Precision Measurements at the ILC

Iain Stewart
MIT

Linear Collider Workshop of the Americas,
Albuquerque, Sept 2009
The ILC is being designed with precision measurements in mind.

- **Standard Model:**
  - Higgs mass & couplings,
  - Precision electroweak, Weak boson couplings, $\alpha_s(Q)$
  - Top Width, $m_t$, top couplings, . . .

- **Beyond the SM:**

  The most exciting precision measurements are of the mass and couplings of particles we have not yet seen. In this regard the ILC is crucial to decipher the new physics we “plan” to observed at the LHC.

  This talk is not a review of all possible precision measurements.

  Rather I will focus in detail on two:

  $\alpha_s(Q)$ and $m_t$ from $e^+e^-$ Colliders
Measure $\alpha_s(Q)$ and $m_t$ from $e^+e^-$ colliders

Using:

- $e^+e^- \rightarrow \text{jets}$, event shape measurements of $\alpha_s(m_Z)$
- $e^+e^- \rightarrow t\bar{t}$ at threshold $Q \approx 2m_t$
- $e^+e^- \rightarrow t\bar{t}$ above threshold $Q > 2m_t$

Discuss recent theoretical advances in QCD that have an impact on precision physics at the ILC:

i) fixed order computations,
ii) resummation,
iii) improved theoretical framework for computations

*Factorization & Soft-Collinear Effective Theory (SCET)
$\alpha_s (Q)$
Motivation

- $\alpha_s(m_Z)$ enters the analysis of all collider data (LHC, ILC, ...)
- It also plays a role in searches for new physics
  - indirectly in precision electroweak analyses, $B \rightarrow X_s \gamma$, etc.
  - directly through the unification of couplings:

\[
\alpha_s(m_Z) = 0.1186(26) \\
\sin^2 \theta_W = 0.23136(16)
\]

mSUGRA (from Boer & Sander ‘03)

Allanach et.al. ‘04
Event Shapes are a classic method for determining $\alpha_s (m_Z)$.

