Renormalization in the Complex MSSM and ILC Implications

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based on collaboration with
T. Fritzsche, H. Rzehak, C. Schappacher, G. Weiglein

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2. Renormalization schemes
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1. Introduction

$\tilde{t}/\tilde{b}$ sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ($X_t = A_t - \mu^*/\tan\beta$, $X_b = A_b - \mu^* \tan\beta$):

$$\mathbf{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathbf{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

soft SUSY-breaking parameters A_t, A_b also appear in ϕ-$\tilde{t}/\tilde{b}$ couplings

$$\boxed{SU(2) \text{ relation} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}}$$

$\Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

Effects of complex parameters in the Higgs sector:

Complex parameters enter via loop corrections:

– μ : Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings $\Rightarrow X_{t,b,\tau} = A_{t,b} - \mu^* \{\cot\beta, \tan\beta\}$ complex

– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– $m_{\tilde{g}}$: gluino mass

$\Rightarrow$ can induce $\mathcal{CP}$-violating effects

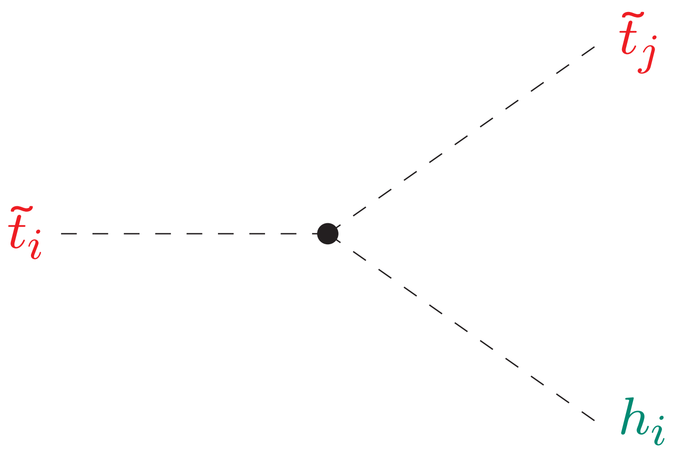
Result:

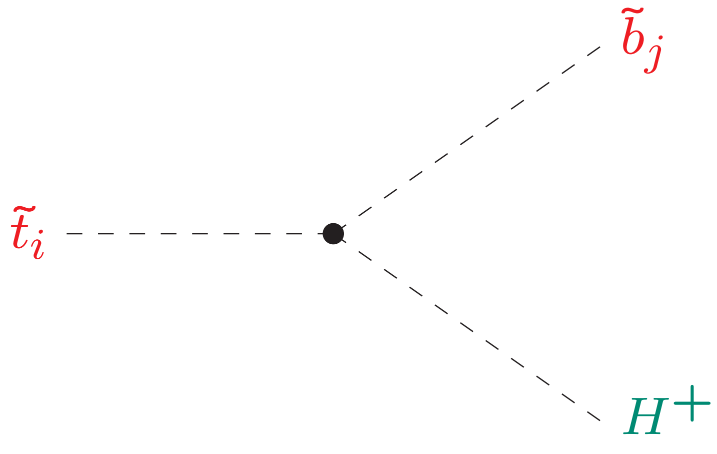
$$(A, H, h) \to (h_3, h_2, h_1 (= \phi))$$

with

$$M_{h_3} > M_{h_2} > M_{h_1}$$

Examples for processes with (external) stops and Higgs bosons:





- important decay modes of stops
- A_t and A_b directly enter the vertex
- possible source of Higgs bosons at the LHC/ILC
- . . .

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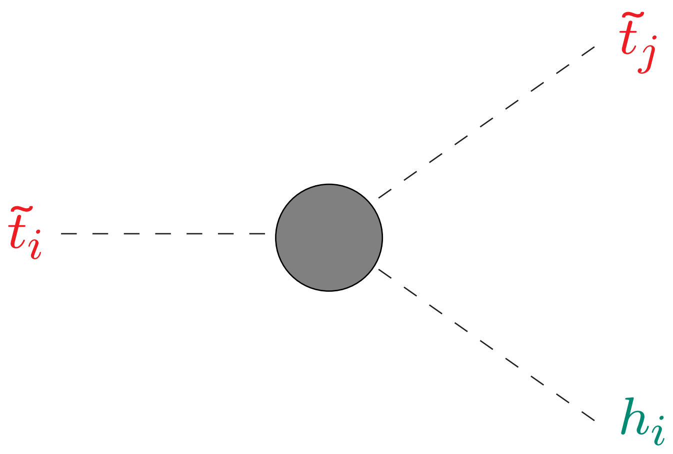

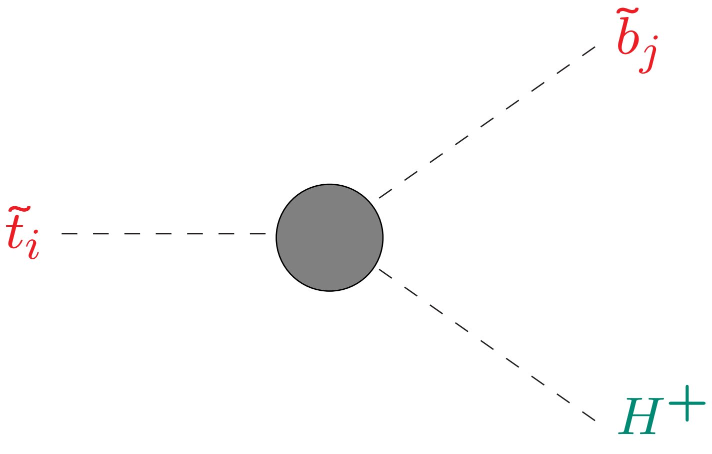

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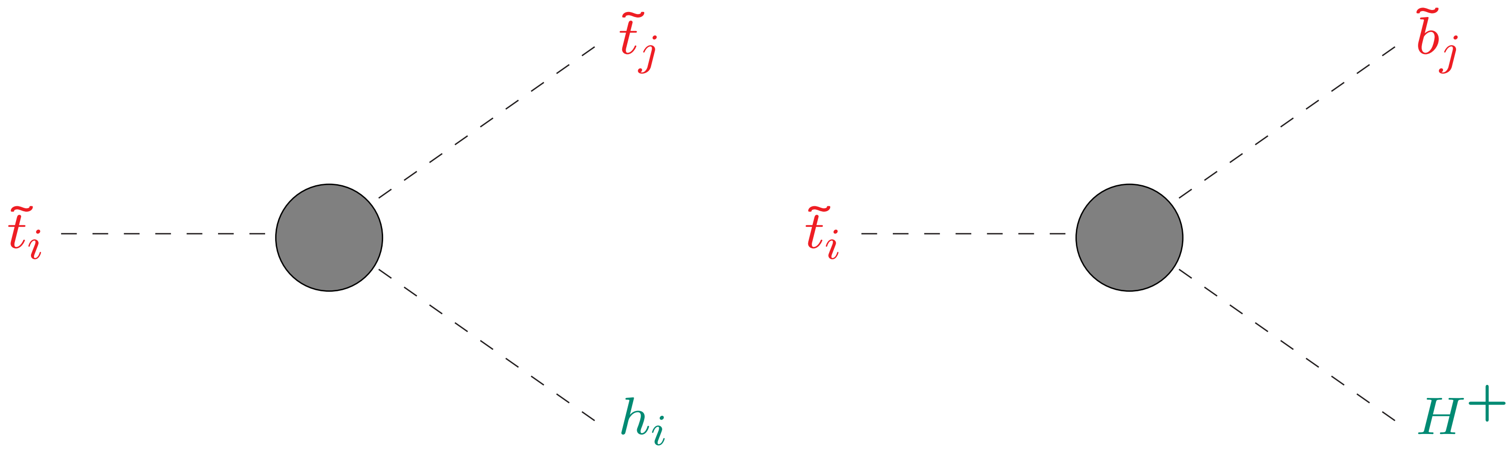

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⇒ simultaneous renormalization of stop and sbottom sector required!

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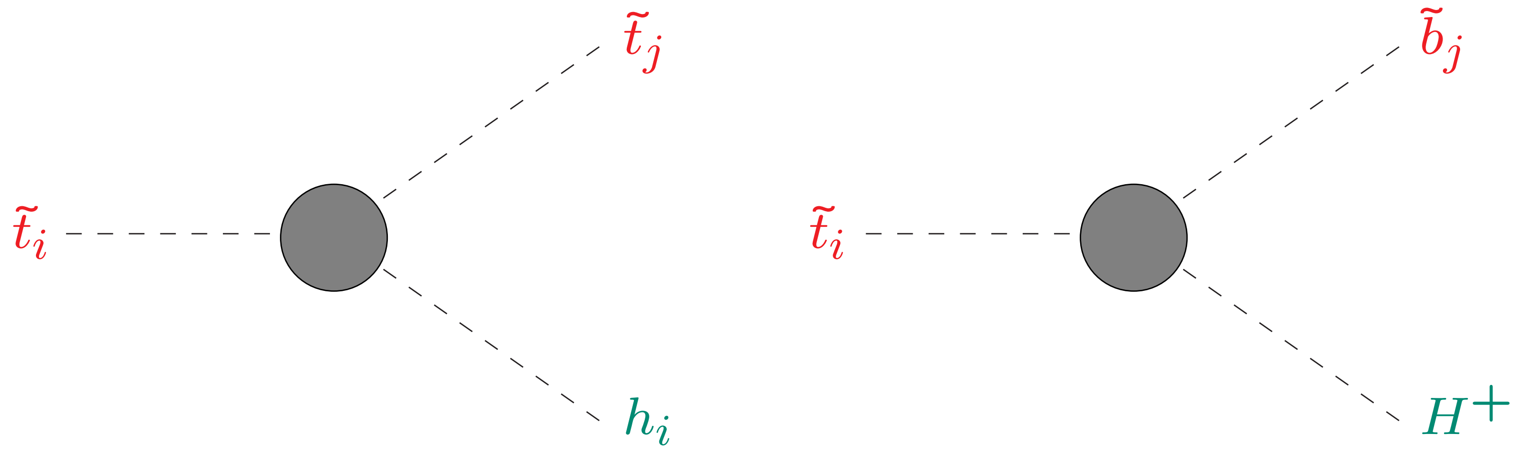

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– A_t and A_b directly enter the vertex
– possible source of Higgs bosons at the LHC/ILC
– . . .

⇒ higher-order corrections important!

⇒ simultaneous renormalization of stop and sbottom sector required!
⇒ with on-shell properties for external particles!

