Review of Spin Rotators for LC

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- Spin dynamics and design criteria
- Spin rotator location
- Spin rotator options
- CLIC case: two options
- Summary and conclusions

Spin Dynamics Summary

⇒ The precession motion for the magnetic moment of an accelerating relativistic particle is given by the solution of the Thomas-BMT equation, see for instance Bryan W. Montague, Phys. Rep., **113**, No. 1, 8-13 (1984)

- Spin Precession

$$\phi_s = G\,\gamma_0\,\alpha$$

- Mean polarization:

$$< P_z >= P_0 e^{\frac{-(G\gamma_0\alpha\sigma_\delta)^2}{2}}$$

- Relative depolarization:

$$1 - \frac{< P_z >}{P_0}$$

- Where

Symbol	Value	Description
G	0.00115965219	anomalous momentum of the electron
α	-	arc bending angle
γ_0	-	relativistic factor
σ_δ	-	energy spread

Spin Depolarization

- In the damping rings, if the spin direction is not perpendicular to the horizontal plane, spin precedes during the storage

- Because the precession frequency depends on the beam energy, the precession phase is randomized by energy spread

- This randomization causes a significant depolarization. The spin direction has to be perpendicular to the horizontal plane to avoid this depolarization effect by the precession

Spin Rotator Design Criteria

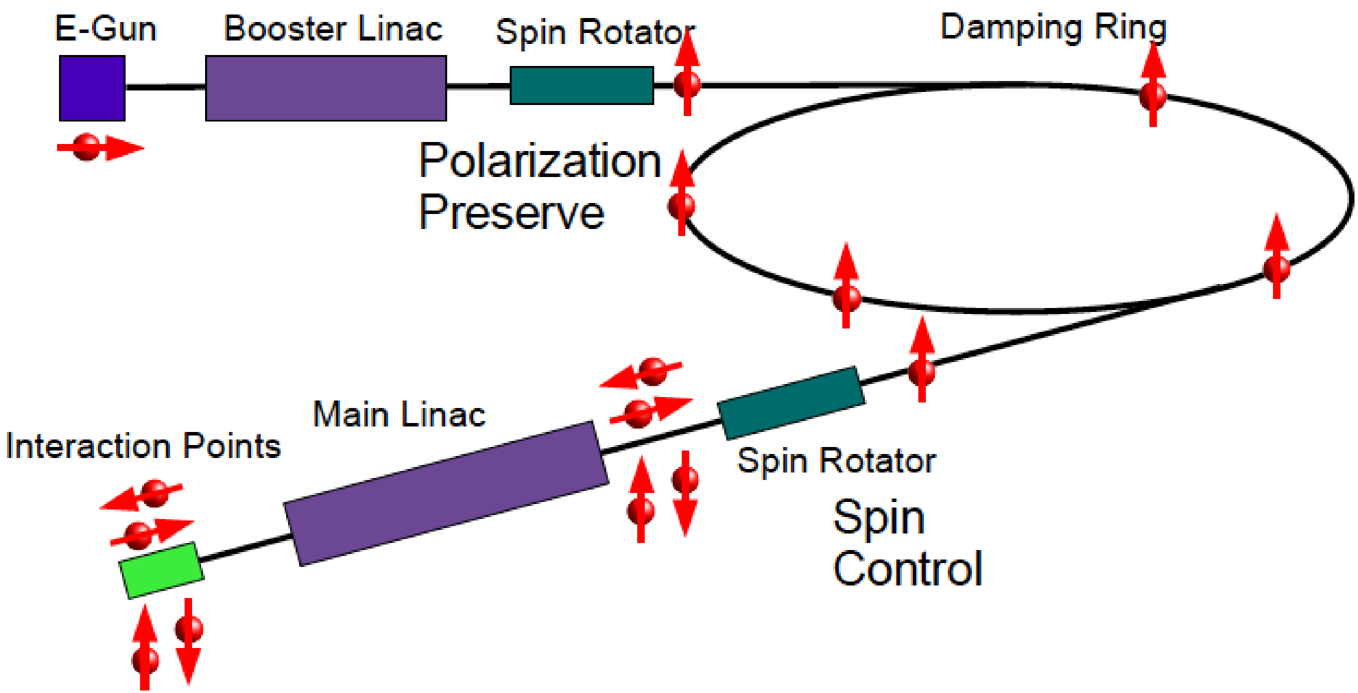
- Design Criteria (P. Emma for NLC, 1994)
  - *Spin should be orientable in any direction*
  - *Net momentum compaction must be small such that energy fluctuations do not become longitudinal position fluctuations (less than 100 μm bunch length @ IP for NLC)*
  - *It should be located such that total spin diffusion due to energy spread is small*
  - *System should not dilute significantly the beam transverse emittance (small energy spread)*
  - *System should be short, simple and robust*

