Alternative 1-loop Calculations

R. Pittau (U. of Granada)
Geneva, October 20th, 2010

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2. New techniques (OPP)
3. OPP vs Generalized Unitarity
4. Results (I skip this!)

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1. Why 1(2)-loop calculations? (I skip this!)
2. New techniques (OPP)
3. OPP vs Generalized Unitarity
4. Results (I skip this!)
5. Open problems and outlooks

A typical $2 \to m$ process at 1-loop

$$\sigma^{NLO} = \int_m d\sigma^B + \int_m \left(d\sigma^V + \int_1 d\sigma^A \right) + \int_{m+1} \left(d\sigma^R - d\sigma^A \right)$$

1. $d\sigma^B$ is the Born cross section
2. $d\sigma^V$ is the Virtual correction (loop diagrams)
3. $d\sigma^R$ is the Real correction
4. $d\sigma^A$ and $\int_1 d\sigma^A$ are *unintegrated* and *integrated* counterterms (allowing to compute the Real emission of massless particles in 4 dimensions)

The Virtual corrections $d\sigma^V$

The decomposition of any 1-loop amplitude

$$
\begin{aligned}
A \quad = \quad & \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}\bar{D}_{i_3}} \\
& + \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}} \\
& + \sum_{i_0<i_1}^{m-1} b(i_0 i_1) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}} \\
& + \sum_{i_0}^{m-1} a(i_0) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}} + R
\end{aligned}
$$

The problem is getting the set $\mathcal{S} = \left\{ \begin{array}{lll} d(i_0 i_1 i_2 i_3), & c(i_0 i_1 i_2), & \\ b(i_0 i_1), & a(i_0), & R \end{array} \right.$

The OPP Method (Ossola, Papadopoulos, Pittau, 2007)

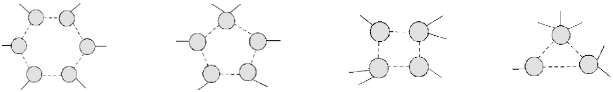
Working at the *integrand* level

$$A = \int d^n\bar{q}\;\left[\mathcal{A}(q) + \tilde{A}(q,\tilde{q},\epsilon)\right]$$

$$\left(\begin{array}{c} \bar{q} = q + \tilde{q} \\ n = 4 + \epsilon \end{array}\right)$$

- For example, in the case of $2 \to 6$

$$\mathcal{A}(q) = \sum \underbrace{\frac{N_i^{(6)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_5}}} + \underbrace{\frac{N_i^{(5)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_4}}} + \underbrace{\frac{N_i^{(4)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_3}}} + \underbrace{\frac{N_i^{(3)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}}} + \cdots$$



The function to be sampled *numerically* to extract the coefficients

$$
\begin{aligned}
N_i^{(6)}(q) &= \sum_{i_0<i_1<i_2<i_3}^{5} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] D_{i_4} D_{i_5} \\
&+ \sum_{i_0<i_1<i_2}^{5} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] D_{i_3} D_{i_4} D_{i_5} \\
&+ \sum_{i_0<i_1}^{5} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] D_{i_2} D_{i_3} D_{i_4} D_{i_5} \\
&+ \sum_{i_0}^{5} \left[a(i_0) + \tilde{a}(q; i_0)\right] D_{i_1} D_{i_2} D_{i_3} D_{i_4} D_{i_5} \\
&+ \tilde{P}(q) D_{i_0} D_{i_1} D_{i_2} D_{i_3} D_{i_4} D_{i_5}
\end{aligned}
$$

Solving the OPP Equation 1

- The functional form of the *spurious* terms should be known
  Ossola, Papadopoulos, R. P., Nucl.Phys.B763:147-169,2007
  del Aguila, R. P., JHEP 0407:017,2004

Example ($p_0 = 0$)

$$\tilde{d}(q;0123) = \tilde{d}(0123)\,\epsilon(qp_1p_2p_3)$$

$$\int d^n\bar{q}\,\frac{\tilde{d}(q;0123)}{\bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3} = \tilde{d}(0123)\int d^n\bar{q}\,\frac{\epsilon(qp_1p_2p_3)}{\bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3} = 0$$

- The coefficients $\{d_i, c_i, b_i, a_i\}$ and $\{\tilde{d}_i, \tilde{c}_i, \tilde{b}_i, \tilde{a}_i\}$ are extracted by solving linear systems of equations

Solving the OPP Equation 2

The use of special values of q helps (Unitarity)

$$D_0(q^\pm) = D_1(q^\pm) = D_2(q^\pm) = D_3(q^\pm) = 0$$

$$N^{(m-1)}(q^\pm) \quad = \left[d(0123) + \tilde{d}(q^\pm; 0123) \right] \prod_{i \neq 0,1,2,3}^{m-1} D_i(q^\pm)$$

$$d(0123) = \frac{1}{2} \left[\frac{N^{(m-1)}(q^+)}{\prod_{i \neq 0,1,2,3}^{m-1} D_i(q^+)} + \frac{N^{(m-1)}(q^-)}{\prod_{i \neq 0,1,2,3}^{m-1} D_i(q^-)} \right]$$

What about R $(= R_1 + R_2)$?

