Top pair and single top cross sections at Tevatron and LHC energies

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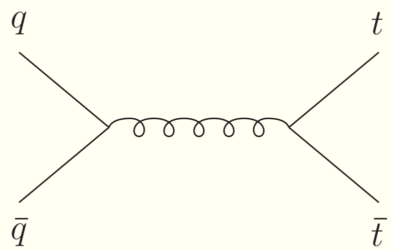
- $t\bar{t}$ **and single top production channels**
- **Higher-order two-loop corrections**
- $t\bar{t}$ **cross section at Tevatron and LHC**
- **Top quark** p_T **and** Y **distribution at Tevatron and LHC**
- t**-channel production at Tevatron and LHC**
- s**-channel production at Tevatron and LHC**
- **Associated production of a top with a** W^- **or** H^-

Partonic processes at LO

Top-antitop pair production

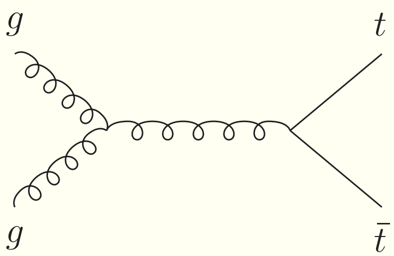
- $q\bar{q} \longrightarrow t\bar{t}$

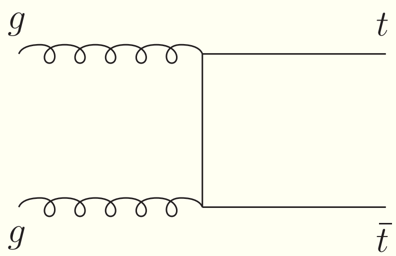
dominant at Tevatron

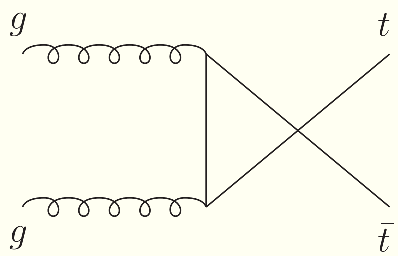


- $gg \longrightarrow t\bar{t}$

dominant at LHC



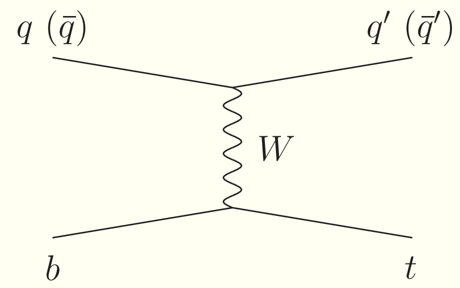




Single top quark production

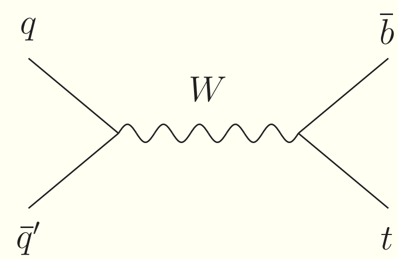
- t **channel:** $qb \longrightarrow q' t$ **and** $\bar{q}b \longrightarrow \bar{q}' t$

dominant at Tevatron and LHC



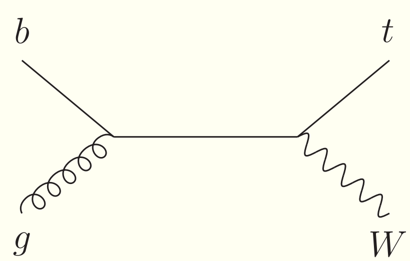
- s **channel:** $q\bar{q}' \longrightarrow \bar{b}t$

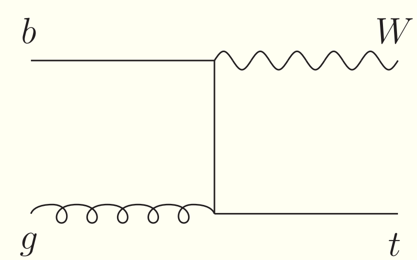
small at Tevatron and LHC



- **associated** tW **production:** $bg \longrightarrow tW^-$

very small at Tevatron, significant at LHC





Related process: $bg \longrightarrow tH^-$

Higher-order corrections

QCD corrections significant for top pair and single top quark production

NLO corrections fully known

Soft-gluon corrections from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (low-energy) gluons

Soft corrections $\left[\frac{\ln^k(s_4/m^2)}{s_4}\right]_+$ **with $k \le 2n-1$ and s_4 distance from threshold**

Soft-gluon corrections are dominant near threshold

Resum (exponentiate) these soft corrections

At NLL accuracy requires one-loop calculations in the eikonal approximation

New results at NNLL–two-loop calculations completed

Approximate NNLO cross section from expansion of resummed cross section

Essential ingredient: two-loop soft anomalous dimension

Allows NNLL resummation

Resummed cross section

Resummation follows from factorization properties of the cross section
- performed in moment space

Use RGE to evolve function associated with soft-gluon emission

H**: hard-scattering function** S**: soft-gluon function**

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N)\right] \exp\left[\sum_j E'_j(N)\right] \mathrm{tr}\left\{H(\alpha_s) \right.$$
$$\left. \times \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu} \Gamma_S^\dagger(\alpha_s(\mu))\right] S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu} \Gamma_S(\alpha_s(\mu))\right]\right\}$$

where

Γ_S **is the soft anomalous dimension - a matrix in color space**

and a function of kinematical invariants s, t, u

Calculate Γ_S **in eikonal approximation**

Calculation is at differential cross section level

kinematics refer to partonic threshold (not just absolute threshold)

Eikonal approximation

Feynman rules for soft gluon emission simplify

$$\bar{u}(p)\,(-ig_s T_F^c)\,\gamma^\mu \frac{i(\not{p}+\not{k}+m)}{(p+k)^2-m^2+i\epsilon} \rightarrow \bar{u}(p)\,g_s T_F^c\,\gamma^\mu \frac{\not{p}+m}{2p\cdot k+i\epsilon} = \bar{u}(p)\,g_s T_F^c\,\frac{v^\mu}{v\cdot k+i\epsilon}$$

with $p \propto v$, T_F^c generators of SU(3)

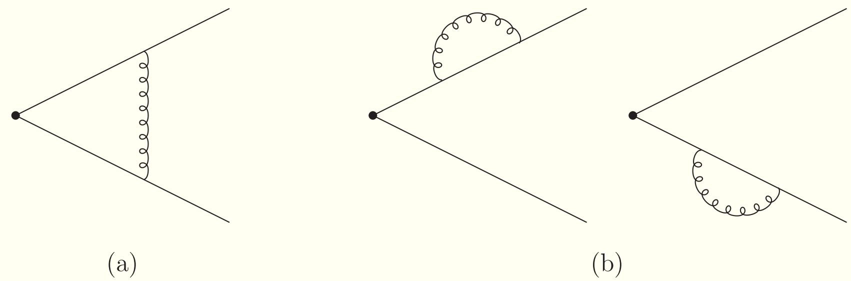
Perform calculation in momentum space and Feynman gauge

Complete two-loop results for

- **soft (cusp) anomalous dimension for $e^+e^- \rightarrow t\bar{t}$**
- **$t\bar{t}$ hadroproduction**
- **t-channel single top production**
- **s-channel single top production**
- **$bg \rightarrow tW^-$ and $bg \rightarrow tH^-$**