Spin Rotator Location

- Spin precedes around the magnetic field

$\Rightarrow$ Longitudinal Polarization should be perpendicular before DR injection

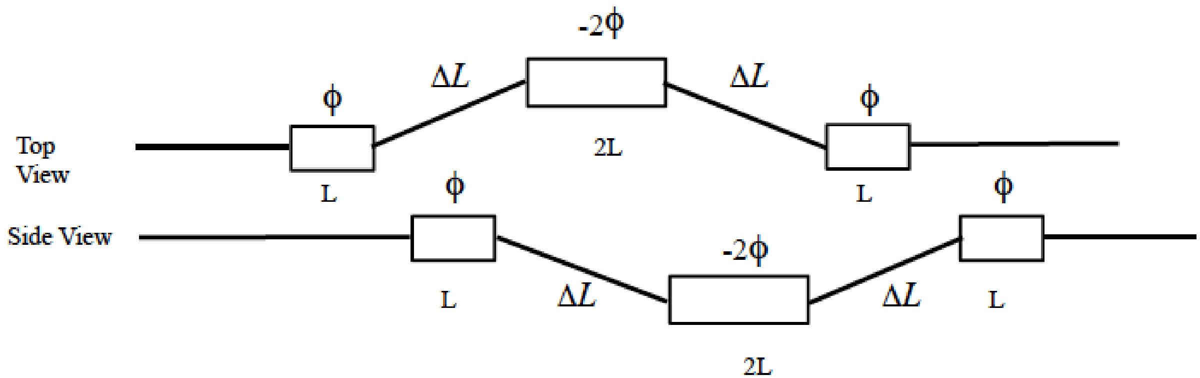
$\Rightarrow$ Polarization control after DR



Half Serpent Spin Rotator

⇒ Very simple schema: the system requires only nested horizontal and vertical chicanes; but they inevitably dilute the transverse emittances through synchrotron radiation emission

⇒ But a vertical bending schemes are not feasible:



Disadvantages

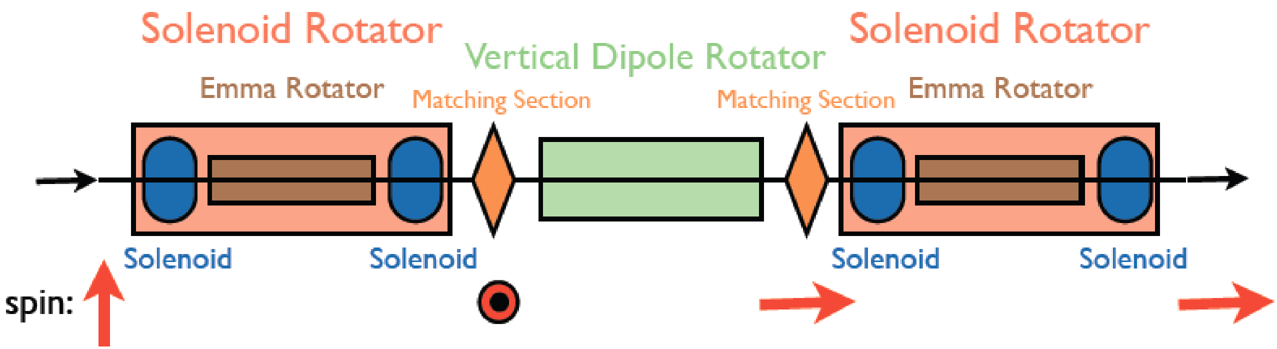
- Each vertical bend would have to be about 1000 meters long to keep vertical emittance from growing even 2%
- R_{56} 800 meters in such a setup – totally unacceptable
- Spin rotation is fixed, we want full variability in exiting polarization

