First and second order spin amplitudes for precision of PHOTOS Monte Carlo

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- **(1)** From semileptonic B and K mesons decays (measurements of quark mixing), properties of W and Z decays at LHC, to signatures for discovery and properties of New Physics particles bremsstrahlung must be taken into account.

- **(2)** PHOTOS Monte Carlo is used in such studies. Essential: Input from spin calculations was necessary for design of the program and for tests of its physical precision.

- **(3)** *I will mention:* phase space parametrization, crude distribution in single photon emission, double photon emisssion, and multiple emission; all modes needed for tests.

- **(4)** *Technical points like:* event record type HEPEVT, HepMC; intermediate particles explicitly stored in it or not; numerical tests for user installation will be skipped.

- **(5)** *How QED FSR is separated from the rest (precision):* genuine weak corrections, ISR, ISR$\times$PS, ISR-FSR interference. This important topic, I hope, will be covered in other talks.

PHOTOS: short presentation

Presentation

- PHOTOS (by E.Barberio, B. van Eijk, Z. W., P.Golonka) is used to simulate the effect of radiatiative corrections in decays, since 1989.
- many citations from experiments $\longrightarrow$ responsability
- Full events combining complicated tree structure of production and subsequent decays are fed into PHOTOS, usually with the help of HEPEVT event record of F77
- PHOTOS version for HepMC event record used in C++ applications is ready for tests now.
- At every event decay branching, PHOTOS intervene. With certain probability extra photon may be added and kinematics of other particles adjusted.

Main References

- E. Barberio, B. van Eijk and Z. Was, Comput. Phys. Commun. **66**, 115 (1991): single emission
- E. Barberio and Z. Was, Comput. Phys. Commun. **79**, 291 (1994). double emission introduced, tests with second order matrix elements
- P. Golonka and Z. Was, EPJC 45 (2006) 97 multiple photon emisson introduced, tests with precioson second order exponentiation MC.
- P. Golonka and Z. Was, EPJC 50 (2007) 53 complete matrix element for Z decay, and further tests
- G. Nanava, Z. Was, Eur.Phys.J.C51:569-583,2007, best description of phase space
- G. Nanava, Z. Was, Q. Xu, arXiv:0906.4052. EPJC in print complete matrix element for W decay
- N. Davidson, T. Przedzinski, Z. Was, IFJPAN-IV-2010-6, Presently main web-page for program C++ version: http://www.ph.unimelb.edu.au/˜ndavidson/photos/doxygen/index.html HepMC interface

Phase space and crude distribution

Phase Space: must be exact to discuss matrix elements

Orthodox exact Lorentz-invariant phase space (*Lips*) is in use in PHOTOS!

$$dLips_{n+1}(P) =$$
$$\frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3}\frac{d^3q}{2q^0(2\pi)^3}(2\pi)^4\delta^4\Big(P-\sum_1^n k_i - q\Big)$$
$$= \quad d^4p\delta^4(P-p-q)\frac{d^3q}{2q^0(2\pi)^3}\frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3}(2\pi)^4\delta^4\Big(p-\sum_1^n k_i\Big)$$
$$= \quad d^4p\delta^4(P-p-q)\frac{d^3q}{2q^0(2\pi)^3}dLips_n(p \to k_1 ... k_n).$$

Integration variables, the four-vector p, compensated with $\delta^4\big(p-\sum_1^n k_i\big)$, and another integration variable M_1 compensated with $\delta\big(p^2-M_1^2\big)$ are introduced.

Phase space and crude distribution

Phase Space Formula of Photos

$$dLips_{n+1}(P \to k_1...k_n, k_{n+1}) = dLips_n^{+1\ tangent} \times W_n^{n+1},$$

$$dLips_n^{+1\ tangent} = dk_\gamma d\cos\theta d\phi \times dLips_n(P \to \bar{k}_1...\bar{k}_n),$$
$$\{k_1, \ldots, k_{n+1}\} = \mathbf{T}\big(k_\gamma, \theta, \phi, \{\bar{k}_1, \ldots, \bar{k}_n\}\big). \quad (1)$$

1. One can verify that if $dLips_n(P)$ was exact, then this formula lead to exact parametrization of $dLips_{n+1}(P)$
2. Practical implementation: Take completely construced n-body phase space point (event).
3. Reconstruct coordinate variables, any parametrization can be used.
4. Construct new kinematical configuration from those variables and $k_\gamma \theta \phi$.
5. **Forget about temporary $k_\gamma \theta \phi$. Now, only weight and new four vectors count.**
6. A lot depend on $\mathbf{T}$. Options depend on matrix element: must tangent at singularities. Simultaneous use of several $\mathbf{T}$ is necessary/convenient if more than one charge is present in final state.

Phase space and crude distribution

Phase Space: (main formula)

If we choose

$$G_n \;:\; M^2_{2\ldots n}, \theta_1, \phi_1, M^2_{3\ldots n}, \theta_2, \phi_2, \ldots, \theta_{n-1}, \phi_{n-1} \;\longrightarrow\; \bar{k}_1 \ldots \bar{k}_n \tag{2}$$

and

$$G_{n+1} \;:\; k_\gamma, \theta, \phi, M^2_{2\ldots n}, \theta_1, \phi_1, M^2_{3\ldots n}, \theta_2, \phi_2, \ldots, \theta_{n-1}, \phi_{n-1} \;\longrightarrow\; k_1 \ldots k_n, k_{n+1} \tag{3}$$

then

$$\mathbf{T} = G_{n+1}(k_\gamma, \theta, \phi, G_n^{-1}(\bar{k}_1, \ldots, \bar{k}_n)). \tag{4}$$

The ratio of the Jacobians form the phase space weight W_n^{n+1} for the transformation. Such solution is universal and valid for any choice of G's. However, G_{n+1} and G_n has to match matrix element, otherwise algorithm will be inefficient (factor 10^{10} ...).

In case of PHOTOS G_n's

$$W_n^{n+1} = k_\gamma \frac{1}{2(2\pi)^3} \times \frac{\lambda^{1/2}(1, m_1^2/M^2_{1\ldots n}, M^2_{2\ldots n}/M^2_{1\ldots n})}{\lambda^{1/2}(1, m_1^2/M^2, M^2_{2\ldots n}/M^2)}, \tag{5}$$

Phase space and crude distribution

Phase Space: (multiply iterated)

By iteration, we can generalize formula (1) and add l particles:

$$dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l}) = \frac{1}{l!} \prod_{i=1}^{l} \Big[dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W^{n+i}_{n+i-1} \Big]$$

$$\times dLips_n(P \to \bar{k}_1...\bar{k}_n), \tag{6}$$

$$\{k_1, \dots, k_{n+l}\} = \mathbf{T}\big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}\big(\dots, \mathbf{T}\big(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \dots, \bar{k}_n\}\big) \dots\big).$$

Note that variables $k_{\gamma_m}, \theta_{\gamma_m}, \phi_{\gamma_m}$ are used at a time of the $m-$th step of iteration only, and are not needed elsewhere in construction of the physical phase space; the same is true for invariants and angles $M^2_{2\dots n}, \theta_1, \phi_1, \dots, \theta_{n-1}, \phi_{n-1} \;\; \to \;\; \bar{k}_1 \dots \bar{k}_n$ of (2,3), which are also redefined at each step of the iteration. Also intermediate steps require explicit construction of temporary $\bar{k}'_1 \dots \bar{k}'_n \dots \bar{k}'_{n+m}$, statistical factor $\frac{1}{l!}$ added.

We have **exact distribution** of **weighted** events over l and $n+l$ body phase spaces.