LEP 2 jet event

OPAL 3 jet event

SLD 3 jet event

Three jet events are proportional to $\alpha_s$, good sensitivity

$$q^\mu \rightarrow \gamma, Z$$

$$e^+$$

$$e^-$$

$$Q^2 = q^2$$

$$\bar{q}$$

$$q$$

$$g$$
<table>
<thead>
<tr>
<th>Process</th>
<th>$Q$ [GeV]</th>
<th>$\alpha_s(Q)$</th>
<th>$\alpha_s(M_{Z^0})$</th>
<th>$\Delta\alpha_s(M_{Z^0})$</th>
<th>Theory</th>
<th>refs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIS [pol. SF]</td>
<td>0.7 - 8</td>
<td>$0.375 \pm 0.062$</td>
<td>$0.113 \pm 0.010$</td>
<td>$\pm 0.004$</td>
<td>NLO</td>
<td>[76]</td>
</tr>
<tr>
<td>DIS [Bj-SR]</td>
<td>1.58</td>
<td>$0.280 \pm 0.070$</td>
<td>$0.121 \pm 0.006$</td>
<td>$\pm 0.009$</td>
<td>NNLO</td>
<td>[77]</td>
</tr>
<tr>
<td>DIS [GLS-SR]</td>
<td>1.73</td>
<td>$0.345 \pm 0.010$</td>
<td>$0.112 \pm 0.012$</td>
<td>$\pm 0.006$</td>
<td>NNLO</td>
<td>[78]</td>
</tr>
<tr>
<td>$\tau$-decays</td>
<td>2.8 - 11</td>
<td>$0.119 \pm 0.006$</td>
<td>$0.1215 \pm 0.0012$</td>
<td>$0.0004$</td>
<td>NNLO</td>
<td>[70]</td>
</tr>
<tr>
<td>DIS [$\nu; xF_3$]</td>
<td>2 - 15</td>
<td>$0.1166 \pm 0.0022$</td>
<td>$0.0009$</td>
<td>$0.0020$</td>
<td>NNLO</td>
<td>[80, 81]</td>
</tr>
<tr>
<td>DIS [$e/\mu; F_2$]</td>
<td>6 - 100</td>
<td>$0.1186 \pm 0.0051$</td>
<td>$0.0011$</td>
<td>$0.0050$</td>
<td>NLO</td>
<td>[67]</td>
</tr>
<tr>
<td>$\tau$ decays jets &amp; shps</td>
<td>1.4 - 28</td>
<td>$0.217 \pm 0.021$</td>
<td>$0.118 \pm 0.006$</td>
<td>$\pm 0.009$</td>
<td>NLO</td>
<td>[82]</td>
</tr>
<tr>
<td>$e^+e^-$ [$F_2$]</td>
<td>7.5</td>
<td>$0.1886 \pm 0.0032$</td>
<td>$0.1170 \pm 0.0012$</td>
<td>$0.0000$</td>
<td>NLO</td>
<td>[83]</td>
</tr>
<tr>
<td>$e^+e^-$ $[\sigma_{had}]$</td>
<td>10.52</td>
<td>$0.20 \pm 0.06$</td>
<td>$0.130 \pm 0.021$</td>
<td>$\pm 0.009$</td>
<td>NLO</td>
<td>[84]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>14.0</td>
<td>$0.170 \pm 0.017$</td>
<td>$0.120 \pm 0.010$</td>
<td>$0.002$</td>
<td>resum</td>
<td>[85]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>22.0</td>
<td>$0.151 \pm 0.015$</td>
<td>$0.118 \pm 0.009$</td>
<td>$0.003$</td>
<td>resum</td>
<td>[85]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>35.0</td>
<td>$0.145 \pm 0.012$</td>
<td>$0.123 \pm 0.009$</td>
<td>$0.002$</td>
<td>resum</td>
<td>[85]</td>
</tr>
<tr>
<td>$e^+e^-$ $[\sigma_{had}]$</td>
<td>42.4</td>
<td>$0.144 \pm 0.029$</td>
<td>$0.126 \pm 0.022$</td>
<td>$0.002$</td>
<td>NLO</td>
<td>[86, 32]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>44.0</td>
<td>$0.139 \pm 0.018$</td>
<td>$0.123 \pm 0.006$</td>
<td>$0.003$</td>
<td>resum</td>
<td>[85]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>58.0</td>
<td>$0.132 \pm 0.008$</td>
<td>$0.123 \pm 0.007$</td>
<td>$0.003$</td>
<td>resum</td>
<td>[87]</td>
</tr>
<tr>
<td>p$p \to b\bar{b}X$</td>
<td>20.0</td>
<td>$0.145 \pm 0.019$</td>
<td>$0.113 \pm 0.011$</td>
<td>$\pm 0.007$</td>
<td>NLO</td>
<td>[88]</td>
</tr>
<tr>
<td>p$p, p\to \gamma X$</td>
<td>24.3</td>
<td>$0.135 \pm 0.012$</td>
<td>$0.110 \pm 0.008$</td>
<td>$\pm 0.004$</td>
<td>NLO</td>
<td>[89]</td>
</tr>
<tr>
<td>$\sigma(p\bar{p} \to jets)$</td>
<td>40 - 250</td>
<td>$0.118 \pm 0.012$</td>
<td>$\pm 0.010$</td>
<td>$\pm 0.009$</td>
<td>NLO</td>
<td>[90]</td>
</tr>
<tr>
<td>$e^+e^- \Gamma(Z \to had)$</td>
<td>91.2</td>
<td>$0.1226 \pm 0.0058$</td>
<td>$0.1226 \pm 0.0058$</td>
<td>$\pm 0.0038$</td>
<td>NNLO</td>
<td>[91]</td>
</tr>
<tr>
<td>$e^+e^- 4$-jet rate</td>
<td>91.2</td>
<td>$0.1176 \pm 0.0022$</td>
<td>$0.1176 \pm 0.0022$</td>
<td>$0.0010$</td>
<td>NLO</td>
<td>[92]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>91.2</td>
<td>$0.121 \pm 0.006$</td>
<td>$0.121 \pm 0.006$</td>
<td>$0.001$</td>
<td>resum</td>
<td>[32]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>133</td>
<td>$0.113 \pm 0.008$</td>
<td>$0.120 \pm 0.007$</td>
<td>$0.003$</td>
<td>resum</td>
<td>[32]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>161</td>
<td>$0.109 \pm 0.007$</td>
<td>$0.118 \pm 0.008$</td>
<td>$0.005$</td>
<td>resum</td>
<td>[32]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>172</td>
<td>$0.104 \pm 0.007$</td>
<td>$0.114 \pm 0.008$</td>
<td>$0.005$</td>
<td>resum</td>
<td>[32]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>183</td>
<td>$0.109 \pm 0.005$</td>
<td>$0.121 \pm 0.006$</td>
<td>$0.002$</td>
<td>resum</td>
<td>[32]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>189</td>
<td>$0.109 \pm 0.004$</td>
<td>$0.121 \pm 0.005$</td>
<td>$0.001$</td>
<td>resum</td>
<td>[32]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>195</td>
<td>$0.109 \pm 0.005$</td>
<td>$0.122 \pm 0.006$</td>
<td>$0.001$</td>
<td>resum</td>
<td>[32]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>201</td>
<td>$0.110 \pm 0.005$</td>
<td>$0.124 \pm 0.006$</td>
<td>$0.002$</td>
<td>resum</td>
<td>[32]</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shps</td>
<td>206</td>
<td>$0.110 \pm 0.005$</td>
<td>$0.124 \pm 0.006$</td>
<td>$0.001$</td>
<td>resum</td>
<td>[32]</td>
</tr>
</tbody>
</table>
I will show that by
i) improving the theory and
ii) performing a global fit,
that LEP data already gives a precision comparable to the lattice result.