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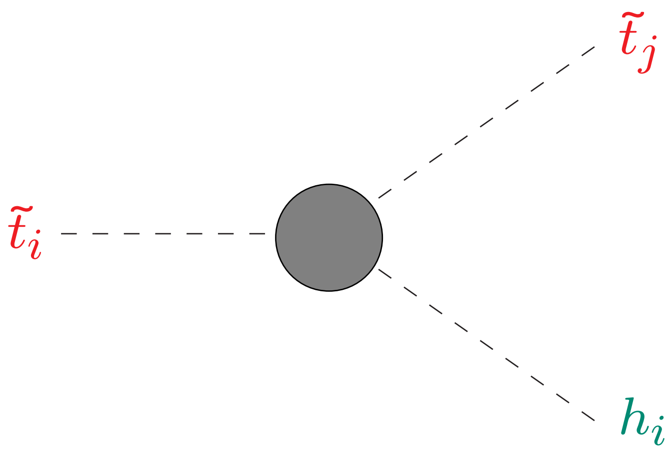
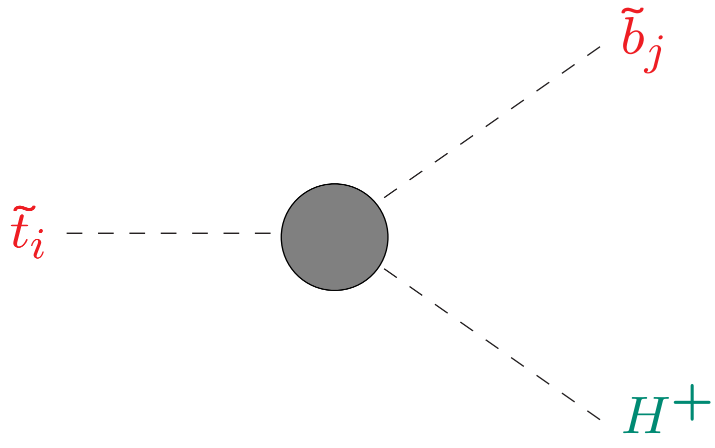


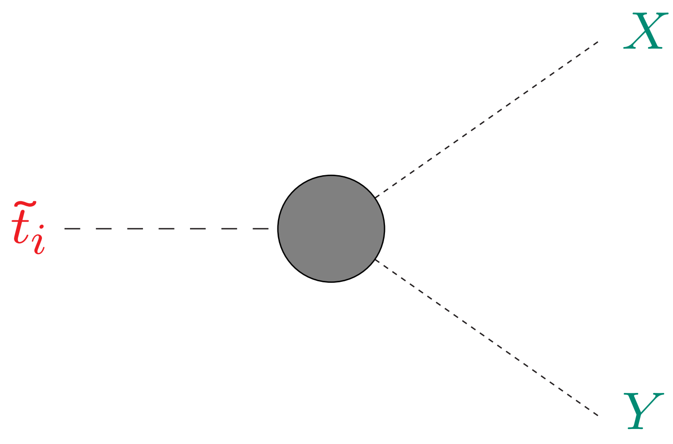


– important decay modes of stops
– A_t and A_b directly enter the vertex incl. complex phases!
– possible source of Higgs bosons at the LHC/ILC
– ...

⇒ higher-order corrections important!

⇒ simultaneous renormalization of stop and sbottom sector required!
⇒ including complex phases!

The bigger picture: stop decays in the cMSSM

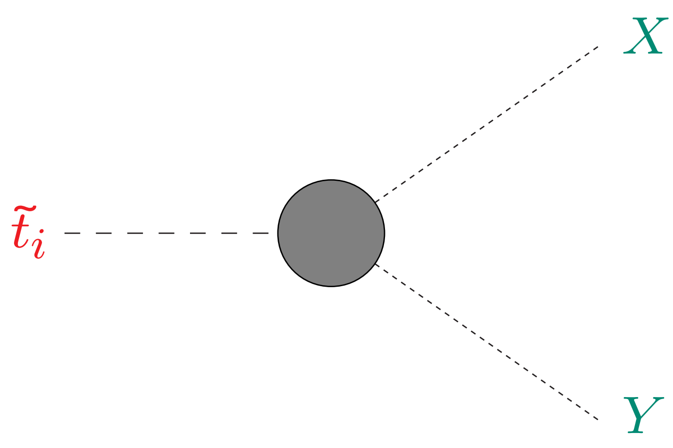


$\Rightarrow$ to get BRs right $\Rightarrow$ all decays needed

$\Rightarrow$ (nearly) all sectors of the cMSSM enter as external particles

$\Rightarrow$ (nearly) all sectors of the cMSSM have to be renormalized simultaneously

The bigger picture: stop decays in the cMSSM



⇒ to get BRs right ⇒ all decays needed

⇒ (nearly) all sectors of the cMSSM enter as external particles

⇒ (nearly) all sectors of the cMSSM have to be renormalized simultaneously

⇒ nearly ready

⇒ focus here on stop/sbottom sector

2. Renormalization schemes

Generic parameter and field renormalization for scalar quarks:

$$\mathbf{D}_{\tilde{q}} = \mathbf{U}_{\tilde{q}} \mathbf{M}_{\tilde{q}} \mathbf{U}_{\tilde{q}}^{\dagger} \quad (\tilde{q} = \tilde{t}, \tilde{b})$$

$$\mathbf{U}_{\tilde{q}} \mathbf{M}_{\tilde{q}} \mathbf{U}_{\tilde{q}}^{\dagger} \to \mathbf{U}_{\tilde{q}} \mathbf{M}_{\tilde{q}} \mathbf{U}_{\tilde{q}}^{\dagger} + \mathbf{U}_{\tilde{q}} \delta\mathbf{M}_{\tilde{q}} \mathbf{U}_{\tilde{q}}^{\dagger} = \begin{pmatrix} m_{\tilde{q}_1}^2 & Y_q \\ Y_q^* & m_{\tilde{q}_2}^2 \end{pmatrix} + \begin{pmatrix} \delta m_{\tilde{q}_1}^2 & \delta Y_q \\ \delta Y_q^* & \delta m_{\tilde{q}_2}^2 \end{pmatrix}$$

$$\delta\mathbf{M}_{\tilde{q}_{12}} = U_{\tilde{q}_{11}}^* U_{\tilde{q}_{12}} (\delta m_{\tilde{q}_1}^2 - \delta m_{\tilde{q}_2}^2) + U_{\tilde{q}_{11}}^* U_{\tilde{q}_{22}} \delta Y_q + U_{\tilde{q}_{12}} U_{\tilde{q}_{21}}^* \delta Y_q^*$$

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} \to \left(1\!\!1 + \frac{1}{2} \delta\mathbf{Z}_{\tilde{q}} \right) \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} \quad \text{with} \quad \delta\mathbf{Z}_{\tilde{q}} = \begin{pmatrix} \delta Z_{\tilde{q}_{11}} & \delta Z_{\tilde{q}_{12}} \\ \delta Z_{\tilde{q}_{21}} & \delta Z_{\tilde{q}_{22}} \end{pmatrix}$$

Renormalization of the $t/\tilde{t}$ sector

$\longrightarrow$ employ the widely used on-shell renormalization

$$\delta m_t = \frac{1}{2}\widetilde{\mathrm{Re}}\left\{m_t\left[\Sigma_t^L(m_t^2) + \Sigma_t^R(m_t^2)\right] + \left[\Sigma_t^{SL}(m_t^2) + \Sigma_t^{SR}(m_t^2)\right]\right\}$$

$$\delta m_{\tilde{t}_i}^2 = \widetilde{\mathrm{Re}}\Sigma_{\tilde{t}_{ii}}(m_{\tilde{t}_i}^2) \qquad (i = 1, 2)$$

$$\delta Y_t = \frac{1}{2}\widetilde{\mathrm{Re}}\left\{\Sigma_{\tilde{t}_{12}}(m_{\tilde{t}_1}^2) + \Sigma_{\tilde{t}_{12}}(m_{\tilde{t}_2}^2)\right\}$$

[*W. Hollik, H. Rzehak '03*]

This defines the counter term for A_t:

$$\delta A_t = \frac{1}{m_t}\Big[U_{\tilde{t}_{11}}U^*_{\tilde{t}_{12}}(\delta m_{\tilde{t}_1}^2 - \delta m_{\tilde{t}_2}^2) + U_{\tilde{t}_{11}}U^*_{\tilde{t}_{22}}\delta Y_t^* + U^*_{\tilde{t}_{12}}U_{\tilde{t}_{21}}\delta Y_t - (A_t - \mu^* \cot\beta)\,\delta m_t\Big] + (\delta\mu^* \cot\beta - \mu^* \cot^2\beta\,\delta\tan\beta)$$

(with $\delta\mu$ from chargino/neutralino sector, $\delta\tan\beta$ from Higgs sector)

Field renormalization for on-shell squarks ($\tilde{t}$, $\tilde{b}$, . . .):

Diagonal Z factors:

the real part of the residua of propagators is set to unity:

$$\widetilde{\text{Re}}\frac{\partial\hat{\Sigma}_{\tilde{q}_{ii}}(k^2)}{\partial k^2}\Big|_{k^2=m^2_{\tilde{q}_i}} = 0 \qquad (i = 1, 2)$$

yielding

$$\text{Re}\delta Z_{\tilde{q}_{ii}} = -\widetilde{\text{Re}}\frac{\partial\Sigma_{\tilde{q}_{ii}}(k^2)}{\partial k^2}\Big|_{k^2=m^2_{\tilde{q}_i}} \qquad \text{Im}\,\delta Z_{\tilde{q}_{ii}} = 0 \qquad (i = 1, 2)$$

Off-diagonal Z factors:

no transition from one squark to the other occurs:

$$\widetilde{\text{Re}}\hat{\Sigma}_{\tilde{q}_{12}}(m^2_{\tilde{q}_1}) = 0 \qquad \widetilde{\text{Re}}\hat{\Sigma}_{\tilde{q}_{12}}(m^2_{\tilde{q}_2}) = 0$$

yielding

$$\delta Z_{\tilde{q}_{12}} = +2\frac{\widetilde{\text{Re}}\Sigma_{\tilde{q}_{12}}(m^2_{\tilde{q}_2}) - \delta Y_q}{(m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2})} \qquad \delta Z_{\tilde{q}_{21}} = -2\frac{\widetilde{\text{Re}}\Sigma_{\tilde{q}_{21}}(m^2_{\tilde{q}_1}) - \delta Y^*_q}{(m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2})}$$

$$\boxed{SU(2) \text{ relation} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}}$$

"LL" soft SUSY-breaking term for $\tilde{q} = \{\tilde{t}, \tilde{b}\}$:

$$M^2_{\tilde{Q}_L}(\tilde{q}) = |U_{\tilde{q}11}|^2 m^2_{\tilde{q}_1} + |U_{\tilde{q}12}|^2 m^2_{\tilde{q}_2} - M_Z^2 c_{2\beta}(T_q^3 - Q_q s_\mathrm{W}^2) - m_q^2$$

Keeping $SU(2)$ relation at the one-loop level leads to a shift in the soft SUSY-breaking parameters

[*A. Bartl, H. Eberl, K. Hidaka, S. Kraml, W. Majerotto, W. Porod, Y. Yamada '97, '98*]

[*A. Djouadi, P. Gambino, S.H., W. Hollik, C. Jünger, G. Weiglein '98*]

$$M^2_{\tilde{Q}_L}(\tilde{b}) = M^2_{\tilde{Q}_L}(\tilde{t}) + \delta M^2_{\tilde{Q}_L}(\tilde{t}) - \delta M^2_{\tilde{Q}_L}(\tilde{b})$$

with

$$\delta M^2_{\tilde{Q}_L}(\tilde{q}) = |U_{\tilde{q}11}|^2 \delta m^2_{\tilde{q}_1} + |U_{\tilde{q}12}|^2 \delta m^2_{\tilde{q}_2} - U_{\tilde{q}22} U^*_{\tilde{q}12} \delta Y_q - U_{\tilde{q}12} U^*_{\tilde{q}22} \delta Y_q^* - 2 m_q \delta m_q$$
$$+ M_Z^2\, c_{2\beta}\, Q_q\, \delta s_\mathrm{W}^2 - (T_q^3 - Q_q s_\mathrm{W}^2)(c_{2\beta}\, \delta M_Z^2 + M_Z^2\, \delta c_{2\beta})$$

$\longrightarrow$ under control

Renormalizations of the $b/\tilde{b}$ sector in the complex MSSM:

scheme	$m_{\tilde{b}_{1,2}}$	m_b	A_b	Y_b	name
analogous to the $t/\tilde{t}$ sector: "OS"	OS	OS	—	OS	RS1
"m_b, A_b $\overline{\text{DR}}$"	OS	$\overline{\text{DR}}$	$\overline{\text{DR}}$	—	RS2
"m_b, Y_b $\overline{\text{DR}}$"	OS	$\overline{\text{DR}}$	—	$\overline{\text{DR}}$	RS3
"m_b $\overline{\text{DR}}$, Y_b OS"	OS	$\overline{\text{DR}}$	—	OS	RS4
"A_b $\overline{\text{DR}}$, ReY_b OS"	OS	—	$\overline{\text{DR}}$	ReY_b: OS	RS5
"A_b vertex, ReY_b OS"	OS	—	vertex	ReY_b: OS	RS6

