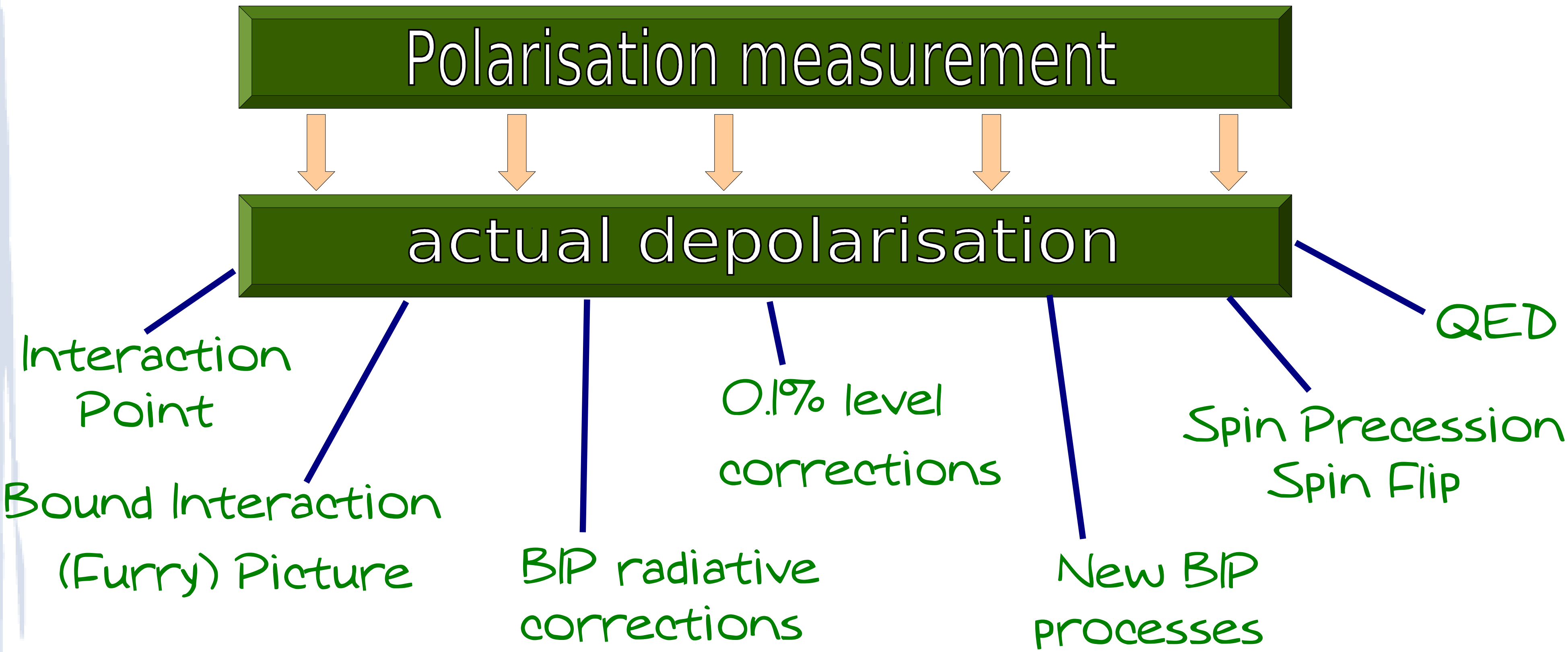
Strong field physics in beam-beam interactions at a linear colliders

Tony Hartin, IWLC2010, 20 October 2010



Motivation I – IP depolarisation

There is depolarization (spin flip) due to the QED process of Beamsstrahlung, given by the Sokolov-Ternov equation

$$\frac{dW}{d\omega_f} = \frac{\alpha m}{\sqrt{3}\pi\gamma^2}\int_z^\infty K_{5/3}(x)\,dx + \frac{y^2}{1-y}K_{2/3}(z) \;,\quad z \propto \omega_f(1-\cos\theta_f)$$

Generally, the simulation programs ignore the dependence on the radiation angle, and smear it withing the radiation cone

The fermion spin can also precess in the bunch fields. Equation of motion of the spin given by the T-BMT equation

$$\frac{d\vec{S}}{dt} = -\frac{e}{m\gamma}\left[(\gamma a+1)\vec{B}_T + (\gamma a+1)\vec{B}_L - \gamma/c^2\left(a+\frac{1}{\gamma+1}\right)\vec{v}\,x\,\vec{E}\right]x\,\vec{S}$$

At the IP, the anomalous magnetic moment has to be calculated using exact solutions in the strong bunch fields – in the Bound Interaction (Furry) Picture

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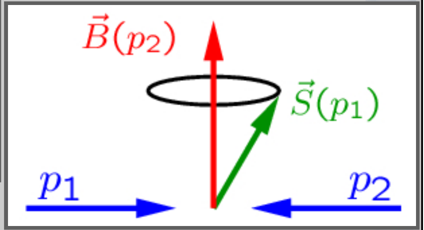
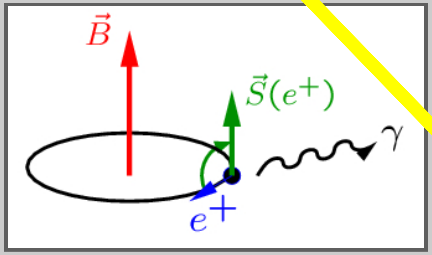
Classical spin precession in inhomogeneous external fields: T-BMT equation