With an ILC we can do even better.
Thrust is a classic example of an "event-shape"

\[
T = \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \quad \tau = 1 - T
\]

Almost all event shape fits cut on \( \tau \), eg. keep \( \tau \in \{0.09, 0.25\} \).
For $\tau > 0$

$$\frac{1}{\sigma} \frac{d\sigma}{d\tau} = \sum_{n,m} \alpha_s^n \ln^m \tau + \sum_{n,m} \alpha_s^n \ln^m \tau + \sum_{n,m} \alpha_s^n f_m(\tau)$$

$$+ f(\tau, \Lambda_{\text{QCD}}/Q)$$

**Factorization Theorem:**

- **Hard Function**
  $$\frac{d\sigma}{d\tau} = \sigma_0 H(Q, m_Z, \mu) Q \int d\ell J_T(Q^2 \tau - Q\ell, \mu) S_T(\ell, \mu)$$
- **Jet Function**
  encodes dominant power corrections by a universal function
- **Soft Function**
  $$+ \left( \frac{d\sigma}{d\tau} \right)_{\text{nonsingular}}$$

Renormalization group evolution sums logs of $\tau$

**Subleading SCET factorization theorems** tells us how power corrections enter here too.

**Singular terms**

**Non-singular**

**Non-perturbative power corrections**
eg. $e^+ e^- \rightarrow Z \rightarrow 2\text{ jets} + X_{\text{soft}}$

$$m_Z^2 \gg M_{\text{jet}}^2 \gg E_{\text{soft}}^2$$

$$\mu_Q \simeq m_Z = 91.2 \text{ GeV}$$

$$\mu_J \simeq M_{\text{jet}} \simeq 20 \text{ GeV}$$

$$\mu_S \simeq E_{\text{soft}} \simeq 5 \text{ GeV or smaller, down to } \Lambda_{\text{QCD}}$$

$$Q^2 \gg Q^2 \tau \gg (Q \tau)^2$$

hard jet soft

$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q, \mu) Q \int d\ell J_T \left( Q^2 \tau - Q\ell, \mu \right) S_T(\ell, \mu)$$
Our Three Regions:

- **peak**
  
  \[ Q^2 \gg Q^2\tau \gg (Q\tau)^2 \sim \Lambda_{QCD}^2 \]

- **tail**
  
  \[ Q^2 \sim Q^2\tau \sim (Q\tau)^2 \gg \Lambda_{QCD}^2 \]

- **multijet**

**nonperturbative \( S_\tau \)**

**universal \( \Omega_1/Q\tau \)** power correction

**sum the logs**

**small power corrections**

do not sum the logs(!)
i) $\mathcal{O}(\alpha_s^3)$ fixed order results (numerical)

ii) summation of large logs to $N^3\text{LL}$
    (analytic with Soft-Collinear EFT)

iii) power corrections

iv) All together, a Global Thrust Fit for alphas

Recent Literature

Gehrmann, Gehrmann-De Ridder, Glover, Heinrich
S. Weinzierl

Becher and Schwartz

Davison & Webber; Lee & Sterman;
Hoang & I.S.; Ligeti, I.S., Tackmann.

Abbate, Fickinger, Hoang, Mateu, I.S.
fixed order results

- $O(\alpha_s^3)$

Two-loop matrix elements
$|\mathcal{M}|^2_{2\text{-loop},3\text{ partons}}$

One-loop matrix elements
$|\mathcal{M}|^2_{1\text{-loop},4\text{ partons}}$

Tree level matrix elements
$|\mathcal{M}|^2_{\text{tree},5\text{ partons}}$

Infrared Poles cancel in the sum

\[ Q = M_Z \]
\[ \alpha_s(M_Z) = 0.1189 \]

convergence? $\mu$ dependence?
• summation of large logs to $N^3LL$ (analytic with SCET)

Becher and Schwartz

Catani et.al.

