A Framework for Precision 2HDM Studies at the ILC/CLIC

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Outline

- What can we learn from precision MSSM Higgs studies?
  - Radiatively-corrected MSSM Higgs Yukawa couplings
  - Approaching the decoupling limit

- Framework for the general Two-Higgs-Doublet Model (2HDM)
  - Basis-independent formalism
  - The Higgs-fermion interaction
  - The banishment of $\tan\beta$
  - Experimental tests for Type-I and Type-II 2HDM Yukawa couplings

- Decoupling limit of the general 2HDM

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What can we learn from precision MSSM Higgs studies?

At the ILC, it may be possible to measure the $h^0 b\bar{b}$ coupling to an accuracy of a few percent or less. What is that good for?

In the MSSM, the tree-level Higgs–quark Yukawa Lagrangian is supersymmetry-conserving and is given by:

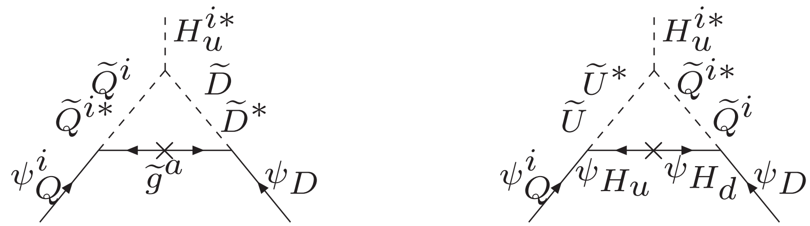
$$\mathcal{L}_{\rm yuk}^{\rm tree} = -\epsilon_{ij} h_b H_d^i \psi_Q^j \psi_D + \epsilon_{ij} h_t H_u^i \psi_Q^j \psi_U + \text{h.c.}$$

Two other possible dimension-four gauge-invariant non-holomorphic Higgs-quark interactions terms, the so-called wrong-Higgs interactions,

$$H_u^{k*} \psi_D \psi_Q^k \qquad \text{and} \qquad H_d^{k*} \psi_U \psi_Q^k \,,$$

are not supersymmetric (since the dimension-four supersymmetric Yukawa interactions must be holomorphic), and hence are absent from the tree-level Yukawa Lagrangian.

Nevertheless, the wrong-Higgs interactions can be generated in the effective low-energy theory below the scale of SUSY-breaking. In particular, one-loop radiative corrections, in which supersymmetric particles (squarks, higgsinos and gauginos) propagate inside the loop can generate the wrong-Higgs interactions.



(a) (b)

One-loop diagrams contributing to the wrong-Higgs Yukawa effective operators. In (a), the cross ($\times$) corresponds to a factor of the gluino mass M_3. In (b), the cross corresponds to a factor of the higgsino Majorana mass parameter μ. Field labels correspond to annihilation of the corresponding particle at each vertex of the triangle.

If the superpartners are heavy, then one can derive an effective field theory description of the Higgs-quark Yukawa couplings below the scale of SUSY-breaking ($M_{\rm SUSY}$), where one has integrated out the heavy SUSY particles propagating in the loops.

The resulting effective Lagrangian is:

$$\mathcal{L}_{\rm yuk}^{\rm eff} = -\epsilon_{ij}(h_b + \delta h_b)\psi_b H_d^i \psi_Q^j + \Delta h_b \psi_b H_u^{k*} \psi_Q^k$$
$$+\epsilon_{ij}(h_t + \delta h_t)\psi_t H_u^i \psi_Q^j + \Delta h_t \psi_t H_d^{k*} \psi_Q^k + {\rm h.c.}$$

In the limit of $M_{\rm SUSY} \gg m_Z$,

$$\Delta h_b = h_b \left[\frac{2\alpha_s}{3\pi} \mu M_3 \mathcal{I}(M_{\tilde{b}_1}, M_{\tilde{b}_2}, M_g) + \frac{h_t^2}{16\pi^2} \mu A_t \mathcal{I}(M_{\tilde{t}_1}, M_{\tilde{t}_2}, \mu) \right] ,$$

where, M_3 is the Majorana gluino mass, μ is the supersymmetric Higgs-mass parameter, and $\widetilde{b}_{1,2}$ and $\widetilde{t}_{1,2}$ are the mass-eigenstate bottom squarks and top squarks, respectively. The loop integral $\mathcal{I}(a,b,c) \sim 1/\max(a^2,b^2,c^2)$ in the limit where at least one of the arguments of $\mathcal{I}(a,b,c)$ is large.*

Thus, in the limit where $M_3 \sim \mu \sim A_t \sim M_{\tilde{b}} \sim M_{\tilde{t}} \sim M_{\rm SUSY} \gg m_Z$, the one-loop contributions to Δh_b do *not* decouple.