Soft (cusp) anomalous dimension

One-loop eikonal diagrams



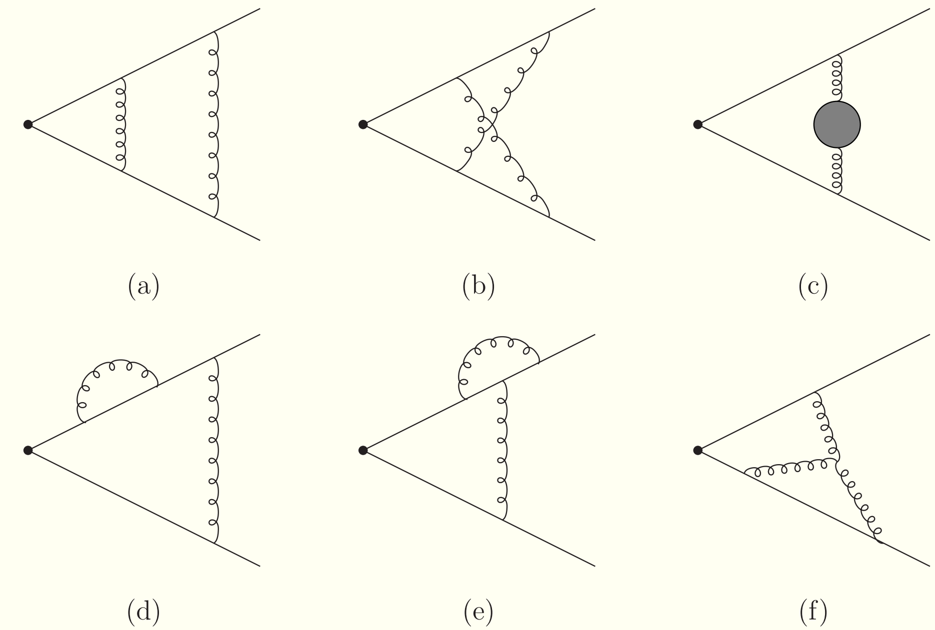
$$\Gamma_S = \frac{\alpha_s}{\pi}\Gamma_S^{(1)} + \frac{\alpha_s^2}{\pi^2}\Gamma_S^{(2)} + \cdots$$

The one-loop soft anomalous dimension, $\Gamma_S^{(1)}$, can be read off the coefficient of the ultraviolet (UV) pole of the one-loop diagrams

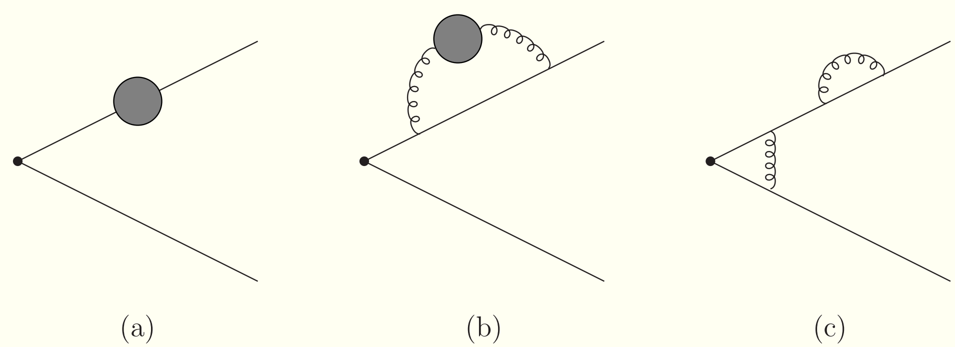
$$\Gamma_S^{(1)} = C_F \left[-\frac{(1+\beta^2)}{2\beta} \ln\left(\frac{1-\beta}{1+\beta}\right) - 1 \right] \quad \text{with} \quad \beta = \sqrt{1 - \frac{4m^2}{s}}$$

Two-loop eikonal diagrams

Vertex correction graphs



Heavy-quark self-energy graphs



Include counterterms for all graphs and multiply with corresponding color factors

Determine two-loop soft anomalous dimension from UV poles of the sum of the graphs

$$\Gamma_S^{(2)}=\frac{K}{2}\,\Gamma_S^{(1)}+C_F C_A M_\beta=\frac{K}{2}\,\Gamma_S^{(1)}+C_F C_A\left\{\frac{1}{2}+\frac{\zeta_2}{2}+\frac{1}{2}\ln^2\left(\frac{1-\beta}{1+\beta}\right)\right.$$
$$-\frac{(1+\beta^2)^2}{8\beta^2}\left[\zeta_3+\zeta_2\ln\left(\frac{1-\beta}{1+\beta}\right)+\frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right)+\ln\left(\frac{1-\beta}{1+\beta}\right)\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)-\mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]$$
$$-\frac{(1+\beta^2)}{4\beta}\left[\zeta_2-\zeta_2\ln\left(\frac{1-\beta}{1+\beta}\right)+\ln^2\left(\frac{1-\beta}{1+\beta}\right)-\frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right)+2\ln\left(\frac{1-\beta}{1+\beta}\right)\ln\left(\frac{(1+\beta)^2}{4\beta}\right)\right.$$
$$\left.\left.-\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]\right\}$$

where $K=C_A(67/18-\zeta_2)-5n_f/9$

N. Kidonakis, Phys. Rev. Lett. 102, 232003 (2009), arXiv:0903.2561 [hep-ph]

$\Gamma_S^{(2)}$ **vanishes at** $\beta=0$**, the threshold limit, and diverges at** $\beta=1$**, the massless limit**

If one quark is massless and one is massive

$$\Gamma_S^{(2)}=\frac{K}{2}\Gamma_S^{(1)}+C_F C_A\frac{(1-\zeta_3)}{4}$$

QCD processes: **Color structure gets more complicated with more than two colored partons in the process - Cusp anomalous dimension an essential component of other calculations**

Top-antitop production in hadron colliders

The soft anomalous dimension matrix for $q\bar{q} \rightarrow t\bar{t}$ is

$$\Gamma_{S\,q\bar{q}} = \left[\begin{array}{cc} \Gamma_{q\bar{q}\,11} & \Gamma_{q\bar{q}\,12} \\ \Gamma_{q\bar{q}\,21} & \Gamma_{q\bar{q}\,22} \end{array}\right]$$

At one loop

$$\Gamma^{(1)}_{q\bar{q}\,11} = -C_F\,[L_\beta + 1] \qquad \Gamma^{(1)}_{q\bar{q}\,21} = 2\ln\left(\frac{u_1}{t_1}\right) \qquad \Gamma^{(1)}_{q\bar{q}\,12} = \frac{C_F}{C_A}\ln\left(\frac{u_1}{t_1}\right)$$

$$\Gamma^{(1)}_{q\bar{q}\,22} = C_F\left[4\ln\left(\frac{u_1}{t_1}\right) - L_\beta - 1\right] + \frac{C_A}{2}\left[-3\ln\left(\frac{u_1}{t_1}\right) + \ln\left(\frac{t_1 u_1}{s m^2}\right) + L_\beta\right]$$

where $L_\beta = \frac{1+\beta^2}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right)$ **with** $\beta = \sqrt{1-4m^2/s}$

Write the two-loop cusp anomalous dimension as $\Gamma_S^{(2)} = \frac{K}{2}\,\Gamma_S^{(1)} + C_F C_A M_\beta$**. Then at two loops**