Crude Distribution for multiple emission

If we add arbitrary factors $f(k_{\gamma_i},\theta_{\gamma_i},\phi_{\gamma_i})$ and sum over l we obtain:

$$\sum_{l=0} \exp(-F)\frac{1}{l!}\prod_{i=1}^{l} f(k_{\gamma_i},\theta_{\gamma_i},\phi_{\gamma_i}) dLips_{n+l}(P\to k_1...k_n, k_{n+1}...k_{n+l}) =$$

$$\sum_{l=0} \exp(-F)\frac{1}{l!}\prod_{i=1}^{l}\Big[f(k_{\gamma_i},\theta_{\gamma_i},\phi_{\gamma_i}) dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W^{n+i}_{n+i-1}\Big]\times$$

$$dLips_n(P\to \bar{k}_1...\bar{k}_n), \tag{7}$$

$$\{k_1,\ldots,k_{n+l}\} = \mathbf{T}\Big(k_{\gamma_l},\theta_{\gamma_l},\phi_{\gamma_l},\mathbf{T}\Big(\ldots,\mathbf{T}\Big(k_{\gamma_1},\theta_{\gamma_1},\phi_{\gamma_1},\{\bar{k}_1,\ldots,\bar{k}_n\}\Big)\ldots\Big),$$

$$F=\int_{k_{min}}^{k_{max}} dk_\gamma d\cos\theta_\gamma d\phi_\gamma f(k_\gamma,\theta_\gamma,\phi_\gamma).$$

- The Green parts of rhs. alone, give crude distribution over tangent space (orthogonal set of variables k_i,θ_i,ϕ_i).

Phase space and crude distribution

- Factors f (W' ignored) must be integrable over coordinates. Regulators of singularities necessary, but simple.

- If we request from infrared regulators, f and F that

$$\sigma_{tangent} = 1 =$$
$$\sum_{l=0} \exp(-F)\frac{1}{l!}\prod_{i=1}^{l}\Big[f(k_{\gamma_i},\theta_{\gamma_i},\phi_{\gamma_i})dk_{\gamma_i}d\cos\theta_{\gamma_i}d\phi_{\gamma_i}\Big]$$

we get Poissonian distribution in l.

- Sum rules originating from perturbative approach (KLM theorem) are necessary to inccorporate dominant part of virtual corrections, into the scheme. We get Monte Carlo solution of PHOTOS type.

- For that to work, real emission and virtual corrections need to be calculated and their factorization properties analyzed. Choice for f and G are fixed from that.

- If such conditions are fulfilled construction of Monte Carlo algorithm is prepared.

- Truncate $\sigma_{tangent}|_{\mathcal{O}(\alpha),\mathcal{O}(\alpha^2)}$, $\longrightarrow$ phase space in single/double photon mode.

Important property of fully differential distribution

- Fully differential single photon emission formula in Z decay reads:

$$X_f = \frac{Q'^2 \alpha(1-\Delta)}{4\pi^2 s} s^2 \quad \left\{ \frac{1}{(k'_+ k'_-)} \left[\frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t,u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t',u) \right] \right\}$$

- Variables in use:

$$s = 2p_+ \cdot p_-, \quad s' = 2q_+ \cdot q_-, \quad t = 2p_+ \cdot q_+, \quad t' = 2p_+ \cdot q_-,$$
$$u = 2p_+ \cdot q_-, \quad u' = 2_- \cdot q_+, \quad k'_\pm = q_\pm \cdot k, \quad x_k = 2E_\gamma/\sqrt{s}$$

- The Δ term is responsable for final state mass dependent terms, p_+, p_-, q_+, q_-, k denote four-momenta of incoming positron, electron beams, outcoming muons and bremsstrahlung photon.

- Factorization of first order matrix element and fully differential distribution breaks at the level $\frac{\alpha^2}{\pi^2} \simeq 10^{-4}$

Important property of fully differential distribution

- after trivial manipulation it can be written as:

$$X_f = \frac{Q'^2 \alpha(1-\Delta)}{4\pi^2 s} s^2 \quad \Bigg\{ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_-} \left[\frac{d\sigma_B}{d\Omega}(s,t,u') + \frac{d\sigma_B}{d\Omega}(s,t',u) \right]$$

$$+ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_+} \left[\frac{d\sigma_B}{d\Omega}(s,t,u') + \frac{d\sigma_B}{d\Omega}(s,t',u) \right] \Bigg\}$$

- In PHOTOS the following kernel is used (decay channel, decay particle orientation, independent, (essential: universal interference wt introduced too):

$$X_f^{PHOTOS} = \frac{Q'^2 \alpha(1-\Delta)}{4\pi^2 s} s^2 \Bigg\{$$

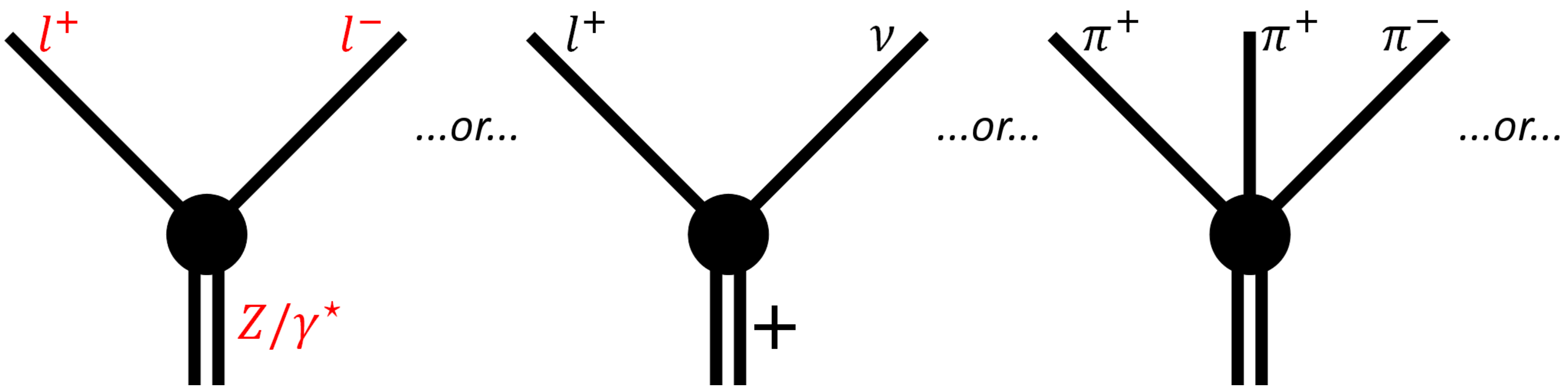
$$\frac{1}{k'_+ + k'_-} \frac{1}{k'_-} \left[\left(1 + (1-x_k)^2\right) \frac{d\sigma_B}{d\Omega}\left(s, \frac{s(1-\cos\Theta_+)}{2}, \frac{s(1+\cos\Theta_+)}{2}\right) \right] \frac{(1+\beta\cos\Theta_\gamma)}{2}$$

$$+ \frac{1}{k'_+ + k'_-} \frac{1}{k'_+} \left[\left(1 + (1-x_k)^2\right) \frac{d\sigma_B}{d\Omega}\left(s, \frac{s(1-\cos\Theta_-)}{2}, \frac{s(1+\cos\Theta_-)}{2}\right) \right] \frac{(1-\beta\cos\Theta_\gamma)}{2} \Bigg\}$$

$$\text{where}: \Theta_+ = \angle(p_+, q_+),\ \Theta_- = \angle(p_-, q_-)$$

$\Theta_\gamma = \angle(\gamma, \mu^-)$ are defined in (μ^+, μ^-)-pair rest frame

First order spin amplitudes



- *The formula which we had on previous slide could be constructed because the Born level matrix elemet (and resulting Born level distribution) relates with the one of first order in α_{QED} through convolution of positively defined function (I will use it as emission kernel) (Berends Kleiss Jadach 1982).*