*$\mathcal{I}(a,b,c) = \left[a^2 b^2 \ln(a^2/b^2) + b^2 c^2 \ln(b^2/c^2) + c^2 a^2 \ln(c^2/a^2) \right] / [(a^2-b^2)(b^2-c^2)(a^2-c^2)]$.

Phenomenological consequences of the wrong-Higgs Yukawas

The effect of the wrong-Higgs couplings is a $\tan\beta$-enhanced modification of a physical observable. To see this, rewrite the Higgs fields in terms of the physical mass-eigenstates (and the Goldstone bosons):

$$H_d^1 = \frac{1}{\sqrt{2}}(v\cos\beta + H^0\cos\alpha - h^0\sin\alpha + iA^0\sin\beta - iG^0\cos\beta)\,,$$

$$H_u^2 = \frac{1}{\sqrt{2}}(v\sin\beta + H^0\sin\alpha + h^0\cos\alpha + iA^0\cos\beta + iG^0\sin\beta)\,,$$

$$H_d^2 = H^-\sin\beta - G^-\cos\beta\,,$$

$$H_u^1 = H^+\cos\beta + G^+\sin\beta\,,$$

with $v^2 \equiv v_u^2 + v_d^2 = (246~\text{GeV})^2$ and $\tan\beta \equiv v_u/v_d$. The b-quark mass is:

$$m_b = \frac{h_b v}{\sqrt{2}}\cos\beta\left(1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b \tan\beta}{h_b}\right) \equiv \frac{h_b v}{\sqrt{2}}\cos\beta(1+\Delta_b)\,,$$

which defines the quantity Δ_b.

In the limit of large $\tan\beta$ the term proportional to δh_b can be neglected, in which case,

$$\Delta_b \simeq (\Delta h_b/h_b)\tan\beta\,.$$

Thus, Δ_b is $\tan\beta$–enhanced if $\tan\beta \gg 1$. As previously noted, Δ_b survives in the limit of large $M_{\rm SUSY}$; this effect does not decouple. It can generate measurable shifts in the decay rate for $h^0 \to b\bar{b}$:

$$g_{h^\circ b\bar{b}} = -\frac{m_b}{v}\frac{\sin\alpha}{\cos\beta}\left[1+\frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b}-\Delta_b\right)(1+\cot\alpha\cot\beta)\right]\,.$$

At large $\tan\beta \sim 20$—50, Δ_b can be as large as 0.5 in magnitude and of either sign, leading to a significant enhancement or suppression of the Higgs decay rate to $b\bar{b}$. If $m_{H^\pm} \gg m_Z$ (the Higgs decoupling limit), then

$$-\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha) = 1 - \frac{2m_Z^2\sin^2\beta\cos 2\beta}{m_{H^\pm}^2} + \mathcal{O}\left(\frac{m_Z^4}{m_{H^\pm}^4}\right),$$

$$1+\cot\alpha\cot\beta = -\frac{2m_Z^2}{m_{H^\pm}^2}\cos 2\beta + \mathcal{O}\left(\frac{m_Z^4}{m_{H^\pm}^4}\right)\,.$$

The decoupling limit of the MSSM Higgs sector

In a significant fraction of the MSSM Higgs sector parameter space, one finds a neutral CP Higgs boson with SM-like tree-level couplings and additional scalar states that are somewhat heavier in mass (of order $m_{H^\pm}$), with small mass splittings of order $m_Z^2/m_{H^\pm}^2$. Below the scale $m_{H^\pm}$, the effective Higgs theory coincides with that of the Standard Model (SM).

In the limit of $m_{H^\pm} \gg m_Z$, the expressions for the tree-level MSSM Higgs masses and CP-even neutral Higgs mixing angle α simplify:

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta\,, \qquad m_H^2 \simeq m_A^2 + m_Z^2 \sin^2 2\beta\,,$$

$$m_{H^\pm}^2 = m_A^2 + m_W^2\,, \qquad \cos^2(\beta-\alpha) \simeq \frac{m_Z^4 \sin^2 4\beta}{4m_{H^\pm}^4}\,.$$

Including radiative corrections does not alter the following conclusions:

1. The two neutral heavy Higgs states and $H^\pm$ are approximately mass-degenerate up to corrections of $\mathcal{O}(m_Z^2/m_{H^\pm}^2)$.
2. $\cos(\beta-\alpha)=0$ up to corrections of $\mathcal{O}(m_Z^2/m_{H^\pm}^2)$.