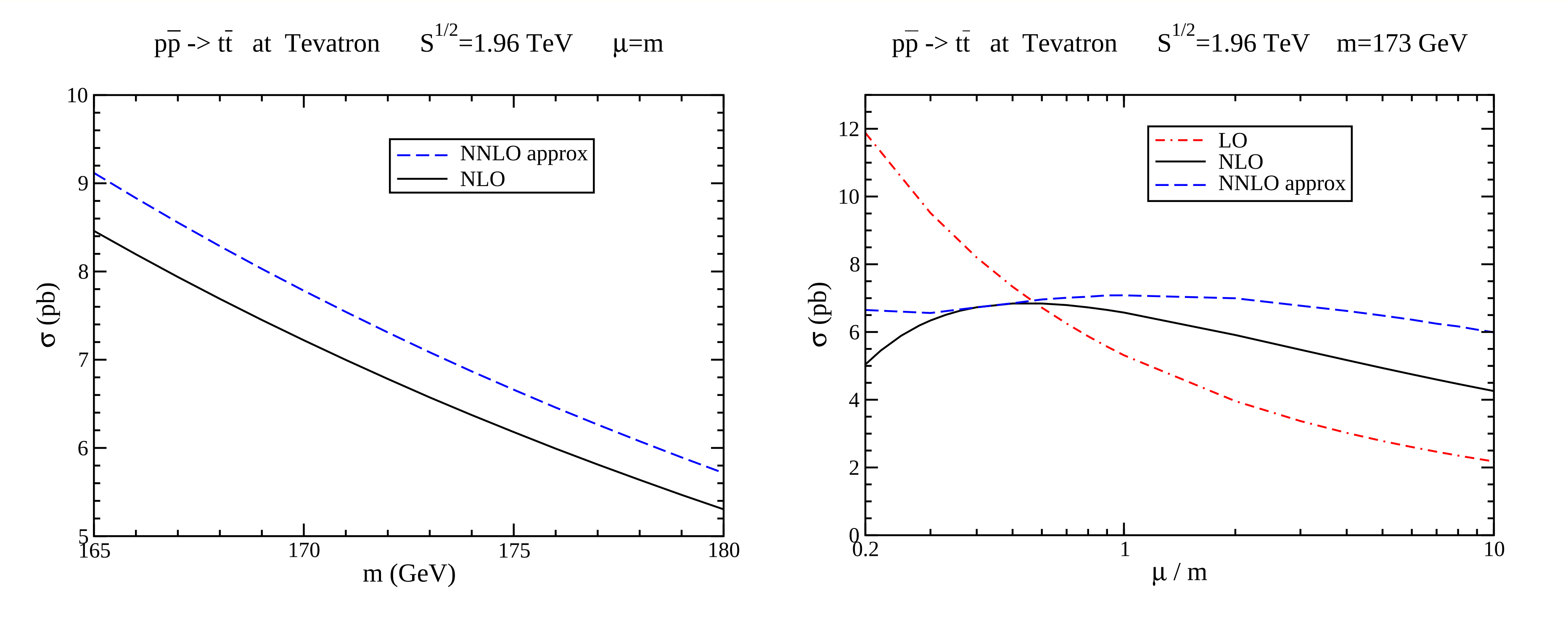
$$\Gamma^{(2)}_{q\bar{q}\,11} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,11} + C_F C_A\,M_\beta \qquad \Gamma^{(2)}_{q\bar{q}\,22} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,22} + C_A\left(C_F - \frac{C_A}{2}\right)M_\beta$$

$$\Gamma^{(2)}_{q\bar{q}\,21} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,21} + C_A N_\beta \ln\left(\frac{u_1}{t_1}\right) \qquad \Gamma^{(2)}_{q\bar{q}\,12} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,12} - \frac{C_F}{2}N_\beta \ln\left(\frac{u_1}{t_1}\right)$$

with N_β a subset of terms of M_β

Similar results for $gg \rightarrow t\bar{t}$ channel N. Kidonakis, Phys. Rev. D 82, 114030 (2010), arXiv:1009.4935 [hep-ph]

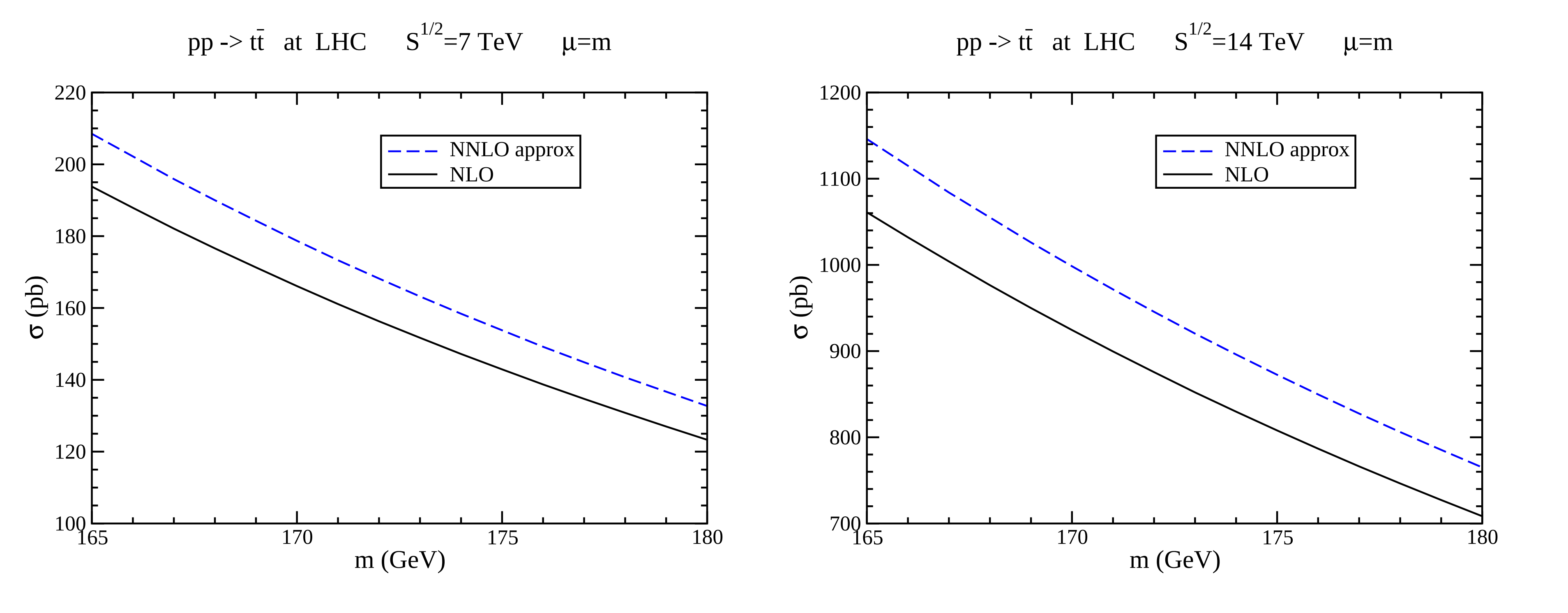
$t\bar{t}$ cross section at the Tevatron



$$\sigma_{t\bar{t}}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV},\, 1.96\,\mathrm{TeV}) \quad = \quad 7.08^{+0.00\,+0.36}_{-0.24\,-0.27}\ \mathrm{pb}$$