Solenoid Based Spin Rotator

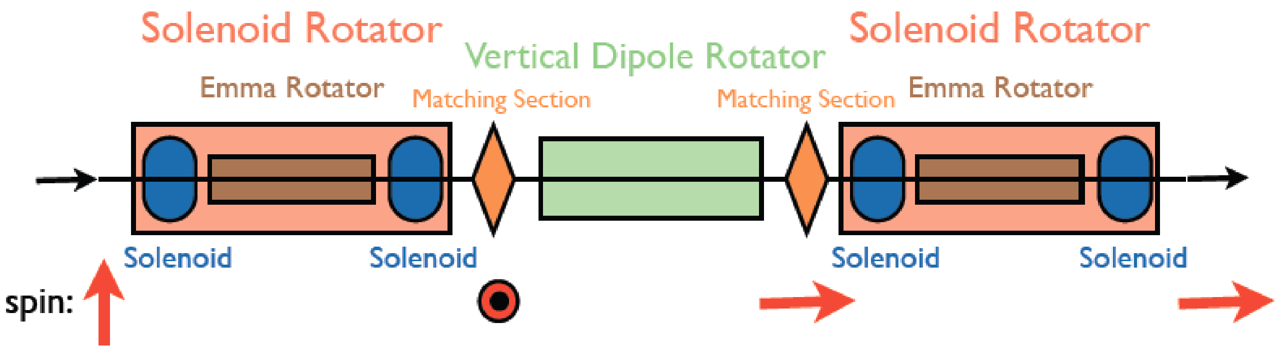
$\Rightarrow$ First designed by Paul Emma for NLC

- Spin Rotation is achieved by two solenoids with a bending magnet in between
- Each solenoid is split in two parts separated by a *reflector* $\begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}$ to correct for couplings $\Rightarrow$ there are four solenoids in total
- The central bending section must rotate the spin by 90 degrees
- This configuration allows arbitrary spin orientation