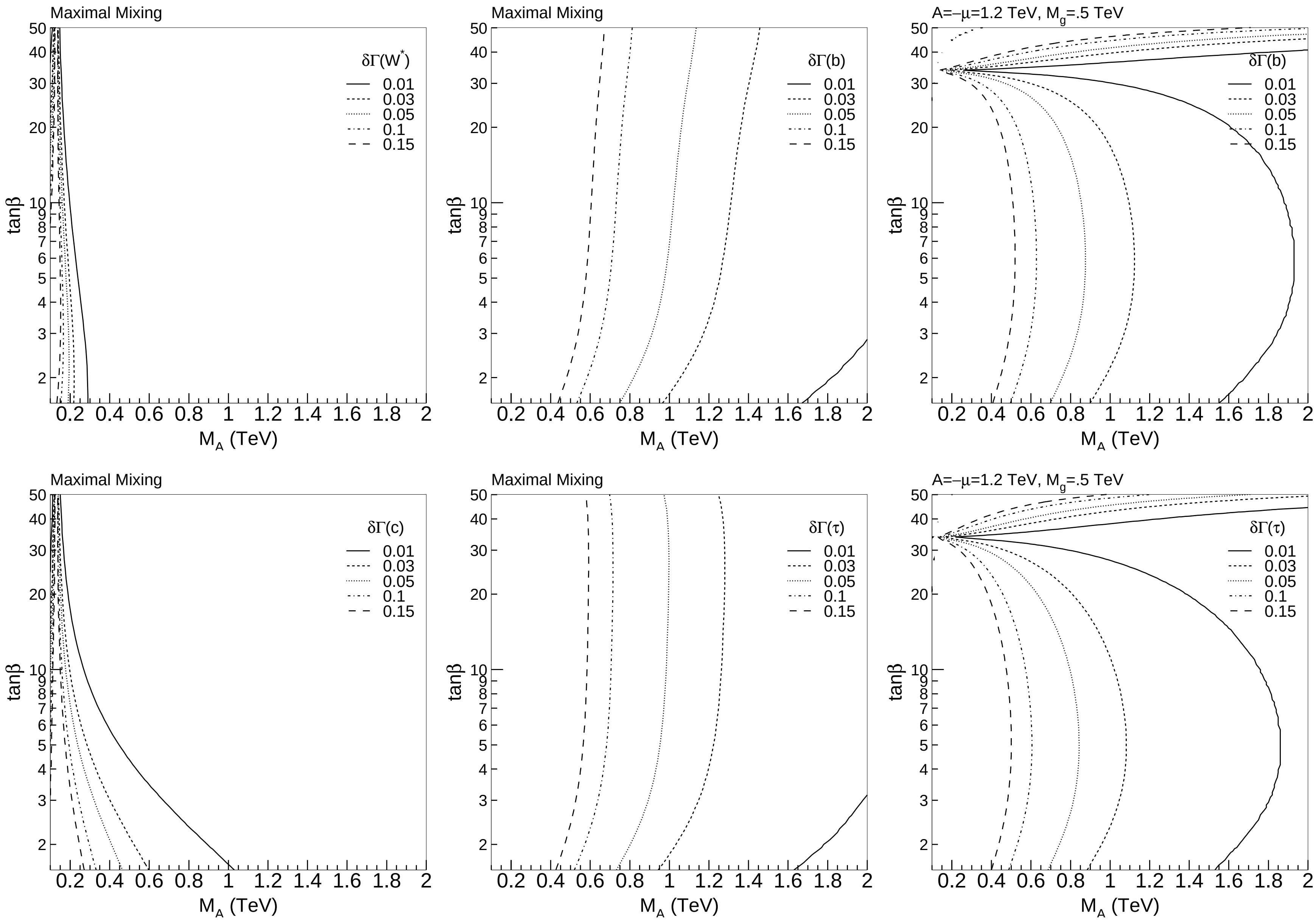
In general, in the limit of $\cos(\beta - \alpha) \to 0$, all the h^0 couplings to SM particles approach their SM limits. For example, if we keep only the leading $\tan\beta$-enhanced radiative corrections, then for $m_A \gg m_Z$ (approaching the decoupling limit),

$$\frac{g^2_{hVV}}{g^2_{h_{\rm SM}VV}} \simeq 1 - \frac{c^2 m_Z^4 \sin^2 4\beta}{4m^4_{H^\pm}}\,,$$

$$\frac{g^2_{htt}}{g^2_{h_{\rm SM}tt}} \simeq 1 + \frac{c m_Z^2 \sin 4\beta \cot\beta}{m^2_{H^\pm}}\,,$$

