New Developments in Loop Calculations and Their Implications

Carola F. Berger

CTP, MIT

ALCPG09, Sept 30th 2009
Perturbative Calculations

- Parton distribution functions (not for LC)
- Matrix elements
- Parton showers, resummation
- Monte Carlo models (also for hadronization)
Instead of an Outline

2007

- technique well established
- partial results/special cases

A. Juste at ALCPG07

- 0 legs
- 1 loop
- 2 loops
- 3 loops
- 4 loops
- 5 loops
- 6 loops
- 7 loops
- 8 loops

A. Juste at ALCPG07

Carola F. Berger
ALCPG09, Sept 30th 2009
New Developments in Loop Calculations - 4/28
Instead of an Outline

- Perturbative Calculations
  - NNLO
  - NLO
- Implications
- Conclusions

2007

- technique well established
- partial results/special cases
- required for LC physics
- □ = leading effects

A. Juste at ALCPG07
Instead of an Outline

2009

- technique well established
- partial results/special cases

# loops

# legs

- 0
- 1
- 2
- 3
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- 5
- 6
- 7
- 8

- 1
- 2
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- 1
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Introduction

NNLO

- NNLO
- LHC and ILC

Processes Known at NNLO

- Example of State-of-the-Art NNLO: Higgs
- $e^+e^- \rightarrow 3\text{ Jets}$ at NNLO

NLO

Implications

Conclusions

Carola F. Berger  ALCPG09, Sept 30th 2009  New Developments in Loop Calculations - 5/28
For certain processes, NNLO is needed
- when the **NLO corrections are large**, e.g. Higgs production
- for **benchmark measurements** where experimental errors are small or to facilitate calibration of detectors and determine efficiencies
- to minimize **PDF and luminosity uncertainties**
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- to minimize **PDF and luminosity uncertainties**

From the updated Les Houches wishlist 2007:

<table>
<thead>
<tr>
<th>process wanted at/beyond NNLO</th>
<th>(gg \rightarrow W^<em>W^</em>\mathcal{O}(\alpha^2\alpha_s^3))</th>
<th>background to Higgs benchmark process Higgs couplings, SM benchmark SM benchmark</th>
</tr>
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<tbody>
<tr>
<td>10. (pp \rightarrow t\bar{t})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. VBF, (Z/\gamma + \text{jet})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. (W/Z) production at NNLO QCD, NLO EW</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
LHC and ILC Processes Known at NNLO

- (differential) $Z, W$
  
  Anastasiou, Dixon, Melnikov, Petriello; Catani, Cieri, Ferrera, de Florian, Grazzini

- (differential) Higgs
  
  Ravindran, Smith, van Neerven; Kilgore, Harlander; Anastasiou, Melnikov;
  
  Anastasiou, Dixon, Melnikov, Petriello; Anastasiou, Dissertori, Grazzini, Stoeckli,
  
  Webber; Catani, Grazzini; Harlander, Ozeren; Pak, Rogal, Steinhauser

- $e^+ e^- \rightarrow 3 \text{ jets, event shapes}$
  
  Gehrmann-De Ridder, Gehrmann, Glover, Heinrich; Weinzierl

- DGLAP splitting kernels
  
  Moch, Vermaseren, Vogt

- NNLO parton distributions
  
  Martin, Stirling, Thorne, Watt; Alekhin, Blümlein, Klein, Moch; Jimenez-Delgado, Reya
### Example of State-of-the-Art NNLO: Higgs

\[ p\bar{p} \rightarrow H + X \rightarrow WW + X \rightarrow \mu^+\nu \mu^-\nu + X \]

\[ \frac{m_H}{2} \leq m_Z = \mu_Z \leq 2 m_H \]

\[ m_H = 160 \text{ GeV} \]

\[ \sigma \text{ (fb)} \]

\[ m_H \text{ (GeV)} \]