$\Rightarrow$ **Sketch**



Emma Rotator

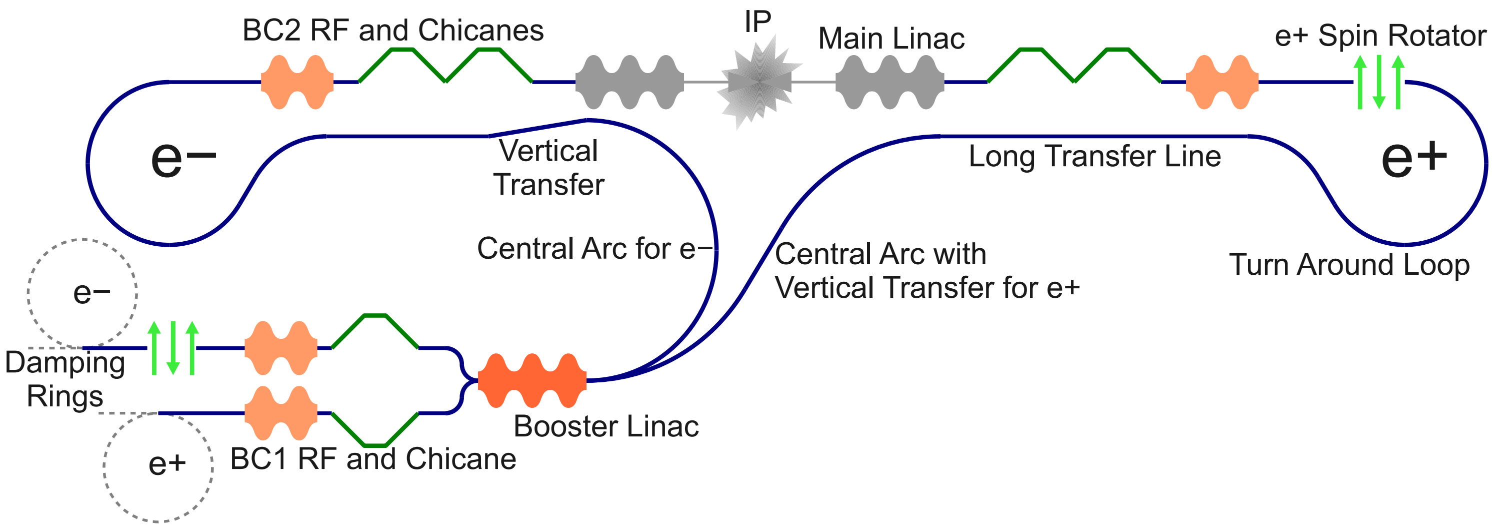


Description

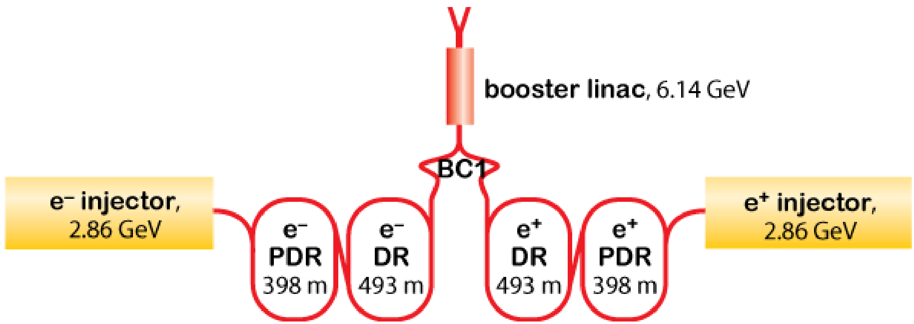
- Reflector beamline : four FODO cells with 90 degrees phase advance in X and 45 degrees phase advance in Y
- Bend section : mini arc composed by three FODO cells with 90 degrees phase advance in X and Y (can be shortened)

Spin Rotator Location in CLIC

CLIC RTML layout (F.Stulle)



Previous layout



Spin Precession and Depolarization in CLIC

region	E_0 [GeV]	σ_δ	$\alpha_{\text{electrons}}$ **[rad]**	$\alpha_{\text{positrons}}$ **[rad]**
exit of damping rings to bc1	2.86	0.13%	0	0
exit of bc1 to booster	2.86	1.04%	0	0
exit of booster to bc2	9	0.33%	$\pi - \pi$+HV-doglegs$= 0$	π+HV-doglegs$= \pi$
exit of bc2 to bds	9	1.64%	0	0
exit of main linac to ip	1500	0.35%	$\mathbf{1 \cdot 10^{-3}}$	$\mathbf{1 \cdot 10^{-3}}$

region	E_0 [GeV]	σ_δ	$1 - \frac{<P_z>}{P_0}$ [%]	$\phi_s = a\,\gamma_0\,\alpha$ [deg]	*n*-turns
exit of damping rings to bc1	2.86	0.13%	0	0	0
exit of bc1 to booster	2.86	1.04%	0	0	0
exit of booster to bc2 entrance	9	0.33%	0 / 2.2	$0 \;/\; 3676.4 \equiv 76.4$	0 / 10.2
exit of bc2 to bds	9	1.64%	0	0	0
exit of main linac to ip	1500	0.35%	**0.007**	**195**	**0.54**

$\Rightarrow$ From the point of view of the spin dynamics, ideal location for the spin rotators would probably be: before bc1 for the electrons, before bc2 for the positrons

$\Rightarrow$ Notice that, in case of a symmetric RTML where both spin rotators are placed before bc1 and assuming that the beam experiences a total bending angle $\alpha = \pi/2_{\text{booster}\to\text{bc2}}$ for each line, the total depolarization per beam is 0.56% per line. (with a precession of 5.1 n-turns)