$LL, NLL, NNLL, N^3LL$

\[
\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \ldots
\]

\( y = \text{Fourier transform of } \tau \)

<table>
<thead>
<tr>
<th></th>
<th>cusp</th>
<th>non-cusp</th>
<th>matching</th>
<th>alphas</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LL$</td>
<td>1</td>
<td>–</td>
<td>tree</td>
<td>1</td>
</tr>
<tr>
<td>$NLL$</td>
<td>2</td>
<td>1</td>
<td>tree</td>
<td>2</td>
</tr>
<tr>
<td>$NNLL$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$N^3LL$</td>
<td>$4^{pade}$</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$LL'$</td>
<td>1</td>
<td>–</td>
<td>tree</td>
<td>1</td>
</tr>
<tr>
<td>$NLL'$</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$NNLL'$</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$N^3LL'$</td>
<td>$4^{pade}$</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

when fixed order results are important primed counting is better
• summation of large logs to $\text{N}^3\text{LL}$ (analytic with SCET) Becher and Schwartz

$$\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \ldots$$

LL, NLL, NNLL, N$^3$LL

better convergence

close-to-$\mu$ dependence

anti-correlated scale variation

Wednesday, September 30, 2009
\[ \ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s (\alpha_s \ln)^k + \alpha_s^2 (\alpha_s \ln)^k + \ldots \]

\begin{align*}
\ln & \text{LL} \quad \ln \text{NLL} \quad \ln \text{NNLL} \quad \ln \text{N}^3 \text{LL} \\
\end{align*}

\[ \alpha_s(m_Z) = 0.1172 \pm 0.0022 \]

error competitive with WA

- Nonperturbative corrections not included in central value

- summation of large logs to $N^3$LL (analytic with SCET)
Nonperturbative Corrections

\[ S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_s^+) \delta(\ell^- - k_s^-) \langle 0 | Y_n^\dagger Y_n(0) | X_s \rangle \langle X_s | Y_n^\dagger Y_n^\dagger(0) | 0 \rangle \]

\[ S_T(\tau) \] is symmetric projection

OPE:

\[ S_T(\tau) = S_{\text{pert}}(\tau) - S'_{\text{pert}}(\tau) \frac{2\Omega_1}{Q} + \ldots \]

\[ = S'_{\text{pert}}(\tau - 2\Omega_1/Q) + \ldots \]

\[ \Omega_1 \sim \Lambda_{\text{QCD}} \] a universal parameter

Universal Soft Function

Korchemsky, Sterman, Lee & Sterman

Dokshitzer & Webber;

shifts distributions to the right
Nonperturbative Corrections

\[ S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_s^+) \delta(\ell^- - k_s^-) \langle 0 | Y_n \ Y_n(0) | X_s \rangle \langle X_s | Y_n \ Y_n(0) | 0 \rangle \]

\[ S_T(\tau) \text{ is symmetric projection} \]

Perturbative & Nonperturbative parts:

\[ S(\ell, \mu) = \int d\ell' \ S_{\text{part}}(\ell - \ell', \mu) \ F(\ell') \]

- Partonic soft function at fixed order
- Normalized model function, complete basis (must have exponential fall off!)

In general, Pert. and Nonpert. parts are hard to separate (renormalons).

Use renormalon free scheme for parameters in \( F \), such as \( \Omega_1 \)

Universal Soft Function

Hoang & I.S.;
Ligeti, I.S., Tackmann
**Thrust Data Sets**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Values of Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>{91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0}</td>
</tr>
<tr>
<td>DELPHI</td>
<td>{45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0}</td>
</tr>
<tr>
<td>OPAL</td>
<td>{91.0, 133.0, 177.0, 197.0}</td>
</tr>
<tr>
<td>L3</td>
<td>{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2}</td>
</tr>
<tr>
<td>SLD</td>
<td>{91.2}</td>
</tr>
<tr>
<td>TASSO</td>
<td>{14.0, 22.0, 35.0, 44.0}</td>
</tr>
<tr>
<td>JADE</td>
<td>{35.0, 44.0}</td>
</tr>
<tr>
<td>AMY</td>
<td>{55.2}</td>
</tr>
</tbody>
</table>

At each Q there is a distribution in $\tau$

Lots of Data: 807 bins
Ingredients for Global Analysis

• SCET Factorization Theorems, Sum Large Logs: \[ \sum_k (\alpha_s \ln^2)^k \]
  LL, NLL, NNLL, N^3LL and/or LL', NLL', NNLL', N^3LL'
  \[ \frac{\Lambda_{QCD}}{\mu_S}, \frac{\Lambda_{QCD}}{\mu_J}, \frac{\Lambda_{QCD}}{\mu_h}, \frac{\mu_S}{\mu_J} \]