- Anastasiou, Dissertori, Grazzini, Stöckli, Webber

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The (In)Famous Wishlist

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<tr>
<td>$(V \in {Z, W, \gamma})$</td>
<td></td>
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<tr>
<td>1. $pp \rightarrow VV + \text{jet}$</td>
<td>$ttH$, new physics</td>
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<tr>
<td>2. $pp \rightarrow H + 2\text{ jets}$</td>
<td>$H$ production by vector boson fusion (VBF)</td>
</tr>
<tr>
<td>3. $pp \rightarrow tt\bar{b}\bar{b}$</td>
<td>$ttH$</td>
</tr>
<tr>
<td>4. $pp \rightarrow \bar{t}t + 2\text{ jets}$</td>
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</tr>
<tr>
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<td>VBF $\rightarrow H \rightarrow VV$, $ttH$, new physics</td>
</tr>
<tr>
<td>6. $pp \rightarrow VV + 2\text{ jets}$</td>
<td>VBF $\rightarrow H \rightarrow VV$</td>
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<td>(gg: ) Campbell, Ellis, Zanderighi</td>
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<tr>
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<td>Higgs and new physics</td>
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partially completed, via standard methods
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Ingredients:
- One-loop (virtual) matrix elements
- Tree-level matrix elements for real emission
- Both have IR divergences, which cancel in the full cross section $\Rightarrow$ subtraction terms
- Convolution with PDFs (only for hadronic collisions)
- Integration over final state phase space (with cuts)

Bottleneck up until now: 1-loop matrix elements
Any (massless) one-loop integral can be decomposed into

\[ M = \sum_i d^D_i I^D_{4i} + \sum_i c^D_i I^D_{3i} + \sum_i b^D_i I^D_{2i} \]

\[ = \sum_i d^{D=4}_i I^{D=4}_{4i} + \sum_i c^{D=4}_i I^{D=4}_{3i} + \sum_i b^{D=4}_i I^{D=4}_{2i} + R \]

Integrals are known, task is to determine the coefficients

Integrals tabulated in: Bern, Dixon, Dunbar, Kosower; Ellis, Zanderighi
Any (massless) one-loop integral can be decomposed into

\[ \mathcal{M} = \sum_i d_i^D I_{4i}^D + \sum_i c_i^D I_{3i}^D + \sum_i b_i^D I_{2i}^D \]

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Integrals are known, task is to determine the coefficients

Standard procedure:
- Generate all Feynman diagrams \( \Rightarrow \) many terms
- Translate into equations \( \Rightarrow \) many more terms
- Reduce to known Master integrals \( \Rightarrow \) large cancellations between spurious singularities
*$pp \rightarrow t\bar{t}b\bar{b}$*

**Important background to $pp \rightarrow t\bar{t}H$, with $H \rightarrow b\bar{b}$**

![Graph showing cross-sections and $K$ factors for $pp \rightarrow t\bar{t}b\bar{b}$.](image)

- $\sigma [\text{fb}]$ vs. $m_{b\bar{b},\text{cut}} [\text{GeV}]$ for $pp \rightarrow t\bar{t}b\bar{b} + X$.
- $K$ factor vs. $m_{b\bar{b}} [\text{GeV}]$.

- $m_t = 172.6 \text{ GeV}$
- $\mu_0/2 < \mu < 2\mu_0$

- Left: Bredenstein, Denner, Dittmaier, Pozzorini; right: Bevilaqua, Czakon, Papadopoulos, Pittau, Worek
New Ideas

\[ \mathcal{M} = \sum_i d_i^{D=4} I_{4i}^D + \sum_i c_i^{D=4} I_{3i}^D + \sum_i b_i^{D=4} I_{2i}^D + R \]
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- **Generalized unitarity**

  Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng

  \[ \Rightarrow \textbf{BlackHat} \quad \text{CFB, Bern, Dixon, Forde, Febres Cordero, Ita, Kosower, Maitre} \]

  \[ \Rightarrow \textbf{Rocket} \quad \text{Ellis, Giele, Kunszt, Melnikov, Zanderighi} \]
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\[ \mathcal{M} = \sum_i d_{i}^{D=4} I_{4i}^D + \sum_i c_{i}^{D=4} I_{3i}^D + \sum_i b_{i}^{D=4} I_{2i}^D + R \]