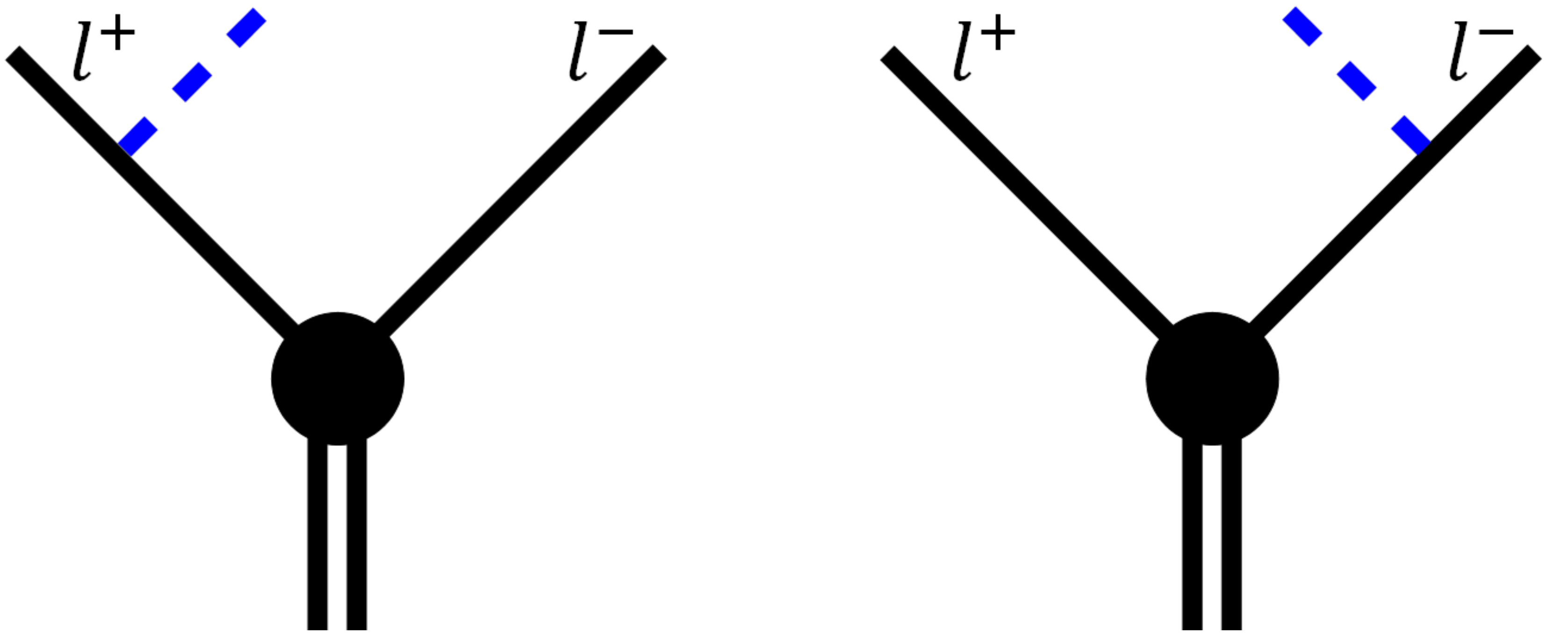
- *Does such convolution hold for other processes, even if we are concerned with the first order only?*

- *Paper by R. Kleiss from 1992 tells us that it will not hold at level of $(\frac{\alpha}{\pi})^2 \simeq 10^{-5}$.*

- *Comment, these properties are important for all variants of NLO factorizations.*

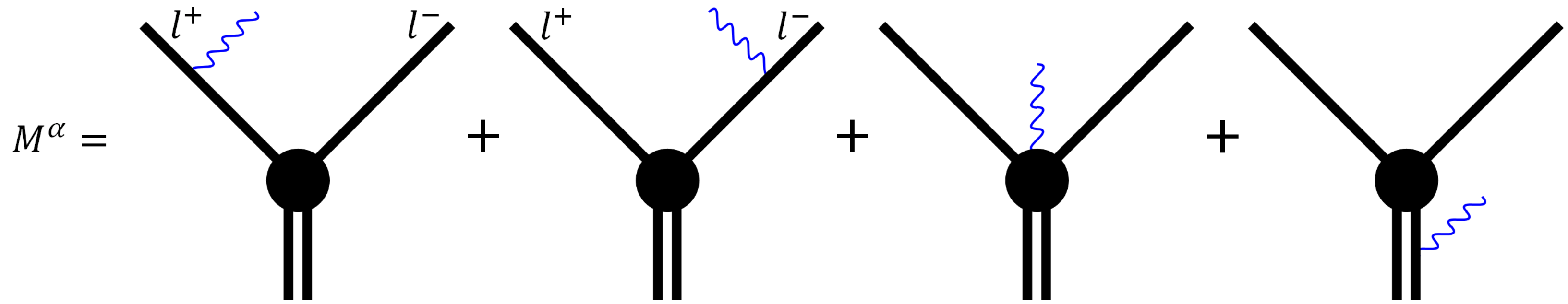
- *All these issues can be solved with studies of matrix elements only.*

First order spin amplitudes



- *Structure of singularities for the first order corrections to decay of Z/γ^* which we will use as an example.*
- *Two kinematical branches need to be taken into account.*
- *Fortunately kinematical parametrizations for the two branches have identical phase space Jacobians. It simplifies tasks for multiphoton configurations.*

First order spin amplitudes



- *Feynman diagrams for FSR in* Z/γ^* *decays*
- *Out of the first two diagrams distribution for* Z/γ *decay was obtained.*
- *Other two diagrams appear e.g. in scalar QED, and/or in decays of W's or B mesons.*
- *Let us look into sub-structure of these amplitudes.*

First order spin amplitudes

Matrix Element Z/γ^* decay, (formalism $\sim$ Kleiss-Stirling methods):

- $$I = I^A \qquad\qquad + I^B \qquad\qquad + I^C$$
- $$I = \not{J}\left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\right] - \left[\frac{1}{2}\frac{\not{e}_1\not{k}_1}{p\cdot k_1}\right]\not{J} + \not{J}\left[\frac{1}{2}\frac{\not{e}_1\not{k}_1}{q\cdot k_1}\right]$$
- Decomposes into 3 parts. Each is independently gauge invariant, valid for "any" $\not{J}$.
- Only $|I^A|^2$ contributes to infrared singularities.
- Terms I^B and I^C contribute to collinear big logarithms.
- We could expect another term I^D which would not contribute neither to collinear nor soft divergent/large logarithms (once integration is performed)

structure of singularities apparent already at amplitude level

First order spin amplitudes

What happens for other decays

1. $W \to l\nu_l\gamma$: I^A, I^B and I^D dependent on electroweak calculation scheme.
2. $B^0 \to \pi^+ K^- \gamma$: I^A only
3. $B^+ \to \pi^0 K^+ \gamma$: I^A only
4. $\gamma^* \to \pi^+ \pi^- \gamma$: I^A, and I^D
5. $\tau^+ \to \pi^+ \nu_\tau \gamma$: I^A and I^D
6. ...

It is important that in all cases, and not only for processes of QED, amplitudes can be constructed from the same building blocks.

These properties of amplitudes translate into properties of distributions and that is why exact PHOTOS algorithm for single photon emission can be constructed.

If non dominat terms can be neglected algorithm simlifies and process dependent weights can be replaced by the ones dependinch on charges and spins of outgoing particles.