Solenoid Strength

- Each of the four solenoids must be capable of providing a maximum of ± 45 degrees spin rotation

$$\psi_{\rm spin} = \pi/4, \qquad \text{with} \qquad \psi_{\rm beam} = \psi_{\rm spin}/2$$

- Solenoid strength

$$k = \frac{\psi_{\rm spin}}{2L} = \frac{B_z}{2(B_0\rho)}$$

Assuming 2.6 meters long solenoids (like ILC)

$$k = \frac{\pi/4}{2}\,\frac{1}{(L = 2.6\ \rm m)} = 0.15104\ \rm m^{-1}$$

$\Rightarrow$ The maximum longitudinal field is:

$$B_{z,max} \;=\; 2 \cdot k \cdot (B_0\rho) \;=\; 2 \cdot k \cdot \frac{E_0}{ec} = \; 2 \cdot 0.15104\ \rm m^{-1} \cdot \frac{E_0}{ec}$$

required magnetic field at 2.86 or 9 GeV is:

$$B_{z,max} \text{ @ } 2.86\ \rm GeV = \mathbf{2.9}\ T$$

$$B_{z,max} \text{ @ } 9\ \rm GeV = \mathbf{9.1}\ T$$

Bending Arc

- The bending section should rotate the spin by 90 degrees

$$\phi_s = a\,\gamma_0\,\alpha = \frac{\pi}{2}$$

$$\alpha \text{ @ } 2.86 \text{ GeV} = \frac{\pi/2}{a\ (\gamma_0 = 2.86e3/0.511)} = 0.24202 \text{ rad} = 13.867 \text{ degrees}$$

$$\alpha \text{ @ } 9 \text{ GeV} = \frac{\pi/2}{a\ (\gamma_0 = 9e3/0.511)} = 0.076908 \text{ rad} = 4.4065 \text{ degrees}$$

- Magnetic strength:

$$B\rho \text{ @ } 2.86 \text{ GeV} = \frac{pc}{ec} = \frac{2.86 \text{ GV}}{c} = \frac{2.86 \text{ GV}}{2.997925 \cdot 10^8 \text{ m/s}} = 9.5 \text{ T m}$$

$$B\rho \text{ @ } 9 \text{ GeV} = \frac{pc}{ec} = \frac{9 \text{ GV}}{c} = \frac{9 \text{ GV}}{2.997925 \cdot 10^8 \text{ m/s}} = 30 \text{ T m}$$

Bending Magnets and Longitudinal Motion

- Assuming to be using 6, 1 meter long magnets, this corresponds to a bending radius

$$\rho \text{ @ } 2.86 \text{ GeV} = \frac{L}{\alpha} = \frac{6 \cdot 1 \text{ m}}{0.24202 \text{ rad}} = 24.792 \text{ m}$$

$$\rho \text{ @ } 9 \text{ GeV} = \frac{L}{\alpha} = \frac{6 \cdot 1 \text{ m}}{0.076908 \text{ rad}} = 78.015 \text{ m}$$

⇒ Magnetic field

$$B \text{ @ } 2.86 \text{ GeV} = \frac{9.5 \text{ T m}}{24.792 \text{ m}} = 0.38319 \text{ T}$$

$$B \text{ @ } 9 \text{ GeV} = \frac{30 \text{ T m}}{78.015 \text{ m}} = 0.38454 \text{ T}$$

⇒ R_{56} for the bending section is:

$$R_{56} \text{ @ } 2.86 \text{ GeV} = 60.0 \text{ mm}$$

$$R_{56} \text{ @ } 9 \text{ GeV} = 6.0 \text{ mm}$$

ISR-Induced Emittance Growth

The effect of incoherent synchrotron radiation (ISR) emission on the emittance growth can be estimated using

$$\Delta\gamma\epsilon = 4 \times 10^{-8} E^6 \text{ [GeV] } I_5 \text{ [m}^{-1}\text{]}$$

where

$$I_5 = \frac{4L}{|\rho|^3} \cdot \frac{\eta^2 + (\eta\alpha + \eta'\beta)^2}{\beta}$$