$$\frac{g^2_{hbb}}{g^2_{h_{\rm SM}bb}} \simeq 1 - \frac{4c m_Z^2 \cos 2\beta}{m^2_{H^\pm}}\left[\sin^2\beta - \frac{\Delta_b}{1+\Delta_b}\right]\,,$$

where $c \equiv 1 + \mathcal{O}(g^2)$ and $\Delta_b \equiv \tan\beta \times \mathcal{O}(g^2)$ [g is a generic gauge or Yukawa coupling]. The quantities c and Δ_b depend on the MSSM spectrum. The approach to decoupling is fastest for the h^0 couplings to vector boson pairs and slowest for the couplings to down-type quarks.



Deviations of Higgs partial widths from their SM values in two different MSSM scenarios (Carena, Haber, Logan and Mrenna).

Take home message

- In the approach to the decoupling limit, it is critical to measure deviations of the SM-like Higgs boson couplings from the predicted SM values to detect evidence of an extended Higgs sector (or other manifestations of new physics beyond the SM).

- Precision Higgs measurements can be sensitive to mass scales of new physics that lie beyond the reach of the collider.

- Given a 2HDM with symmetries that restrict the form of the Higgs interactions (e.g. discrete symmetries or supersymmetry†), the breaking of these symmetries can yield an effective low-energy 2HDM that contains all possible dimension ≤ 4 gauge-invariant interaction terms (e.g. the "wrong Higgs" couplings of the MSSM). That is, the effective low-energy Higgs sector may be a general 2HDM.

† These are needed, e.g., to avoid tree-level Higgs mediated flavor-changing neutral currents (FCNCs).

The general 2HDM

Consider the most general 2HDM potential,

$$\mathcal{V} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} .$$

In a general 2HDM, Φ_1 and Φ_2 are indistinguishable fields. A basis change consists of a global U(2) transformation $\Phi_a \to U_{a\bar{b}} \Phi_b$ (and $\Phi_{\bar{a}}^\dagger = \Phi_{\bar{b}}^\dagger U_{b\bar{a}}^\dagger$). Note that the gauge-covariant kinetic energy terms of the scalar fields are invariant with respect to U(2), whereas the scalar potential squared-masses and couplings change under U(2) transformations and thus are *basis-dependent* quantities.

Physical quantities that can be measured in the laboratory must be basis-independent. Thus, any model-independent experimental study of 2HDM phenomena must employ basis-independent methods for analyzing data associated with 2HDM physics.

Caveats

- The most general 2HDM contains large tree-level Higgs-mediated FCNC and CP-violating effects, which are inconsistent with present experimental data over a large range of the 2HDM parameter space. This can be rectified by either
  - fine-tuning of 2HDM parameters to reduce the size of the FCNC and CP-violating effects below the experimentally allowed limits; or
  - imposing additional symmetries (discrete and/or continuous) on the Higgs Lagrangian to eliminate tree-level Higgs-mediated FCNCs and CP-violation. The latter can distinguish between Φ_1 and Φ_2, in which case a choice of basis acquires physical significance.

- Basis-independent methods can be employed to experimentally identify and distinguish among possible symmetries. In cases where these symmetries are broken, it is again useful to regard the Higgs theory as a general 2HDM.

The basis-independent formalism

The scalar potential can be rewritten in U(2)-covariant notation:

$$\mathcal{V} = Y_{a\bar{b}}\Phi^\dagger_{\bar{a}}\Phi_b + \tfrac{1}{2}Z_{a\bar{b}c\bar{d}}(\Phi^\dagger_{\bar{a}}\Phi_b)(\Phi^\dagger_{\bar{c}}\Phi_d)\,, \qquad a,b,c,d = 1,2\,,$$

where $Z_{a\bar{b}c\bar{d}} = Z_{c\bar{d}a\bar{b}}$ and hermiticity implies $Y_{a\bar{b}} = (Y_{b\bar{a}})^*$ and $Z_{a\bar{b}c\bar{d}} = (Z_{b\bar{a}d\bar{c}})^*$. The barred indices help keep track of which indices transform with U and which transform with $U^\dagger$. For example, $Y_{a\bar{b}} \to U_{a\bar{c}}Y_{c\bar{d}}U^\dagger_{d\bar{b}}$ and $Z_{a\bar{b}c\bar{d}} \to U_{a\bar{e}}U^\dagger_{f\bar{b}}U_{c\bar{g}}U^\dagger_{h\bar{d}}Z_{e\bar{f}g\bar{h}}$.