Single emission

1. **Solution for single emission works perfect.**
2. **Technical precision controlled to precision better than statistical error of 100 Mevts.**
3. **An example where interference between emission from two charged lines is hidden in exact process dependent kernel, but must be added if basically identical one is used.**
4. **Web page with multitude of automated tests (RECOMENDATION: to be repeated after installation in collaboration software): http://mc-tester.web.cern.ch/MC-TESTER/**
5. **Let us go to iteration, used in solution for double and muliple photon emission modes.**

Elementary test of principle

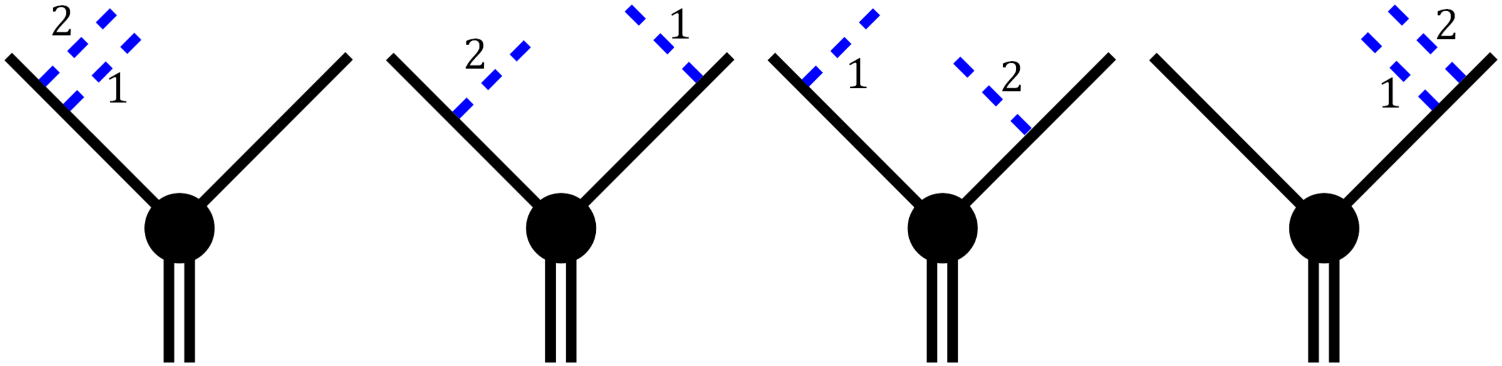
- Do PHOTOS generate the LL contribution to lepton spectra?
- Formal solution of QED evolution equation can be written as:

$$D(x,\beta_{ch}) = \delta(1-x)+\beta_{ch}P(x)+\frac{1}{2!}\beta_{ch}^2\{P\times P\}(x)+\frac{1}{3!}\beta_{ch}^3\{P\times P\times P\}(x)+\dots \tag{8}$$

where $P(x) = \delta(1-x)(\ln\varepsilon + 3/4) + \Theta(1-x-\varepsilon)\frac{1}{x}(1+x^2)/(1-x)$
and $\{P \times P\}(x) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(x - x_1x_2)P(x_1)P(x_2)$.

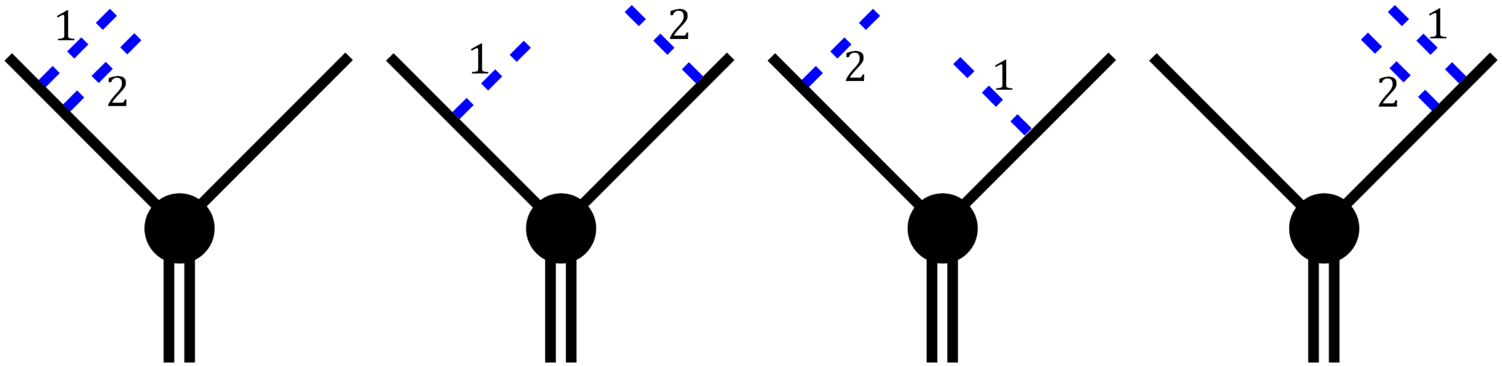
- In LL contributing regions, phase space Jacobian's of PHOTOS trivialize (CPC 1994). and the expression given above is obtained in a straightforward manner. In fact for each of the outcoming charged lines simultaneously.
- But it is only a limit! **PHOTOS treat phase space corners exactly.** We had to understand at spin amplitude, and exact distribution, levels why formula (8) work, keeping in mind what happens with amplitudes non leading parts.

Multiple photon emission



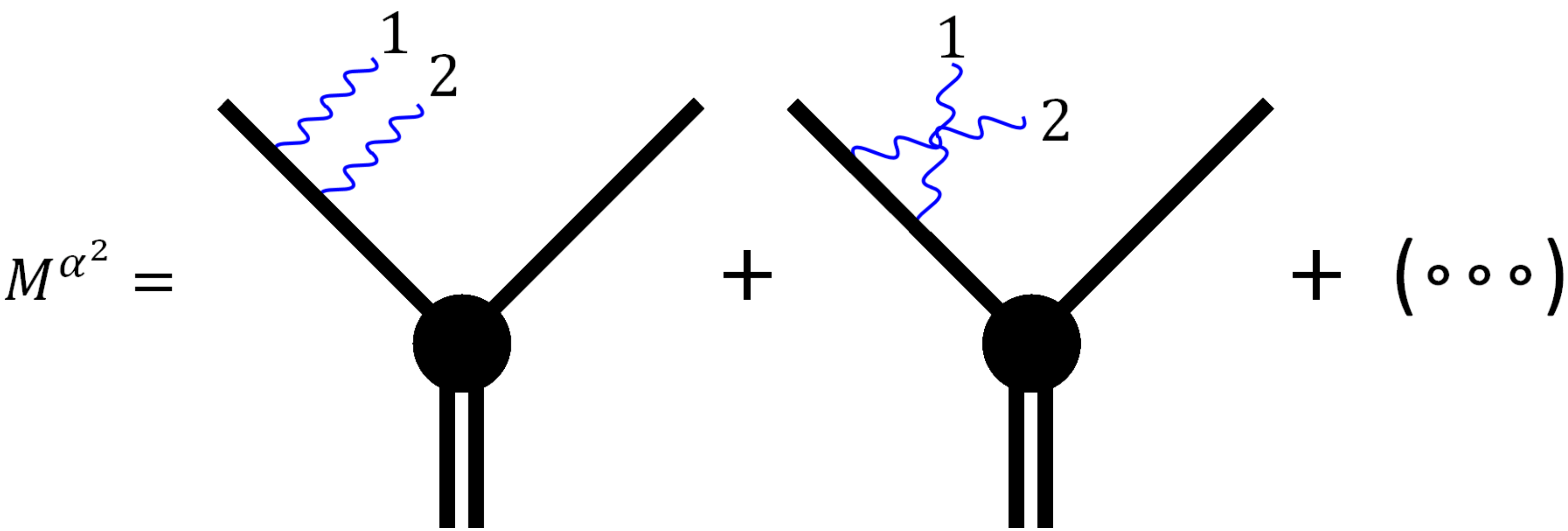
- *To generate consecutive photons, PHOTOS simply iterates its single photon algorithm.*
- *Previously generated photons are treated a any other decay products.*
- *We generate photon 1 (each leg one after another)*
- *We include interference or matrix element weight*
- *And in the same way photon 2.*
- *previously generated photon(s) we remove from kinematical configuration, using reduction procedure.*

Multiple photon emission



- *We can produce such point in phase space starting with generation of photon 2 and continuing with 1.*
- *Each of the two generation chains cover all phase space. There is no phase space ordering in use. Instead we have statistical factor* $\frac{1}{l!}$ *from*
- *Such solution must be confronted with distributions obtained from matrix elements.*
- *Comparisons with distributions obtained from double and triple photon amplitudes were performed in 1994.*
- *Now let us look at properties of spin amplitudes.*

Multiple photon emission



- *We have to check if description given in two previous slides justifies with properties of spin amplitudes.*
- *Iterative algorithm? What with interferences of consecutive emissions?*
- *It is important to check if such properties are process dependent or generalize.*
- *My decade long work under leadership of S. Jadach on* e^+e^- *generators provided help.*
- *Is double photon emission amplitude build from terms we know from first order?*
- *From calculation it is clear that the structure of* $Z/\gamma^* \to l^+l^-\gamma\gamma$ *generalizes to other processes.*

Multiple photon emission

$\mathcal{E}$xact $\mathcal{M}$atrix $\mathcal{E}$lement: $Z \to \mu^+\mu^-\gamma\gamma$ written explicitly

- We use conventions from paper A. van Hameren, Z.W., EPJC 61 (2009) 33. Expressions are valid for any current J, (also for QCD part proportional to $\{T^A T^B\}$, T^A is for first T^B for second gluon.

