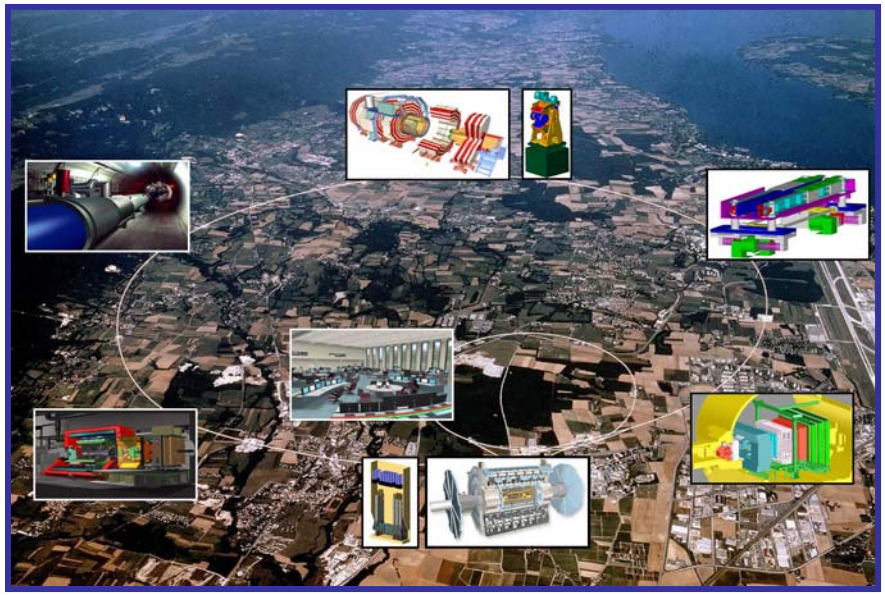
Thrust distribution at N^3LL with power corrections and precision determination of $\alpha_s(m_Z)$



Vicent Mateu

Max-Plank-Institute for Physics

Munich

IWLC 2010

Geneva/CERN

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Taskforce : A. Hoang & V. Mateu - MPI [AFHMS]

R. Abbate & I. Stewart - MIT

M. Fickinger - University of Arizona

arXiv: 1006.3080 [hep-ph]

Outline

- **Motivations**
  - Recent world average for $\alpha_s(m_Z)$.
  - Experimental data.
- **Theoretical developments**
  - SCET: factorization & resummation & power corrections.
  - Nonsingular terms.
- **Results**
  - Tail fits: a two parameter fit.
  - Final value for $\alpha_s(m_Z)$.
- **Applications to a linear collider**
  - Size of non-perturbative effects
  - Sensitivity to $\alpha_s(m_Z)$.

Motivations

Determinations of $\alpha_s(m_Z)$

$\alpha_s(m_Z)$ is a key parameter for the analysis of all collider experiments, flavour observables and new physics searches

$$\alpha_s(m_Z) = 0.1184(7)$$

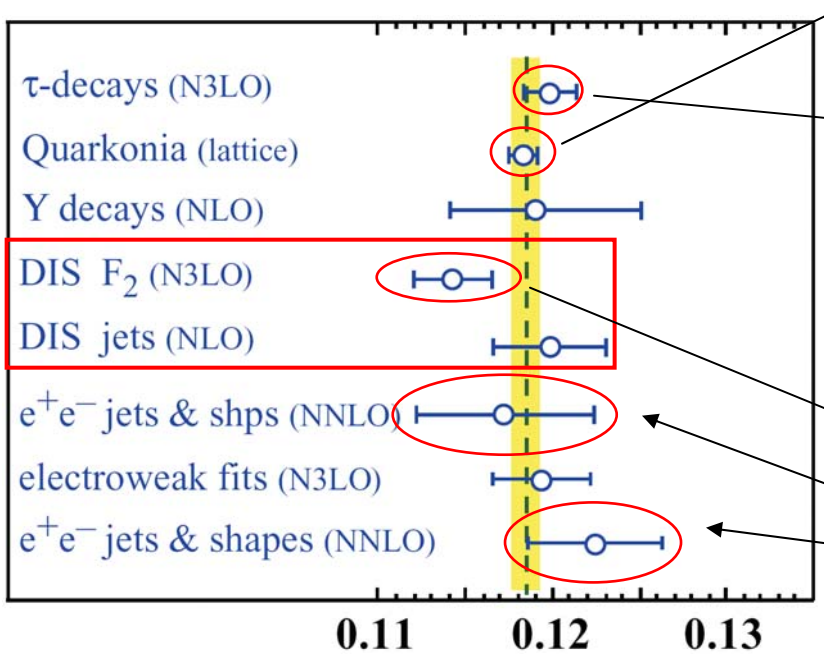
EPJC64 689 [Bethke '09]

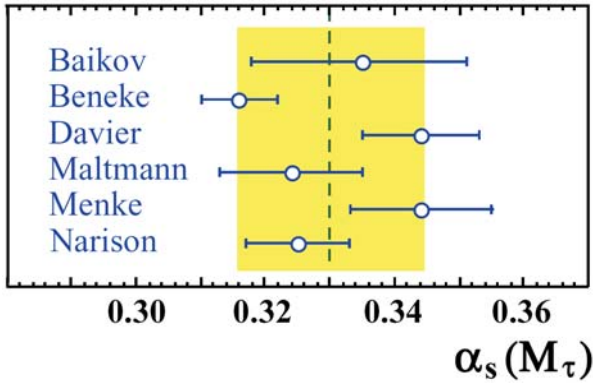
Ever decreasing error from averaging

$$\alpha_s(m_Z) = 0.1183(8)$$

PRD78 114507 [HPQCD '08]

Fit to Υ-splittings, Wilson loops





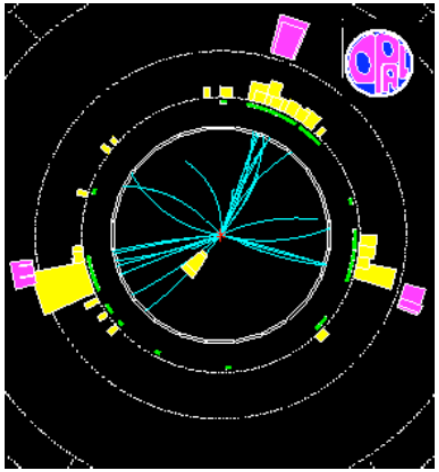
Substantially lower than world average!

Event shape results at $O(\alpha_s^3)$ (fixed order)