"— " = dependent parameter

$\Rightarrow$ often very involved analytical dependences

$\rightarrow$ more combinations possible

. . . also tested

. . . upcoming results remain unchanged

Renormalization of the sbottom masses:

OS renormalization:

$$\delta m^2_{\tilde{b}_i} = \widetilde{\text{Re}}\Sigma_{\tilde{b}_{ii}}(m^2_{\tilde{b}_i}) \qquad (i = 1, 2)$$

Renormalization of the bottom mass:

OS renormalization:

$$\delta m_b = \frac{1}{2}\widetilde{\text{Re}}\left\{m_b\left[\Sigma_b^L(m_b^2) + \Sigma_b^R(m_b^2)\right] + \left[\Sigma_b^{SL}(m_b^2) + \Sigma_b^{SL}(m_b^2)\right]\right\}$$

$\overline{\text{DR}}$ renormalization:

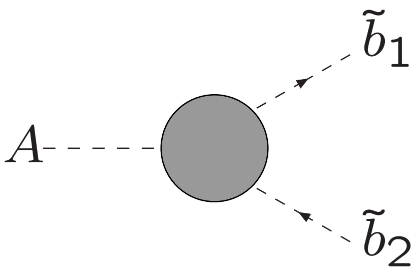
$$\delta m_b = \frac{1}{2}\widetilde{\text{Re}}\left\{m_b\left[\Sigma_b^L(m_b^2) + \Sigma_b^R(m_b^2)\right]_{\text{div}} + \left[\Sigma_b^{SL}(m_b^2) + \Sigma_b^{SL}(m_b^2)\right]_{\text{div}}\right\}$$

Renormalization of A_b:

$\overline{\text{DR}}$ renormalization: analogous to A_t:

$$\delta A_b = \frac{1}{m_b}\Big[U_{\tilde{b}_{11}}U^*_{\tilde{b}_{12}}\left(\widetilde{\text{Re}}\Sigma_{\tilde{b}_{11}}(m^2_{\tilde{b}_1})|_{\text{div}} - \widetilde{\text{Re}}\Sigma_{\tilde{b}_{22}}(m^2_{\tilde{b}_2})|_{\text{div}}\right)$$
$$+\frac{1}{2}U^*_{\tilde{b}_{12}}U_{\tilde{b}_{21}}\left(\widetilde{\text{Re}}\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_1})|_{\text{div}} + \widetilde{\text{Re}}\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_2})|_{\text{div}}\right)$$
$$+\frac{1}{2}U_{\tilde{b}_{11}}U^*_{\tilde{b}_{22}}\left(\widetilde{\text{Re}}\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_1})|_{\text{div}} + \widetilde{\text{Re}}\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_2})|_{\text{div}}\right)^*$$
$$-\frac{1}{2}(A_b - \mu^* \tan\beta)\,\widetilde{\text{Re}}\Big\{m_b\left[\Sigma^L_b(m^2_b) + \Sigma^R_b(m^2_b)\right]_{\text{div}}$$
$$+\left[\Sigma^{SL}_b(m^2_b) + \Sigma^{SR}_b(m^2_b)\right]_{\text{div}}\Big\}\Big] + \delta\mu^*|_{\text{div}}\tan\beta + \mu^*\,\delta\tan\beta$$

Vertex renormalization:



$$\widehat{=} \quad i\,\hat{\Lambda}(p^2_A, p^2_{\tilde{b}_1}, p^2_{\tilde{b}_2})$$

via $\quad \widetilde{\text{Re}}\hat{\Lambda}(0, m^2_{\tilde{b}_1}, m^2_{\tilde{b}_1}) + \widetilde{\text{Re}}\hat{\Lambda}(0, m^2_{\tilde{b}_2}, m^2_{\tilde{b}_2}) \stackrel{!}{=} 0$

Renormalization of Y_b:

OS renormalization:

$$\delta Y_b = \frac{1}{2}\widetilde{\mathrm{Re}}\left\{\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_1}) + \Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_2})\right\}$$

$\overline{\mathrm{DR}}$ renormalization:

$$\delta Y_b = \frac{1}{2}\widetilde{\mathrm{Re}}\left\{\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_1})|_{\mathrm{div}} + \Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_2})|_{\mathrm{div}}\right\}$$

ReY_b OS renormalization

$$\mathrm{Re}\delta Y_b = \frac{1}{2}\mathrm{Re}\left\{\widetilde{\mathrm{Re}}\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_1}) + \widetilde{\mathrm{Re}}\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_2})\right\}$$

Existing analyses all in the real MSSM:

- [*A. Bartl et al. '98*] [*L. Jin, C. Li '01*]
  "OS" used for stop and sbottom decays
  ($\to$ implemented into SDecay)

- [*C. Weber, K. Kovarik, H. Eberl, W. Majerotto '07*]
  similar to "m_b, A_b $\overline{\text{DR}}$" used for Higgs decays to sfermions

- [*A. Arhrib, R. Benbrik '04*]
  an "OS" scheme used for $\tilde{f} \to \tilde{f}'V$

- [*Q. Li, L. Jin, C. Li '02*]
  an "OS" scheme with running m_t, m_b, A_t, A_b used for $\tilde{t}_2 \to \tilde{t}_1\phi$

- [*H. Eberl et al. '10*]
  pure $\overline{\text{DR}}$ scheme used for stop decays

- [*A. Brignole, G. Degrassi, P. Slavich and F. Zwirner '02*]
  [*S.H., W. Hollik, H. Rzehak, G. Weiglein '04*]
  real "A_b vertex, ReY_b OS" used for two-loop Higgs self-energies

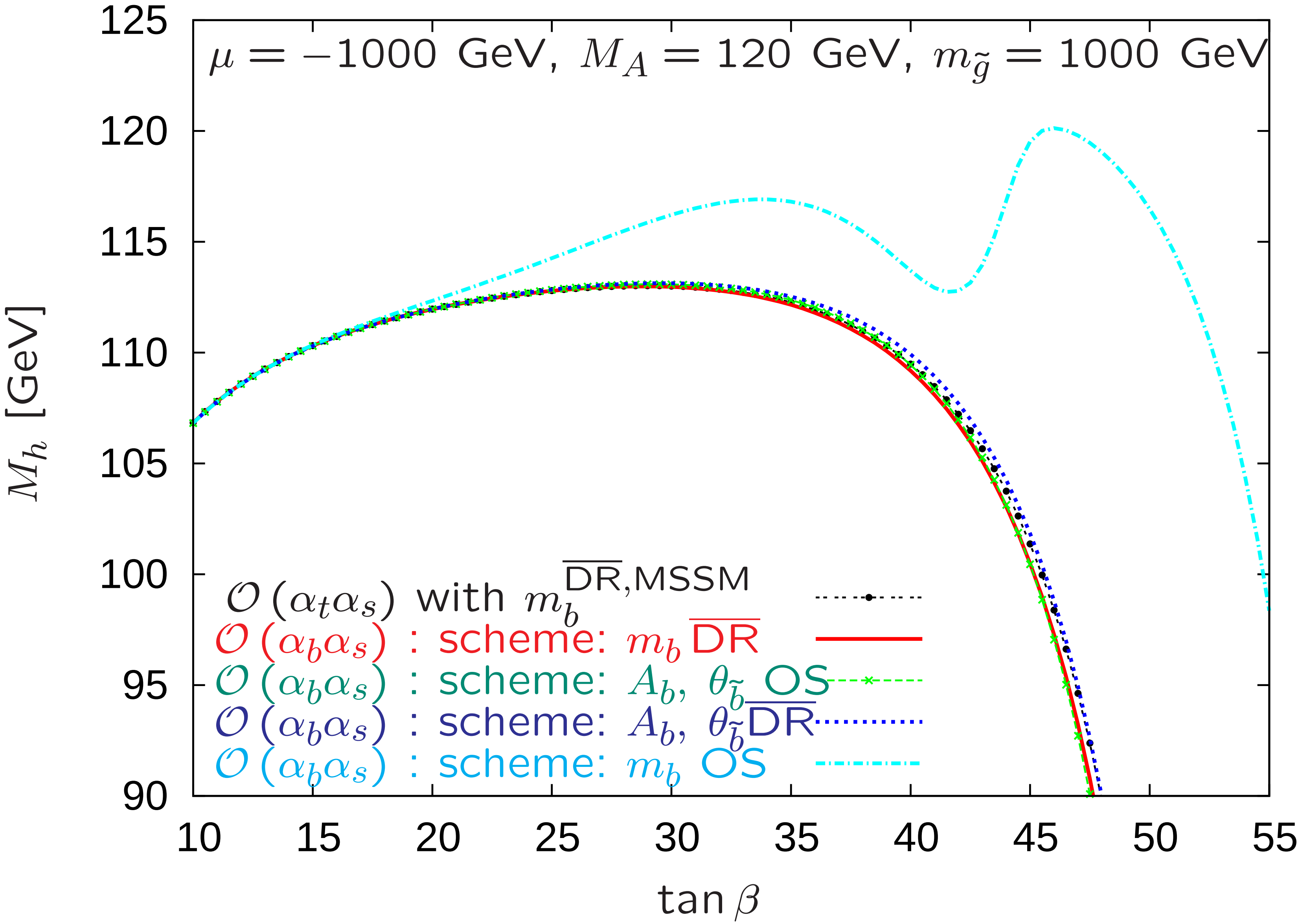
3. Analysis of the renormalization schemes

Numerical scenarios:

Scen.	$M_{H^\pm}$	$m_{\tilde{t}_2}$	μ	A_t	A_b	M_1	M_2	M_3
S1	150	600	200	900	400	200	300	800
S2	180	900	300	1800	1600	150	200	400

Scen.	$\tan\beta$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$
S1	2	293.391	600.000	441.987	447.168
	20	235.073	600.000	418.824	439.226
	50	230.662	600.000	400.815	449.638
S2	2	495.014	900.000	702.522	707.598
	20	445.885	900.000	678.531	695.180
	50	442.416	900.000	628.615	697.202

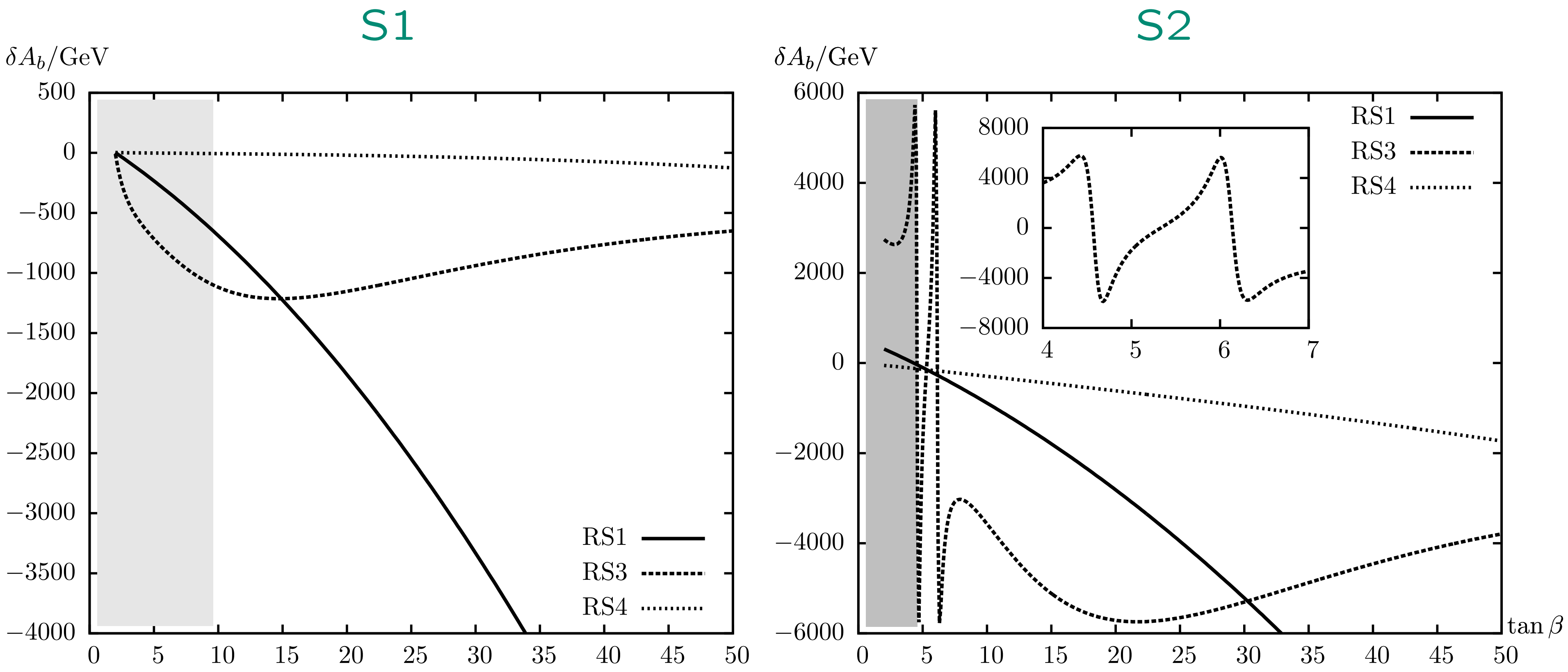
"OS" scheme: $\delta A_b = \frac{1}{m_b}\left[-(A_b - \mu^* \tan\beta)\,\delta m_b + \ldots\right]$