Depol sims with CLIC parameters (I Bailey) change in polarization vector magnitude

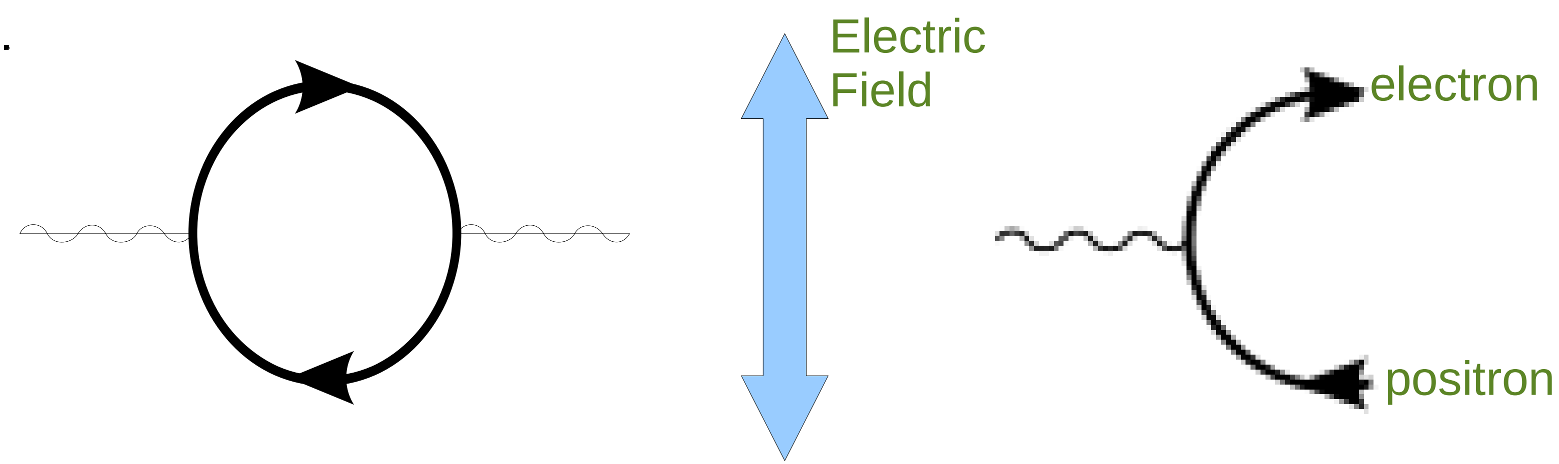
	CLIC-G	ILC nom	ILC (80/30%)
T-BMT	0.10%	0.17%	0.14%
Beamstr.	3.40%	0.05%	0.03%
incoherent	0.06%	0.00%	0.00%
coherent	1.30%	0.00%	0.00%
total	**4.80%**	**0.22%**	**0.17%**



Stochastic spin diffusion from photon emission: Sokolov-Ternov effect, etc.

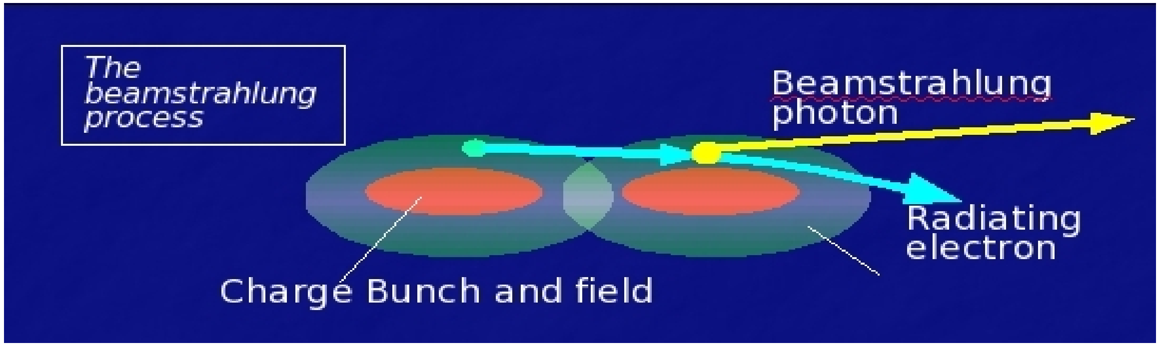
Motivation II – Vacuum Polarisation

Within $\Delta t = \frac{\hbar}{m} c^2$ a field strong enough to separate a virtual pair by a compton wavelength. Threshold field is the Schwinger critical field E_c (1.3x10[16] V/cm)



- First observed at SLAC E144 in the late 1990's. 46 Gev e- beam collides with laser 1.4x10[19] W/cm
- Linear Collider: field associated with charge bunches at the point of collision are $0.13\,E_c$ (ILC SB2009) and $2.6\,E_c$ (CLIC) leads to coherent pair production (2x10[7] per bunch crossing at CLIC)