NNLO approx: 7.8% enhancement over NLO

$t\bar{t}$ cross section at the LHC

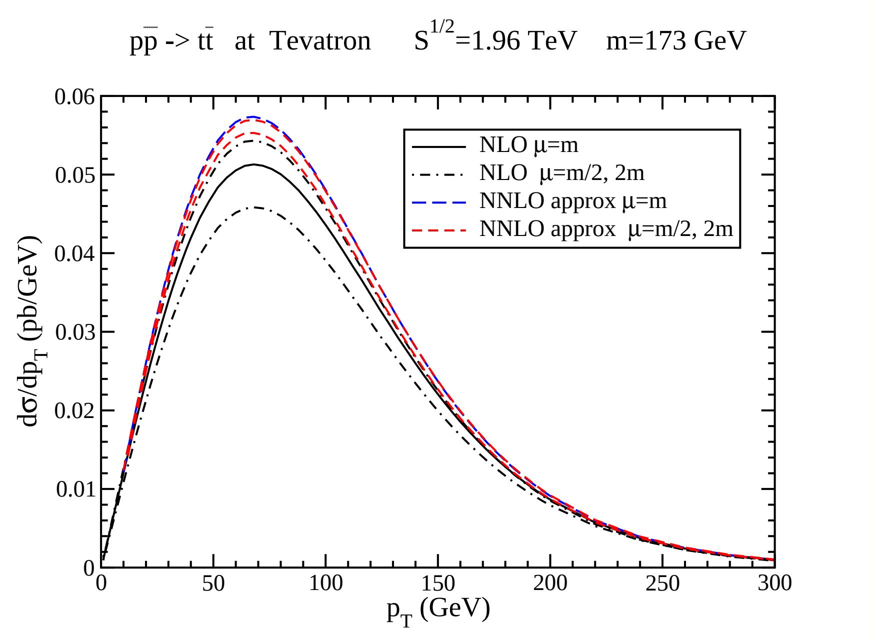


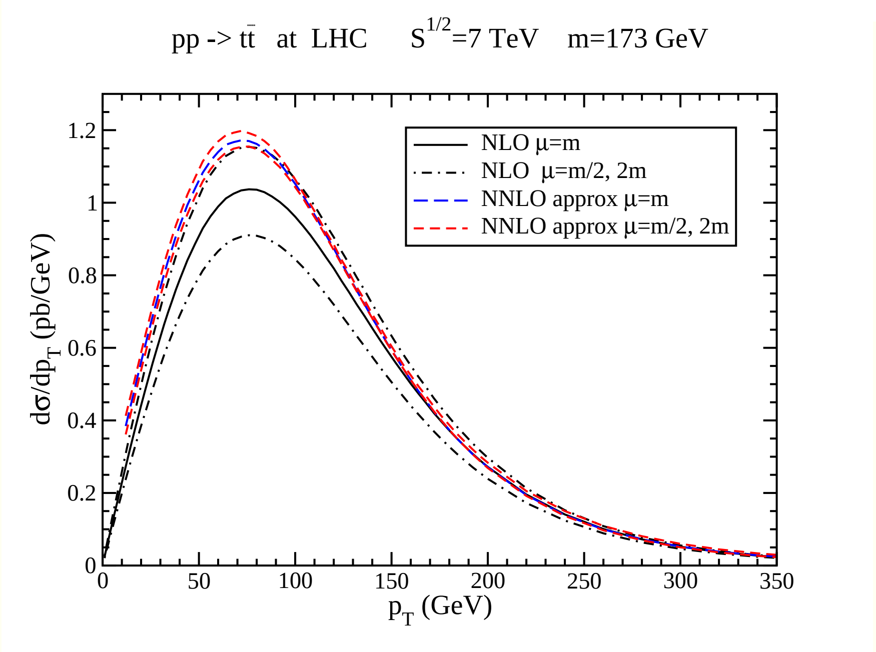
$$\sigma_{t\bar{t}}^{\text{NNLOapprox}}(m_t = 173\,\text{GeV}, 7\,\text{TeV}) \quad = \quad 163^{+7}_{-5}{}^{+9}_{-9}\ \text{pb}$$

$$\sigma_{t\bar{t}}^{\text{NNLOapprox}}(m_t = 173\,\text{GeV}, 14\,\text{TeV}) \quad = \quad 920^{+50}_{-39}{}^{+33}_{-35}\ \text{pb}$$

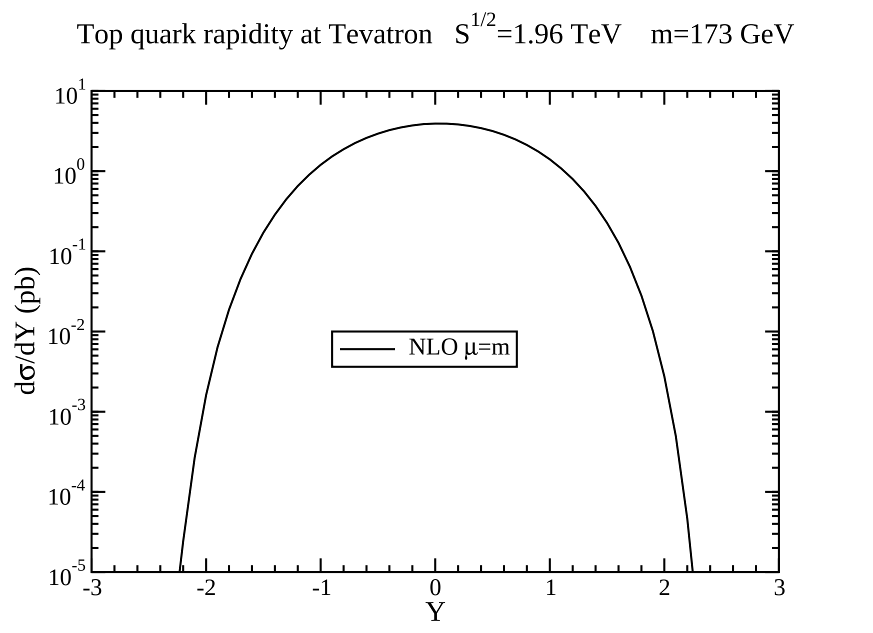
NNLO approx: enhancement over NLO is 7.6% at 7 TeV; 8.0% at 14 TeV

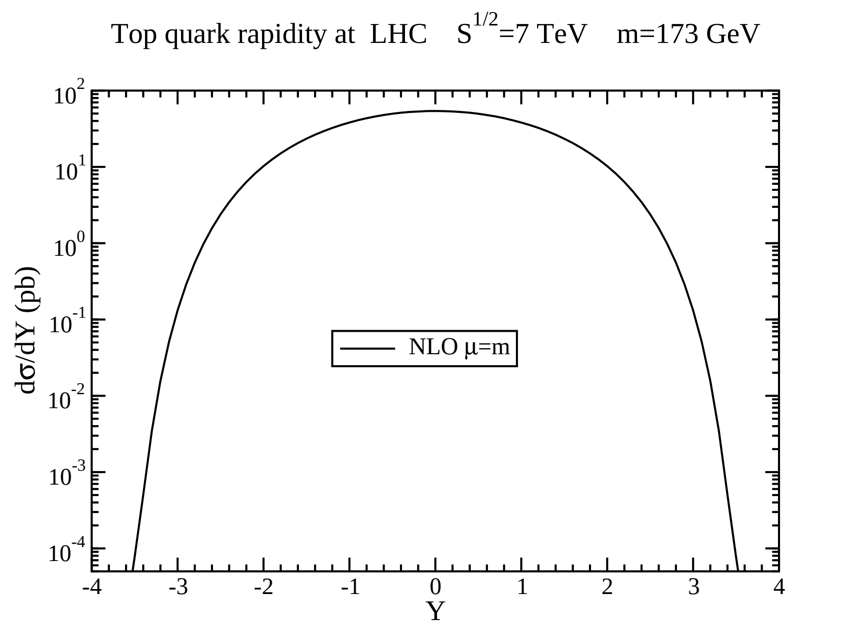
Top quark p_T distribution at Tevatron and LHC