$\Rightarrow$ fails already for Higgs boson self-energies

Problems of non-A_b renormalizations:

$$\delta A_b|_{\text{fin}} = \frac{1}{m_b}\left[U_{\tilde{b}_{11}} U^*_{\tilde{b}_{12}} \left(\delta m^2_{\tilde{b}_1} - \delta m^2_{\tilde{b}_2}\right)\right]_{\text{fin}} + \dots$$

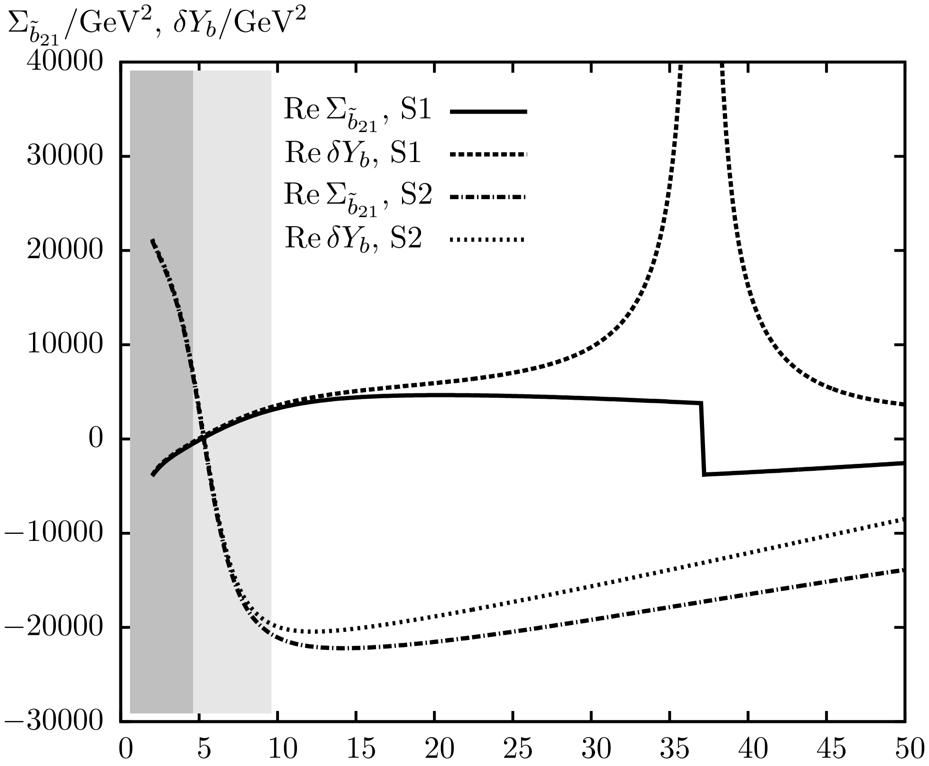


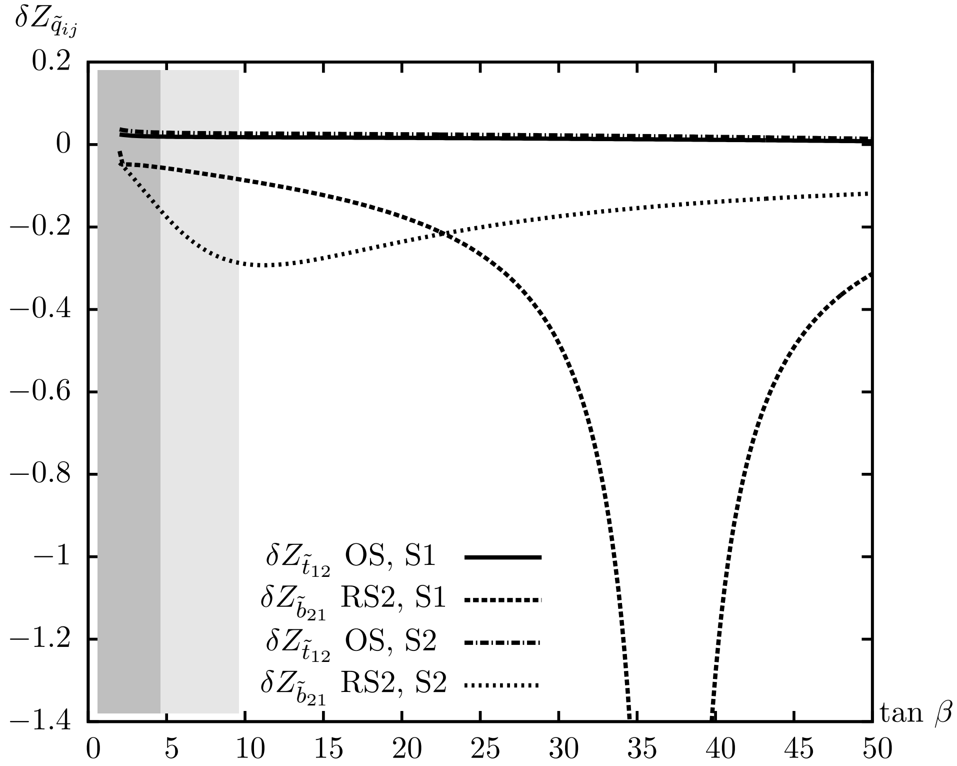
⇒ too large contributions to A_b are induced

Problems of m_b–A_b renormalizations:

$$\delta Y_b = \frac{U_{\tilde{b}_{11}} U_{\tilde{b}_{21}}}{|U_{\tilde{b}_{11}}|^2 - |U_{\tilde{b}_{12}}|^2} \left(\delta m^2_{\tilde{b}_1} - \delta m^2_{\tilde{b}_2}\right) + \ldots, \qquad \delta Z_{\tilde{b}_{21}} = -2 \frac{\mathrm{Re}\Sigma_{\tilde{b}_{21}}(m^2_{\tilde{b}_2}) - \delta Y_b}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}$$

$\Rightarrow$ divergence for $|U_{\tilde{b}_{11}}| = |U_{\tilde{b}_{12}}|$ reached for $\tan\beta \approx 37$ in S1:





Problems of non-m_b renormalizations:

"A_b $\overline{\text{DR}}$, ReY_b OS" (RS5): (rMSSM)

$$\delta m_b = -\frac{m_b\,\delta A_b + \delta S}{(A_b - \mu \tan\beta)}$$

$\Rightarrow$ divergent for $A_b = \mu \tan\beta$

"A_b vertex, ReY_b OS" (RS6): (rMSSM)

$$\delta m_b = \frac{\delta S + F}{\mu\,(\tan\beta + 1/\tan\beta)}$$

$\Rightarrow$ no problem in the rMSSM!

"A_b vertex, ReY_b OS" (RS6): (cMSSM: $U_- = U_{\tilde{b}_{11}} U^*_{\tilde{b}_{22}} - U_{\tilde{b}_{12}} U^*_{\tilde{b}_{21}}$)

$$\frac{1}{\delta m_b} \sim 4\,\mu\,\tan^3\beta \left[\mathrm{Re} U_- \left(|U_{\tilde{b}_{11}}|^2 - |U_{\tilde{b}_{12}}|^2 \right) + \mathrm{Im}\, U_- \frac{4\,m_b}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}} \mathrm{Im} \left(U^*_{\tilde{b}_{11}} U_{\tilde{b}_{12}} A_b \right) \right]$$

$\Rightarrow$ divergences appear depending on ϕ_{A_b}!

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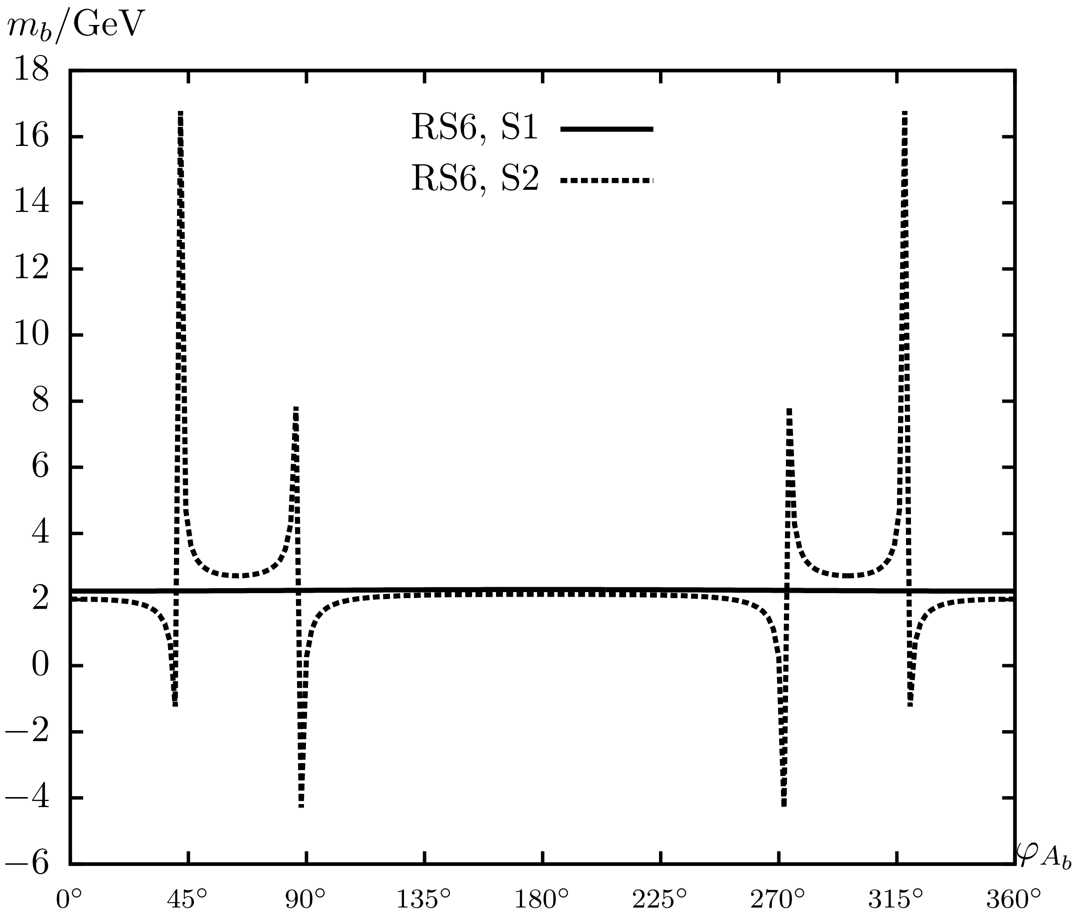