• Power Corrections: \[ \bar{\Lambda}_{QCD}, \bar{\Lambda}_{J} \]

• Multiple Regions: 
  i) peak: \[ \mu_h \gg \mu_J \gg \mu_S \sim \Lambda_{QCD} \]
  ii) tail: \[ \mu_h \gg \mu_J \gg \mu_S \gg \Lambda_{QCD} \]
  iii) far tail: \[ \mu_h \sim \mu_J \sim \mu_S \gg \Lambda_{QCD} \]
  (multi jet)

• Renormalon Subtractions (Mass, Gap), R-RGE

• Complete Basis for modeling Hadronic functions

• Final State QED radiation, with resummation of Sudakov

• Rigorous treatment of b-quark mass effects (factorization)
Perturbation Theory
+ Sum Logs Only

NLL' \ NNLL \ NNLL'
N^3LL \ N^3LL'

Add Soft Radiation
Basis Function

\[ F \]

Include renormalon
subtractions from gap

AFHMS

stable peaks
predictions

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \]
Tail Predictions with Scan over Theory Uncertainties

\[ \frac{1}{\sigma} \int d\sigma \frac{d\sigma}{d\tau} \]

Perturbation Theory Only (Sums Logs)

Include F & renormalon subtractions from gap
Tail Predictions with Scan over Theory Uncertainties

\[
\frac{1}{\sigma} \frac{d\sigma}{d\tau} \quad \frac{d\sigma}{d\tau}
\]

Perturbation Theory Only (Sums Logs)

Include F & renormalon subtractions from gap

NLL' NNLL
Tail Predictions with Scan over Theory Uncertainties

NLL'  NNLL  NNLL'

\[ \frac{1}{\sigma T} \frac{d\sigma}{dT} \]

Perturbation Theory Only (Sums Logs)

Include F & renormalon subtractions from gap

Wednesday, September 30, 2009
Tail Predictions with Scan over Theory Uncertainties

NLL'  NNLL  NNLL'  N^3LL

\[ \frac{1}{\tau} \frac{d\sigma}{d\tau} \]

Perturbation Theory Only (Sums Logs)

Include F & renormalon subtractions from gap
Tail Predictions with Scan over Theory Uncertainties

NLL'  NNLL  NNLL'  N^3LL  N^3LL'

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \]

Perturbation Theory Only (Sums Logs)

Include F & renormalon subtractions from gap

Wednesday, September 30, 2009
What Parameters to fit?

- **vary $\alpha_s(m_Z)$**
  - peak

- **vary first moment**
  - $2\Omega_1$

- **vary $\alpha_s(m_Z)$**
  - tail

- **vary $c[2]$, with 1st moment fixed**
Sample Fit results:

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \]

0.34
0.36
0.38
0.40
0.42

0.00
0.02
0.04
0.06
0.08

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \]

0.15
0.20
0.25
0.30

0.0
0.2
0.4
0.6
0.8
1.0

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \]

0.10
0.15
0.20
0.25
0.30

0.0
0.5
1.0
1.5
2.0
2.5
3.0

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \]

0.10
0.15
0.20
0.25
0.30

0.0
0.1
0.2
0.3
0.4

Here

3 parameters:

\[ \alpha_s(m_Z), \Omega_1, c_2, [\Delta_0] \]
A Tail Fit

For \( \tau \) in the tail region \((Q = 91, \tau \in [0.09, 0.33], \text{etc.})\) we can safely do a two parameter fit

\[
\{ \alpha_s(m_Z), \Omega_1 \}
\]
Fit Uncertainties:

Statistical Error + Systematic Error
+ Hadronization (2Ω₁)

Error Ellipse from Fit

Theory Uncertainties

We do a flat scan over unknown theory parameters, fitting each time and take the range of central values

μ₀ n₁ τ₂ εₖ rₜ = μₜ/Q nₛ

mu dependence:  s₁ ε₂ ε₃ Γ₃\text{cusp} H₃ J₃ S₃

2, 3 loop uncertainties:  theory MC statistics
$2\Omega_1$ (GeV)

Perturbation Theory, Sums Logs + add F

$\chi^2$/dof

2.108
1.561
1.570
1.228

without $2\Omega_1$

$\chi^2$/dof $\gtrsim 2$
Include F & renormalon subtractions from gap

\[ \chi^2 / \text{dof} \]