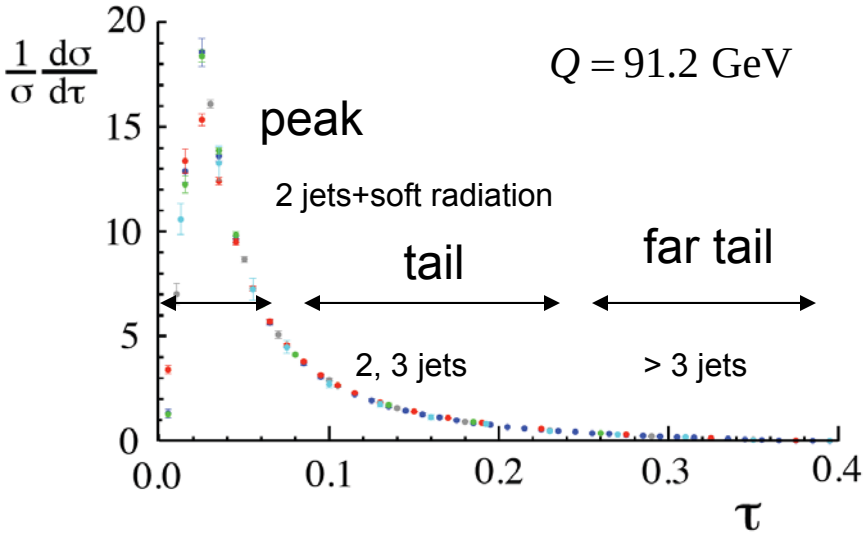
Thrust experimental data

$$e^+e^- \xrightarrow{Q} \text{jets}$$

LEP 2 jet event



OPAL 3 jet event

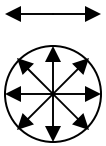


$$1 - \tau = \max_{\hat{n}} \frac{\sum_i |\hat{n} \cdot \vec{p}_i|}{Q}$$

τ is an event-shape variable

$\tau \to 0 \quad \Rightarrow$ dijet

$\tau \to 0.5 \Rightarrow$ spherical



At each Q there is a distribution in τ

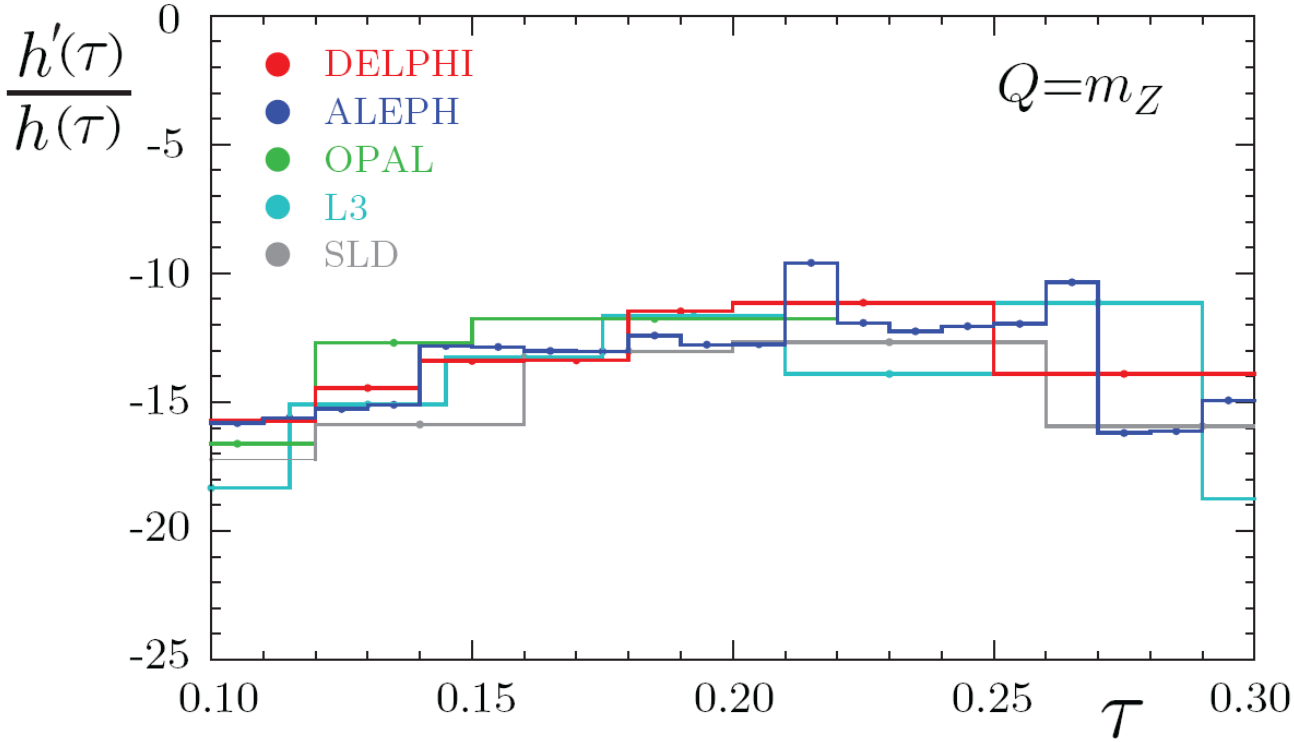
Theoretical motivation

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = h\left(\tau - \frac{2\Lambda}{Q}\right)$$

Perturbative expression

Power correction

assuming that $h \propto \alpha_s$ $\longrightarrow$ $\frac{\delta\alpha_s}{\alpha_s} \approx \frac{2\Lambda}{Q}\frac{h'(\tau)}{h(\tau)}$



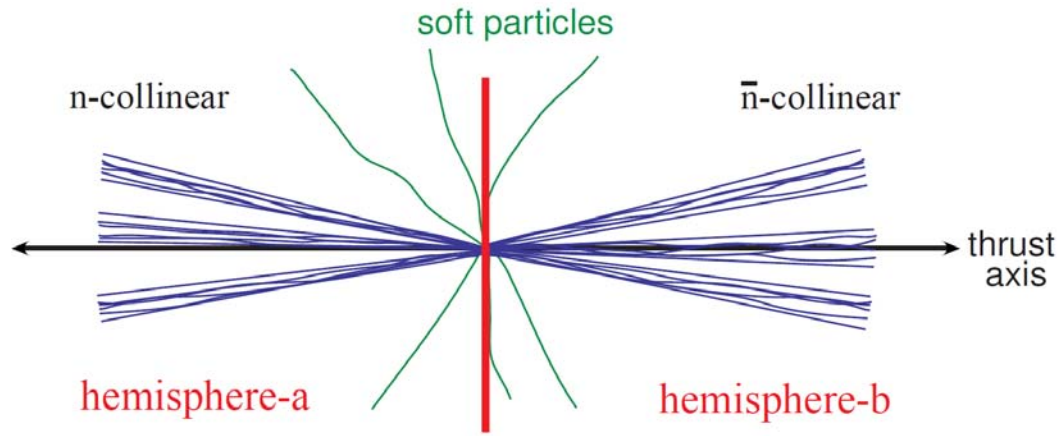
$$\frac{h'(\tau)}{h(\tau)} \approx -14 \pm 4$$

assuming $\Lambda \sim 0.3\,\text{GeV}$ $\longrightarrow$ $\boxed{\frac{\delta\alpha_s}{\alpha_s} \approx -(9 \pm 3)\%}$

Theoretical developements

Jet production

$$e^+e^- \xrightarrow{\gamma,Z} 2 \text{ jets} + X_{soft}$$



Particles in each hemisphere have

$$p_+ = E + p_n \sim O(Q)$$

$$p_- = E - p_n \sim O(\Lambda_{QCD})$$

$$p_\perp \sim O(Q\Lambda_{QCD})$$

$$Q^2 = \left(E_{e^+} + E_{e^-}\right)^2 = \left(\sum_{hem\ a+b} p_i^\mu\right)^2 \approx 2\sum_{hem\ a} E_i^2 \sim (100\text{ GeV})^2$$

$$\mu_{Jet}^2 = \left(\sum_{hem\ a} p_i^\mu\right)^2 \approx \sum_{hem\ a} (E_i^2 - \vec{p}_i^{\,2}) \sim \sum_{hem\ a} p_i^+ p_i^- \sim Q\,\Lambda_{QCD} \sim (20\text{ GeV})^2$$

$$\mu_{soft}^2 = p_{hadron}^2 = \Lambda_{QCD}^2 \sim (2\text{ GeV})^2$$

large logs !

3 energy scales $\longrightarrow$ EFT

In this case the small SCET parameter is either τ or $\dfrac{\Lambda_{QCD}}{Q}$

Factorization theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\mathrm{QCD}}}{Q} \right)$$

b mass corrections

nonperturbative soft function

sets our accuracy

$$\frac{\Delta\alpha_s}{\alpha_s} \sim 0.5\%$$

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l} \alpha_s^n \frac{\ln^l \tau}{\tau}$$

Singular partonic for massless quarks QCD+QED final states

$$\frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} = \sum_{n,l} \alpha_s^n \ln^l \tau + \sum_n \alpha_s^n f_n(\tau)$$

Nonsingular partonic

Resummation for singular partonic

$$\ln\left(\int_0^\tau \mathrm{d}\tau \frac{1}{\sigma} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} \right) \sim (\ln\tau) \sum_{k=0} (\alpha_s \ln\tau)^{k+1} + \sum_{k=0} (\alpha_s \ln\tau)^{k+1} + \alpha_s \sum_{k=0} (\alpha_s \ln\tau)^k + \alpha_s^2 \sum_{k=0} (\alpha_s \ln\tau)^k + \ldots$$

LL NLL NNLL N^3LL

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\boxed{\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\mathrm{QCD}}}{Q} \right)$$

Singular terms

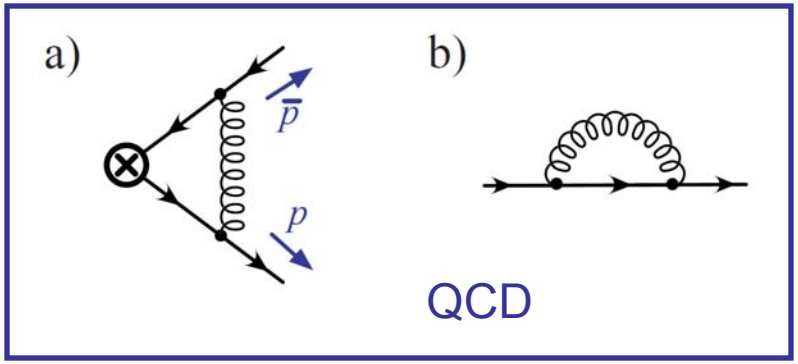
$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = Q \sum_i \sigma_0^I \, H_Q^I(Q,\mu) \int \mathrm{ds} \; J_\tau(s,\mu) \, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} S_\tau^{\mathrm{part}} \left(Q\tau - \frac{s}{Q}, \mu \right)$$