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Most "robust" behavior:

– RS2: "m_b, A_b $\overline{\mathrm{DR}}$"
  $\Rightarrow$ problems only for maximal sbottom mixing
– RS6: "A_b vertex, ReY_b OS"
  $\Rightarrow$ problems depending on ϕ_{A_b}

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Most "robust" behavior:

– RS2: "m_b, A_b $\overline{\text{DR}}$"
  $\Rightarrow$ problems only for maximal sbottom mixing
– RS6: "A_b vertex, ReY_b OS"
  $\Rightarrow$ problems depending on ϕ_{A_b}

All problems could be avoided in a pure $\overline{\text{DR}}$ scheme
$\Rightarrow$ not suited for external stops and sbottoms

What is the "preferred" renormalization scheme?

one counterterm is a "dependent" quantity

$\Rightarrow$ no scheme can be identified that shows "good" behavior
over the full cMSSM parameter space

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All problems could be avoided in a pure $\overline{\text{DR}}$ scheme
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$\Rightarrow$ we choose RS2: "m_b, A_b $\overline{\text{DR}}$" as our "preferred" scheme

4. Numerical results in the favored scheme: "m_b, A_b $\overline{\mathrm{DR}}$"

Calculation of partial widths:

– all diagrams created with FeynArts $\to$ TT

$\to$ model file with all counterterms in the cMSSM

– including all soft/hard QED/QCD diagrams

– further evaluation with FormCalc

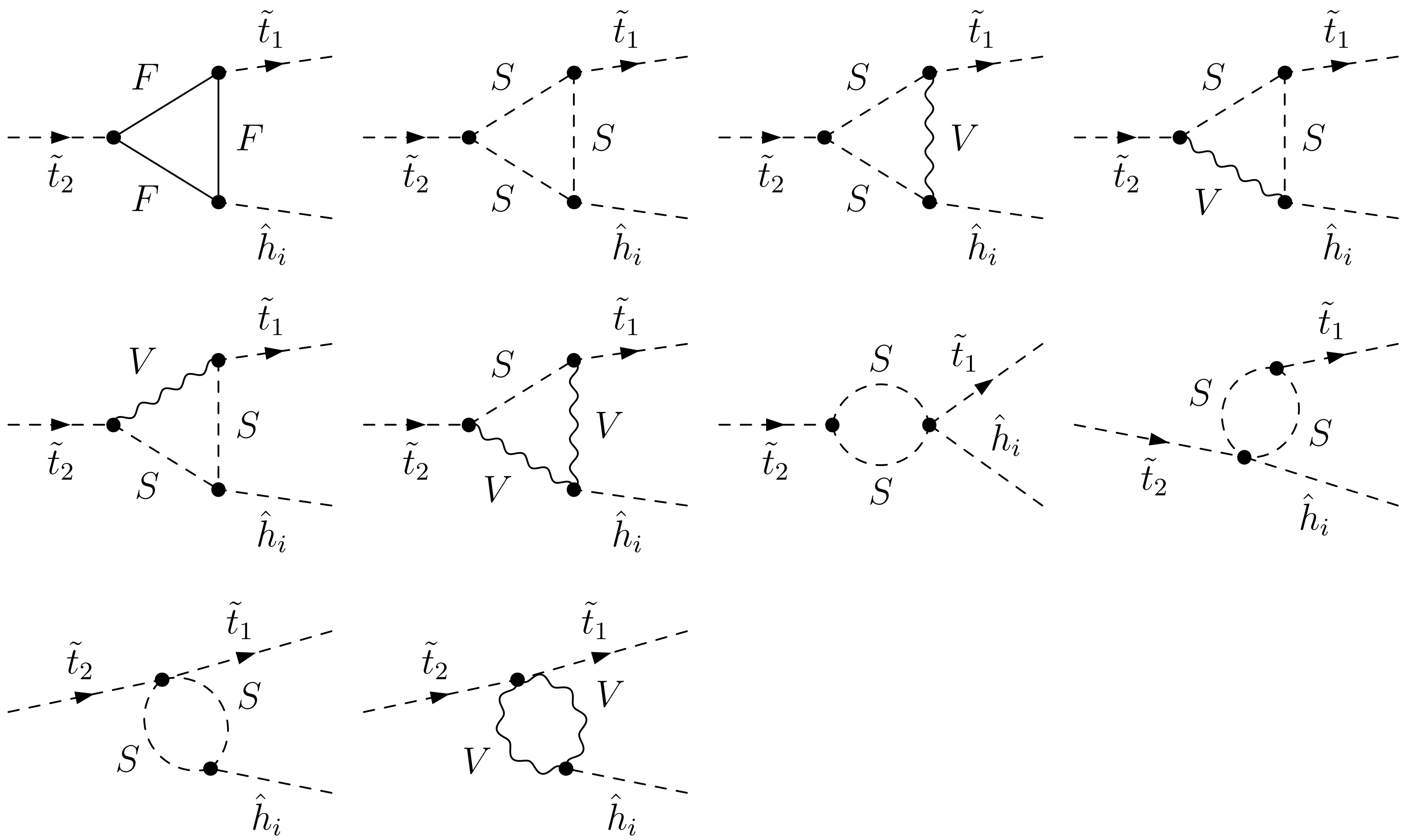
– Dimensional REDuction

– all UV and IR divergences cancel

– results will be included into FeynHiggs (www.feynhiggs.de)

$\to$ example plots will focus on $\mathrm{BR}(\tilde{t}_2 \to \tilde{t}_1 h_1)$

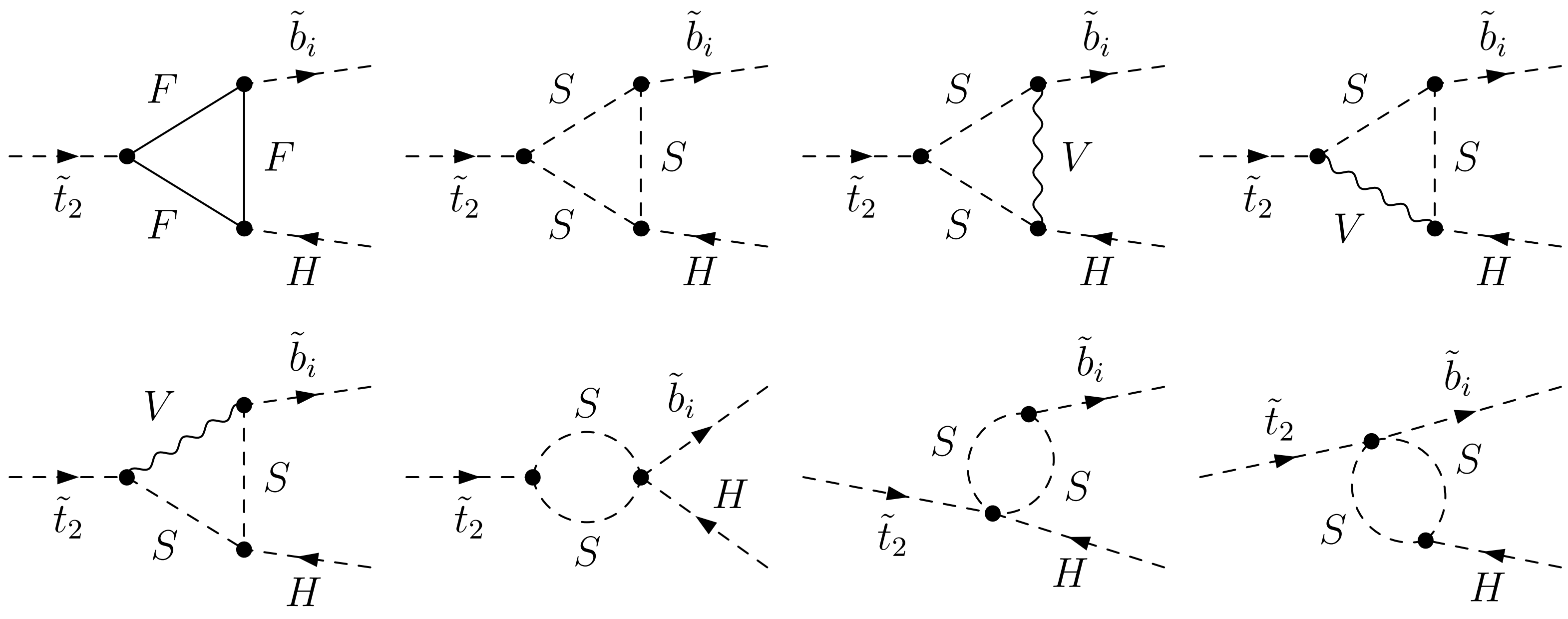
Feynman diagrams for $\tilde{t}_2 \to \tilde{t}_1 h_i$



– including Z–A or G–A transition contribution on the external Higgs boson leg

– including all soft/hard QED/QCD diagrams

Feynman diagrams for $\tilde{t}_2 \to \tilde{b}_i H^+$



– including W^+–H^+ or G^+–H^+ transition contribution on the external Higgs boson leg

– including all soft/hard QED/QCD diagrams

Numerical scenarios:

Scen.	$M_{H^\pm}$	$m_{\tilde{t}_2}$	$m_{\tilde{t}_1}$	$m_{\tilde{b}_2}$	μ	A_t	A_b	M_1	M_2	M_3
S1	150	650	$0.4\, m_{\tilde{t}_2}$	$0.7\, m_{\tilde{t}_2}$	200	900	400	200	300	800
S2	180	1200	$0.6\, m_{\tilde{t}_2}$	$0.8\, m_{\tilde{t}_2}$	300	1800	1600	150	200	400

Scen.	$\tan\beta$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$
S1	2	260.000	650.000	305.436	455.000
	20	260.000	650.000	333.572	455.000
	50	260.000	650.000	329.755	455.000
S2	2	720.000	1200.000	769.801	960.000
	20	720.000	1200.000	783.300	960.000
	50	720.000	1200.000	783.094	960.000

S1: $e^+e^- \to \tilde{t}_2\tilde{t}_1 \to \tilde{t}_1\phi\,\tilde{t}_1$ possible at the ILC(1000)

S1: $e^+e^- \to \tilde{t}_2\tilde{t}_1 \to \tilde{t}_1\phi\,\tilde{t}_1$ possible at the ILC(1000)

S1, S2: all decay modes are open

For $m_{\tilde{t}_1} \approx 600$ GeV:

$\sigma(e^+e^- \to \tilde{t}_2\tilde{t}_1) \approx 1.5$ fb^{-1}

1 ab^{-1} $\Rightarrow \sim 1500\tilde{t}_2$

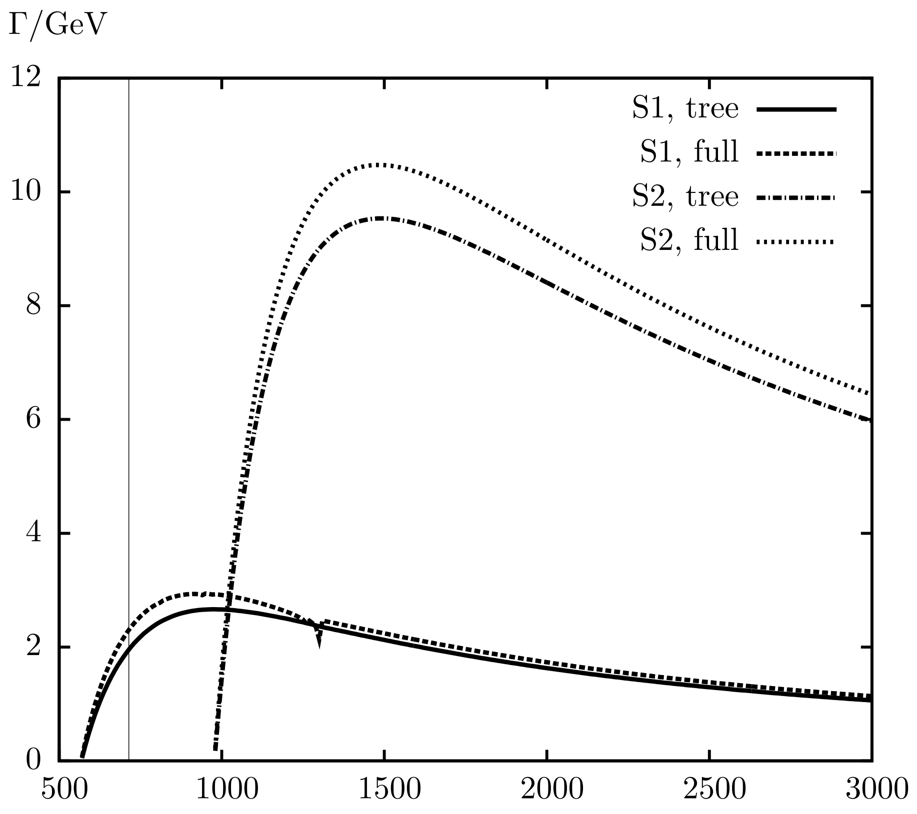
ILC will permit to measure the BRs close to the statistical uncertainty

