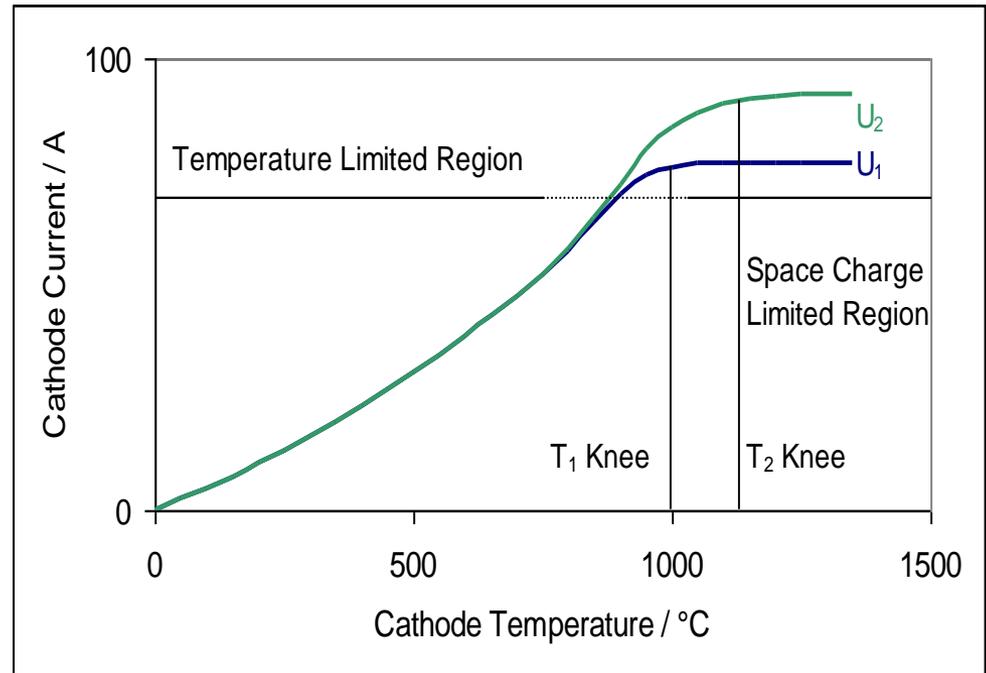
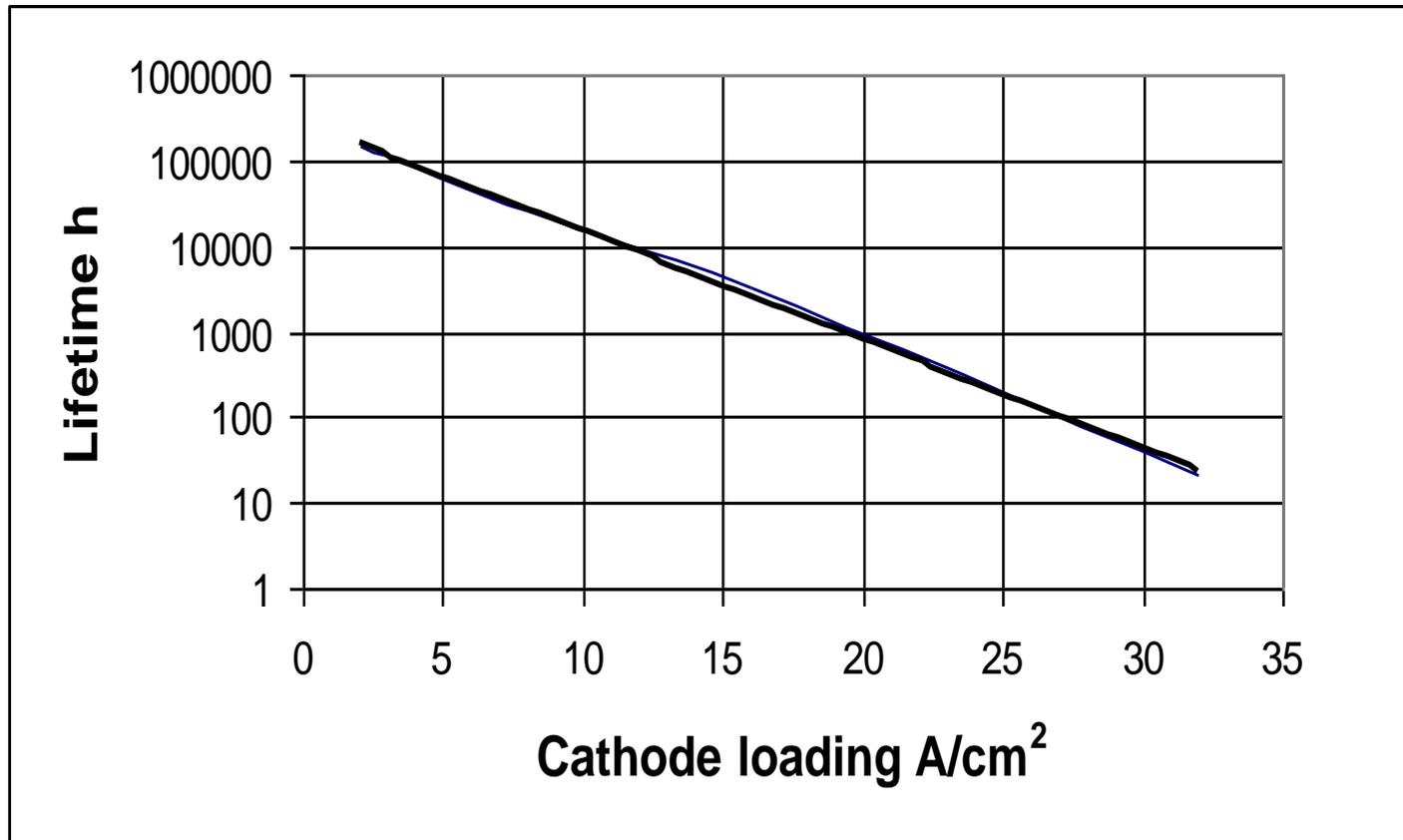