The OPP Solution:

The origin of R_1

$$\frac{1}{\bar{D}_i} = \frac{1}{D_i}\left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right)$$

⇒ predicted within OPP
if the denominator structure is known

The origin of R_2

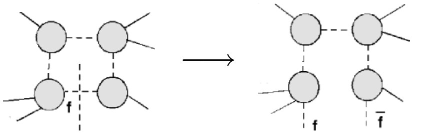
$$R_2 = \int d^n \bar{q} \frac{\tilde{N}(q, \tilde{q}, \epsilon)}{\bar{D}_0 \cdots \bar{D}_{m-1}}$$

⇒ effective tree-level Feynman Rules
up to 4 points *

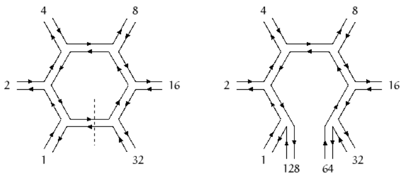
* QCD: Draggiotis, Garzelli, Papadopoulos, R. P., JHEP 0904:072,2009
EW: Garzelli, Malamos, R. P., JHEP 1001:040,2010
EW in the R_ξ and Unitary gauges:
Garzelli, Malamos, R. P.,arXiv:1009.4302 [hep-ph]

Recursion Relations at 1-loop (cutting 1 arbitrary leg)

- OPP + 1 hard-cut allow to use *the same tree-level Recursion Relations* for $m + 2$ tree-like structures



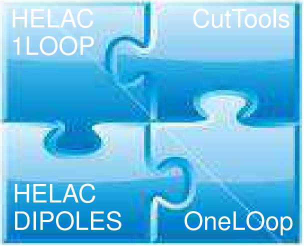
- The color can be treated *as at the tree level*



$\Rightarrow$ Tree level codes can be *transformed* into 1-loop ones $\Rightarrow$

An Example in QCD: The Helac-NLO System

1. CutTools
   $\{d_i, c_i, b_i, a_i\}$ and R_1
2. HELAC-1LOOP
   $N(q)$ and R_2
3. OneLOop
   scalar 1-loop integrals
4. HELAC-DIPOLES
   Real correction and CS dipoles



(figure by G. Bevilacqua)

- Ossola, Papadopoulos, R. P., JHEP 0803 (2008) 042
- van Hameren, Papadopoulos, R. P., JHEP 0909 (2009) 106
- van Hameren, e-Print: arXiv:1007.4716 [hep-ph]
- Czakon, Papadopoulos, Worek, JHEP 0908 (2009) 085

The HELAC-NLO group *

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R. P.	C.G. Papadopoulos	M. Worek

*

Contributors

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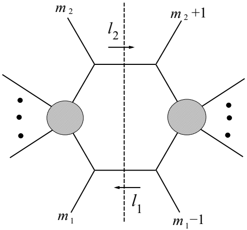
Unitarity: Cutting (Gluing)

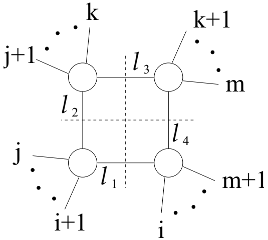
1. Double cuts ⇔ gluing 2 tree-level, *gauge invariant*, amplitudes (Bern, Dixon, Dunbar, Kosower 1994)



2. Different double cuts are applied to disentangle 1-loop scalar functions *by looking at the analytic structure of the result*
3. R is reconstructed by looking at collinear and infrared limits

Generalized Unitarity: more Cutting (more Gluing)

1. Quadruple cuts ⇔ gluing 4 tree-level, *gauge invariant*, amplitudes (Britto, Cachazo, Feng, hep-th/0412103)



2. q integration frozen ⇒ *coefficient d_i of the box extracted*
3. 3 bubbles are connected together, the box contributions subtracted and the *coefficients c_i of the triangles extracted*
4. . . .

⇒

Generalized Unitarity (Relevant References)

- Bern, Dixon, Dunbar, Kosower, hep-ph/9403226 and hep-ph/9409265;
- Forde,0704.1835 [hep-ph];
- Ellis, Giele, Kunszt, 0708.2398 [hep-ph];
- Ellis, Giele, Kunszt, Melnikov, 0806.3467 [hep-ph];
- Berger et al. (`BlackHat`), 0803.4180 [hep-ph].