Top quark rapidity distribution at Tevatron and LHC





Single top quark production - t channel

Dominant single top production channel at both Tevatron and LHC energies

Soft anomalous dimension for t-channel single top production

One loop

$$\Gamma_{S\,11}^{(1)}=C_F\left[\ln\left(\frac{-t}{s}\right)+\ln\left(\frac{m_t^2-t}{m_t\sqrt{s}}\right)-\frac{1}{2}\right]$$

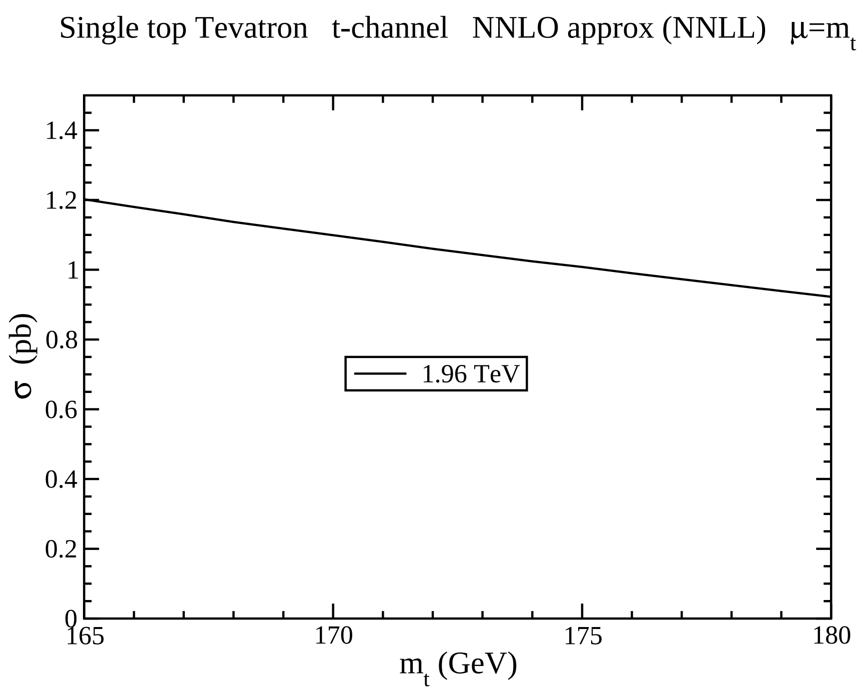
$$\Gamma_{S\,21}^{(1)}=\ln\left(\frac{u(u-m_t^2)}{s(s-m_t^2)}\right) \qquad \Gamma_{S\,12}^{(1)}=\frac{C_F}{2N_c}\,\Gamma_{S\,21}^{(1)}$$

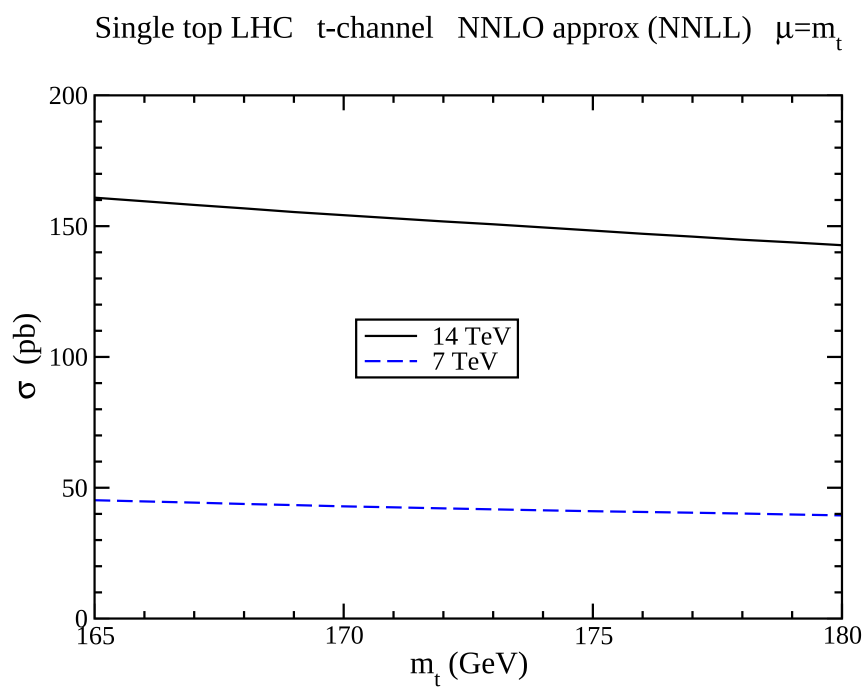
Two loops

$$\Gamma_{S\,11}^{(2)}=\frac{K}{2}\Gamma_{S\,11}^{(1)}+C_F C_A\,\frac{(1-\zeta_3)}{4}$$

N. Kidonakis, arXiv:1103.2792 [hep-ph]

Single top quark production at Tevatron and LHC - t channel





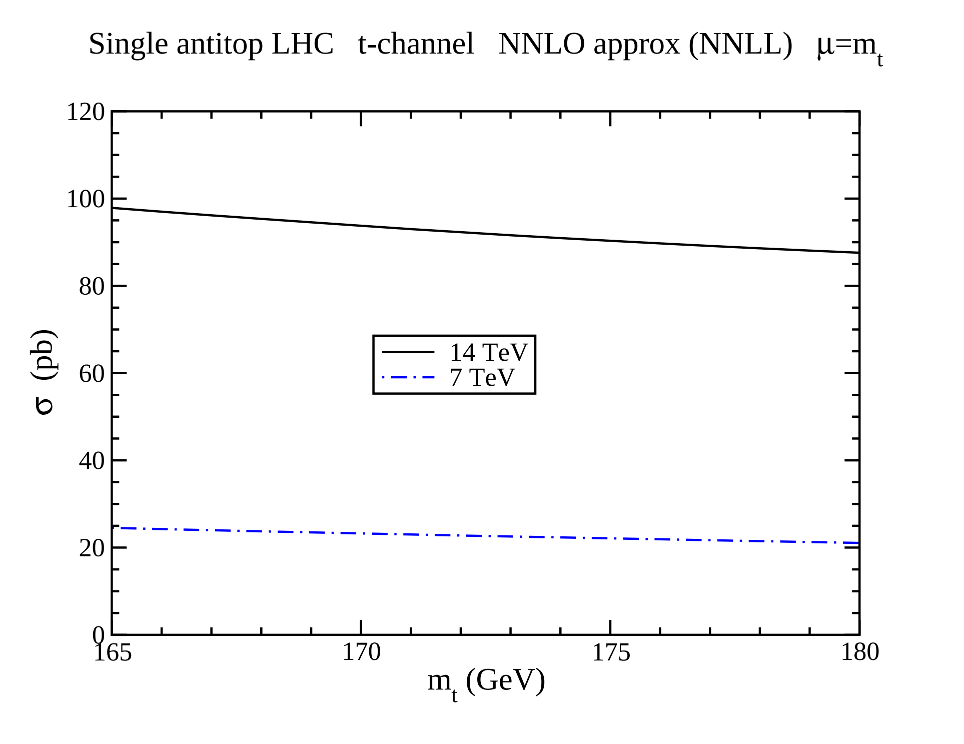
$$\sigma^{\rm NNLOapprox,\, top}_{t-{\rm channel}}(m_t = 173\,{\rm GeV},\, 1.96\,{\rm TeV}) = 1.04^{+0.00}_{-0.02} \pm 0.06 \ {\rm pb}$$