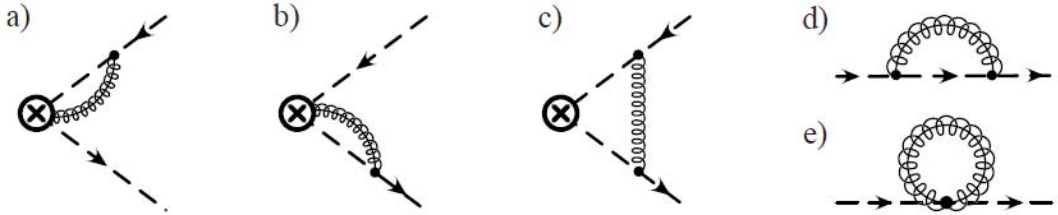
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\boxed{\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\mathrm{QCD}}}{Q} \right)$$

Singular terms

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = Q \sum_i \sigma_0^I \boxed{H_Q^I(Q,\mu)} \int \mathrm{ds}\ J_\tau(s,\mu)\, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} S_\tau^{\mathrm{part}}\left(Q\tau - \frac{s}{Q}, \mu \right)$$

Hard Wilson coefficient (function)



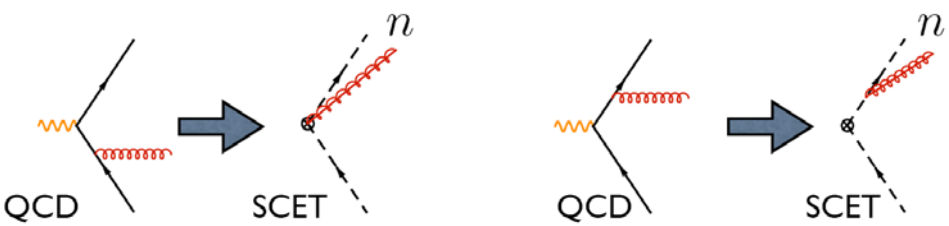


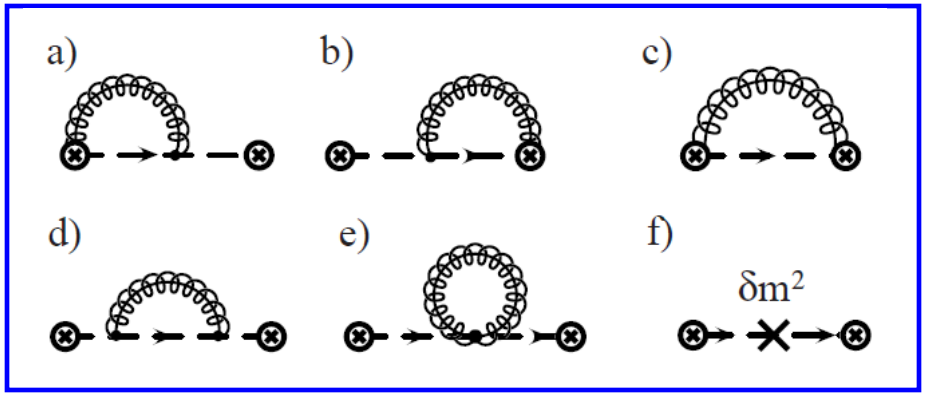
$$\frac{d\sigma}{d\tau} = \int dk \left(\boxed{\frac{d\hat{\sigma}}{d\tau}} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)$$

Singular terms

$$\frac{d\hat{\sigma}}{d\tau} = Q \sum_i \sigma_0^I \, H_Q^I(Q,\mu) \int ds \, \boxed{J_\tau(s,\mu)} \, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} S_\tau^{part}\left(Q\tau - \frac{s}{Q}, \mu \right)$$

Jet function



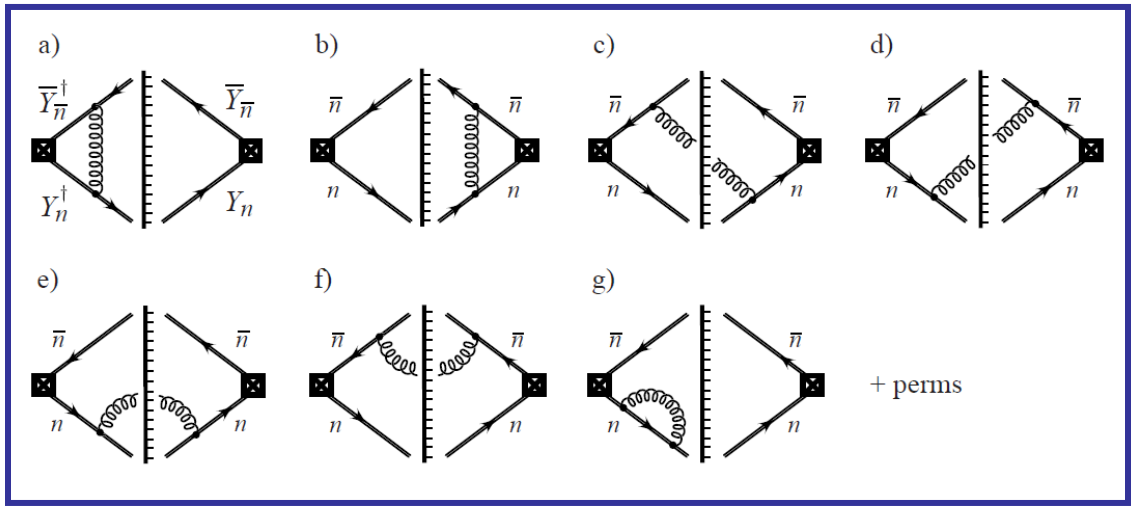


$$\frac{d\sigma}{d\tau} = \int dk \left(\boxed{\frac{d\hat{\sigma}}{d\tau}} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)$$

Singular terms

$$\frac{d\hat{\sigma}}{d\tau} = Q \sum_i \sigma_0^I \, H_Q^I(Q,\mu) \int ds \; J_\tau(s,\mu) \, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} \boxed{S_\tau^{part}\left(Q\tau - \frac{s}{Q}, \mu \right)}$$

Soft Function



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\boxed{\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\mathrm{QCD}}}{Q} \right)$$

Singular terms

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = Q \sum_i \sigma_0^I \, H_Q^I(Q,\mu) \int \mathrm{ds} \;\; J_\tau(s,\mu) \boxed{e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}}} S_\tau^{\mathrm{part}} \left(Q\tau - \frac{s}{Q}, \mu \right)$$

Renormalon subtraction

- Reduces sensitivity to low momenta in the soft function
- Removes an $O(\Lambda_{\mathrm{QCD}})$ renormalon from the first moment of the soft function

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\boxed{\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\mathrm{QCD}}}{Q} \right)$$

Singular terms

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = Q \sum_i \sigma_0^I \, H_Q^I(Q,\mu) \int \mathrm{d}s \;\; J_\tau(s,\mu) \, e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} S_\tau^{\mathrm{part}} \left(Q\tau - \frac{s}{Q}, \mu \right)$$

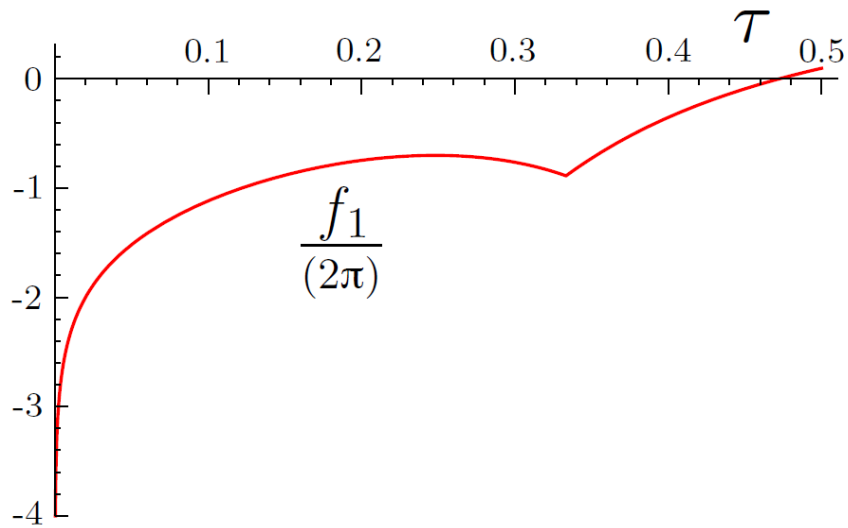
Still has large logs

Resummation of large logs!