- 1.294
- 0.888
- 0.892
- 0.895
Tail Fit
Result

we use LEP working group’s corr. model for syst.errors:

$$\frac{\chi^2}{dof} = \frac{385.9}{433 - 2} = 0.895$$

$$\alpha_s(m_Z) = 0.1135 \pm 0.0008\ ^{+0.0007}_{-0.0013}$$

hadronization + expt. error reduced by a factor of 2-3

comparison to

$$\alpha_s(m_Z) = 0.1172 \pm 0.0010(stat) \pm 0.0008(sys) \pm 0.0012(had) \pm 0.0012(pert)$$

Becher & Schwartz fit
resum

$$\alpha_s(m_Z) = 0.1224 \pm 0.0009(stat) \pm 0.0009(sys) \pm 0.0012(had) \pm 0.0035(theo)$$

Gehrmann, et al.
fixed order
Tail Fit with QED & b-mass

\[
\frac{\chi^2}{dof} = \frac{377.4}{433 - 2} = 0.876
\]

(became a bit smaller)

Global Thrust Analysis

\[\Delta \chi^2 = 1\]

QCD+mass

\[\alpha_s(m_Z) = 0.1132 \pm 0.0013\]
Implications for ILC:

- Further improvements can be made by extending the fit to other event shapes without(!) requiring additional fit parameters.

- At the ILC we will have better statistical errors. And presumably improvements in the systematics. eg. Get full correlation matrix across bins which will lead to better control (perhaps less conservative). Also better data will pin down higher moments, $\Omega_n$, of soft function, which in turn allows more data to be used (a feedback effect).

- Event shapes are complementary and competitive with other ILC methods, like the total Z-decay rate (at Giga-Z).

Together this will yield a systematic program to improve the determination of $\alpha_s(m_Z)$, at an ILC.
Motivation

- The top mass is a fundamental parameter of the Standard Model
  \[ m_t = 173.1 \pm 0.6_{\text{stat}} \pm 1.1_{\text{syst}} \text{GeV} \] (a 0.8% error)
  (theory error? what mass is it?)

- Important for precision e.w. constraints
  eg. \[ m_H = 76^{+33}_{-24} \text{GeV} \]
  \[ m_H < 182 \text{GeV} \] (95% CL)
  A 2 GeV shift in \( m_t \) changes the central values by 15%

- Top Yukawa coupling is large. Top parameters are important for analyzing many new physics models. (eg. Higgs masses in MSSM)

- Top is very unstable, it decays before it has a chance to hadronize. This provides an intrinsic smearing for jet observables.
  \[ \Gamma_t = 1.4 \text{GeV} \]
  from \( t \rightarrow bW \)
  Top provides playground for future analysis of new short lived strongly interacting particles.
Threshold Scan  \( e^+ e^- \rightarrow t\bar{t} \)

\[ \sqrt{s} \simeq 350 \text{ GeV} \]

- count number of \( t\bar{t} \) events
- color singlet state
- background is non-resonant
- physics well understood (renormalons, summations)

- Measure a short-distance top-quark mass, like \( m_t^{1S} \)
  NOT the top pole mass.

- Have smearing by ISR and beamstrahlung, which must be controlled precisely

the classic ILC method

Precision Theory meets precision experiment:

\[ \rightarrow \delta m^\text{exp}_t \simeq 50 \text{ MeV} \]
\[ \rightarrow \delta m^\text{th}_t \simeq 100 \text{ MeV} \]

(“peak” position)

Teubner, AH; Melnikov, Yelkovski; Yakovlev; Beneke, Signer, Smirnov; Sumino, Kiy

\( m_t^{1S} = 175 \text{ GeV} \)

\( Q^2 R^\nu \)

Manohar, Stewart, Teubner, AH
Threshold Theory Status

NRQCD with computable power and radiative corrections.

good is \approx 3\% for \frac{\delta \sigma}{\sigma}

potential  \quad V(r) :  \quad \text{full NNLL}

short-distance coefficients  \quad C(\nu) :  \quad \text{almost NNLL}

unstable top

- complex matching conditions & anomalous dimensions
- effective Lagrangian non-hermitian
- total rates through the optical theorem
- phase space matching

compute electroweak effects
compute non-resonant irreducible bkgnd

\begin{align*}
\text{NLL: } & \text{Luke etal }'99 \\
\text{NNLL(matching): } & \text{Beneke etal; Czarnecki etal }'99 \\
\text{NNLL(non-mixing): } & \text{Hoang }'03 \\
\text{NNLL(mixing): } & \text{mostly known} \\
\text{spin-dependent soft } & \text{Penin etal. }'04 \\
\text{usoft nf } & \text{Stahlhofen, Hoang }'05
\end{align*}

\begin{align*}
\text{Beneke etal. } & '03, '04 \\
\text{Grzadkowski, Kuhn } & '87 \\
\text{Guth, Kuhn } & '92 \\
\text{Reisser, Hoang; } & '05 \\
\text{Reisser, Hoang } & '06
\end{align*}

\begin{align*}
\text{peak shift: } & \approx -10\% \\
\text{30-50 MeV peak shift: } & \approx -3\% \\
\text{} & \approx -2\%
\end{align*}
Above Threshold \( e^+ e^- \rightarrow t\bar{t} \)