$$\sigma^{\rm NNLOapprox,\, top}_{t-{\rm channel}}(m_t = 173\,{\rm GeV},\, 7\,{\rm TeV}) = 41.7^{+1.6}_{-0.2} \pm 0.8 \ {\rm pb}$$

$$\sigma^{\rm NNLOapprox,\, top}_{t-{\rm channel}}(m_t = 173\,{\rm GeV},\, 14\,{\rm TeV}) = 151^{+4}_{-1} \pm 3 \ {\rm pb}$$

NNLO approx: 4% increase at Tevatron; 1% decrease at 7 TeV; 3% decrease at 14 TeV relative to NLO

Single antitop production at LHC - t channel



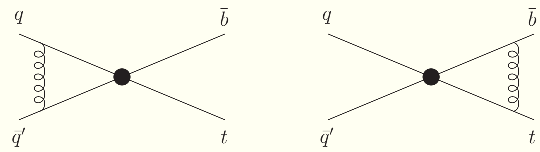
$$\sigma^{\text{NNLOapprox, antitop}}_{t-\text{channel}}(m_t = 173\,\text{GeV},\, 7\,\text{TeV}) = 22.5 \pm 0.5^{+0.7}_{-0.9}\ \text{pb}$$

$$\sigma^{\text{NNLOapprox, antitop}}_{t-\text{channel}}(m_t = 173\,\text{GeV},\, 14\,\text{TeV}) = 92^{+2}_{-1}{}^{+2}_{-3}\ \text{pb}$$

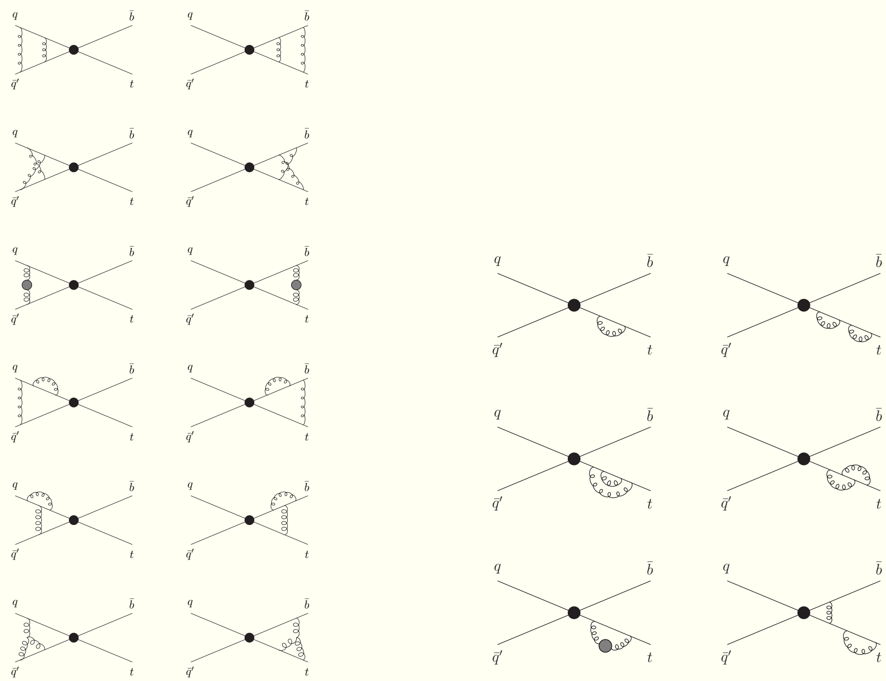
NNLO approx: 1% decrease at 7 TeV; 3% decrease at 14 TeV relative to NLO

Single top quark production - s channel

One-loop eikonal diagrams



Two-loop eikonal diagrams

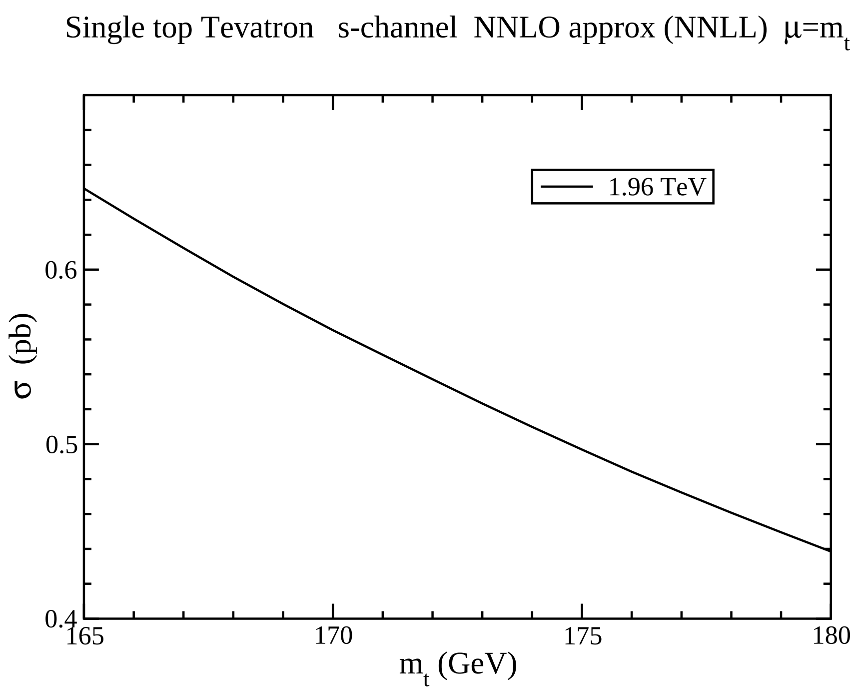


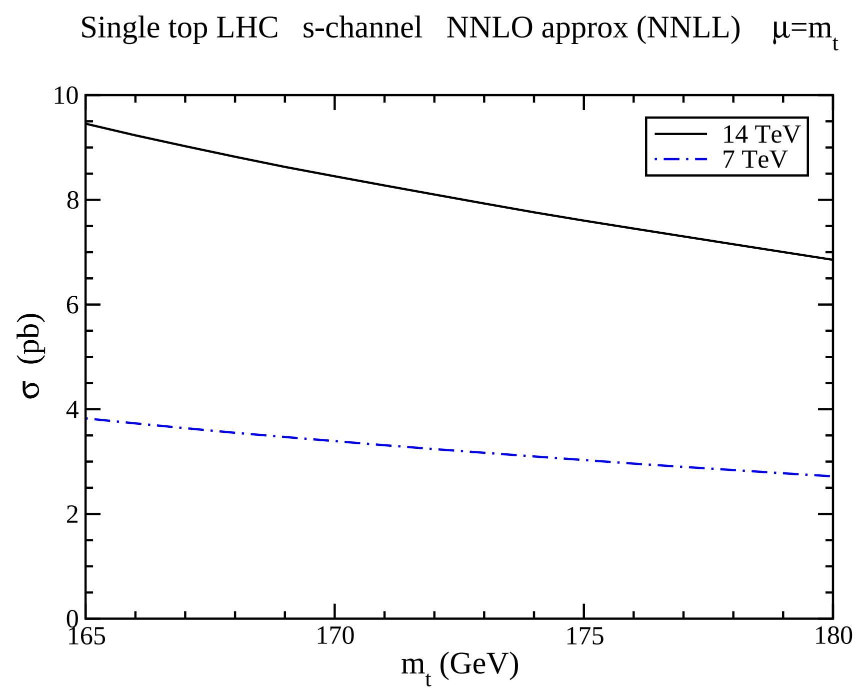
Soft anomalous dimension for s-channel single top production