# *Appendix*

- Cathode typical:
  - A) M-Type: Tungsten-Matrix impregnated with Ba and coated with Os/Ru
  - B) Oxide (BaO, CaO or SO)
- Cathode is operated in the space charge limited region (Child-Langmuir Theory)
 
$$j = (4/9) \epsilon_0 [(2e)/m]^{1/2} U^{3/2} / d$$
- Integration gives:  $I = pU^{3/2}$



## *Klystron: Gun (2)*



For higher cathode loading it is required to operate at higher cathode temperature => the cathode lifetime decreases.



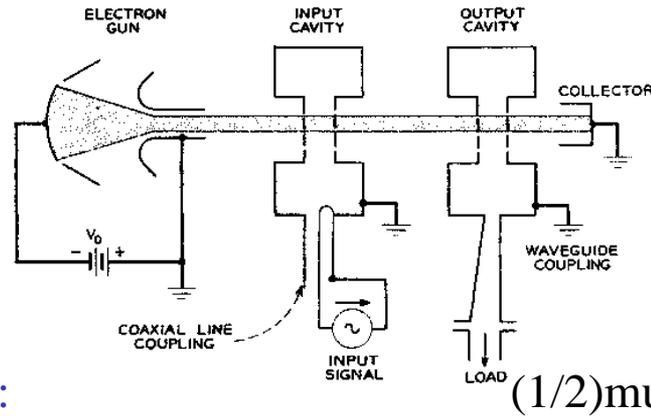
## *Klystron: Beam Focussing*

- Confined flow: The cathode is in the magnetic field of a solenoid (common in travelling wave tubes).
- Brillouin focussing: No magnetic field lines are threading through the cathode. The beam is entering the magnetic field of a (electromagnetic) solenoid around the drift tube section.  
B is  $B = 1.2 - 2 \times B_B$  (typ  $\sim 1000\text{G}$ )  
with  $B_B$  Brillouinfield  
with  $b$  beam radius,  $u_e$  beam velocity,  $I$  beam current
- Focussing can also be done with permanent magnets: Periodic Permanent Magnet focussing (PPM) e.g. pulsed high power X-Band klystrons (SLAC, KEK).