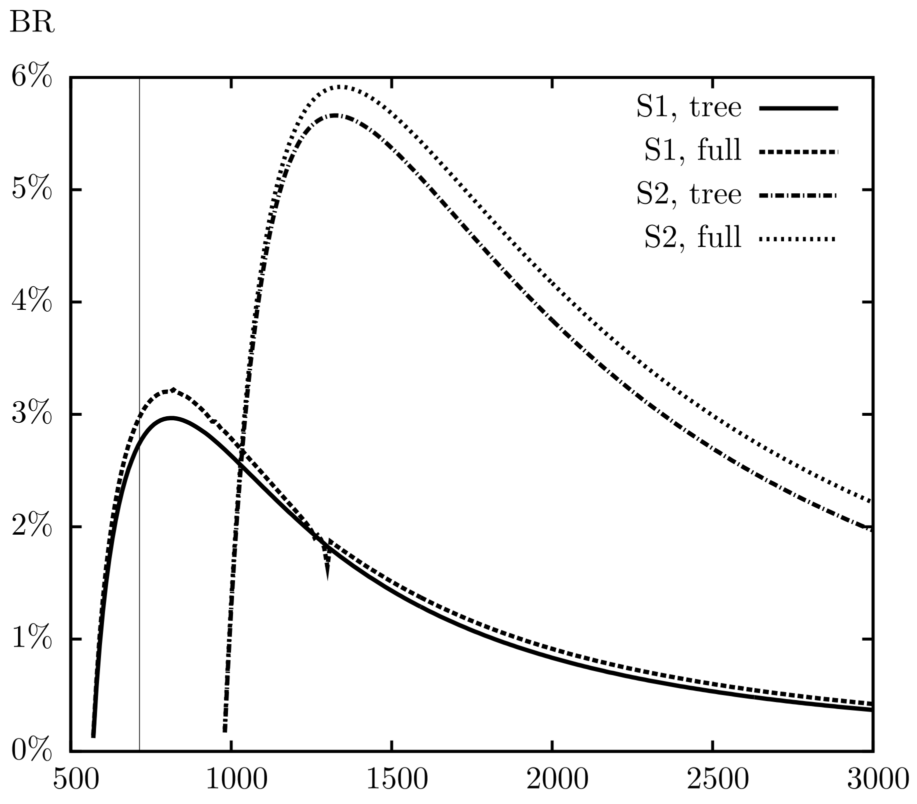
BR = 30% $\Rightarrow$ determination with 5% accuracy

(Worse accuracies for higher $m_{\tilde{t}_2}$ values . . .)

$\Gamma(\tilde{t}_2 \to \tilde{t}_1 h_1)$: dependence on $m_{\tilde{t}_2}$

[PRELIMINARY]



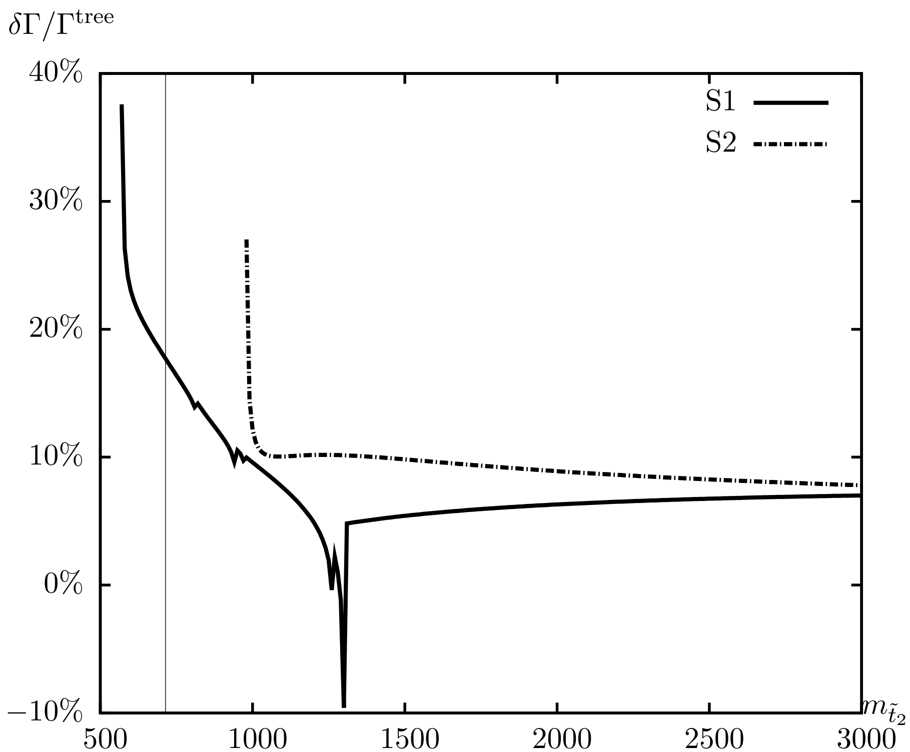


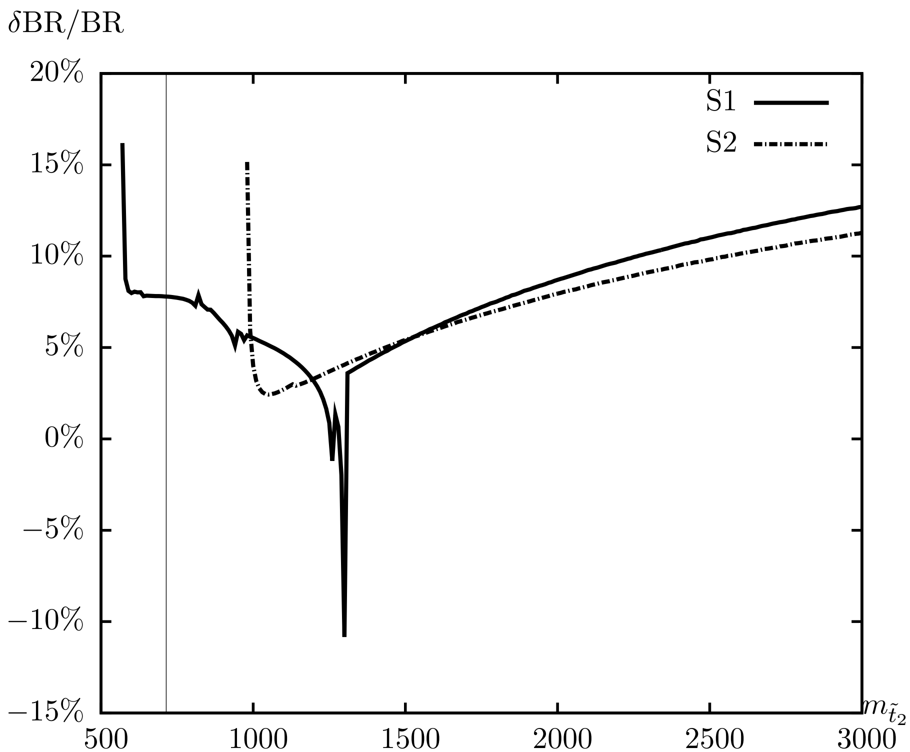
⇒ one-loop corrections under control

⇒ size of BR highly scenario dependent

$\Gamma(\tilde{t}_2 \to \tilde{t}_1 h_1)$: dependence on $m_{\tilde{t}_2}$

[PRELIMINARY]





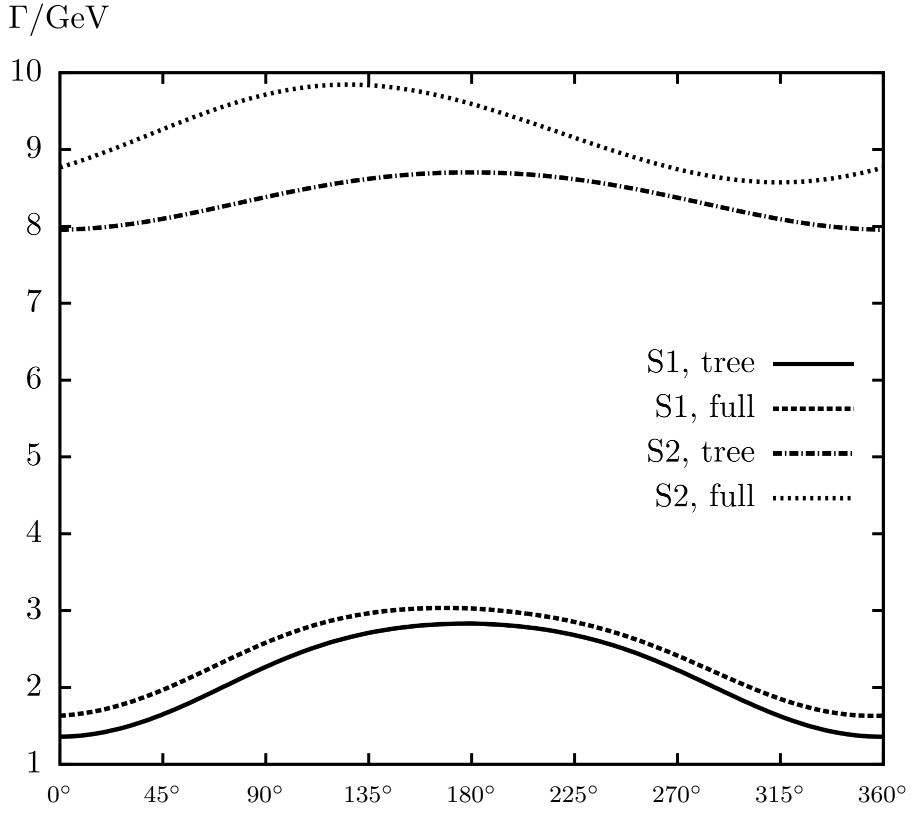
⇒ one-loop corrections under control

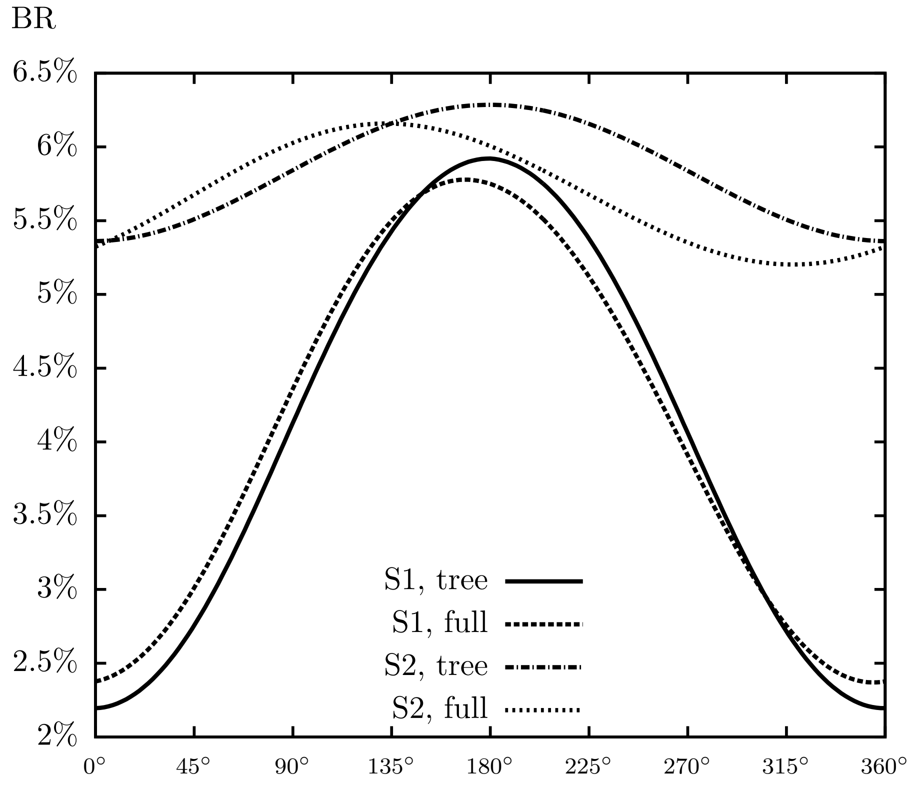
⇒ size of BR highly scenario dependent

⇒ full one-loop corrections crucial for ILC (and even LHC)