- To get complete amplitude sum the gauge invariant parts, add spinors, eg. $\bar{u}(p)$ and $v(q)$; k_1/k_2 e_1/e_2 denotes momenta/polarizations for 1-st/2-nd photon/gluon. Factors of parts coincide with those of first order.

$$I_1^{\{1,2\}} = \frac{1}{2}\, \not{J} \left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right) \qquad eikonal$$

$$I_{2l}^{\{1,2\}} = -\frac{1}{4}\left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\frac{\not{e}_2\not{k}_2}{p\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right)\frac{\not{e}_1\not{k}_1}{p\cdot k_1}\right]\not{J} \qquad \beta_1$$

Multiple photon emission

$$I_{2r}^{\{1,2\}} = \frac{1}{4} \not{J} \left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1} \right) \frac{\not{k}_2 \not{e}_2}{q\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2} \right) \frac{\not{k}_1 \not{e}_1}{q\cdot k_1} \right] \qquad \beta_1$$

$$I_3^{\{1,2\}} = -\frac{1}{8} \left(\frac{\not{e}_1 \not{k}_1}{p\cdot k_1} \not{J} \frac{\not{k}_2 \not{e}_2}{q\cdot k_2} + \frac{\not{e}_2 \not{k}_2}{p\cdot k_2} \not{J} \frac{\not{k}_1 \not{e}_1}{q\cdot k_1} \right) \qquad start for \beta_2 \ldots$$

$$I_{4p}^{\{1,2\}} = \frac{1}{8} \frac{1}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2} \left(\frac{\not{e}_1 \not{k}_1 \not{e}_2 \not{k}_2}{p\cdot k_1} + \frac{\not{e}_2 \not{k}_2 \not{e}_1 \not{k}_1}{p\cdot k_2} \right) \not{J}$$

$$I_{4q}^{\{1,2\}} = \frac{1}{8} \not{J} \frac{1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2} \left(\frac{\not{k}_2 \not{e}_2 \not{k}_1 \not{e}_1}{q\cdot k_1} + \frac{\not{k}_1 \not{e}_1 \not{k}_2 \not{e}_2}{q\cdot k_2} \right)$$

$$I_{5pA}^{\{1,2\}} = \frac{1}{2} \not{J} \frac{k_1\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2} \left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} \right) \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} \right)$$

$$I_{5pB}^{\{1,2\}} = -\frac{1}{2} \not{J} \frac{1}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2} \left(\frac{k_1\cdot e_2 k_2\cdot e_1}{k_1\cdot k_2} - e_1\cdot e_2 \right)$$

$$I_{5qA}^{\{1,2\}} = \frac{1}{2} \not{J} \frac{k_1\cdot k_2}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2} \left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} \right) \left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} \right)$$

Multiple photon emission

$$I_{5qB}^{\{1,2\}} = -\frac{1}{2}\not{J}\frac{1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left(\frac{k_1\cdot e_2 k_2\cdot e_1}{k_1\cdot k_2} - e_1\cdot e_2\right)$$

$$I_{6B}^{\{1,2\}} = -\frac{1}{4}\frac{k_1\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left[+\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_1\cdot k_2}\right)\frac{\not{e}_2\not{k}_2}{p\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2}\right)\frac{\not{e}_1\not{k}_1}{p\cdot k_1}\right]\not{J}$$

$$I_{7B}^{\{1,2\}} = -\frac{1}{4}\not{J}\frac{k_1\cdot k_2}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left[+\left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_1\cdot k_2}\right)\frac{\not{k}_2\not{e}_2}{q\cdot k_2} + \left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2}\right)\frac{\not{k}_1\not{e}_1}{q\cdot k_1}\right]$$

- for **exponentiation** one use **separation** into 3 parts only.
- $I_3^{\{1,2\}}$, $I_{4p}^{\{1,2\}}$, $I_{4q}^{\{1,2\}}$ were studied to improve options for PHOTOS kernel iteration. Things are less transparent, concept of effective fermionic momenta must be used eg. $u((p-k_1)_{long})\bar{u}((p-k_1)_{long}) \simeq \not{p} - \not{k}_1$, it can be interpreted that way in some limits ony. We got what is necessary! Parts for each kinematical branch. In fact sub-structures for amplitudes of other theories proceeses appear as well.
- Separation of β_2 into parts, here of no use. No match with singularities of QED.

Second order amplitudes of QCD.

Matrix Element: $q\bar{q} \to J gg$ *- part proportional to* $T^A T^B$ *fermion spinors dropped*

$$I_{lr}^{(1,2)} = \left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not{e}_1 \not{k}_1}{2p\cdot k_1} \right) \not{J} \left(\frac{\not{k}_2 \not{e}_2}{2q\cdot k_2} + \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2} \right)$$

$$I_{ll}^{(1,2)} = \frac{p\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2} \left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not{e}_1 \not{k}_1}{2p\cdot k_1} \right) \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not{e}_2 \not{k}_2}{2p\cdot k_2} \right) \not{J}$$

$$I_{rr}^{(1,2)} = \not{J} \frac{q\cdot k_1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2} \left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not{k}_1 \not{e}_1}{2q\cdot k_1} \right) \left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not{k}_2 \not{e}_2}{2q\cdot k_2} \right)$$

$$I_{e}^{(1,2)} = \not{J} \left(1 - \frac{p\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2} - \frac{q\cdot k_1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2} \right) \left(\frac{k_1\cdot e_2}{k_1\cdot k_2} \frac{k_2\cdot e_1}{k_1\cdot k_2} - \frac{e_1\cdot e_2}{k_1\cdot k_2} \right)$$

Remainder:

$$I_{p}^{(1,2)} = -\frac{1}{4} \frac{1}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2} \left(\frac{\not{e}_1 \not{k}_1 \not{e}_2 \not{k}_2 - \not{e}_2 \not{k}_2 \not{e}_1 \not{k}_1}{k_1\cdot k_2} \right) \not{J}$$

$$I_{q}^{(1,2)} = -\frac{1}{4} \not{J} \frac{1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2} \left(\frac{\not{k}_1 \not{e}_1 \not{k}_2 \not{e}_2 - \not{k}_2 \not{e}_2 \not{k}_1 \not{e}_1}{k_1\cdot k_2} \right)$$

Second order amplitudes of QCD.