$$B_B = \sqrt{(2I m_0) / (\epsilon_0 \pi b^2 u_e e)}$$

# *Klystron: Ballistic Theory (1)*

## *Treatment of individual electrons without interaction*



Initial electron energy:

$$(1/2)mu_0^2 = eV_0$$

Electron Energy gain in the input cavity:  $(1/2)mu^2 - (1/2)mu_0^2 = eV_1 \sin \omega t$

$$u = u_0 (1 + (mV_1/V_0) \sin \omega t)^{1/2}$$

Assume  $V_1 \ll V_0$  :

$$u = u_0 (1 + (mV_1/2V_0) \sin \omega t)$$

The arrival time  $t_2$  in the second cavity depends on the departure time  $t_1$  in the first cavity with the assumption of an infinite thin gap:

$$t_2 = t_1 + l/u = t_1 + l/u_0 (1 + (mV_1/2V_0) \sin \omega t_1) = t_1 + l/u_0 - (lmV_1/2u_0V_0) \sin \omega t_1$$

or  $\omega t_2 = \omega t_1 + q_0 - X \sin \omega t_1$  with  $q_0 = l/u_0$  and  $X = q_0 mV_1/2V_0$  called **bunching parameter**

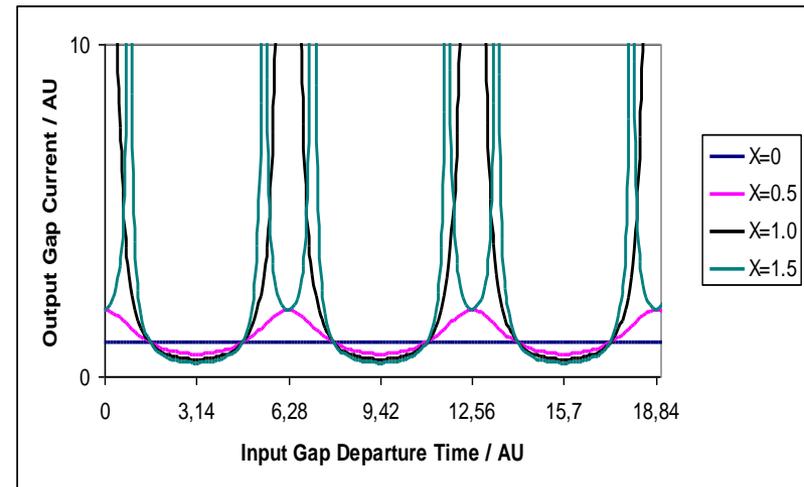
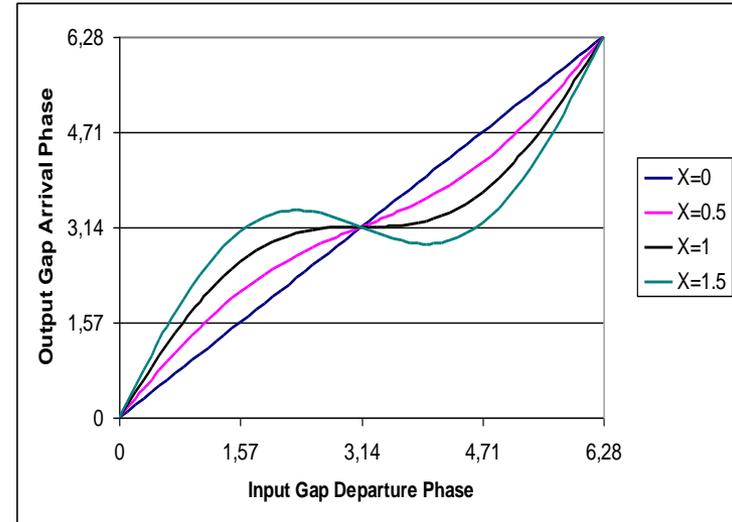
Because of charge conservation:  
 Charge in the input cavity between  
 time  $t_1$  and  $t_1+dt_1$  equals the charge in  
 the output cavity between time  $t_2$  and  
 $t_2+dt_2$

$$I_1 dt_1 = I_2 dt_2$$

With  $dt_2/dt_1 = 1 - X \cos \omega t_1$  and  $I_2 =$   
 $I_1 / (dt_2/dt_1)$  one gets

$$I_2 = I_1 / (1 - X \cos \omega t_1)$$

$$I_2 = I_1 \text{ABS}(1 / (1 - X \cos \omega t_1))$$





## Klystron: Ballistic Theory (3)

Fourier transformation of the current in the output gap  $I_2$

$$I_2 = I_0 + \sum_{n=1}^{\infty} [a_n \cos n(\omega t_2 - \theta_0) + b_n \sin n(\omega t_2 - \theta_0)]$$

$$a_n = (1/\pi) \int_{\theta_0 - \pi}^{\theta_0 + \pi} I_2 \cos n(\omega t_2 - \theta_0) d(\omega t_2) \quad b_n = (1/\pi) \int_{\theta_0 - \pi}^{\theta_0 + \pi} I_2 \sin n(\omega t_2 - \theta_0) d(\omega t_2)$$