Calculation methodologies



- fermions ϵ bend in the field of oncoming bunch and radiate a photon ω_f
- Method of calculation specified by energy ratio $\chi \approx \frac{\omega_f}{\epsilon}$ and field ratio $\Upsilon = \frac{E}{E_c}$

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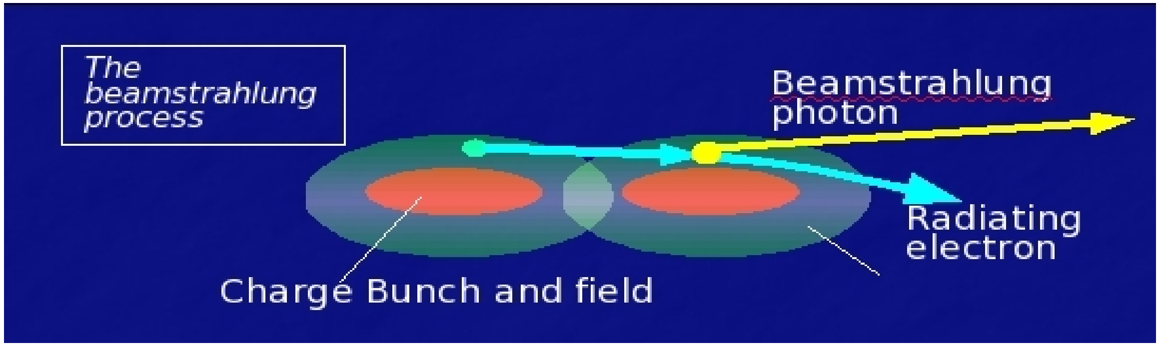

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- $\chi \ll 1, \Upsilon \ll 1$ **Classical electrodynamics**. Radiation within cone $\gamma = \epsilon / m$
- $\chi \ll 1$ **Quasi-classical method** of Baier-Katkov. Quantum interaction between fermion and photon but fermion dynamical variables commute
- $\Upsilon \ll 1$ 1[st] **Born approximation**. Treat bunch field as contributing a single photon to the interaction (a Compton interaction
- *any* χ, Υ **Semi-classical method**. Exact interaction with a classical potential in the Bound Interaction Picture (BIP)

Bound Interaction Picture

$$L_{BIP} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}_V(i\gamma^\mu\partial_\mu + eA^e - m)\psi_V + e\bar{\psi}_V A\psi_V$$

L_M , L_{BD} , L_I

The External classical potential

- Motivated originally by the "Dyson Dilemma" - nonzero radius of convergence in the QED perturbation series. The effect of the external field is treated 'exactly'

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- Exact solution of the Bound Dirac equation for plane wave electromagnetic fields, [Volkov 1935, Bagrov, Gitman et al 1970s]

$$\left[1 + \frac{e}{2(k\cdot p)}\not{k}\not{A}^e\right]\exp\left(-i\int_0^{kx}\left[\frac{e(A^e\cdot p)}{(k\cdot p)} - \frac{e^2A^{e2}}{2(k\cdot p)}\right]d\phi\right)\exp(-ip\cdot x)\,u_s(p)$$

Spin dependent part including a magnetic moment interaction

An additional phase factor

The usual free fermion part

Strong field physics processes I

New odd-vertex processes

- Beamstrahlung
- Coherent pair production
- Trident production
- Photon absorption
- Photon splitting

(Beamstrahlung, Coherent pair production, Trident production:)
- Partially simulated
- Spin effects incomplete

(Photon absorption, Photon splitting →) Not yet implemented

Modified processes

- AMM in external field
- Resonant Compton scattering
- Self energies in external field
- Vertex Correction in ext. field

(AMM in external field →) Implemented, decreases as field intensity increases

(Resonant Compton scattering, Self energies in external field, Vertex Correction in ext. field:)
- Some Analytic forms available
- Not simulated

Strong field physics processes II

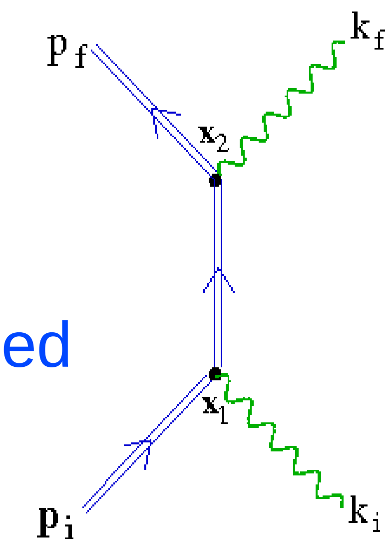
Resonant Compton Scattering

- Modified conservation of momentum

$$\int ds \;\; \delta(p_i + k_i + sk - p_f - k_f)$$

- Mass shell condition for the propagator modified

$$\frac{1}{2(k_i p_i) - 2s(k\, p_i)} \approx \frac{1}{4\,\epsilon_i(\omega_i - s\,\omega)} \quad \textit{instead of} \quad \frac{1}{2(k_i p_i)} \approx \frac{1}{4\,\omega_i \epsilon_i}$$