$\mathcal{M}$atrix $\mathcal{E}$lement: $q\bar{q} \to Jgg$ - part proportional to $T^B T^A$ fermion spinors dropped

$$I_{lr}^{(2,1)} = \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not{e}_2 \not{k}_2}{2p\cdot k_2} \right) \not{J} \left(\frac{\not{k}_1 \not{e}_1}{2q\cdot k_1} + \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1} \right)$$

$$I_{ll}^{(2,1)} = \frac{p\cdot k_1}{p\cdot k_2 + p\cdot k_1 - k_2\cdot k_1} \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not{e}_2 \not{k}_2}{2p\cdot k_2} \right) \left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not{e}_1 \not{k}_1}{2p\cdot k_1} \right) \not{J}$$

$$I_{rr}^{(2,1)} = \not{J} \frac{q\cdot k_2}{q\cdot k_2 + q\cdot k_1 - k_2\cdot k_1} \left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not{k}_2 \not{e}_2}{2q\cdot k_2} \right) \left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not{k}_1 \not{e}_1}{2q\cdot k_1} \right)$$

$$I_{e}^{(2,1)} = \not{J} \left(1 - \frac{p\cdot k_1}{p\cdot k_2 + p\cdot k_1 - k_2\cdot k_1} - \frac{q\cdot k_2}{q\cdot k_2 + q\cdot k_1 - k_2\cdot k_1} \right) \left(\frac{k_2\cdot e_1}{k_2\cdot k_1} \frac{k_1\cdot e_2}{k_2\cdot k_1} - \frac{e_2\cdot e_1}{k_2\cdot k_1} \right)$$

$$I_{p}^{(2,1)} = -\frac{1}{4} \frac{1}{p\cdot k_2 + p\cdot k_1 - k_2\cdot k_1} \left(\frac{\not{e}_2 \not{k}_2 \not{e}_1 \not{k}_1 - \not{e}_1 \not{k}_1 \not{e}_2 \not{k}_2}{k_2\cdot k_1} \right) \not{J}$$

$$I_{q}^{(2,1)} = -\frac{1}{4} \not{J} \frac{1}{q\cdot k_2 + q\cdot k_1 - k_2\cdot k_1} \left(\frac{\not{k}_2 \not{e}_2 \not{k}_1 \not{e}_1 - \not{k}_1 \not{e}_1 \not{k}_2 \not{e}_2}{k_2\cdot k_1} \right)$$

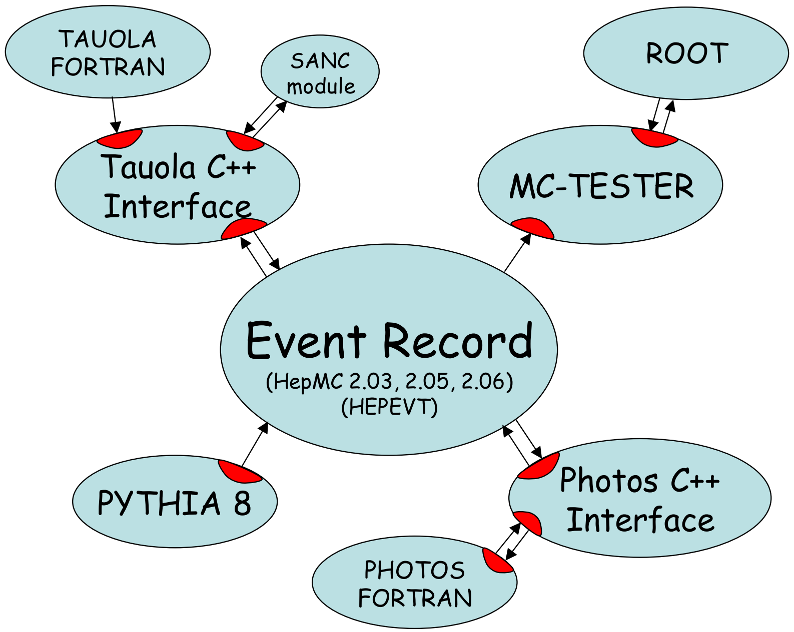
Summary

1. PHOTOS Monte Carlo is for simulation of multiphoton FSR bremsstrahlung.
2. Program is designed to help generate correlated samples: events with and without FSR bremsstrahlung.
3. For processes mediated by Z/γ' and W's high precision is investigated.
4. Important for program construction were presented here studies of spin amplitudes. Structure of their gauge invariant parts is used in definition of photon emission kernel.
5. Remaining parts of amplitudes are needed for discussion of systematic errors, for optimalization of program performance or for construction correcting weights.
6. For some processes eg. where matrix element is obtained from scalar QED introduction of data constrained form factors may be necessary.
7. New version of program, using `HepMC` event record of C++ is available for tests.

EXTRA TRANSPARENCIES

MOSTLY NUMERICAL TESTS

Simulation parts communicate through event record:



- Parts:

- hard process: (Born, weak, new physics),
- parton shower,
- τ decays
- QED bremsstrahlung

- High precision achieved

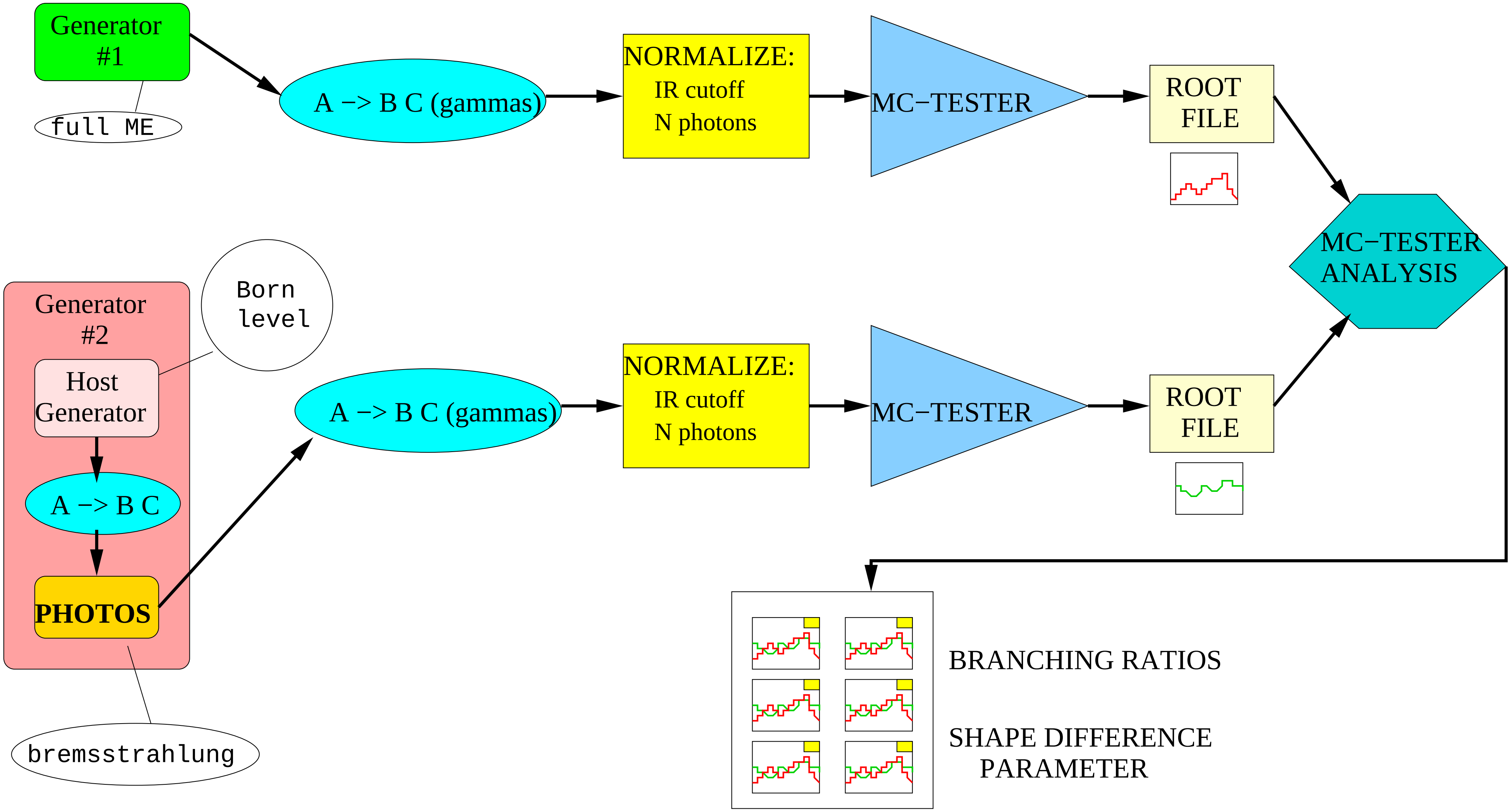
- Detector studies: acceptance, resolution lepton with or wihout photon.