⇒ **Case of E=2.86 GeV**: using L=1 m, ρ = 24.8 m, average dispersion and its derivative η = 0.3 m and η'=0.15 rad, horizontal twiss β=22.5 m and $\alpha = \pm$3.5, and horizontal emittance $\gamma\epsilon = 0.68$ μm:

$$\frac{\Delta\gamma\epsilon}{\gamma\epsilon} = 0.7\%$$

⇒ **Case of E=9 GeV**: using L=1 m, ρ = 78.0 m, average dispersion and its derivative η = 0.1 m and η'=0.05 rad, horizontal twiss β=22.5 m and $\alpha = \pm$3.5, and horizontal emittance $\gamma\epsilon = 0.68$ μm:

$$\frac{\Delta\gamma\epsilon}{\gamma\epsilon} = 0.003\%$$

Spin Rotator and Bunch Compressor

- P. Emma, 1994: *"the rotator system has very little impact on the performance of the bunch compressor"*

- Longitudinal transfer matrix of the bunch compressor

$$R_{\mathrm{BC}} = \begin{pmatrix} 1 + fR_{56} & R_{56} \\ f & 1 \end{pmatrix}$$

- In case of full compression, ie. $1 + fR_{56} = 0$, adding the spin rotator changes the total transfer as follows

$$R_{\mathrm{BC}} \cdot R_{\mathrm{ROT}} = \begin{pmatrix} 1 + fR_{56} & R_{56} \\ f & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & R_{56} \\ f & 1 + \alpha f \end{pmatrix}$$

⇒ Bunch length after compression is unchanged by the rotator and the energy spread after compression is smaller ($f = 2$ m^{-1}, $\alpha = -0.04$ m):

$$\sigma_{z,f} = \sigma_{\delta,i} R_{56}, \qquad \sigma_{\delta,f} = \sqrt{\sigma_{z,i}^2 f^2 + \sigma_{\delta,i}^2 \left(1 + \alpha f\right)}$$

- In our case, as bc1 **does not fully compress**,

$$R_{\mathrm{BC}} \cdot R_{\mathrm{ROT}} = \begin{pmatrix} 1 + fR_{56} & R_{56} + \alpha\,(1 + fR_{56}) \\ f & 1 + \alpha f \end{pmatrix}$$

$\Rightarrow$ Rotator might have an impact on the compression factor

$$\sigma_{z,f} = \sigma_{\delta,i}\left[R_{56} + \alpha\,(1 + fR_{56})\right]$$

$$\sigma_{\delta,f} = \sqrt{\sigma_{z,i}^2 f^2 + \sigma_{\delta,i}^2\,(1 + \alpha f)}$$

Notice that if $\alpha f < 0$ the final energy spread gets reduced

$\Rightarrow$ This problem can be overcome using an isochronous arc.

Summary Table for CLIC

Relevant parameters with the spin rotator location, for electrons and positrons:

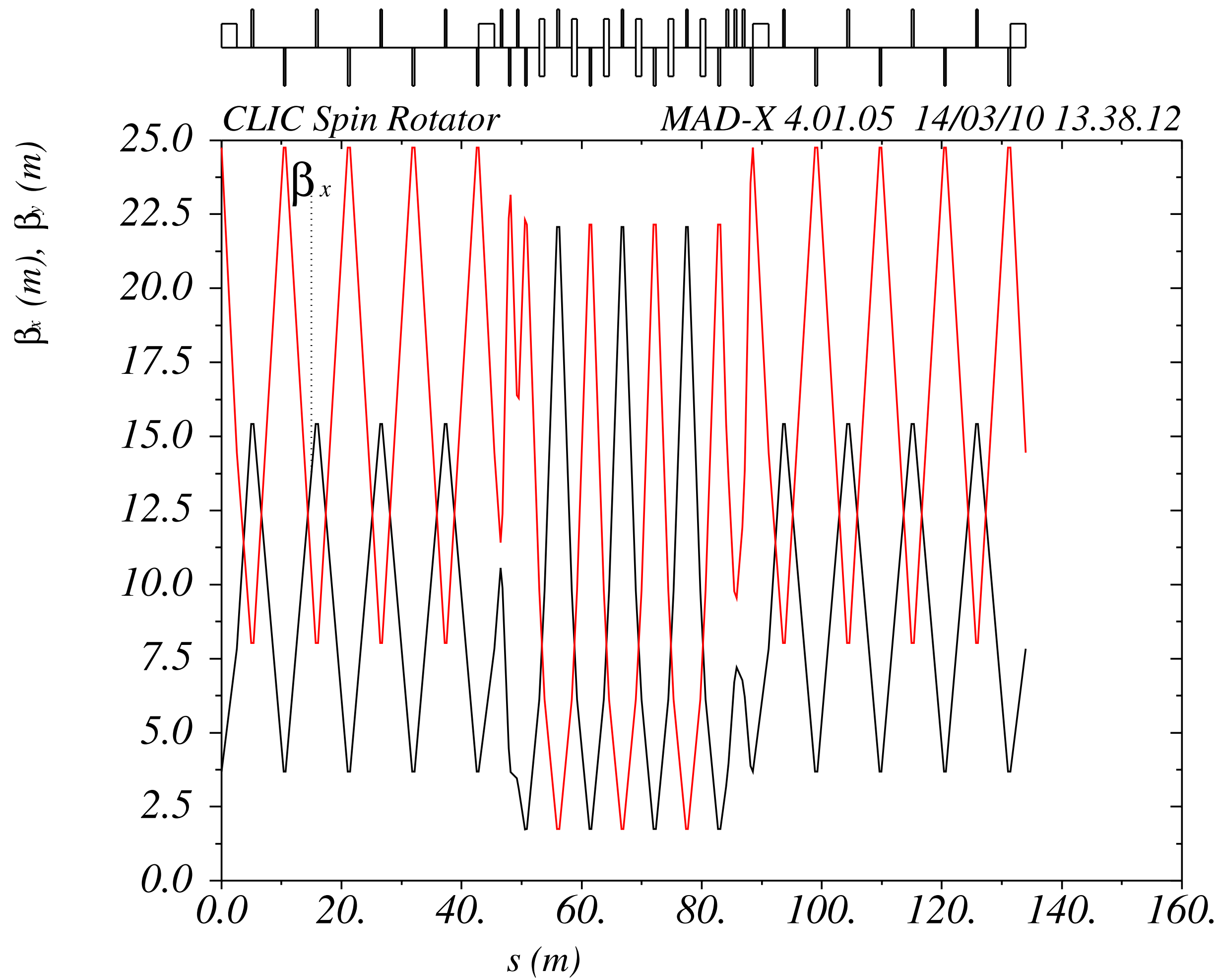
quantity	before bc1$^{(*)}$	before bc2	symm.rtml	unit	remarks
beam energy	2.86	9	2.86	GeV	
bending angle	0 (π)	0	$\pi/2$	rad	
spin depolarization	0 (2.2)	0	0.56	%	bds excluded
spin precession	0 (10.2)	0	5.1	turns	" "
solenoid field	2.9	9.1	like $^{(*)}$	T	L=2.6 m
bending angle	13.9	4.4	like $^{(*)}$	deg	L=1 m
bending magnet	0.38	0.38	like $^{(*)}$	T	" "
R_{56}	60.0	6.0	like $^{(*)}$	mm	
$\Delta\gamma\epsilon_x$ by synrad emission	0.7	0.003	like $^{(*)}$	%	negligible
total length	134.0	longer	like $^{(*)}$	m	scales with the energy

$\Rightarrow$ New RTML layout: potential problem might be the large solenoid field for the positrons; positron spin rotator before bc2 would be longer; positron spin rotator before bc1: 2.2% depolarization seems to me negligible

$\Rightarrow$ Old RTML layout (symmetric): no major problems, negligible depolarization