- jet observable ★★
- suitable top mass for jets ★
- initial state radiation
- final state radiation ★
- color reconnection ★
- sum large logs ★

\[
M_t^{\text{peak}} = m_t + (\text{nonperturbative effects}) + (\text{perturbative effects})
\]

To simplify things we’ll work far above threshold:

\[
Q \gg m_t \gg \Gamma_t
\]

\[
\left( \frac{m_t^2}{Q^2} \right) \text{ dependence can be computed}
\]
Hemisphere Invariant Masses

\[ M_t^2 = \left( \sum_{i \in a} p_i^\mu \right)^2 \]

\[ M_\bar{t}^2 = \left( \sum_{i \in b} p_i^\mu \right)^2 \]

Peak region:

\[ s_t \equiv M_t^2 - m^2 \sim m\Gamma \ll m^2 \]

\[ \hat{s}_t \equiv \frac{M_t^2 - m^2}{m} \sim \Gamma \ll m \]

Breit Wigner:

\[ \frac{m\Gamma}{s_t^2 + (m\Gamma)^2} = \left( \frac{\Gamma}{m} \right) \frac{1}{\hat{s}_t^2 + \Gamma^2} \]
\[ Q \gg m \gg \Gamma \sim \hat{s}_t, \bar{t} \]

Disparate Scales  \rightarrow  Effective Field Theory

QCD  \rightarrow  SCET  \rightarrow  HQET

Integrate out Hard Modes

Factorize Jets, Integrate out energetic collinear gluons

Evolution and decay of top close to mass shell

QCD  \rightarrow  SCET  \rightarrow  HQET

\[ Q \rightarrow n \rightarrow m_t \rightarrow \bar{n} \rightarrow \bar{t} \]

\[ \Gamma_t \]
Factorization Theorem:

\[
\left( \frac{d^2 \sigma}{dM_t^2 \, dM_t^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) \, H_m \left( m, \frac{Q}{m}, \mu_m, \mu \right) \times \int_{-\infty}^{\infty} d\ell^+ \, d\ell^- \, B_+ \left( \hat{s}_t - \frac{Q \ell^+}{m}, \Gamma, \mu \right) \, B_- \left( \hat{s}_{\bar{t}} - \frac{Q \ell^-}{m}, \Gamma, \mu \right) \, S_{\text{hemi}}(\ell^+, \ell^-, \mu).
\]

\[
+ \mathcal{O} \left( \frac{m \alpha_s(m)}{Q} \right) + \mathcal{O} \left( \frac{m^2}{Q^2} \right) + \mathcal{O} \left( \frac{\Gamma_t}{m} \right) + \mathcal{O} \left( \frac{s_t, s_{\bar{t}}}{m^2} \right)
\]

Valid to all orders in \( \alpha_s \)

& includes leading nonperturbative effects
Factorization Theorem:

\[
\left( \frac{d^2 \sigma}{dM_1^2 \ dM_2^2} \right)_{\text{hemi}} = \sigma_0 \ H_Q(Q, \mu_m) \ H_m \left( m, \frac{Q}{m}, \mu_m, \mu \right) \\
\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \ B_+ \left( \hat{s}_t - \frac{Q \ell^+}{m} , \Gamma, \mu \right) B_- \left( \hat{s}_t - \frac{Q \ell^-}{m} , \Gamma, \mu \right) \ S_{\text{hemi}}(\ell^+, \ell^-, \mu).
\]

A useful event shape for massive unstable particles

**Jet Functions**

Evolution and decay of top quark close to mass shell

**Soft Function**

Non-perturbative Cross talk

Fleming, Hoang, Mantry, I.S.
Implications

\[
\left( \frac{d^2 \sigma}{d M_t^2 \, d M_t^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) \, H_m\left( m, \frac{Q}{m}, \mu_m, \mu \right) 
\times \int_{-\infty}^{\infty} d \ell^+ \, d \ell^- \, B_+ \left( \hat{s}_t - \frac{Q \ell^+}{m}, \Gamma, \mu \right) \, B_- \left( \hat{s}_t - \frac{Q \ell^-}{m}, \Gamma, \mu \right) \, S_{\text{hemi}}(\ell^+, \ell^-, \mu).
\]

\[M^\text{peak} = m_t + \Gamma_t (\alpha_s + \alpha_s^2 + \ldots) + \frac{Q \Lambda_{\text{QCD}}}{m_t}\]

measure this 
extract this 
compute this

soft radiation shifts the measured mass

\[\frac{Q \Lambda_{\text{QCD}}}{m_t}\] is predominantly \[\frac{Q \Omega_1}{m_t}\]! known to 10% from fit in part I

\[(\delta m_t)^\Omega_1 \approx 200 \text{ MeV}\]

will be known even better with ILC

\[
\frac{d \sigma}{d M}
\]

