Effect of Cavity Tilt and RF Fluctuation to Transverse Beam Orbit Change in ILC Main Linac

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Abstract:

In ILC Main Linac, cavity tilt (misalignment) and accelerating voltage change of each cavity can cause significant intra-pulse transverse orbit change. Some correction stations may be necessary in the main linac. Here, we give basic evaluations of the effects.

1. Transverse kick by accelerating field of cavity with tilt angle

If a cavity axis deviates from the designed longitudinal direction, beam particles are kicked transversely by the accelerating field of the cavity. For high energy beam, the transverse momentum change in a cavity with voltage V_c and tilt angle θ can be expressed as

$$\Delta p_T = \frac{1}{2}\frac{eV_c}{c}\sin\theta .$$

The factor 1/2 comes from edge fields of the cavity. (See, ref [1].)

2. Random motion of cavity tilt

Let Θ denotes transverse angle change (kick angle) by tilt of a cavity,

$$\Theta = eV_c \sin\theta / 2E \approx eV_c\theta / 2E ,$$

where E the beam energy at the cavity.

Summing effect of all cavities in the linac, the vertical position at the end of linac can be expressed as

$$y_f = \sum_i \sqrt{\beta_f \beta_i} \sin\varphi_i \sqrt{E_i/E_f}\,\Theta_i ,$$

where β_f is the beta-function at the end of linac, β_i beta-function at the i-th cavity, φ_i the phase advance between the cavity and the i-th cavity, E_i the beam energy at the i-th cavity, E_f the beam energy at the end of linac and Θ_i the kick angle at the i-th cavity.

Assuming Θ_i is random and independent for each cavity, expected (average) square of the position is

$$
\begin{aligned}
< y_f^{\,2} > &= \beta_f \sum_i \beta_i \sin^2 \varphi_i \; E_i/E_f \; < \Theta_i^{\,2} > \\
&= \beta_f \sum_i \beta_i \sin^2 \varphi_i \left(E_i/E_f \right) < (eV_c \theta_i / E_i)^2 > /4 . \\
&= \beta_f \sum_i \beta_i \sin^2 \varphi_i \left(1/E_f E_i \right)\left(eV_c \right)^2 < \theta_i^{\,2} > /4
\end{aligned}
$$

For a long linac with many cavities, assuming beta-function does not depend on the beam energy, we can take averages of β_i, $\sin^2 \varphi$, E_i and $< \theta_i^{\,2} >$ separately. We also assume expected tilt angle square is the same for all cavities.

$$
\beta_i \to \overline{\beta}, \; \sin^2 \varphi_i \to 1/2 , < \theta_i^{\,2} > \to < \theta^2 >
$$

and

$$
\sum_i (1/E_i) \to > \int_{E_0}^{E_f} \frac{dE}{eV_c} \frac{1}{E} = \frac{1}{eV_c} \log(E_f / E_0) ,
$$

where E_0 is the initial beam energy. Then,

$$
< y_f^{\,2} > \approx \frac{eV_c}{8E_f} \beta_f \overline{\beta} \log(E_f / E_0) < \theta^2 > ,
$$

It can be compared with square of nominal beam size at the end of linac $\beta_f \varepsilon_{f,0}$, where $\varepsilon_{f,0}$ is the nominal emittance at the end of linac as,

$$
\frac{< y_f^{\,2} >}{\beta_f \varepsilon_{f,0}} \approx \frac{eV_c}{8(\gamma\varepsilon)_0 mc^2} \overline{\beta} \log(E_f / E_0) < \theta^2 > .
$$

where $(\gamma\varepsilon)_0$ is nominal normalized emittance.