$$\Gamma_{S\,11}^{(1)} = C_F \left[\ln\left(\frac{s - m_t^2}{m_t \sqrt{s}} \right) - \frac{1}{2} \right], \qquad \Gamma_{S\,11}^{(2)} = \frac{K}{2} \Gamma_{S\,11}^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$$

N. Kidonakis, Phys. Rev. D 81, 054028 (2010), arXiv:1001.5034 [hep-ph]

Single top quark production at Tevatron and LHC - s channel





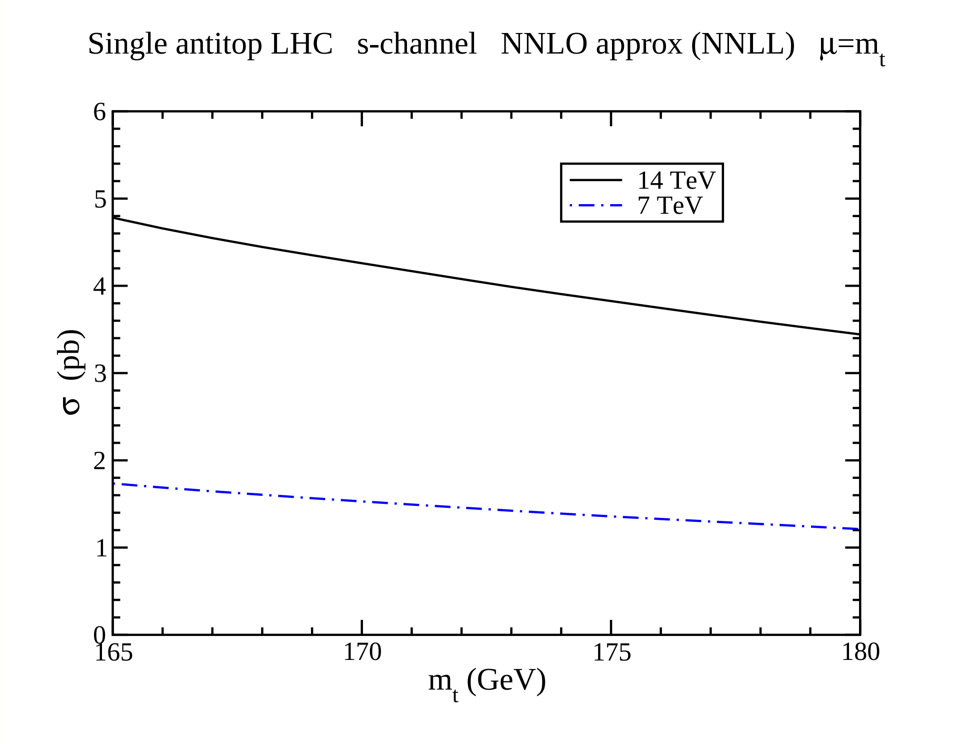
$$\sigma^{\rm NNLOapprox,\, top}_{s-{\rm channel}}(m_t = 173\,{\rm GeV}\, 1.96\,{\rm TeV}) \quad = \quad 0.523^{+0.001\,+0.030}_{-0.005\,-0.028}\ {\rm pb}$$

$$\sigma^{\rm NNLOapprox,\, top}_{s-{\rm channel}}(m_t = 173\,{\rm GeV},\, 7\,{\rm TeV}) \quad = \quad 3.17 \pm 0.06^{+0.13}_{-0.10}\ {\rm pb}$$

$$\sigma^{\rm NNLOapprox,\, top}_{s-{\rm channel}}(m_t = 173\,{\rm GeV}, 14\,{\rm TeV}) \quad = \quad 7.93 \pm 0.14^{+0.31}_{-0.28}\ {\rm pb}$$

NNLO approx: enhancement over NLO is 15% at Tevatron; 13% at LHC

Single antitop production at LHC - s channel

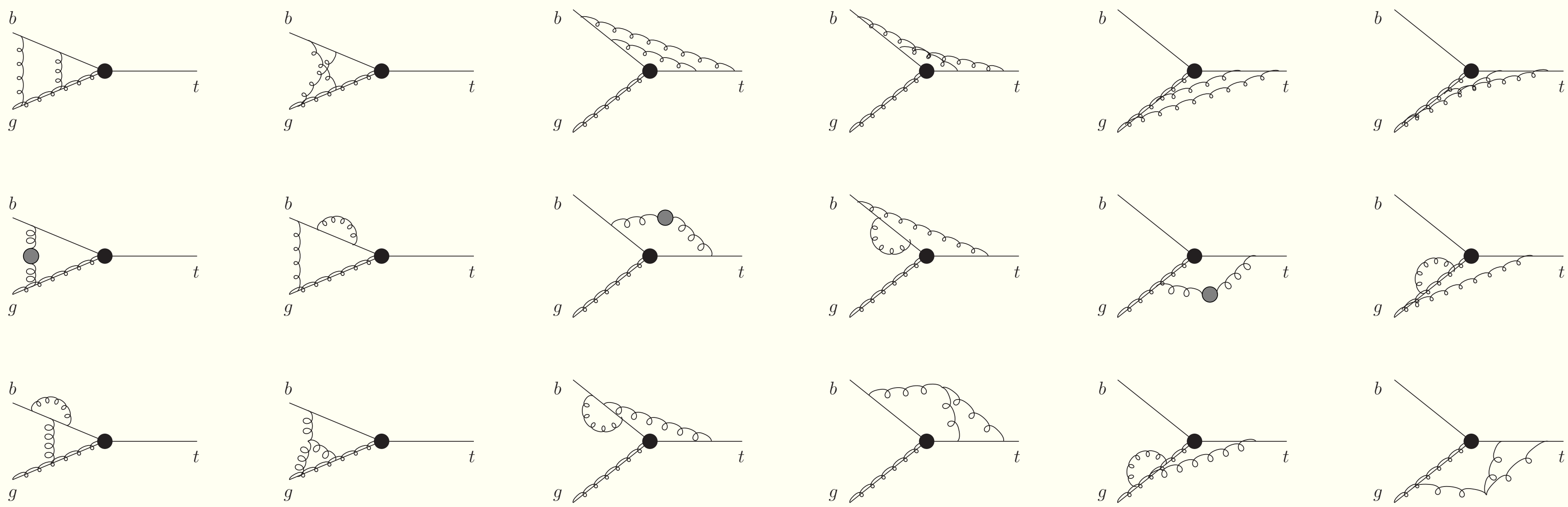


$$\sigma^{\text{NNLOapprox, antitop}}_{s-\text{channel}}(m_t = 173\,\text{GeV},\, 7\,\text{TeV}) = 1.42 \pm 0.01^{+0.06}_{-0.07}\ \text{pb}$$

$$\sigma^{\text{NNLOapprox, antitop}}_{s-\text{channel}}(m_t = 173\,\text{GeV},\, 14\,\text{TeV}) = 3.99 \pm 0.05^{+0.14}_{-0.21}\ \text{pb}$$