Such organization requires:

- Good control of factorization (theory)
- Good understanding of tools on user side.

Extra transparencies: MC-TESTER

MC-TESTER to test PHOTOS/TAUOLA



Extra transparencies: MC-TESTER

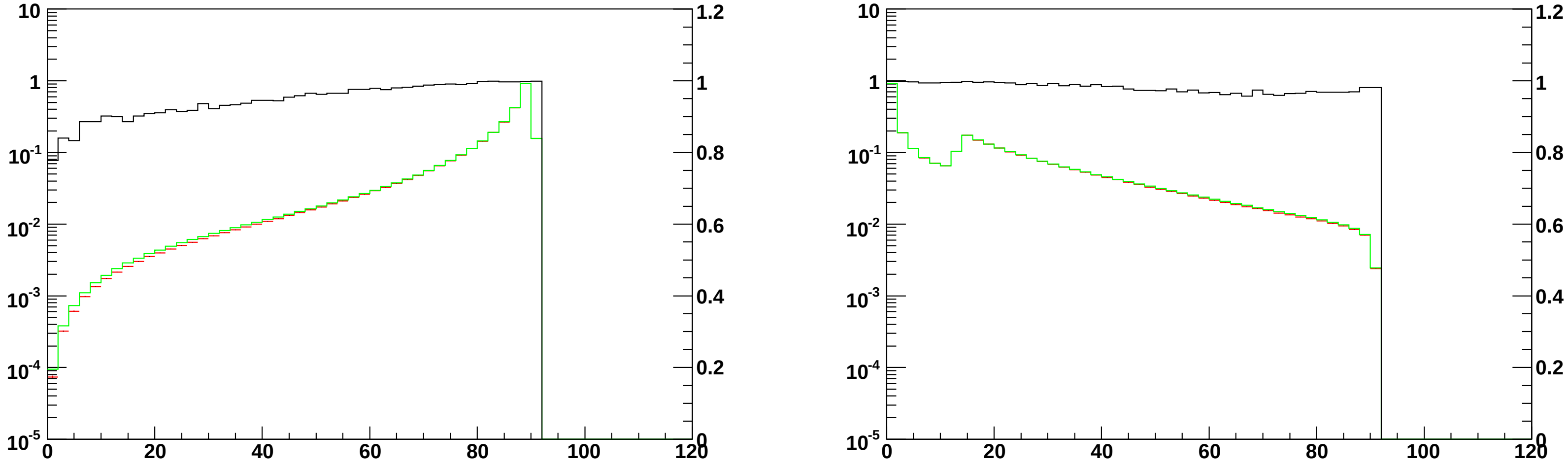


Figure 1: Comparison of standard PHOTOS and KORALZ for single photon emission. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.00534. In the right frame the invariant mass of $\mu^-\gamma$; SDP=0.00296. The histograms produced by the two programs (logarithmic scale) and their ratio (linear scale, black line) are plotted in both frames. The fraction of events with hard photon was 17.4863 $\pm$ 0.0042% for KORALZ and 17.6378 $\pm$ 0.0042% for PHOTOS.

Extra transparencies: MC-TESTER

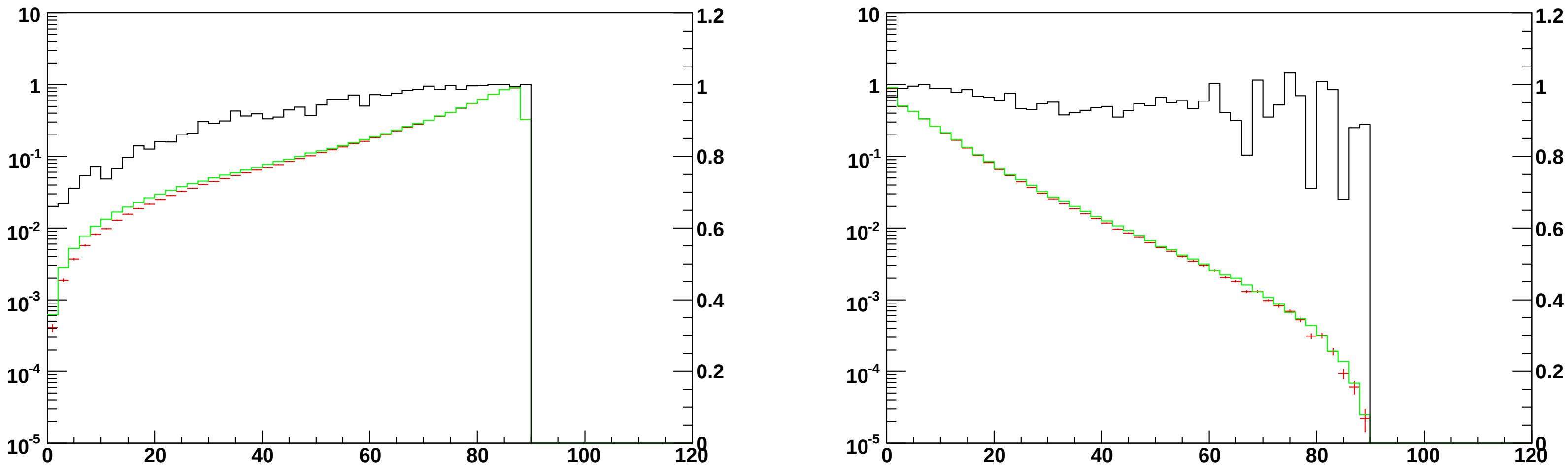


Figure 5: Comparisons of standard PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP= 0.00918 (shape difference parameter). In the right frame the invariant mass of the $\gamma\gamma$ pair; SDP=0.00268. The fraction of events with two hard photons was 1.2659 $\pm$ 0.0011% for KKMC and 1.2952 $\pm$ 0.0011% for PHOTOS.