$\Gamma(\tilde{t}_2 \to \tilde{t}_1 h_1)$: dependence on ϕ_{A_t}

[PRELIMINARY]





⇒ one-loop corrections under control

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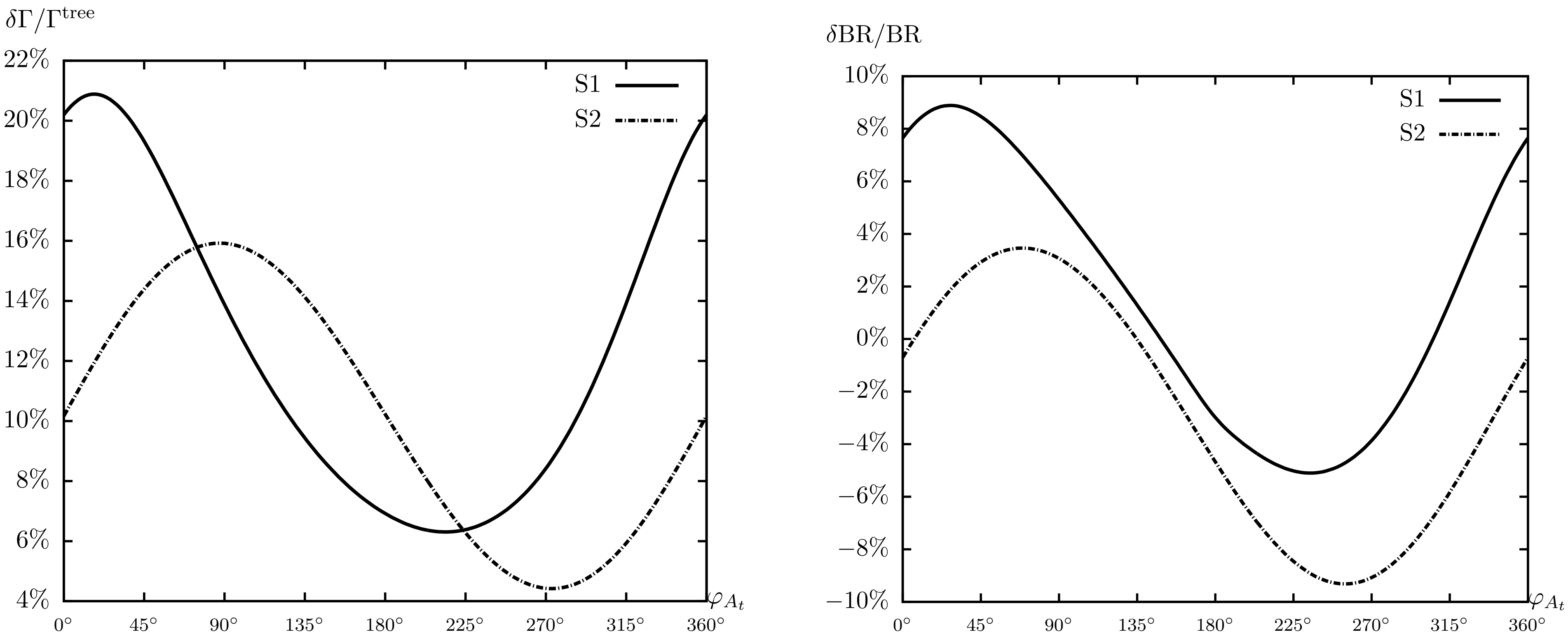


⇒ one-loop corrections under control

⇒ size of BR highly scenario dependent

⇒ full one-loop corrections crucial for ILC (and even LHC)

5. Conclusions & Outlook

- $\tilde{t}$ and $\tilde{b}$ sector important for collider phenomenology

- Simultaneous renormalization of both sectors crucial for higher-order corrections – on-shell properties for external squarks!

  $\Rightarrow$ $\tilde{t}/\tilde{b}$ renormalization in the cMSSM

  $\Rightarrow$ simultaneous renormalization of all sectors in the cMSSM

- Sbottom sector: six (+X) schemes defined and tested
  analytical deficiencies found in all schemes
  most "robust": RS2: "m_b, A_b $\overline{\text{DR}}$" $\leftarrow$ preferred scheme
  RS6: "A_b vertex, ReY_b OS"

- Numerical analysis: RS2: "m_b, A_b $\overline{\text{DR}}$" shows very robust and stable behavior over nearly all (tested) cMSSM parameter space

  Evaluation of $\Gamma(\tilde{t}_2 \to \tilde{t}_1 h_i)$ and BR$(\tilde{t}_2 \to \tilde{t}_1 h_i)$

  $\Rightarrow$ sizable effects in Γ and BR
  $\Rightarrow$ have to be included for ILC analyses (and possibly for LHC)

Back-up

Parameter definition: m_b

$$m_b^{\overline{\mathrm{MS}}}(m_b) = 4.2 \text{ GeV}$$
$$m_b^{\overline{\mathrm{MS}}}(\mu_R) = \Phi^{\mathrm{SM,3-loop}}(m_b^{\overline{\mathrm{MS}}}(m_b))$$

An "on-shell" mass is derived from the $\overline{\mathrm{MS}}$ mass via

$$m_b^{\mathrm{OS}} = m_b^{\overline{\mathrm{MS}}}(\mu_R) \left[1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}(\mu_R)}{\pi} \left(\frac{4}{3} + 2 \ln \frac{\mu_R}{m_b^{\overline{\mathrm{MS}}}(\mu_R)}\right)\right]$$

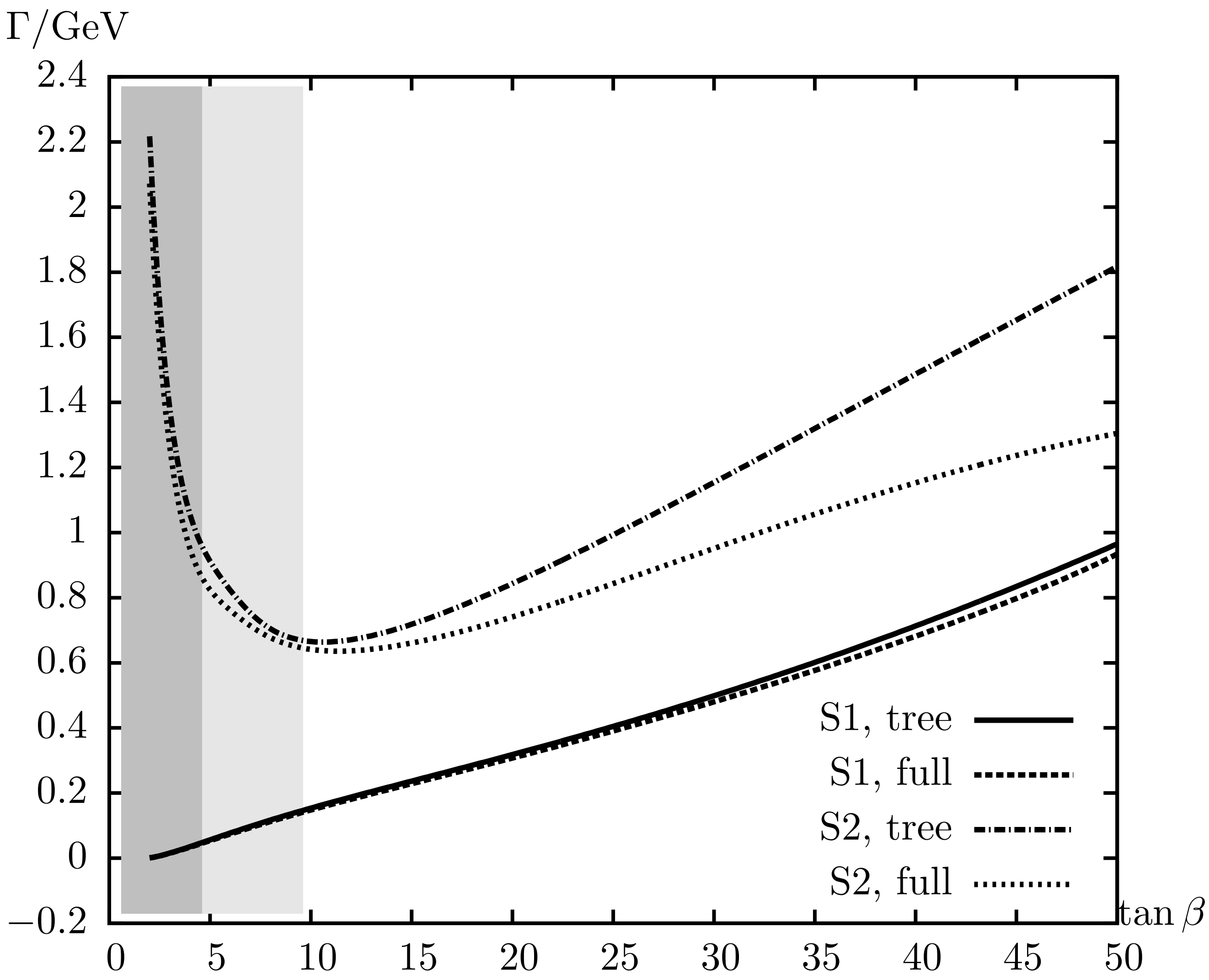
The $\overline{\mathrm{DR}}$ bottom quark mass is calculated iteratively

$$m_b^{\overline{\mathrm{DR}}} = \frac{m_b^{\mathrm{OS}}(1 + \Delta_b) + \delta m_b^{\mathrm{OS}} - \delta m_b^{\overline{\mathrm{DR}}}}{1 + \Delta_b}$$
$$\Delta_b = \frac{2\alpha_s(m_t)}{3\pi} \tan\beta\, M_3^* \mu^* I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) + \ldots$$

The bottom quark mass of a special renormalization scheme:

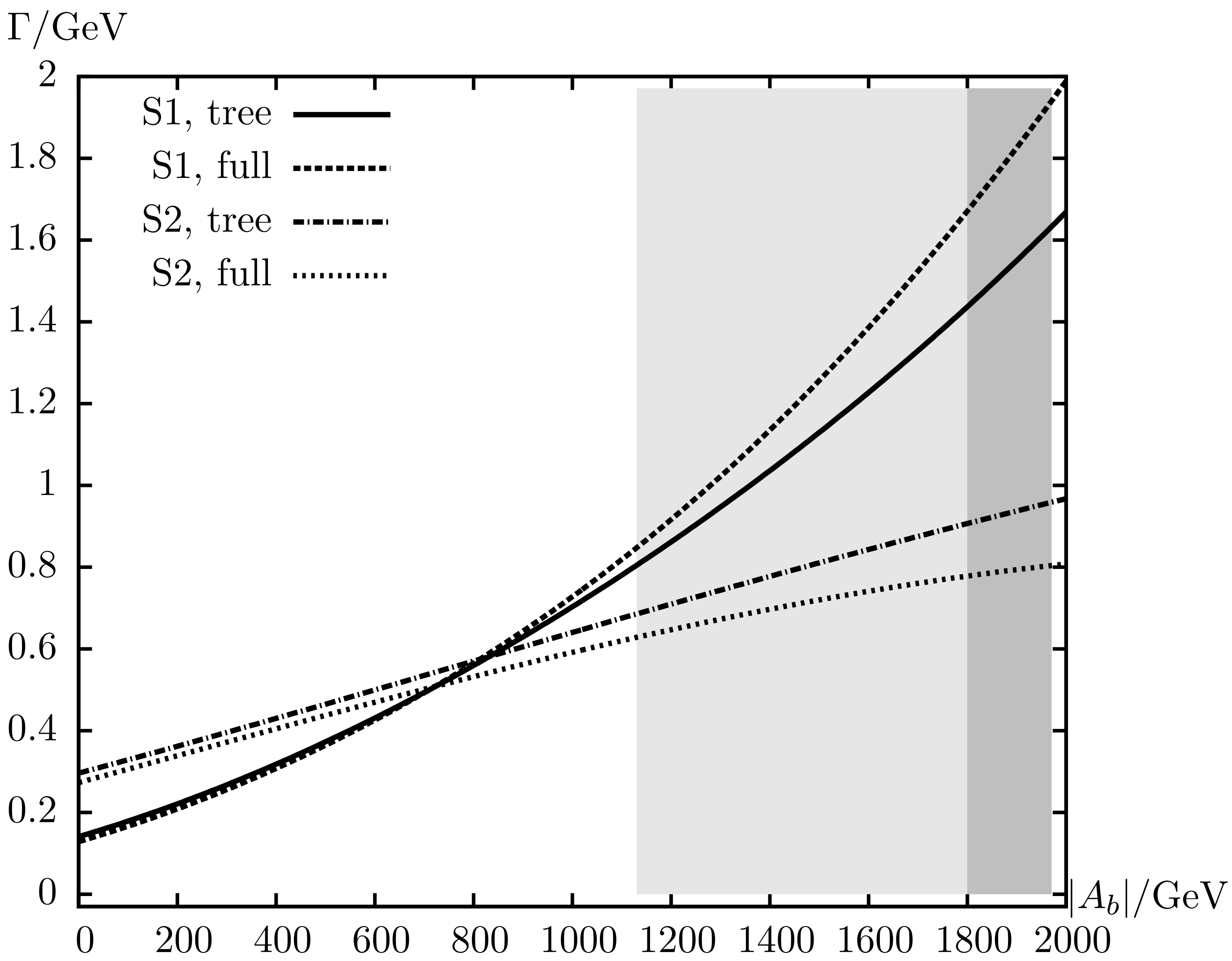
$$m_b = m_b^{\overline{\mathrm{DR}}} + \delta m_b^{\overline{\mathrm{DR}}} - \delta m_b$$

$\Gamma(\tilde{t}_2 \to \tilde{t}_1 h_2)$: dependence on $\tan\beta$



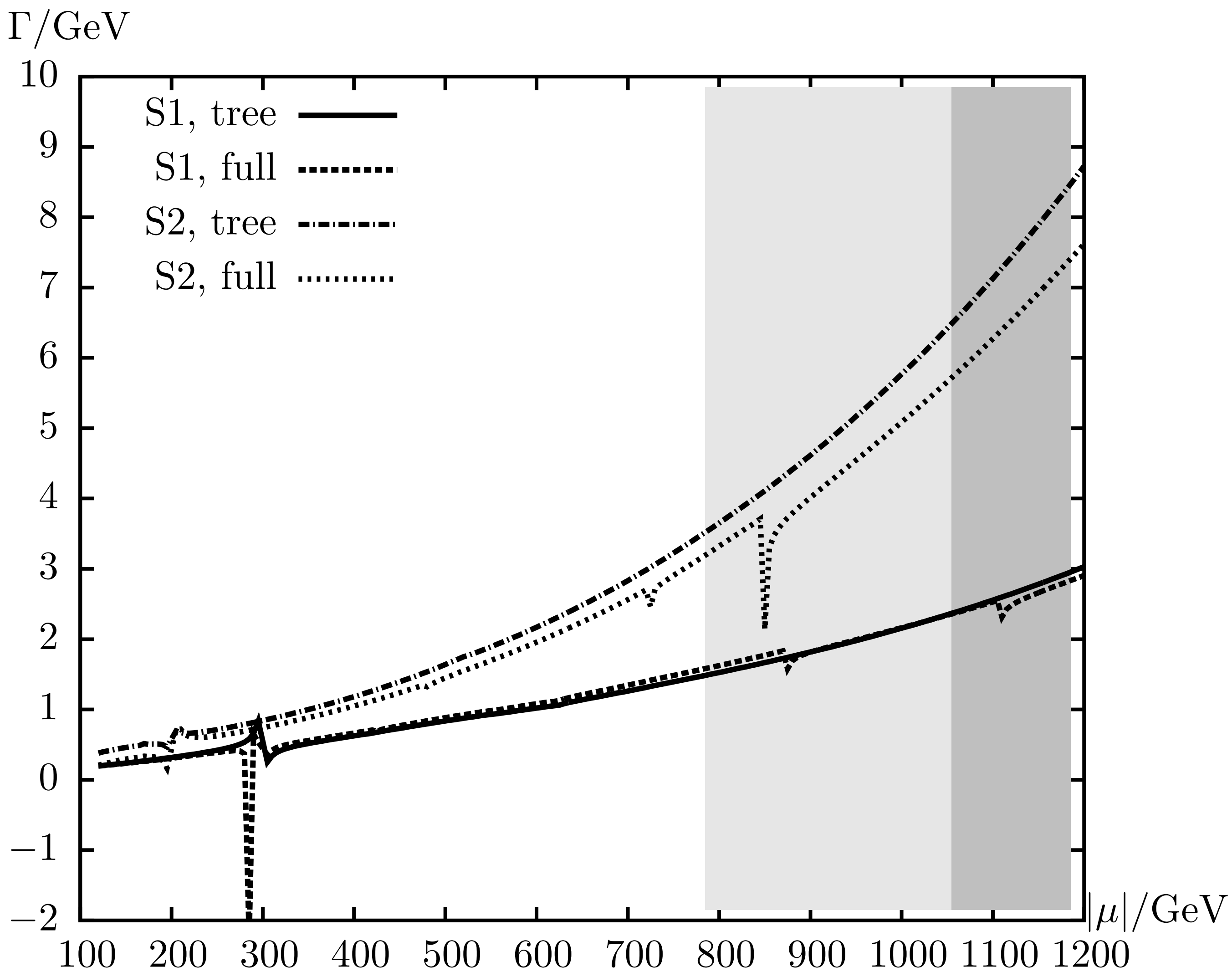
$\Rightarrow$ one-loop corrections under control for all $\tan\beta$ values

$\Gamma(\tilde{t}_2 \to \tilde{t}_1 h_2)$: dependence on A_b ($\tan\beta = 20$)



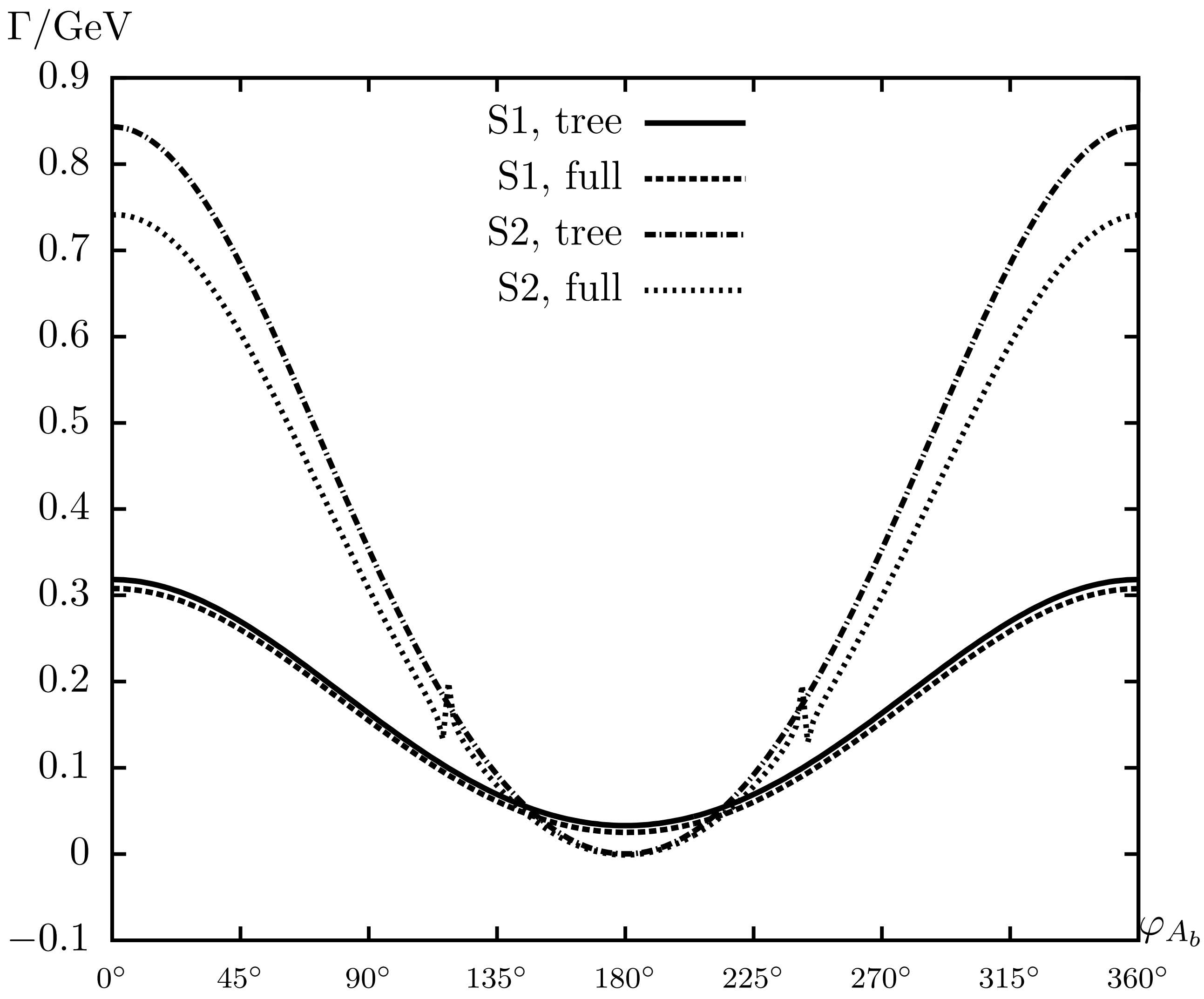
$\Rightarrow$ one-loop corrections under control for all A_b values

$\Gamma(\tilde{t}_2 \to \tilde{t}_1 h_2)$: dependence on μ ($\tan\beta = 20$)



⇒ one-loop corrections under control (but many thresholds)

$\Gamma(\tilde{t}_2 \to \tilde{t}_1 h_2)$: dependence on ϕ_{A_b} ($\tan\beta = 20$)



$\Rightarrow$ one-loop corrections under control except of sharp peaks at $|U_{\tilde{b}_{11}}| \approx |U_{\tilde{b}_{12}}|$