No large logs any more

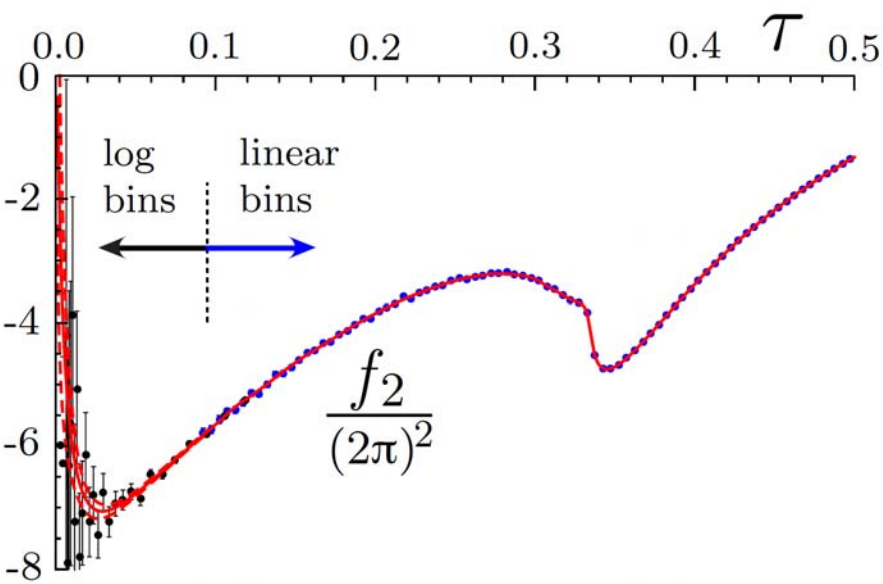
$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = Q \sum_i \sigma_0^I \, H_Q^I(Q,\boxed{\mu_H}) U_Q(Q,\boxed{\mu_H},\mu_S) \int \mathrm{d}s\mathrm{d}s' \, U_J(s-s',\boxed{\mu_S},\mu_J) J_\tau(s',\boxed{\mu_J}) e^{-2\frac{\delta}{Q}\frac{\partial}{\partial\tau}} S_\tau^{\mathrm{part}} \left(Q\tau - \frac{s}{Q}, \boxed{\mu_S} \right)$$

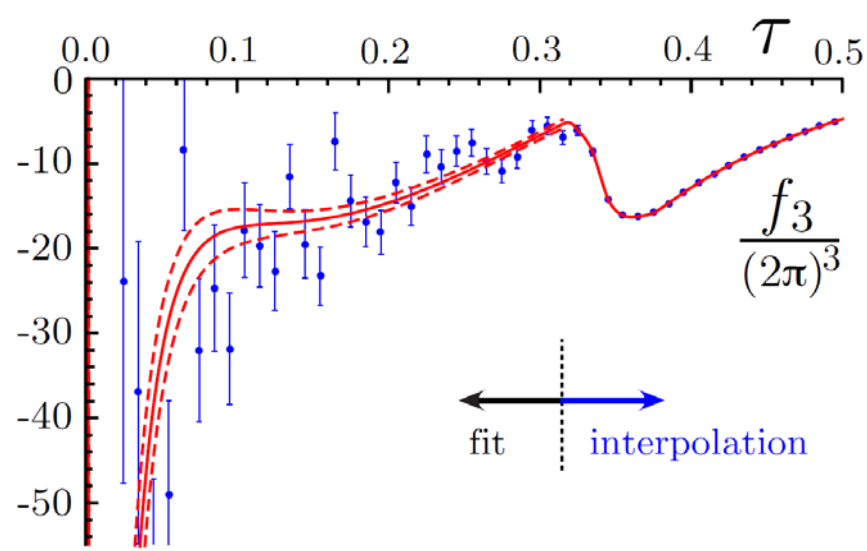
$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}}{d\tau} + \boxed{\frac{d\hat{\sigma}_{ns}}{d\tau}} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)$$



Nonsingular terms

$$\frac{d\hat{\sigma}_{ns}}{d\tau} \equiv \left. \frac{d\hat{\sigma}}{d\tau} \right|_{\text{fixed order}} - \left. \frac{d\hat{\sigma}}{d\tau} \right|_{\text{SCET(no resummation)}}$$





$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_b}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) \boxed{S_\tau^{\mathrm{mod}}(k - 2\bar{\Delta})} + O\left(\sigma_0 \frac{\alpha_s \Lambda_{\mathrm{QCD}}}{Q} \right)$$

Nonperturbative corrections

In the tail region $\ell_{\mathrm{soft}} \sim Q\,\tau \gg \Lambda_{QCD}$

and we can expand the soft function

Shifts distributions to the right !

$$S(\tau) = S_{\mathrm{pert}}(\tau) - S'_{\mathrm{pert}}(\tau) \frac{2\Omega_1}{Q} \approx S_{\mathrm{pert}}\left(\tau - \frac{2\Omega_1}{Q} \right) \qquad \Omega_1 \sim \Lambda_{QCD}$$

Is a nonperturbative parameter defined in field theory

Results

Ingredients for the calculation

Order or the analysis

Anomalous dimension

Matrix elements

α_s running

Fixed order

gap subtractions

		cusp	non-cusp	matching	$\beta[\alpha_s]$	nonsingular	$\gamma_\Delta^{\mu,R}$	δ
standard counting	LL	1	-	tree	1	-	-	-
	NLL	2	1	tree	2	-	-	-
	NNLL	3	2	1	3	1	1	1
	N^3LL	4^{pade}	3	2	4	2	2	2
primed counting	NLL′	2	1	1	2	1	1	1
	NNLL′	3	2	2	3	2	2	2
	N^3LL'	4^{pade}	3	3	$\boxed{4}$	3	3	3

s_2

j_3

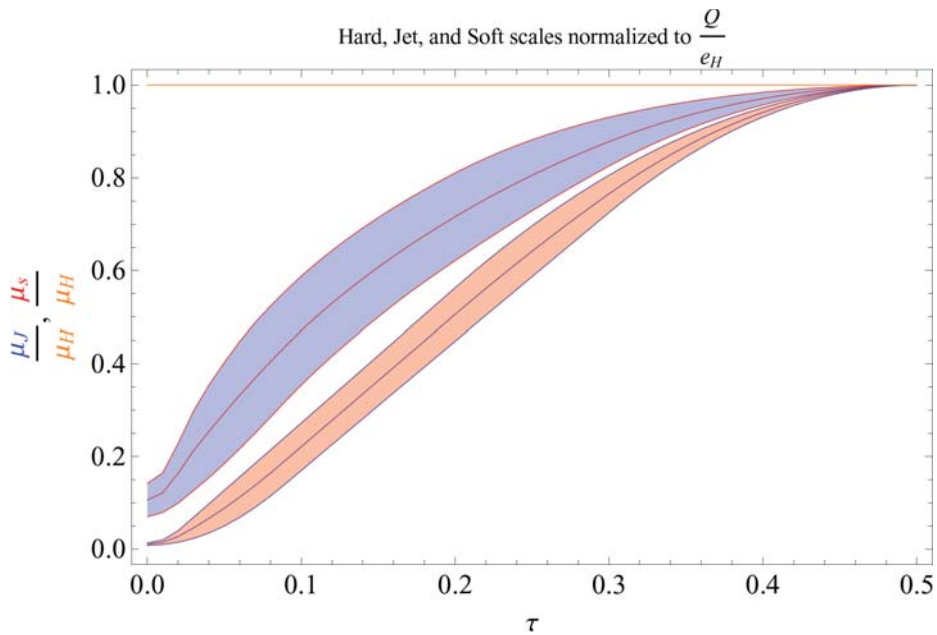
s_3

From a Padè approximant $\Gamma^{(3)}_{\text{cusp}}$

For jet and soft: log information known, and sum of non-log terms known.