Extra transparencies: MC-TESTER

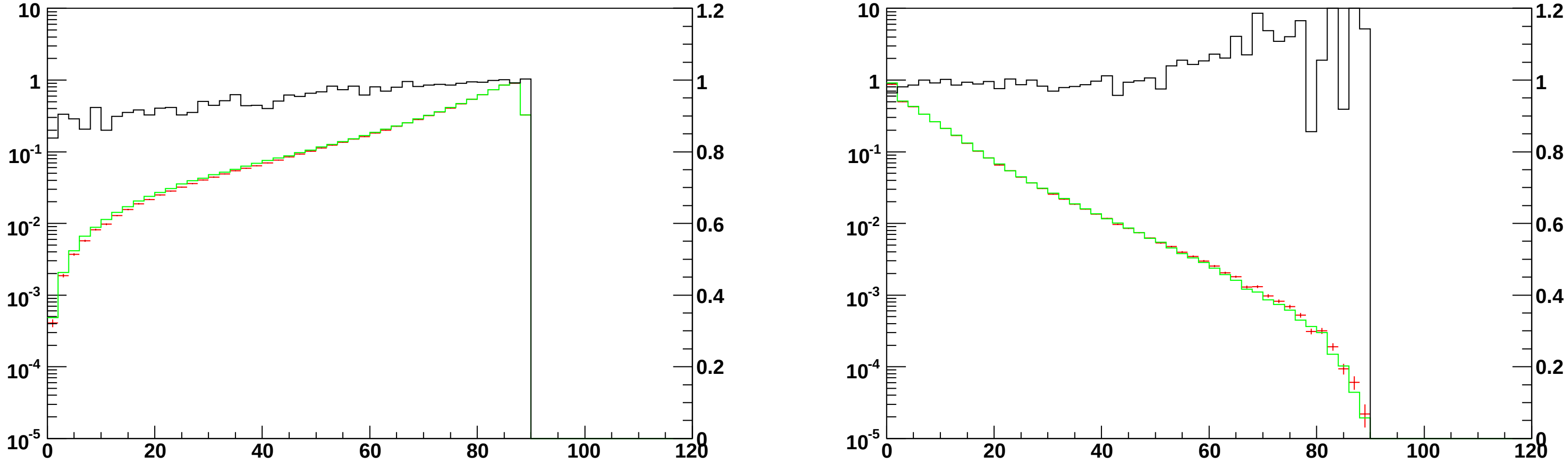
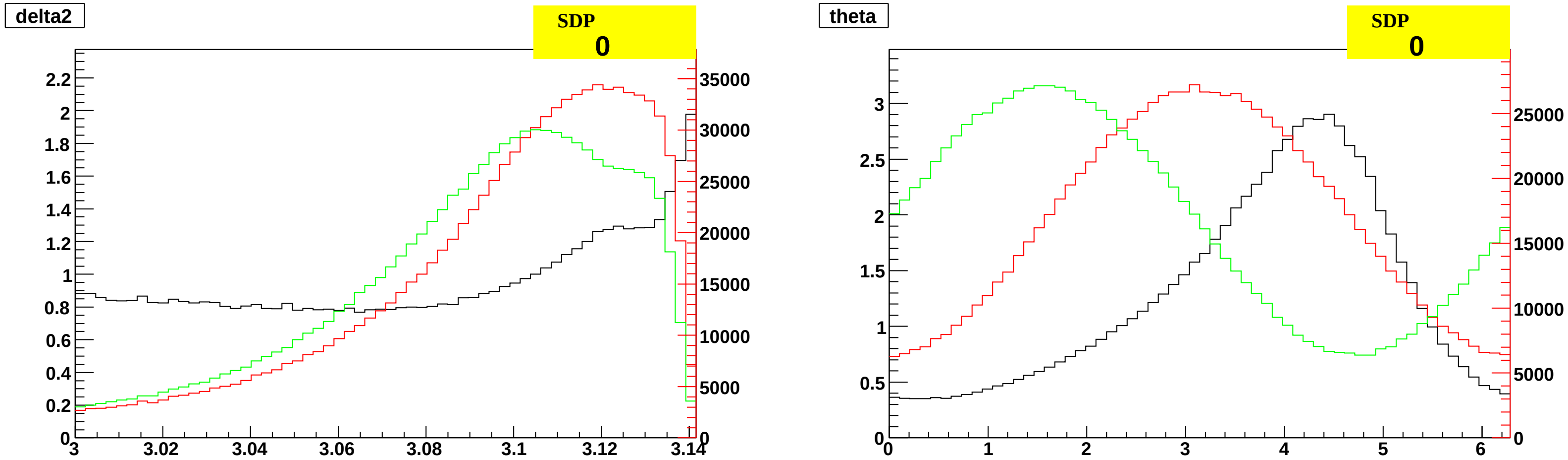


Figure 6: Comparisons of improved PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP= 0.00142. In the right frame the invariant mass of the $\gamma\gamma$; SDP=0.00293. The fraction of events with two hard photons was 1.2659 $\pm$ 0.0011% for KKMC and 1.2868 $\pm$ 0.0011% for PHOTOS.

Extra transparencies: MC-TESTER

Example: Distribution for Higgs parity



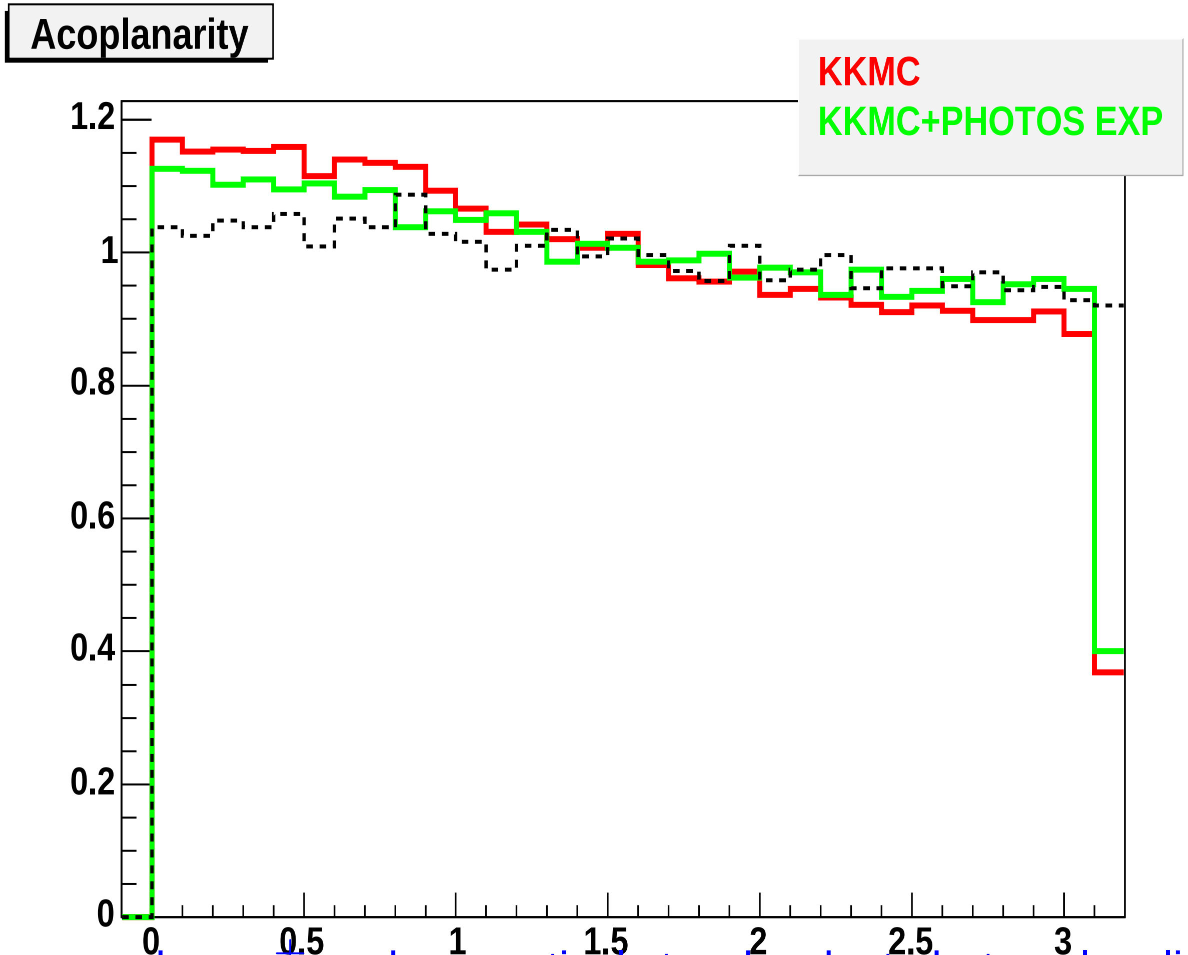
(a) $\pi^+\pi^-$ acollinearity distribution ($\approx \pi$))

(b) $\pi^+\pi^-$ acoplanarity distribution

Figure 1: Transverse spin observables for the H boson for $\tau^\pm \rightarrow \pi^\pm \nu_\tau$. Distributions are shown for scalar higgs (red), scalar-pseudoscalar higgs with mixing angle $\frac{\pi}{4}$ (green) and the ratio between the two (black).

Extra transparencies: MC-TESTER

𝒜coplanarity distribution – ℒooks good



Two plane spanned on μ^+ and respectively two hardest photons localized in the same hemisphere as μ^+. In exlusive exponentiation this asymmetry appears with second order matrix element only.