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- **OPP method**

  \[ \Rightarrow \text{CutTools + HELAC} \]
  
  Bevilaqua, Czakon, van Hameren, Ossola, Papadopoulos, Pittau, Worek
New Ideas

\[ \mathcal{M} = \sum_i d_i^{D=4} I_{4i} + \sum_i c_i^{D=4} I_{3i} + \sum_i b_i^{D=4} I_{2i} + R \]

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- **On-shell recursion at 1 loop** CFB, Bern, Dixon, Forde, Kosower
  - **BlackHat** CFB, Bern, Dixon, Forde, Febres Cordero, Ita, Kosower, Maitre
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  CFB, Bern, Dixon, Forde, Kosower

  ⇒ **BlackHat** CFB, Bern, Dixon, Forde, Febres Cordero, Ita, Kosower, Maitre

Generalized unitarity and recursion reuse **amplitudes**, not Feynman diagrams ⇒ **excellent scaling with number of external legs**
Generalized Unitarity

Determine coefficients without doing explicit reduction by generalized unitarity: put internal propagators on-shell

\[ \frac{1}{p^2 + i\epsilon} \rightarrow i\delta^+(p^2) \]

Thus for boxes, the coefficient collapses into a product of 4 tree amplitudes (in \( D = 4 \))

\[ \left( \int d^4 l \delta^+ (l_1^2) \delta^+ (l_2^2) \delta^+ (l_3^2) \delta^+ (l_4^2) \right) \]
Triangle and bubble coefficients are slightly more complicated – left-over integrals ($< 4$ delta-functions)  
$\Rightarrow$ use special parametrization to extract these  
■ at integrand level – OPP  
■ or at integral level

Ossola, Papadopoulos, Pittau

Forde - BlackHat; Rocket
Triangle and bubble coefficients are slightly more complicated – left-over integrals (< 4 delta-functions)
⇒ use special parametrization to extract these
 ■ at integrand level – OPP
   Ossola, Papadopoulos, Pittau
 ■ or at integral level
   Forde - BlackHat; Rocket

Rational terms:
 ■ Keep full $D$-dimensional information in generalized unitarity
   Ellis, Giele, Kunszt, Melnikov, Zanderighi; Badger
 ■ Rational recursion from lower-point one-loop terms
   CFB, Bern, Dixon, Forde, Kosower
 ■ Special Feynman rules in OPP approach at integrand level
   van Hameren, Ossola, Papadopoulos, Pittau
**Left:** $W + 3$ jets at the Tevatron, comparison to CDF data

**Right:** $W^+ + 3$ jets at the LHC (14 TeV)

$\sqrt{s} = 1.96$ TeV

$E_T^{W} > 20$ GeV, $|\eta^{W}| < 2$

$E_T^{\ell} > 30$ GeV, $M_T^W > 20$ GeV

$R = 0.4$ [siscone]

$\mu_R = \mu_F = H_T$

BlackHat + Sherpa

$\sqrt{s} = 14$ TeV

$E_T^{W} > 30$ GeV, $|\eta^{W}| < 3$

$E_T^{\ell} > 20$ GeV, $|\eta^{\ell}| < 2.5$

$E_T^{\ell} > 30$ GeV, $M_T^W > 20$ GeV

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BlackHat + Sherpa

Ita, Kosower, Maitre
Excellent Scaling with External Legs

Giele, Zanderighi

\[ A^\gamma(\pm \pm \pm \ldots) \]

fit to degree 9 polynom.