When fixed order resuls are important primed counting is better

Estimate of theory uncertainties



Profile functions

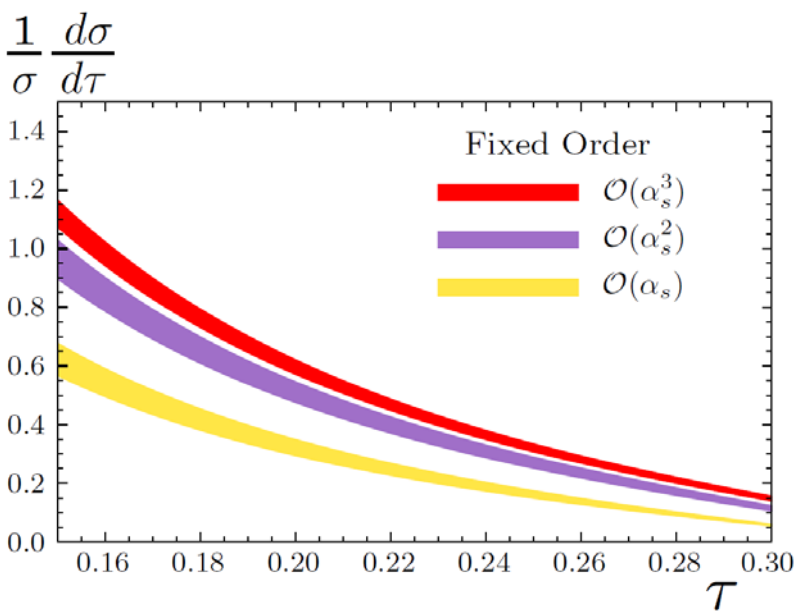
Depend on a family of 6 parameters

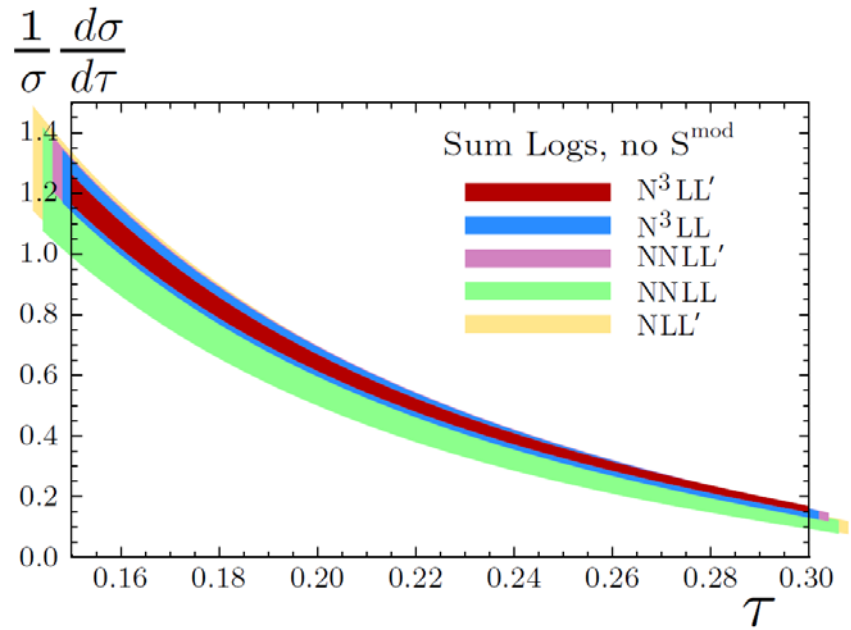
Unknown parameters ⟶ estimated by padé approximants

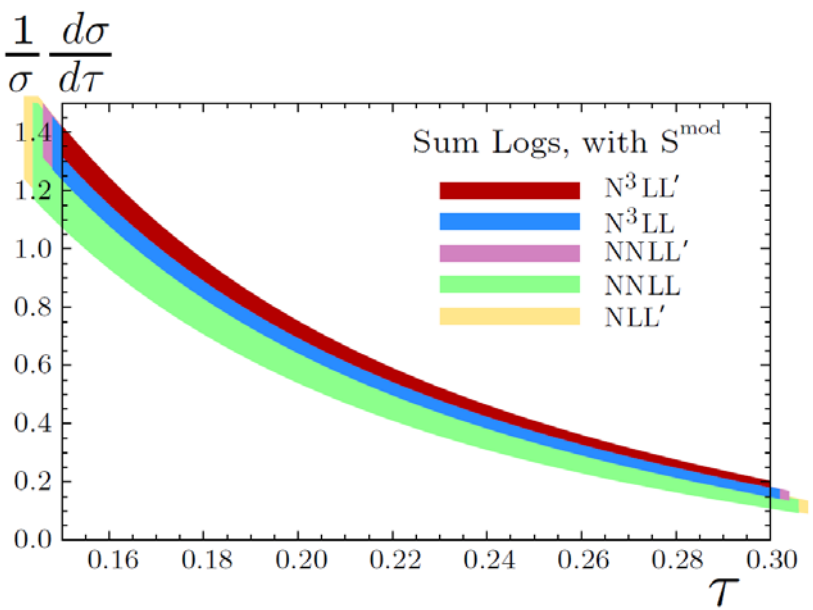
Nonsingular statistical error

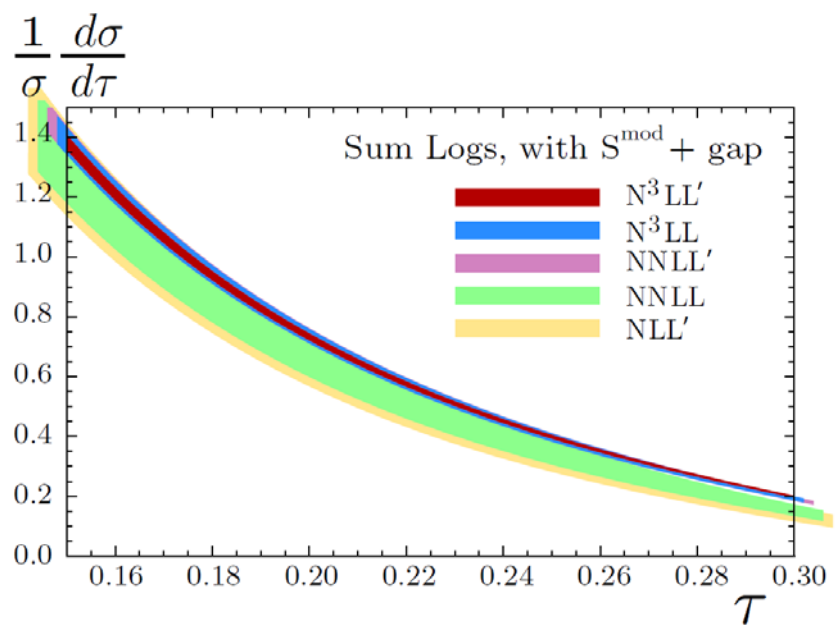
Flat random scan over 500 points to estimate theory uncertainty

Tail predictions, scan over theory uncertainty



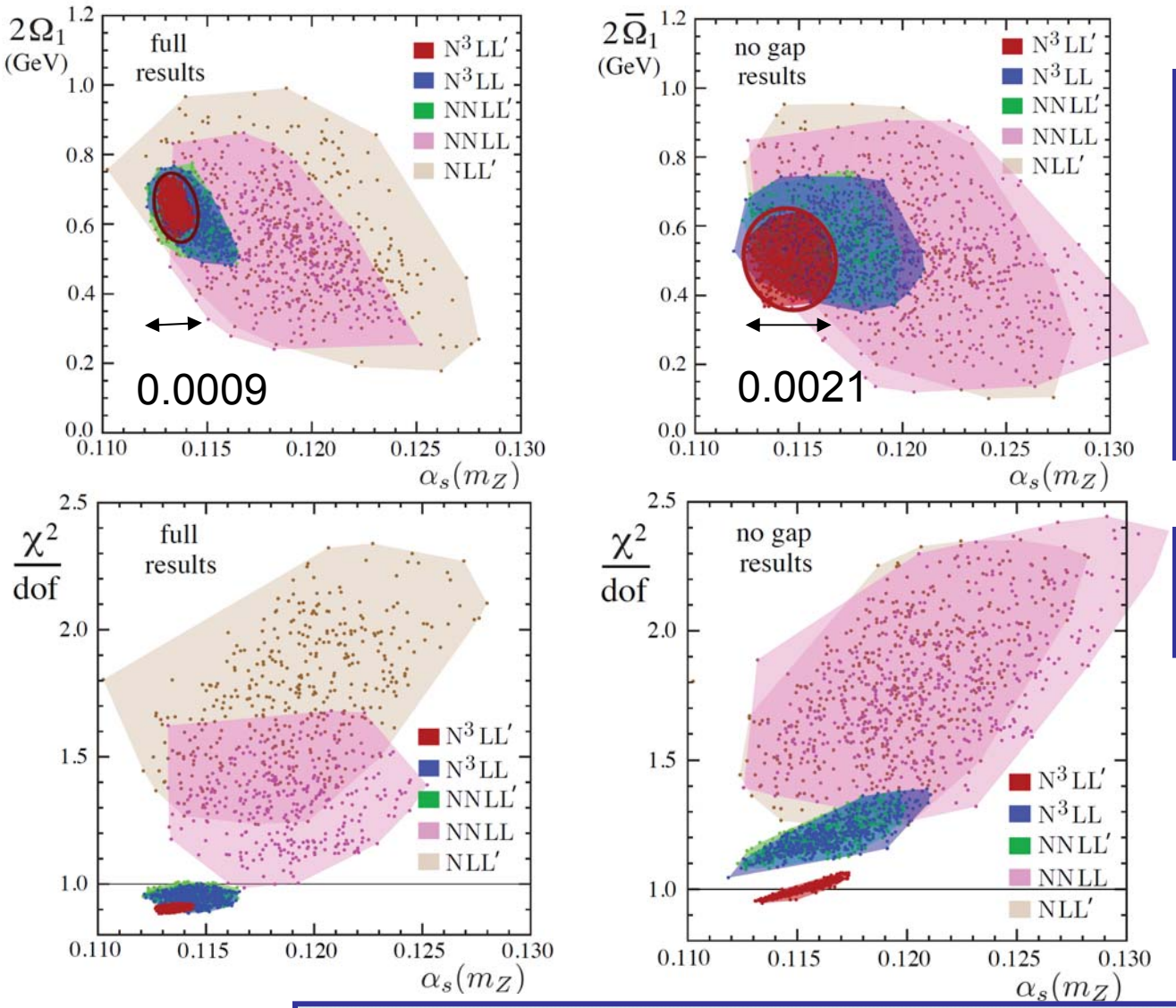






Two parameter fit in the tail: bins

Renormalon free | **With renormalon**



"standard" data set:

$Q > 35\,\text{GeV}$

$$\frac{6\,\text{GeV}}{Q} \le \tau \le 0.33$$

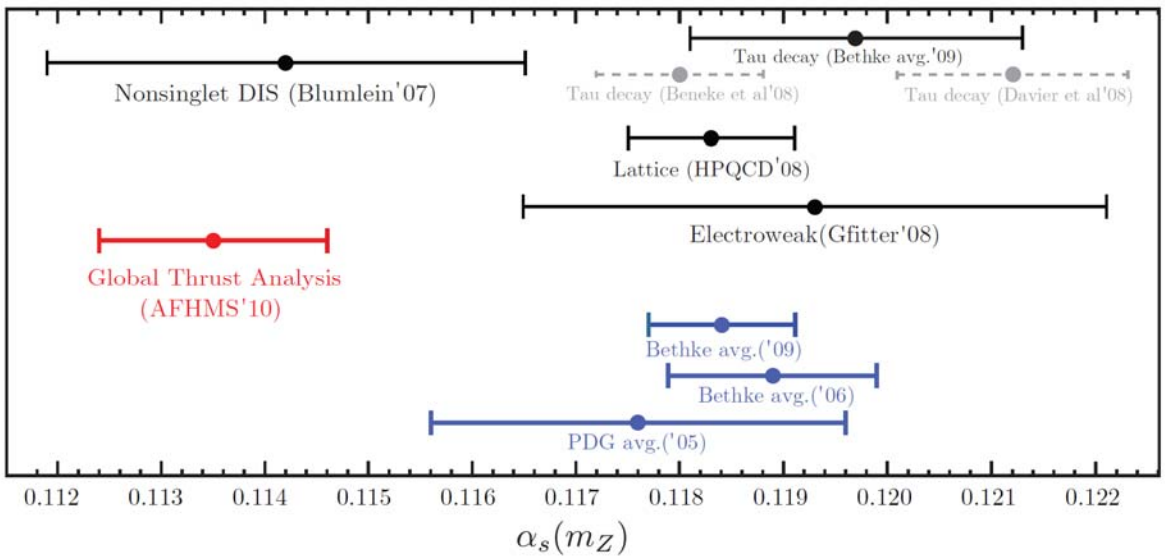
488 bins

Error band method: 0.0004

Renormalon-free results have smaller theory errors and better fits

$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

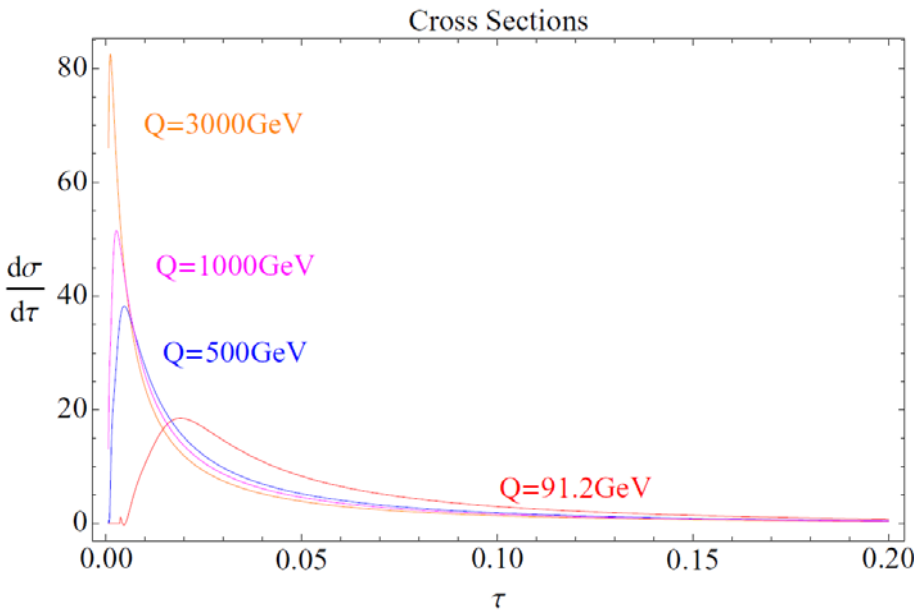
Comparison to recent determinations



Numerical impact of nonperturbative corrections is very important

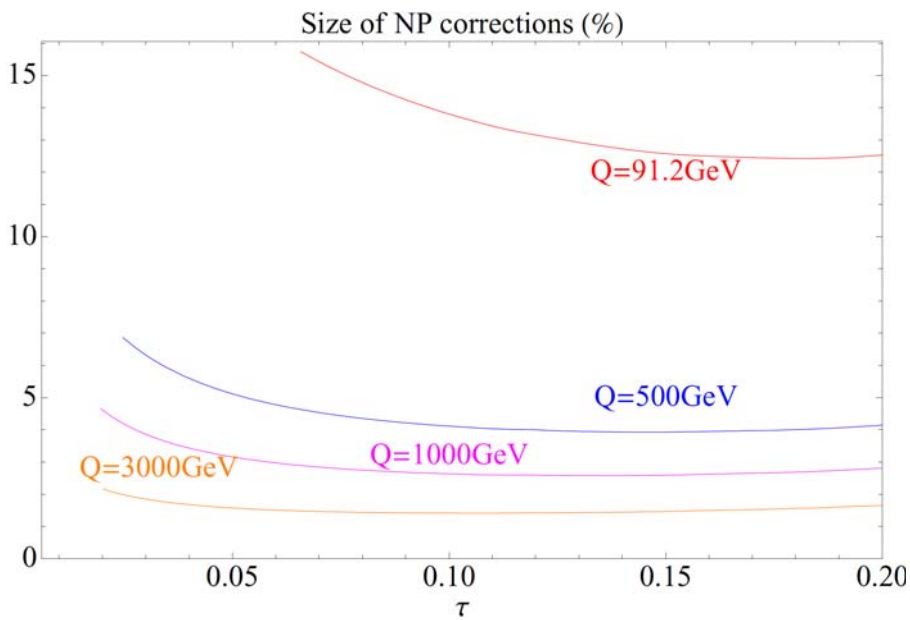
Applications to a linear collider

Size of nonperturbative effects



For increasing Q:

- The peak moves towards smaller τ
- Events tend to accumulate at regions with very small τ
- The tail region becomes larger but less populated

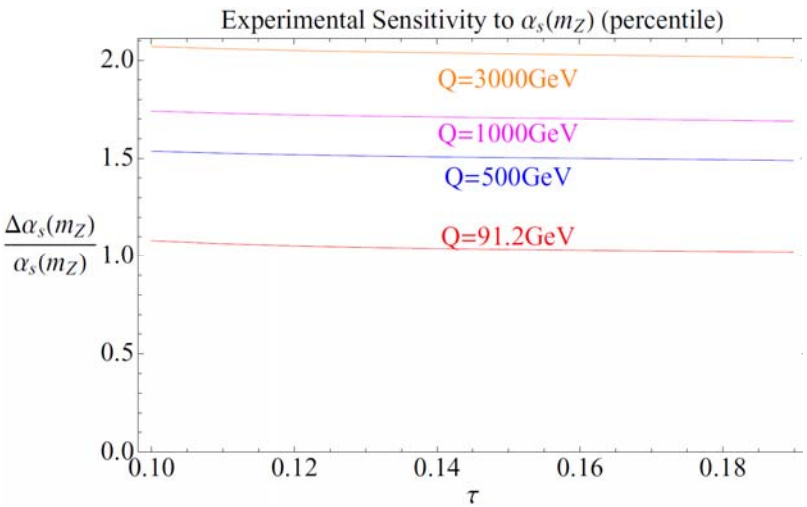


- Size of nonperturbative effects decreases for high Q values
- As expected, it scales as $\frac{\Lambda_{QCD}}{Q}$
- At very large Q, nonperturbative effects can become smaller than theory and experimental errors

$\rightarrow$ might be neglected!