Strong field physics processes II

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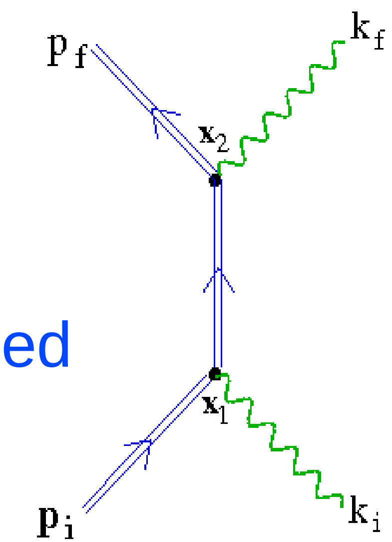

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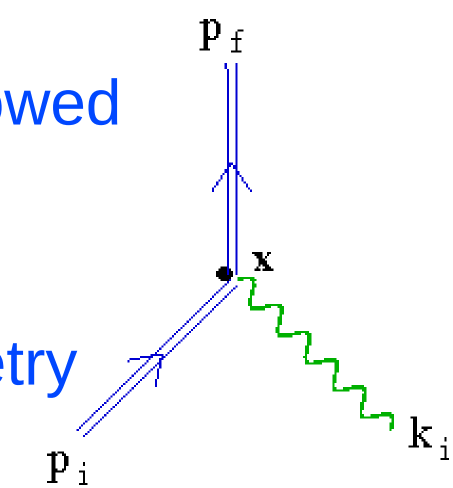
1 Vertex Photon Absorption



- In Furry Picture **all** 1 vertex processes are allowed
- Large flux of oncoming real photons
  (2.2 per electron for 3 TeV CLIC)
- Related to beamstrahlung via crossing symmetry
- Need to analyse w.r.t. kinematics

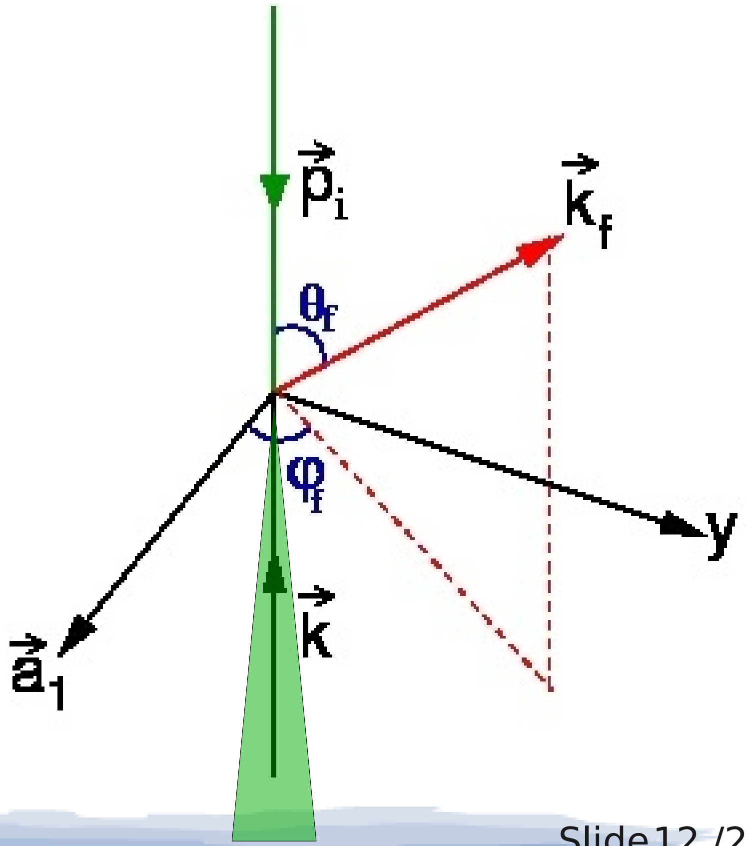
Beamstrahlung radiation angle

- The beamstrahlung (Sokolov-Ternov) equation is written in terms of McDonald's functions

$$\frac{dW}{du} = \frac{\alpha m^2}{\pi\sqrt{3}\epsilon_i}\frac{1}{(1+u)^2}\left[\int_\chi^\infty K_{5/3}(y)dy - \frac{u^2}{1+u}K_{2/3}(\chi)\right]$$