Associated production of a top quark with a W^-

Two-loop eikonal diagrams (+ extra top-quark self-energy graphs)

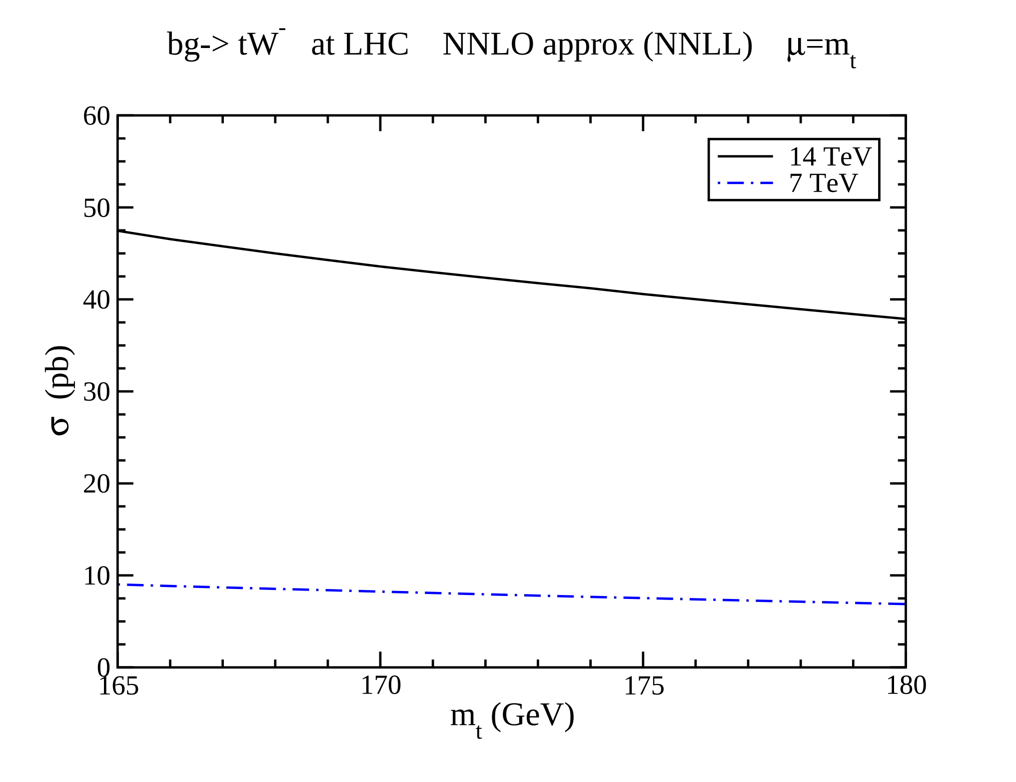


Soft anomalous dimension for $bg \to tW^-$

$$\Gamma^{(1)}_{S,tW^-} = C_F \left[\ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2}\right] + \frac{C_A}{2}\ln\left(\frac{m_t^2 - u}{m_t^2 - t}\right)$$

$$\Gamma^{(2)}_{S,tW^-} = \frac{K}{2}\Gamma^{(1)}_{S,tW^-} + C_F C_A \frac{(1-\zeta_3)}{4}$$

Same analytical result for Γ_S for $bg \to tH^-$



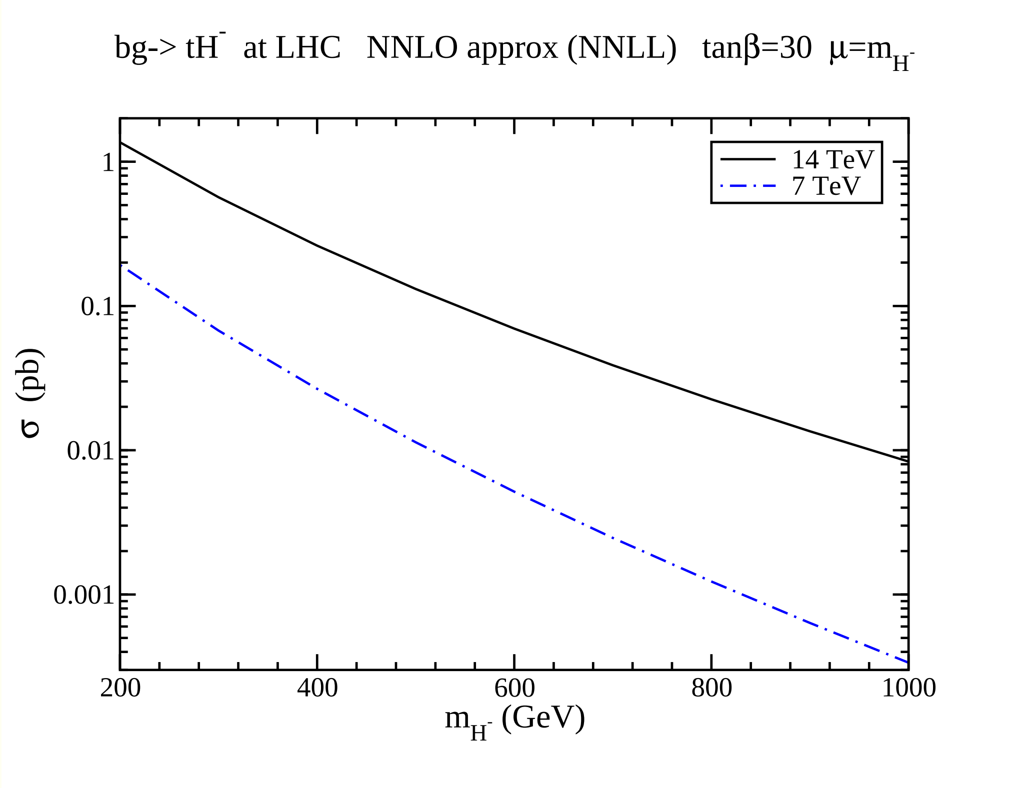
$$\sigma_{tW}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 7\,{\rm TeV}) \quad = \quad 7.8 \pm 0.2^{+0.5}_{-0.6}\ {\rm pb}$$

$$\sigma_{tW}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},14\,{\rm TeV}) \quad = \quad 41.8 \pm 1.0^{+1.5}_{-2.4}\ {\rm pb}$$

NNLO approx corrections increase NLO cross section by $\sim 8\%$

Cross section for $\bar{t}W$ production is identical

Associated production of a top quark with a charged Higgs



NNLO approx corrections increase NLO cross section by ~ 15 to $\sim 20\%$

Summary

- NNLL resummation for top quark pair and single top production
- $t\bar{t}$ production cross section
- top quark p_T and rapidity distributions
- t-channel and s-channel single top production cross section
- $bg \longrightarrow tW^-$ and $bg \longrightarrow tH^-$ at LHC
- NNLO approx corrections for top pair and single top production are significant at Tevatron and LHC