Sensitivity to the strong coupling

Here we assume that one can achieve a 2% experimental accuracy at any energy



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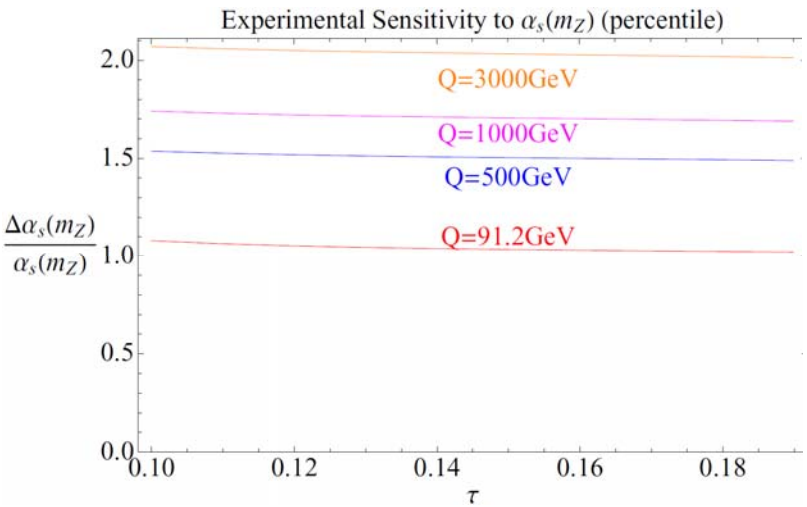
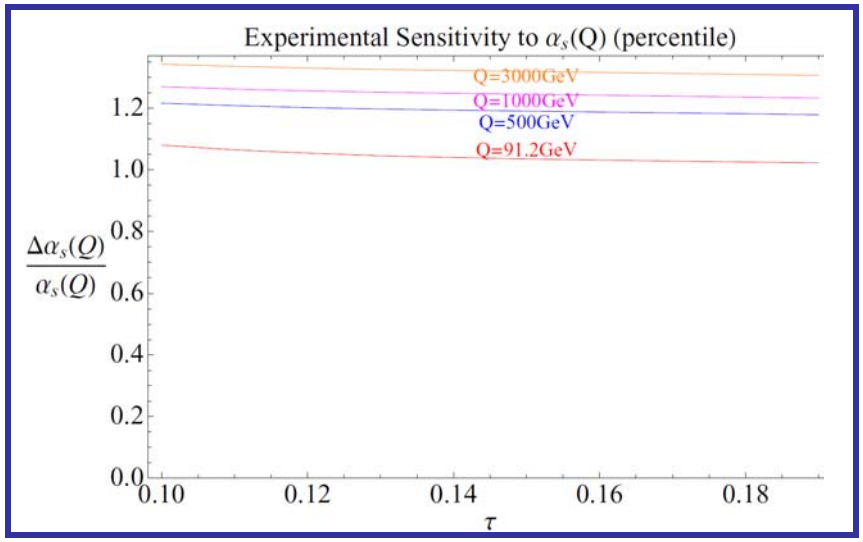
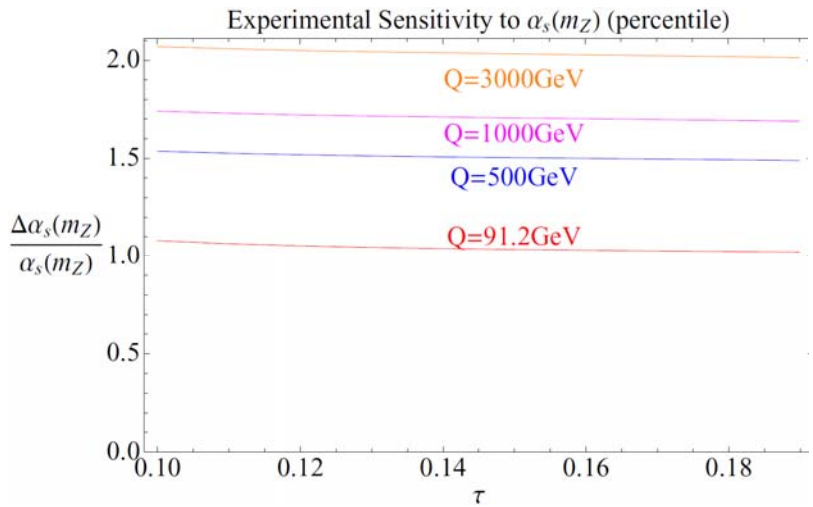




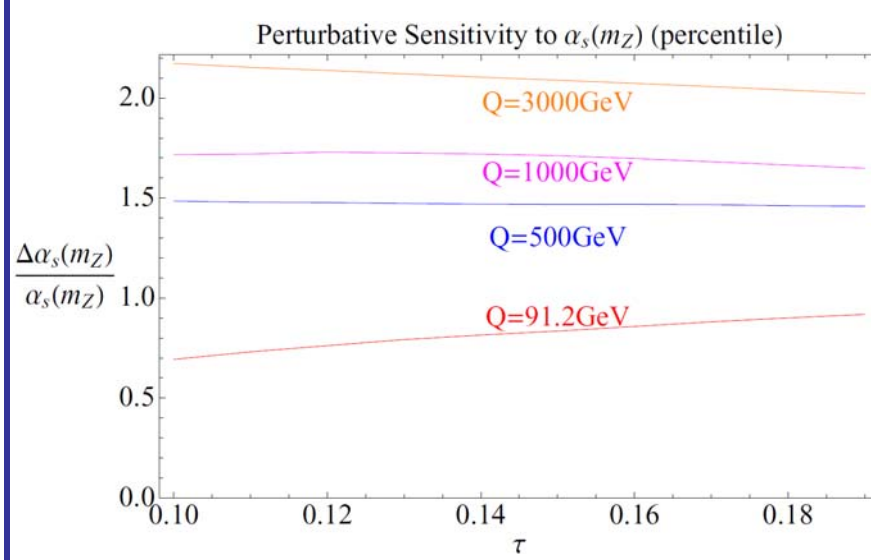
So most of the effect is due to the running of α

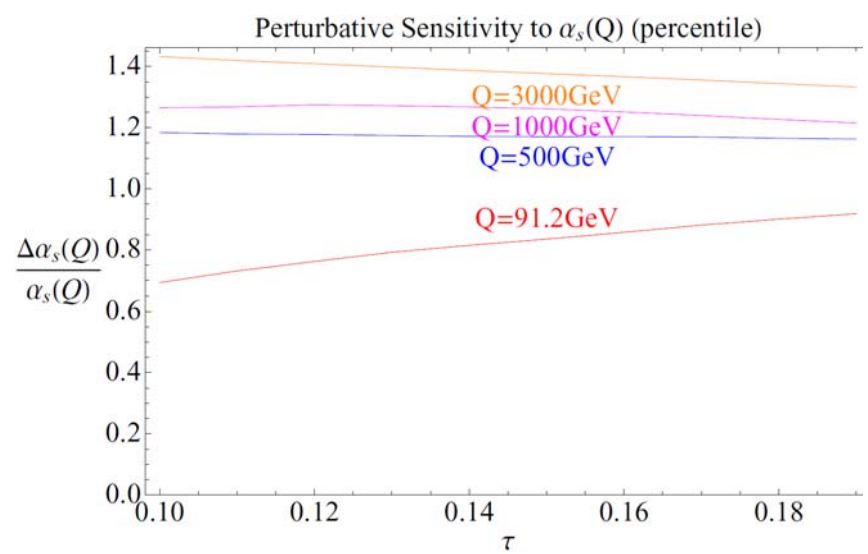
Sensitivity to the strong coupling

Here we assume that one can achieve a 2% experimental accuracy at any energy



Similar effect for perturbative uncertainties





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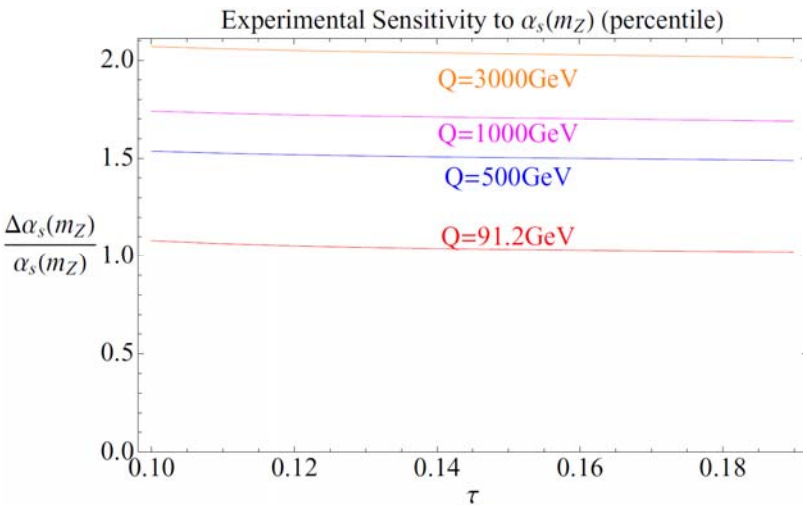