$$\chi, u \propto \omega\,\omega_f(1-\cos\theta_f)$$

  - Radiation in the forward direction as expected



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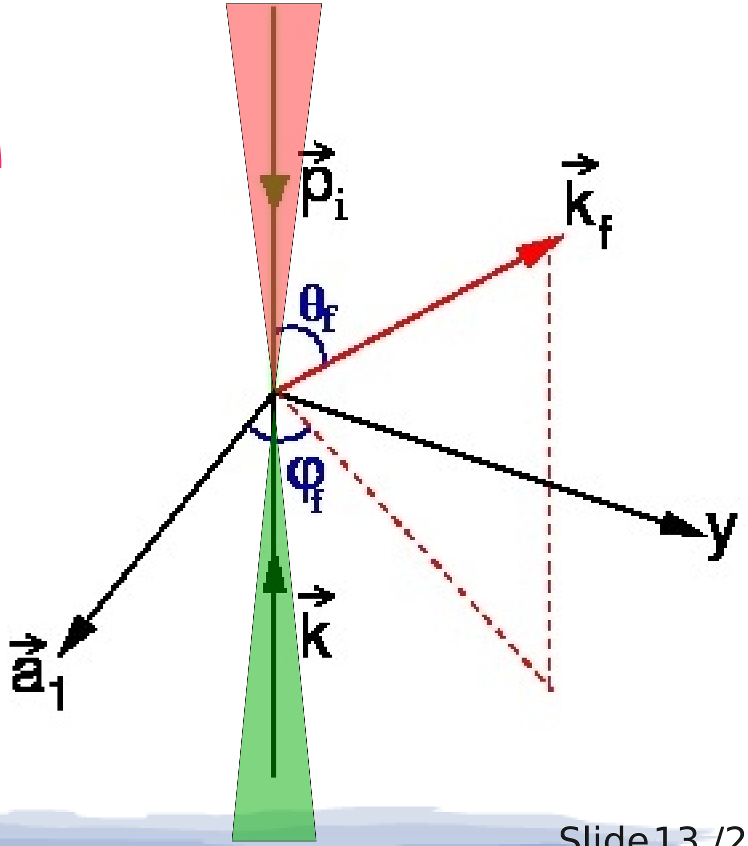
  - dW/du divergent for vanishing u i.e. for IR and radiation in the backwards direction!



Beamstrahlung transition probability

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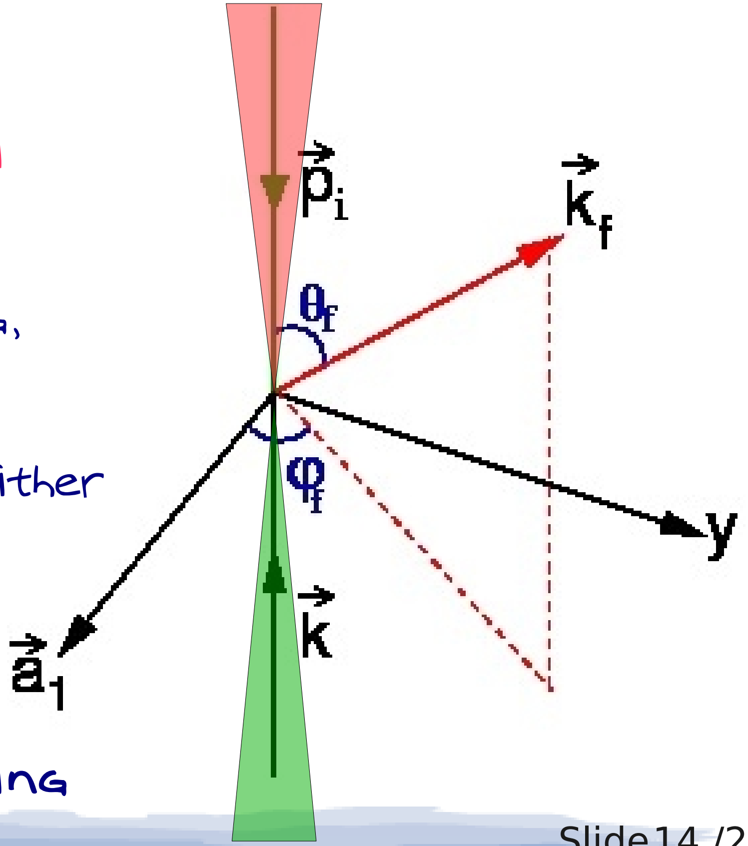
- Radiation in the forward direction as expected
- dW/du divergent for vanishing u i.e. for IR and radiation in the backwards direction!
- Beamstrahlung simulated in CAIN and Guineapig, however....
- No rigorous treatment of radiation angle – either no radiation angle or smeared randomly within expected radiation cone
- Divergences avoided by Monte Carlo
- **Need to address this for precision tracking**



State of the Loops - BIP Radiative corrections

- Electron Self Energy (Ritus 1971)
  - Spin dependent part of helicity amplitude gives AMM in external field

Self Energy

$$\int_0^\infty \frac{du}{(1+u)^2} \left\{ f_1(z) + \frac{2+2u+u^2}{z(1+u)} f'(z) - 2\gamma \frac{zf(z)}{1+u} \right\}$$

Beamstrahlung

$$\int_0^\infty \frac{du}{(1+u)^2} \left\{ \Phi_1(z) + \frac{2+2u+u^2}{z(1+u)} \Phi'(z) - 2\gamma \frac{z\Phi(z)}{1+u} \right\}$$

- Function $\Phi(z)$ (Airy function) is real part of $f(z) = \int_0^\infty \exp(izt + it^3/3)\,dt$
- Analytic properties largely depend on the functions of z
- Needs dimensional Regularization and cancellation of IR divergence