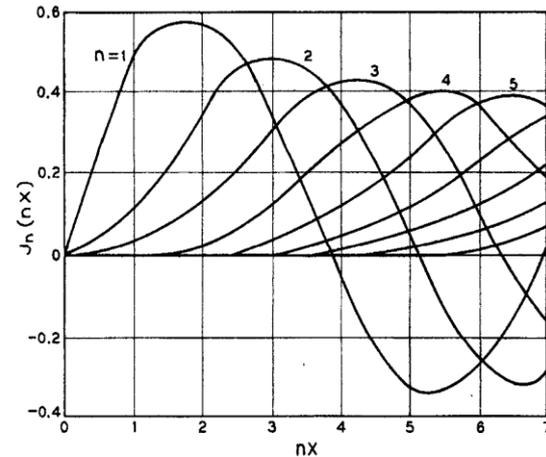
$$a_n = (I_0/\pi) \int_{-\pi}^{\pi} \cos n(\omega t_1 - X \sin \omega t_1) d(\omega t_1)$$

$$b_n = (I_0/\pi) \int_{-\pi}^{\pi} \sin n(\omega t_1 - X \sin \omega t_1) d(\omega t_1) = 0$$

$$a_n = 2 I_0 J_n(nX) \quad \text{with } J_n \text{ Besselfunction of the } n\text{-th order}$$

$$I_2 = I_0 + 2 I_0 \sum_1^{\infty} J_n(nX) \cos n(\omega t_1 - \theta_0)$$

$$I_{\omega} = 2 I_0 J_1(X) \cos(\omega t - \theta_0)$$



Bessel functions of various orders. The maximum value of  $J_1$  occurs at  $X = 1.84$  and is equal to 0.582.

Maximum Output Power:

$$P_{\omega} = \overline{I_{\omega} V_{\omega}} = 2 \times 0.58 (I_0 / \sqrt{2}) (V_0 / \sqrt{2}) = 0.58 P_{Beam}$$



## *Klystron: Space Charge Waves*

- Space charge forces counteract the bunching
- Any perturbation in an electron beam excites an oscillation with the plasma frequency
- Therefore we have 2 waves with the Phase constants
- And therefore
- The group velocity is
- The density modulations appear at a distance of

$$\Omega = \sqrt{\left( (e/m_0)(\rho_0/\epsilon_0) \right)}$$

$$\beta_{e1} = \beta_e (1 + \Omega/\omega)$$

$$\beta_{e2} = \beta_e (1 - \Omega/\omega)$$

$$\beta_e = \omega/u_e \quad u_{e2} = u_e/(1 - \Omega/\omega)$$

$$u_{e1} = u_e/(1 + \Omega/\omega)$$

$$u_g = d\omega/d\beta_e = u_e$$

$$\lambda_p = 2\pi u_e/\Omega$$

This means that the driftspace or the distance between cavities is determined by the plasma frequency (klystron current) and the electron velocity (klystron voltage) and is given by  $\lambda_p/4$



## *Klystron: Coupling (1)*

- Up to now we have neglected the transit time  $t$  in the cavity gap
- The transit angle is:  $f = \omega t$
- The coupling factor is:  $K_1 = (\sin(f/2))/(f/2)$   
e.g.  $K_1 = 1$  max if  $f = 0$  (infinite thin gap)
- In addition there is the transversal coupling factor  
 $K_t = J_0(b_e r) / J_0(b_e b)$  with  $b$  = beam radius and  $r$  = tunnel radius and  $J_0$  modified Besselfunction
- The total coupling factor is  $K = K_1 K_t$  and determines the RF voltage in the cavity gap generated by the RF current
- A typical number is  $K \sim 0.85$  at  $\sim 1$  GHz