Number of gluons

\[ 10^2 \]

\[ 10^1 \]

\[ 10^0 \]

\[ 10^{-1} \]

\[ 10^{-2} \]

\[ 5 \]

\[ 10 \]

\[ 15 \]

\[ 20 \]
Implications

- Lessons Learned from NLO: K-Factors
- Lessons Learned from NLO: Scales I
- Lessons Learned from NLO: Scales II
- Lessons Learned from NLO: IR Safety

Conclusions
## Lessons Learned from NLO: K-Factors

<table>
<thead>
<tr>
<th>Process</th>
<th>Typical scales</th>
<th>Tevatron $K$-factor</th>
<th>LHC $K$-factor</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$\mu_0$</td>
<td>$\mu_1$</td>
<td>$K(\mu_0)$</td>
</tr>
<tr>
<td>$W$</td>
<td>$m_W$</td>
<td>$2m_W$</td>
<td>1.33</td>
</tr>
<tr>
<td>$W+j$</td>
<td>$m_W$</td>
<td>$p_T^j$</td>
<td>1.42</td>
</tr>
<tr>
<td>$W+jj$</td>
<td>$m_W$</td>
<td>$p_T^j$</td>
<td>1.16</td>
</tr>
<tr>
<td>$W+W+j$</td>
<td>$m_W$</td>
<td>$2m_W$</td>
<td>1.19</td>
</tr>
<tr>
<td>$t\bar{t}$</td>
<td>$m_t$</td>
<td>$2m_t$</td>
<td>1.08</td>
</tr>
<tr>
<td>$t\bar{t}+j$</td>
<td>$m_t$</td>
<td>$2m_t$</td>
<td>1.13</td>
</tr>
<tr>
<td>$b\bar{b}$</td>
<td>$m_b$</td>
<td>$2m_b$</td>
<td>1.20</td>
</tr>
<tr>
<td>$H$</td>
<td>$m_H$</td>
<td>$p_T^j$</td>
<td>2.33</td>
</tr>
<tr>
<td>$H+j$</td>
<td>$m_H$</td>
<td>$p_T^j$</td>
<td>2.02</td>
</tr>
<tr>
<td>$H+jj$</td>
<td>$m_H$</td>
<td>$p_T^j$</td>
<td>$-$</td>
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- **Large color annihilation** (e.g. $gg \rightarrow H$) $\Rightarrow$ large $K$-factor
- **Addition of legs in final state** $\Rightarrow$ smaller $K$-factor
Lessons Learned from NLO: Scales I

Fixed scales are in general not a good idea

\[ W^- + 3 \text{ jets} + X \]
\[ \sqrt{s} = 14 \text{ TeV} \]
\[ \mu_0 = 2 M_W = 160.838 \text{ GeV} \]

\[ E_T^{\text{jet}} > 30 \text{ GeV}, \mid \eta^{\text{jet}} \mid < 3 \]
\[ E_T^{\text{jet}} > 20 \text{ GeV}, \mid \eta \mid < 2.5 \]
\[ \mathbf{E}_T > 30 \text{ GeV}, \quad M_W > 20 \text{ GeV} \]
\[ R = 0.4 \quad \text{[siscone]} \]

**BlackHat + Sherpa: CFB, Bern, Dixon, Forde, Febres Cordero, Gleisberg,**

**Ita, Kosower, Maitre**
Lessons Learned from NLO: Scales I

Fixed scales are in general not a good idea

\[ \sigma \text{ [pb]} \]

- \( W^- + 3 \text{ jets} + X \)
- \( \sqrt{s} = 14 \text{ TeV} \)
- \( \mu_0 = \hat{H}_T \)

\( E_T^{\text{jet}} > 30 \text{ GeV}, |\eta^{\text{jet}}| < 3 \)

\( E_T^{\text{e}} > 20 \text{ GeV}, |\eta^{\text{e}}| < 2.5 \)

\( E_T^{l} > 30 \text{ GeV}, M_W > 20 \text{ GeV} \)

\( R = 0.4 \) [siscone]

BlackHat + Sherpa: CFB, Bern, Dixon, Forde, Febres Cordero, Gleisberg, Ita, Kosower, Maitre
Lessons Learned from NLO: Scales II