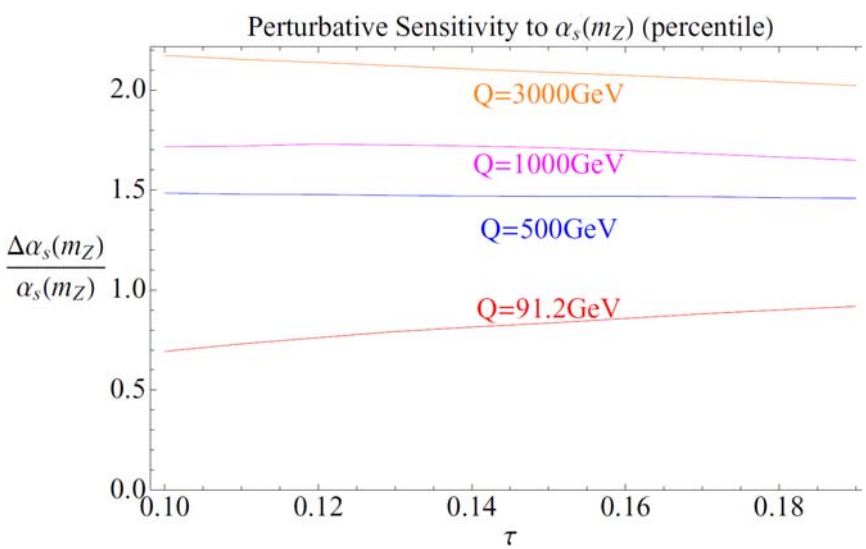

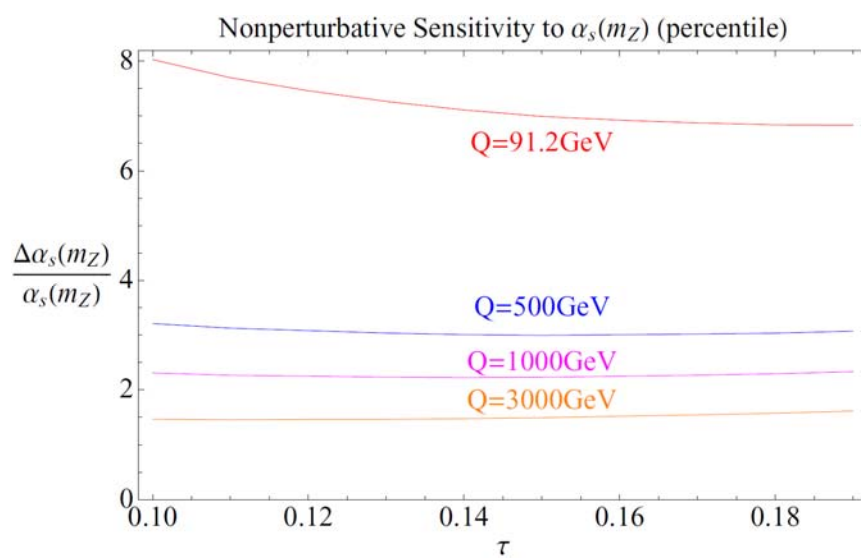

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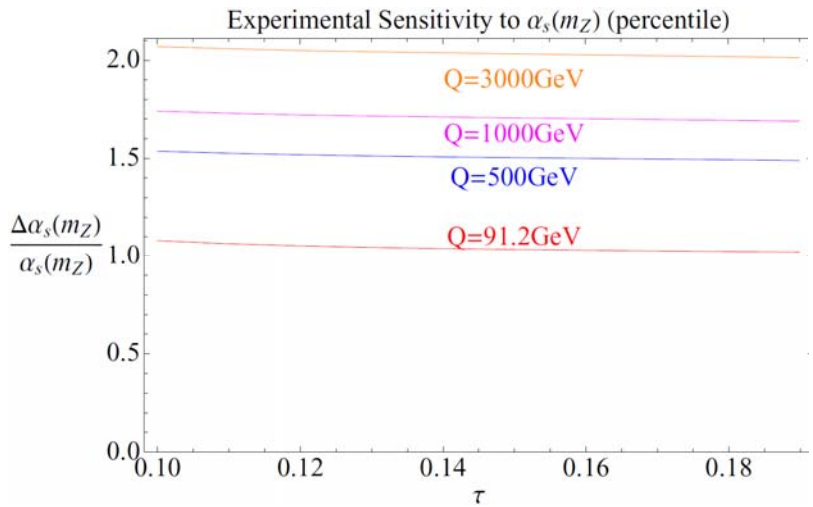
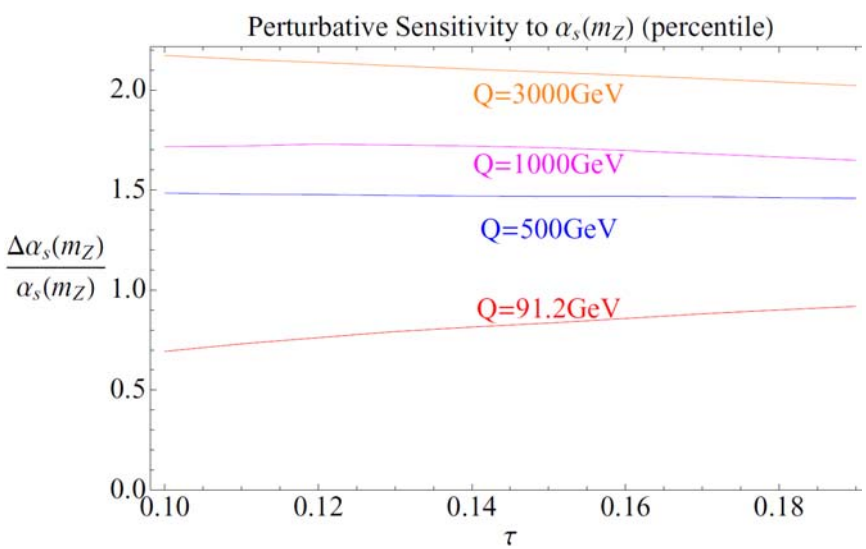
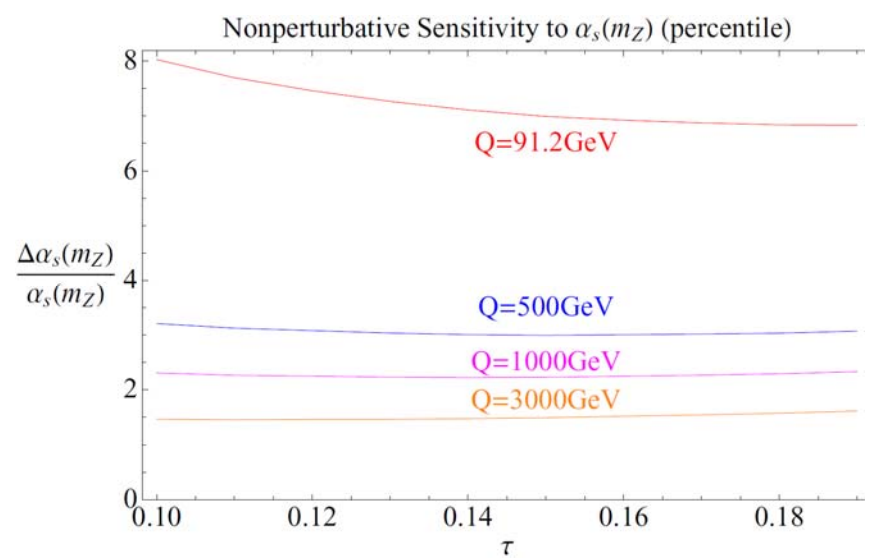


Nonperturbative effects (strong Q dependence) at CLIC energies (3TeV) are smaller than other uncertainties (mild Q dependence)

This would clarify the situation of a value of α much lower than the world average, since NP physics would have no role.





Conclusions

- The Soft-Collinear Effective Theory provides a powerful formalism for deriving factorization theorems and analyzing processes with jets.

- SCET has finally provided theorists with a mean to catch up to the experimental precision of LEP.

$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

- Global fit of all data with all Q's and all τ's .

- Field theoretical treatment of nonperturbative effects (unlike Monte Carlos)

$\longrightarrow$ main reason for a low value of α. Strong motivation to have ILC measurements at as high as possible Q.

ILC outlook

- Increase statistics as the tail regions is much broader.
- Mild decrease of sensitivity at high energies.
- Possibility to complete ignore nonperturbative effects.
- Clarify the situation for the determination of α.

The future for high precision determinations of the strong coupling constant looks good!