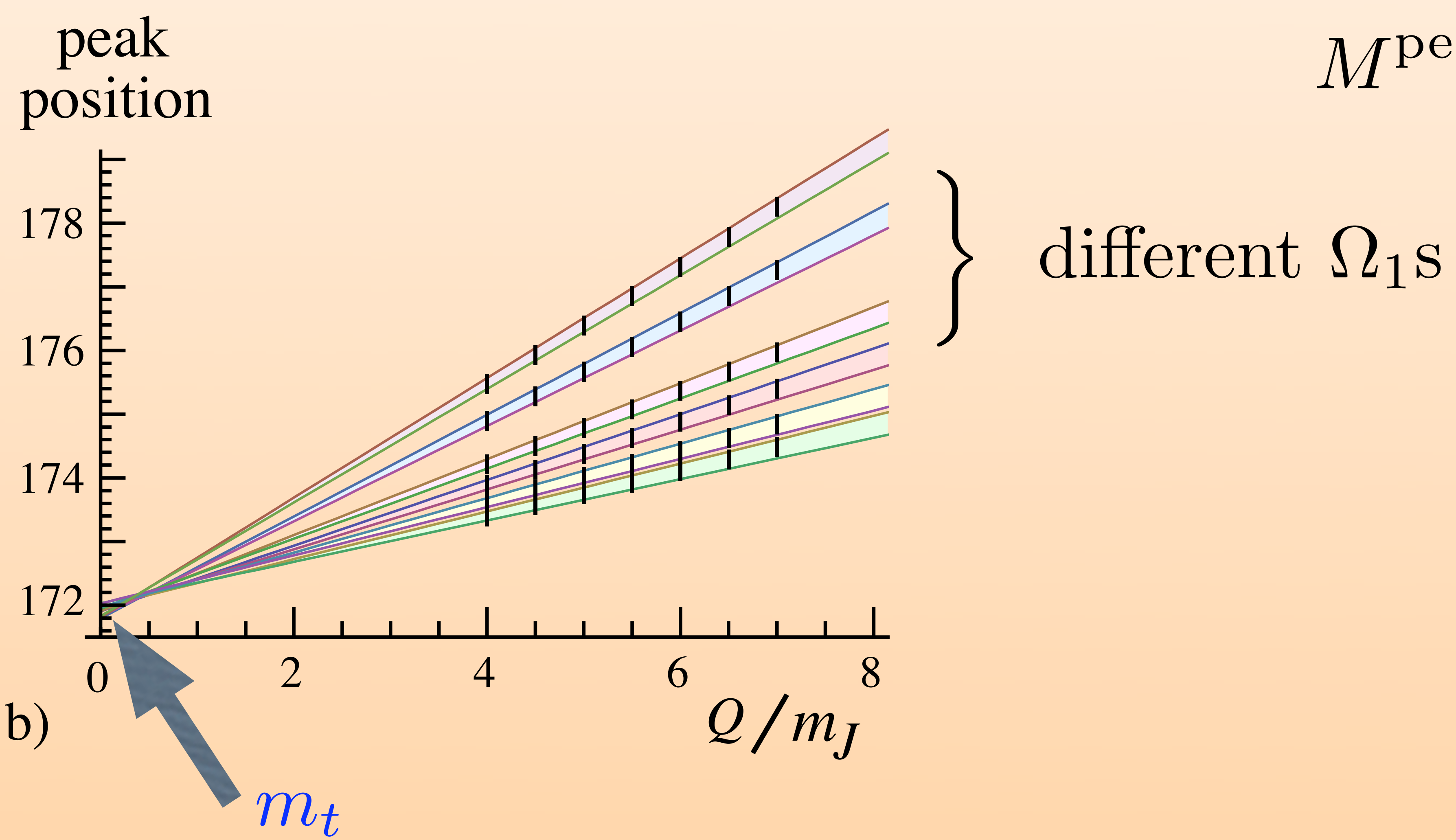
Top mass measurement above threshold at the ILC:

Two options:

I) use the parameter Ω_1 extracted from massless jet data, as advocated above

II) make measurements at multiple Q's and extrapolate linearly

$$M^{\mathrm{peak}} \simeq m_t + \frac{Q\Omega_1}{m_t}$$



While it will be hard to compete with the threshold scan, the above threshold setup is systematically improvable. It will provide a consistency check, and allows measurement at any

$$Q \simeq 0.5 - 1.0\,\mathrm{TeV}$$

With additional hard work (eg. m_t^2/Q^2 corrections, ...) we may get to a $100 - 500\,\mathrm{MeV}$ precision above threshold, by the ILC startup.

Non-perturbative shifts will also be present for measurements of other unstable colored particles, particularly at edges of phase space.

Summary & Outlook

$\alpha_s(m_Z)$

- The Soft-Collinear Effective Theory provides a powerful formalism for deriving factorization theorems and analyzing processes with Jets
- Important to properly account for nonperturbative effects
- Similar computations and fits can (and will) be carried out for other event shapes

The future for high precision determinations of α_s from event shapes at the ILC looks good!

m_t

- Threshold scan hard to beat for a precision top mass. Keeps improving.
- A systematically improvable method exists for measurements above threshold. Perturbative and nonperturbative effects are under control.

QCD factorization and resummation tools for the ILC