Not every dynamical scale is created equal

**Graph:**

- **LO** vs. **NLO**
  - Differential cross section $d\sigma/dE_T$ for $W^- + 3$ jets + $X$
  - $\sqrt{s} = 14$ TeV
  - $p_T = p_T = E_T^W$
  - $E_T^W > 30$ GeV, $|\eta^W| < 3$
  - $E_T^J > 20$ GeV, $|\eta^J| < 2.5$
  - $E_T^J > 30$ GeV, $M_T^W > 20$ GeV
  - $R = 0.4$ [siscone]

**Legend:**
- **BlackHat+Sherpa**
- **LO / NLO**
- **NLO scale dependence**
- **LO scale dependence**

**Equations:**

- $p_T = p_T = E_T^W$
- $E_T^W > 30$ GeV, $|\eta^W| < 3$
- $E_T^J > 20$ GeV, $|\eta^J| < 2.5$
- $E_T^J > 30$ GeV, $M_T^W > 20$ GeV
- $R = 0.4$ [siscone]

**Notes:**
- $\sqrt{s} = 14$ TeV
- $\mu_R = \mu_F = E_T^W$

**Additional Information:**
- BlackHat + Sherpa: CFB, Bern, Dixon, Forde, Febres Cordero, Gleisberg, Ita, Kosower, Maitre
Lessons Learned from NLO: IR Safety

This plot actually doesn’t make sense:

\[ \frac{d\sigma}{dE_T} \text{ [pb/GeV]} \]

\[ \sqrt{s} = 1.96 \text{ TeV} \]

\[ E_T^{W} > 20 \text{ GeV}, \ |\eta^{W}| < 2 \]

\[ E_T^{e} > 20 \text{ GeV}, \ |\eta^{e}| < 1.1 \]

\[ \not{E}_T > 30 \text{ GeV}, \ M_W > 20 \text{ GeV} \]

\[ R = 0.4 \text{ [siscone]} \]

\[ \mu_R = \mu_F = E_T^{W} \]

BlackHat + Sherpa: CFB, Bern, Dixon, Forde, Febres Cordero, Gleisberg, Ita, Kosower, Maitre
Lessons Learned from NLO: IR Safety

This plot actually doesn’t make sense:

BlackHat + Sherpa: CFB, Bern, Dixon, Forde, Febres Cordero, Gleisberg, Ita, Kosower, Maitre

Comparison of infrared-unsafe JetClu (data) with infrared-safe SISCon (BlackHat+Sherpa)
Lessons Learned from NLO: IR Safety

(a) $p_t/\text{GeV}$

(b) $p_t/\text{GeV}$

$y$ values:

- $y = -1$
- $y = 0$
- $y = 1$
- $y = 2$
- $y = 3$

$pt$ values:

- $pt = 0$
- $pt = 100$
- $pt = 200$
- $pt = 300$
- $pt = 400$
Lessons Learned from NLO: IR Safety

- JetClu: 50.1%
- SearchCone: 48.2%
- MidPoint: 16.4%
- Midpoint-3: 15.6%
- PxCone: 9.3%
- Seedless [SM-$p_T$]: 1.6%
- Seedless [SM-MIP]: < 10^{-9}
- Seedless (SISCone): < 10^{-9}

Salam, Soyez
Conclusions and Outlook

- **Progress at NNLO**
  fully differential distributions, several more new calculations soon to be completed

- **Tremendous progress at NLO**
  Feynman diagrams: first $2 \rightarrow 4$ results
  New methods reuse amplitudes instead of Feynman diagrams via generalized unitarity and recursion, OPP reduction

- **General purpose NLO amplitude codes** being developed, progress toward agreement on common interface at Les Houches 2009
  $\Rightarrow$ event generators incl. parton showers at NLO?

- **Lesson learned from NLO calculation for LO simulation**: choose your scale wisely!

- **New jet algorithms**
  Whichever one you use, please choose an infrared safe one!
Omissions

- Parton Distribution Functions
- Shower algorithms, incl. at NLO
- All order conjecture for structure of infrared divergences of amplitudes
- Resummation
- Studies of jet substructure to identify heavy particles
- Omissions from the listed omissions
Outlook