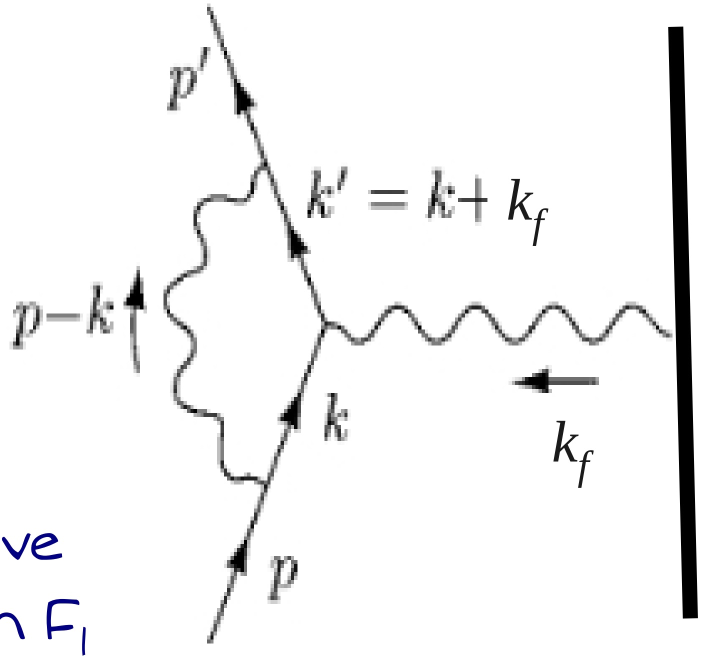
IR divergence removal

via the Vertex Function

$$\Gamma^{\mu} = ie\gamma^{\mu} F_1 + \frac{i\sigma^{\mu\nu} k_{f\nu}}{2m} F_2$$

Correction to the vertex

Correction to the magnetic moment



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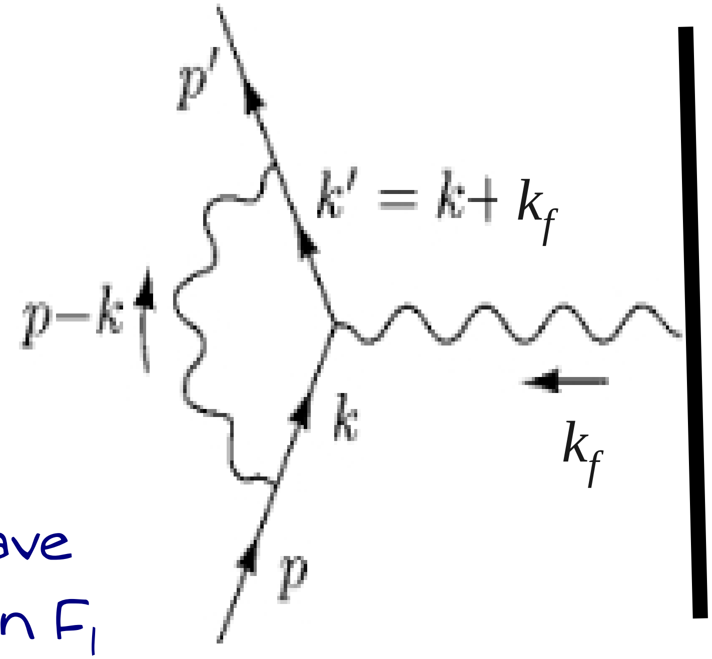
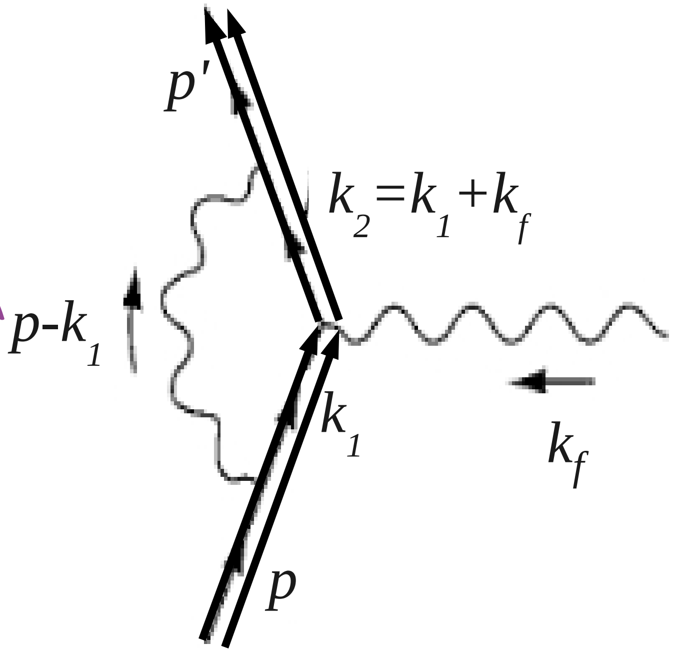


- K_f is a virtual photon
- The vertex function is constrained to have 2 terms, the divergent vertex correction F_1 and the Anomalous Magnetic Moment, F_2
- Regularise F_1 to isolate the UV and IR divergences
- IR divergence is isolated in a logarithm containing a cutoff and cancelled with the IR divergence from soft Bremstrahlung

Vertex function in external Field I

- Fermion lines replaced with exact external field (Volkov) solutions: 256 terms in the numerator, different tensor structure
- External field photons contribute at each vertex: 3 extra integrations (+ 3 Feynman parameters + 3 propagators)
- In the collinear limit, tensor structure is familiar

$$\Gamma^{\mu} = ie\gamma^{\mu} F_1^e + \frac{i\sigma^{\mu\nu} k_{f\nu}}{2m} F_2^e$$