- Detailed beam dynamics studies have to be carried out
- Impact of R_{56} on the bunch compressor must be evaluated / use of an isochronous arc

Spin Rotator Optics



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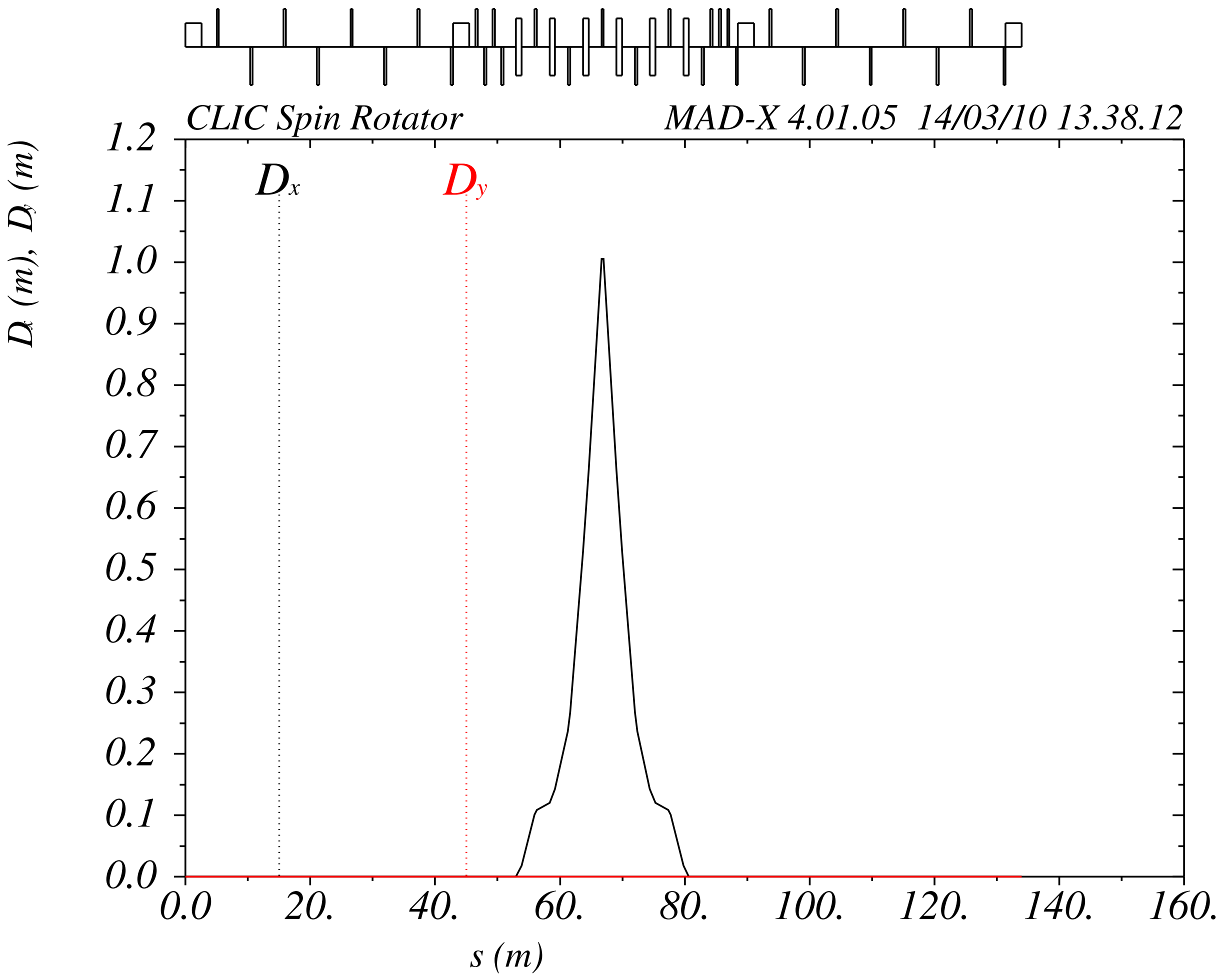

Spin Rotator Optics