"peak region"
Short Distance Mass Scheme for Jets

- top $\overline{\text{MS}}$ mass?
  Can not be treated consistently with Breit-Wigner for decay products

- pole mass?
  Breit-Wigner is fine, but has renormalon problem (instability)

- $1S$ mass?
  Also couples scales in an ugly fashion.

- top jet mass
  Breit-Wigner is fine & no renormalon

  $$m_{\text{pole}} - m_t \sim \alpha_s \Gamma$$

  Use heavy quark jet function $B$ to define the series

Good!
Jet Function Results up to NNLL:

(3 curves vary $\mu_T$)

Jain, Scimemi, I.S.

very stable perturbative peak!
NLL Cross-Section Results

soft non-perturbative radiation shifts the peak $\sim 2\,\text{GeV}$, and broadens the distribution
Symmetric mass projection:

- **Peak Position versus $Q/m$**
  - $M_t^{\text{peak}}$ (GeV) vs. $Q/m$
  - $m_J = 172$
  - $Q/m_J = 4$ to $10$

- **Peak Width versus $Q/m$**
  - Width (GeV) vs. $Q/m$
  - $Q/m_J = 4$ to $10$

**FIG. 9:**
- Effect of a change in $Q$ on the invariant mass distribution.
- Results on the left are generated from $d\sigma/dM_t$ and $d\sigma/d\bar{M}_t$.
- a) Shows the peak position versus $Q/m_J$.
- b) Gives the full width at half-maximum versus $Q/m_J$.
- c) Shows $d\sigma/dM_t$ in units of $2\sigma H_0/\Gamma$ for different values of $Q/m_J$.
- The curves use $m_J = 172$ GeV, $\Gamma = 1.4$ GeV, and the parameters in Eq. (119).

- For different choices of $Q/m_J$, the peak position and width of $F_1(M_t)$ behave in an identical manner to Figs. 9a,b, including having essentially the same slopes. In order to keep the area under the curves constant, the peak height drops as $Q$ is increased. Note that for values $Q/m_J \approx 8–10$ the observed peak location may be as much as 2.0–2.5 GeV above the value of the Lagrangian mass $m_J$ one wants to measure.

- To gain an analytic understanding of this linear behavior, consider the effect of $Q$ on the mean of the cross-section, which is a good approximation to the peak location. Taking the first moment with respect to $\hat{s}/2 = (M_t - m_J)$ over an interval of size $2L \gg Q\Lambda$ and $47$...
Top mass measurement above threshold at the ILC:

Two options:

I) use the parameter $\Omega_1$ extracted from massless jet data, as advocated above

II) make measurements at multiple $Q$’s and extrapolate linearly

$M^{\text{peak}} \simeq m_t + \frac{Q \Omega_1}{m_t}$

\begin{align*}
\text{peak position} & \quad 178 & 176 & 174 & 172 \\
Q/m_J & \quad 0 & 2 & 4 & 6 & 8
\end{align*}
While it will be hard to compete with the threshold scan, the above threshold setup is systematically improvable. It will provide a consistency check, and allows measurement at any 

\[ Q \approx 0.5 - 1.0 \text{ TeV} \]

With additional hard work (eg. \( m_t^2/Q^2 \) corrections, . . .) we may get to a 100 – 500 MeV precision above threshold, by the ILC startup.

Non-perturbative shifts will also be present for measurements of other unstable colored particles, particularly at edges of phase space.
Summary & Outlook

\( \alpha_s(m_Z) \)

- The Soft-Collinear Effective Theory provides a powerful formalism for deriving factorization theorems and analyzing processes with Jets
- Important to properly account for nonperturbative effects
- Similar computations and fits can (and will) be carried out for other event shapes

The future for high precision determinations of \( \alpha_s \) from event shapes at the ILC looks good!

\( m_t \)

- Threshold scan hard to beat for a precision top mass. Keeps improving.
- A systematically improvable method exists for measurements above threshold. Perturbative and nonperturbative effects are under control.

QCD factorization and resummation tools for the ILC