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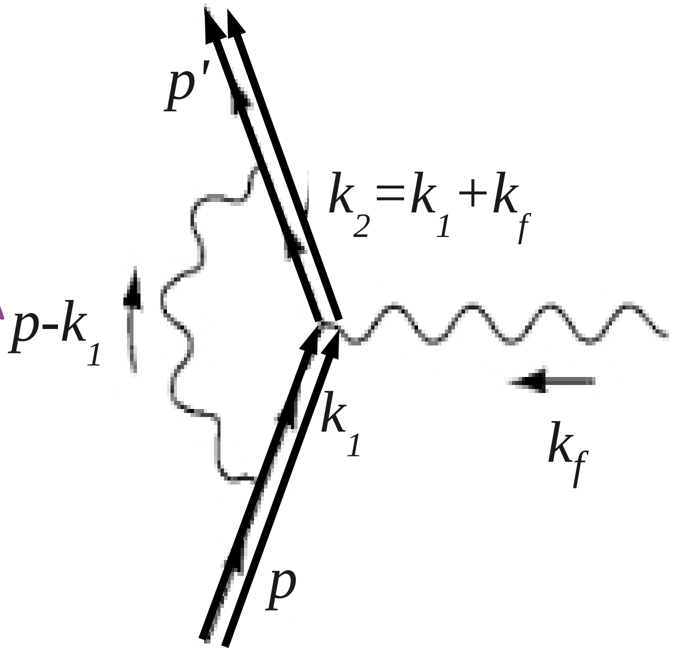

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Analytic simplification:

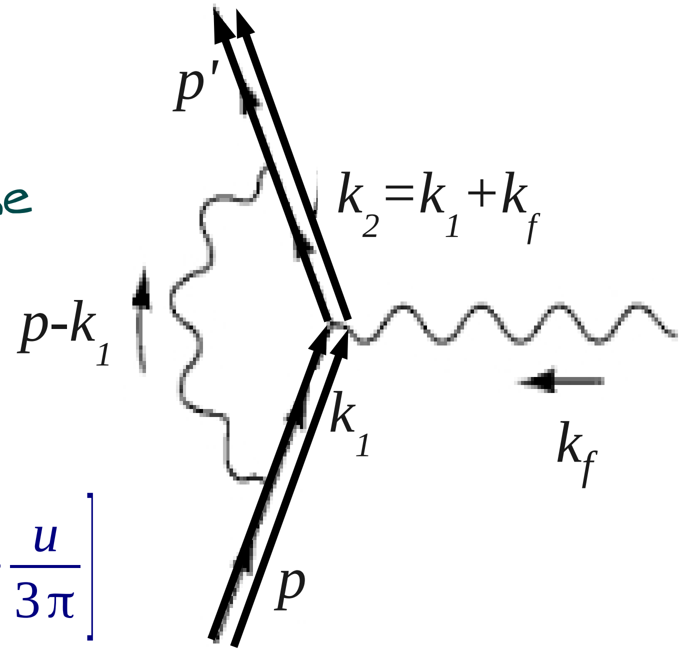
- Need to deal with the extra Volkov phases
- Find analytic solutions to integrals of functions of Airy functions

$$\int ds\, dl\, d^4 k_1 \frac{Ai(s)\, Ai(s-l)}{[-k_1^2 + x_1 s + x_2 + i\epsilon]^3} \exp(i x_3 l) \rightarrow f(z)$$

Same f(z) appearing in self energy

Vertex function in external field II

$$F_2^e \propto \int \frac{z\,du}{(1+u)^3} f(z) \quad , \quad z=(u/2\Upsilon)^{2/3}$$



F^e_2 is the same as the spin-dependent part of the Self Energy in the external field Ritus(1971), Baier(1975) – **Good Crosscheck!**

$$F_1^e \propto \int \frac{du}{(1+u)^3}\left[2u\int f(z)\,dz + z\left(4\frac{1+u}{u}-\frac{5}{3}u\right)f(z) - \frac{u}{3\pi}\right]$$

- F^e_1 has no regularisation yet – Ritus seems to suggest that there remains a UV divergence with respect to the interaction with the vacuum therefore look to isolate an external field-free part

- Need to implement numerically to see what the correction to the Beamstrahlung is

Simulation status vis a vis spin tracking

Feature	CAIN 2.35	Guinea-Pig	Guinea-Pig++
Spin precession	✔	Structure present	✔
Spin Flip	✔		✔
Radiation angle	none	Smeared within γ cone	
Pair backgrounds	Some/all spin components	none	none
Radiative corrections	none	none	none
Higher orders	none	none	none

Highly Desireable: to implement fully in one code and preferably two for cross-check

SpinToolsWS DESY Hamburg 9-11 Nov

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- The analytic form of the two form factors was obtained. The AMM term is the same as that in the literature obtained by other methods