Between OPP and GU:

- Mastrolia, Ossola, Reiter, Tramontano (`Samurai`) arXiv:1006.0710 [hep-ph].

Virtues and drawbacks of OPP and GU

1. **GU:**
   - Deals with Gauge invariant Objects;
   - No general solution for Wave Function Renormalization Corrections (this is not a problem in QCD!);
   - At present, computing R is time consuming;
2. **OPP:**
   - It is s more Feynman Diagram oriented;
   - No gauge invariant building blocks;
   - Wave Function Renormalization can be put by hand;
   - An algorithmic and fast, although not completely general, calculation of R is possible.

Notice that:

At present NO complete 1-loop calculation in the full EW Standard Model has been carried out.

It will be needed for ILC Physics.

Two main open problems:

1. Rational terms:
   No universal and gauge invariant **nor** completely 4-dimensional procedure exists so far!
   - In **GU**: On-shell Recursion Relations and numerical calculation of residues of spurious poles or computation of amplitudes in 5 and 6 dimensions.
   - In **OPP**: Computation of extra Tree level like Feynman rules for the theory at hand (R_2), need to know the denominator structure (R_1). R_1 and R_2 are not separately gauge-invariant.
2. External Self-energy corrections:
   No gauge invariant procedure exists to compute them numerically!

I shall illustrate the situation with the help of the process $e^+e^- \to \mu^+\mu^-$ in QED.

1-loop diagrams giving 3 or 4 point functions in $e^+e^- \to \mu^+\mu^-$

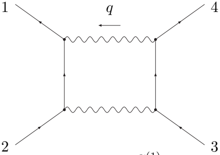


Diagram $G^{(1)}$

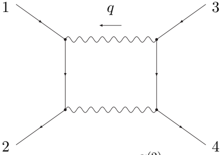


Diagram $G^{(2)}$

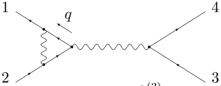


Diagram $G^{(3)}$

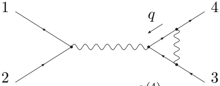
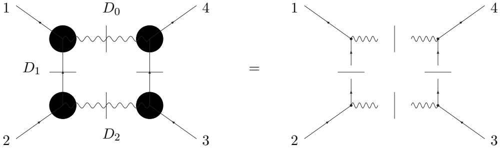


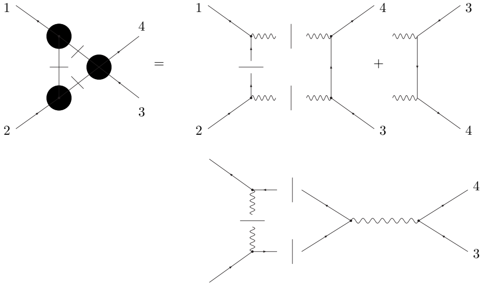
Diagram $G^{(4)}$

The coefficient of a scalar box



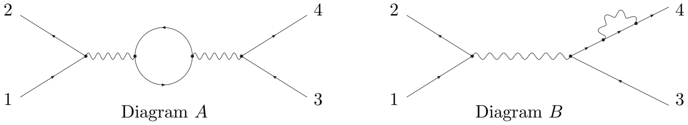
It is obtained by multiplying 4 gauge invariant on shell amplitudes

The coefficient of a scalar triangle



- To one of the 3 on shell amplitudes contribute now different denominator structures (diagrams).
- When using the OPP way to compute R one should disentangle them, breaking, in general, gauge invariance.
- One can keep gauge invariance, at the price of using 5 and 6-dimensional tree level amplitudes. *No tree level generator can be adapted any more to compute 1-loop processes.*

The coefficient of a scalar bubble



- The coefficient of the 2-point function contributing to diagram A can still be computed by multiplying together two on shell amplitudes.
- The coefficient of the 2-point function contributing to diagram B cannot be computed that way, because the on-shell conditions make the internal propagator become singular.
- On the other hand, this diagram should be included to preserve gauge invariance of the tree level amplitudes. Problem still unsolved. In **OPP** it would be put by hand.

Conclusions

1. New techniques and ideas allowed an impressive progress in the field of 1-loop calculations:
   - I (briefly) discussed and compared the **OPP** method and the **GU** techniques;
   - They have been successfully applied in QCD to compute at NLO $pp \to ttbb$, $pp \to ttjj$, $pp \to W + 3\ jets$, $pp \to W + 4\ jets$, $\cdots$;
   - Computing R in both **OPP** and **GU** still unsatisfactory;
   - Wave function renormalization problem unsolved in **GU**. In **OPP** put by hand (but gauge dependent!).
2. No complete EW calculation is available so far with these new techniques.
3. They will be needed for ILC Physics.
4. The final goal should be delivering public 1-loop codes.