ILC parameters are roughly, $V_c \approx 3\times 10^7 V$, $\overline{\beta} \approx 100\ m$, $\log(E_f / E_0) \approx 3$ and $(\gamma\varepsilon)_0 \approx 2\times 10^{-8} m$. Then,

$$
\frac{< y_f^{\,2} >}{\beta_f \varepsilon_{f,0}} \approx 1\times 10^{11} < \theta^2 > \quad \text{or} \quad \sqrt{< \theta^2 >} \approx 3\times 10^{-6} \sqrt{\frac{< y_f^{\,2} >}{\beta_f \varepsilon_{f,0}}} .
$$

It means, 3 micro-radian change of cavity tilt (random and independent) will cause 1-sigma orbit change at the end of linac.

(This is consistent with tracking simulation [2].)

3. Fluctuation of accelerating voltage

Change of accelerating voltage with fixed cavity tilt will have the same effect of the change of tilt. We can replace $<\theta^2>$ by $<(\theta_{\rm fix}\,\Delta V/V)^2>$ and get

$$\frac{<y_f{}^2>}{\beta_f \varepsilon_{f,0}} \approx \frac{eV_c}{8(\gamma\varepsilon)_0 mc^2}\bar{\beta}\log(E_f/E_0) <\theta_{\rm fix}{}^2><(\Delta V/V)^2>$$

where $\Delta V/V$ is relative change of accelerating voltage.
For ILC, roughly,

$$\frac{<y_f{}^2>}{\beta_f \varepsilon_{f,0}} \approx 1\times 10^{11} <\theta_{\rm fix}{}^2><(\Delta V/V)^2>.$$

For "standard" misalignment, $<\theta_{\rm fix}{}^2>=3\times 10^{-4}$,

$$\frac{<y_f{}^2>}{\beta_f \varepsilon_{f,0}} \approx 1\times 10^{4} <(\Delta V/V)^2> \quad \text{or} \quad \sqrt{<(\Delta V/V)^2>} \approx 1\times 10^{-2}\sqrt{\frac{<y_f{}^2>}{\beta_f \varepsilon_{f,0}}}\,.$$

It means, 1% change of cavity voltage (random and independent) will cause 1-sigma orbit change at the end of linac.

4. Conclusion and notes

Required stability of tilt (pitch) of each cavity will be less than 3 micro-radian, which will cause 1-sigma orbit change at the end of linac.
Required stability of accelerating voltage of each cavity will be less than 1%, which will cause 1-sigma orbit change at the end of linac, assuming rms of fixed tilt angle (misalignment) of cavities is 300 micro-radian.

A, Orbit difference in a pulse (head and tail in a pulse)

If cavity voltage changes during one pulse, beam orbit will be changed depending on locations in the pulse. (Bunches in the head and bunches in the tail will have different orbit.)
Supposing one RF source feeds more than one cavity with different RF coupling (different beam loading), voltage of each cavity should change in a pulse even if total voltage (vector sum) is

constant. This can be a problem because setting different coupling for cavities with different gradient is necessary for efficient operation. (And cavities with different gradient should be used for cost efficiency.)

If we allow 20% change of voltage of each cavity in a pulse, orbit at the end of linac will be about 20-sigma, which will not be acceptable. A few orbit correction stations per linac may be necessary, where time dependent (in a pulse) orbit is corrected using kickers. The correction can be fixed for all pulses if the voltage change is also fixed pulse to pulse.

B, Pulse to pulse change of orbit difference in a pulse

There is additional concern. Pulse to pulse fluctuation of time dependent voltage of each cavity will cause fluctuation of orbit difference in a pulse. Such pulse to pulse fluctuation of each cavity seems to be much larger than 1 %, and about 5%, probably due to microphonic [4]. Intra-pulse orbit corrections may be necessary in the linac (one or more correction stations).

5. References:

[1] For example, K. Kubo Lecture 8, “ILC linac beam dynamics” in the First International Accelerator School for Linear Colliders, Hayama, 2006, slide number 74.)

[2] W. Dou, report in ILC10, Beijing BD parallel session]

[3] Daniel Schulte, Private communication.

[4] S. Michizono, Private communication.