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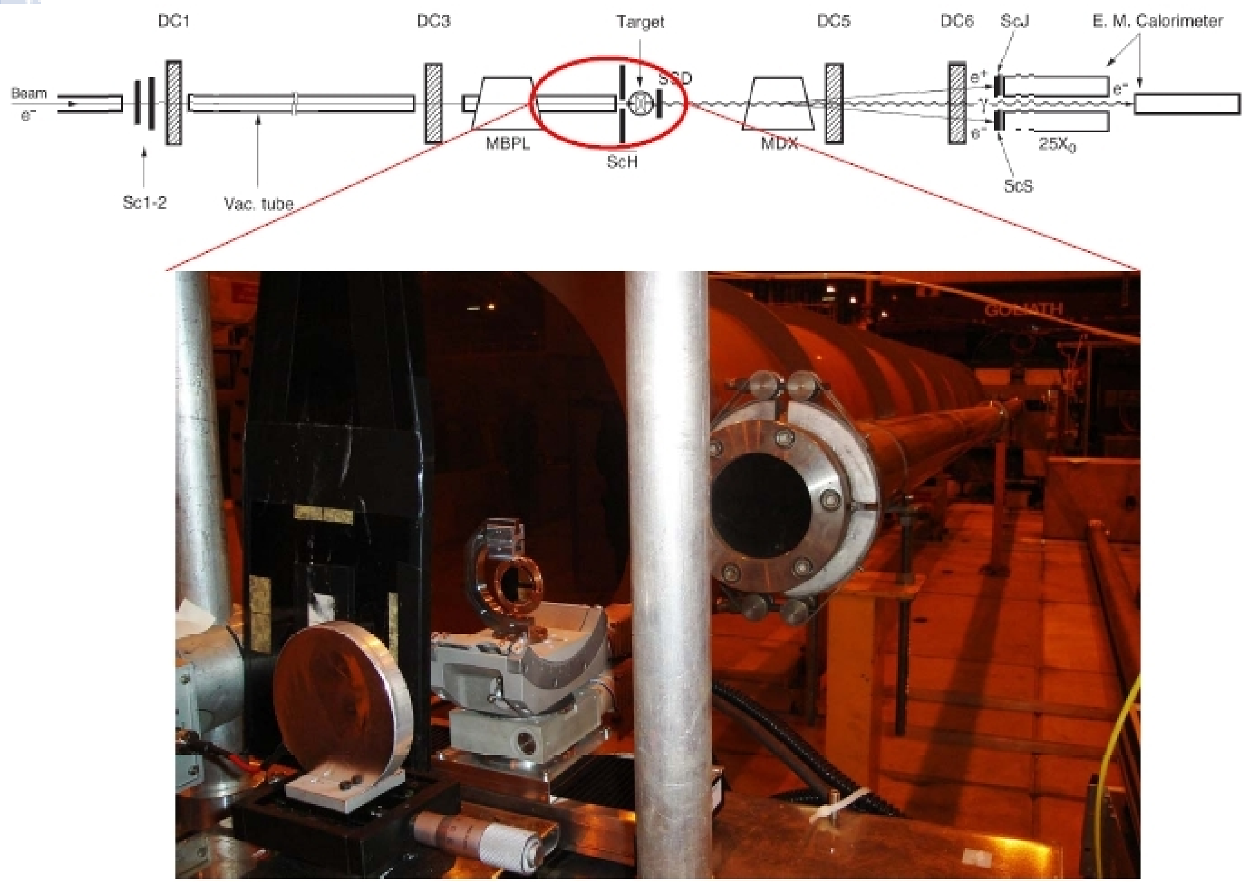
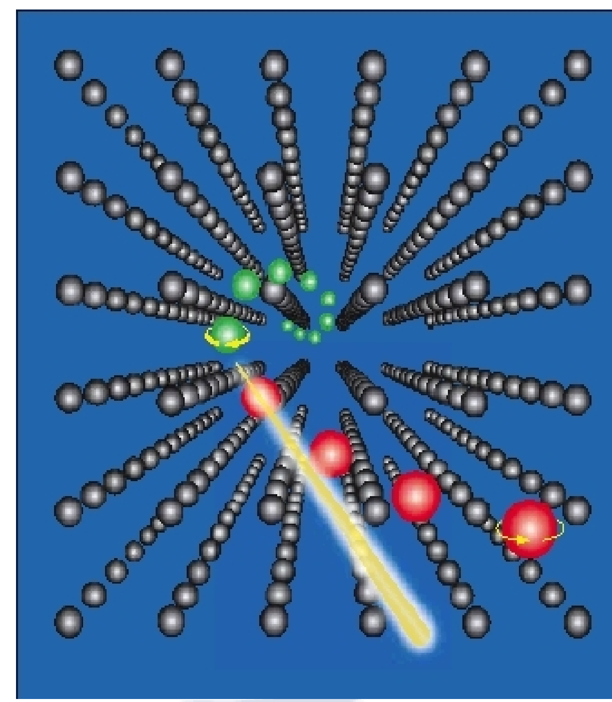
- The analytic form of the two form factors was obtained. The AMM term is the same as that in the literature obtained by other methods
- Regularisation/Renormalisation still to be perfomed
- Numerical simulations to follow

Strong fields in crystals (NA63)

- fields are of the same order and type of those in bunch-bunch collisions
- Can also use polarized beams to study





Other experimental tests

- Mass shift cause by averaged motion in the external field
- Surface of magnetars
- Somnoluminescence
- XFEL – strong laser fields
- See article by McDonald