The vacuum expectation values of the two Higgs fields can be parametrized as

$$\langle\Phi_a\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ \widehat{v}_a \end{pmatrix}, \qquad \text{with} \qquad \widehat{v}_a \equiv e^{i\eta}\begin{pmatrix} \cos\beta \\ e^{i\xi}\sin\beta \end{pmatrix},$$

where $v = 246$ GeV, $0 \leq \beta \leq \frac{1}{2}\pi$ and η is arbitrary. Define $V_{a\bar{b}} \equiv \hat{v}_a\hat{v}^*_{\bar{b}}$, which is hermitian with orthonormal eigenvectors $\hat{v}_b$ and $\widehat{w}_b \equiv \widehat{v}^*_{\bar{c}}\,\epsilon_{cb}$. Under a U(2) transformation,

$$\hat{v}_a \to U_{a\bar{b}}\hat{v}_b\,, \qquad \widehat{w}_a \to e^{-i\chi}\,U_{a\bar{b}}\,\widehat{w}_b\,, \qquad \text{where } e^{i\chi} \equiv \det U\,.$$

That is, $\widehat{w}_a$ is a pseudo-vector with respect to U(2). One can use $\widehat{w}_a$ to construct a proper second-rank tensor: $W_{a\bar{b}} \equiv \hat{w}_a\hat{w}^*_{\bar{b}} \equiv \delta_{a\bar{b}} - V_{a\bar{b}}$. Moreover $\tan\beta \equiv |\hat{v}_2/\hat{v}_1|$ is basis-dependent, and hence is *not* in general a physical parameter.

All 2HDM observables must be invariant under a basis transformation $\Phi_a \to U_{a\bar{b}}\Phi_b$. Examples of manifestly real invariants and potentially complex pseudo-invariants:

$$Y_1 \equiv \mathrm{Tr}\,(YV)\,, \qquad Y_2 \equiv \mathrm{Tr}\,(YW)\,, \qquad Y_3 \equiv Y_{a\bar{b}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,,$$

$$Z_1 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}V_{d\bar{c}}\,, \qquad Z_2 \equiv Z_{a\bar{b}c\bar{d}}\,W_{b\bar{a}}W_{d\bar{c}}\,, \qquad Z_3 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}W_{d\bar{c}}\,,$$

$$Z_4 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{c}}W_{d\bar{a}} \qquad Z_5 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,\widehat{v}^*_{\bar{c}}\,\widehat{w}_d\,,$$

$$Z_6 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{v}_b\,\widehat{v}^*_{\bar{c}}\,\widehat{w}_d\,, \qquad Z_7 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}^*_{\bar{a}}\,\widehat{w}_b\,\widehat{w}^*_{\bar{c}}\,\widehat{w}_d\,.$$

The pseudo-invariants above transform as

$$[Y_3, Z_6, Z_7] \to e^{-i\chi}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to e^{-2i\chi}Z_5\,.$$

Physical quantities must be invariants. For example, the charged Higgs boson mass is $m^2_{H^\pm} = Y_2 + \frac{1}{2}Z_3 v^2$. The potential minimum conditions, $Y_1 = -\frac{1}{2}Z_1 v^2$ and $Y_3 = -\frac{1}{2}Z_6 v^2$, are covariant conditions with respect to U(2). Pseudo-invariants are useful because one can always combine two such quantities to create an invariant.

The Higgs basis

Define new linear combinations of the Higgs doublet fields:

$$H_1 = (H_1^+\,,\,H_1^0) \equiv \widehat{v}_{\bar{a}}^*\Phi_a\,, \qquad\qquad H_2 = (H_2^+\,,\,H_2^0) \equiv \widehat{w}_{\bar{a}}^*\Phi_a\,.$$

Equivalently, $\Phi_a = H_1\hat{v}_a + H_2\hat{w}_a$. It follows that

$$\langle H_1^0\rangle = \frac{v}{\sqrt{2}}\,, \qquad\qquad \langle H_2^0\rangle = 0\,.$$

Under a U(2) transformation, H_1 is *invariant*, whereas $H_2 \to e^{i\chi}H_2$. That is, the Higgs basis is define uniquely up to a possible rephasing of H_2.

In the Higgs basis, the scalar potential is given by:

$$\begin{aligned}\mathcal{V} = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 \\ & + \tfrac{1}{2}Z_2(H_2^\dagger H_2)^2 + Z_3(H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\ & + \left\{\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + \left[Z_6(H_1^\dagger H_1) + Z_7(H_2^\dagger H_2)\right] H_1^\dagger H_2 + \text{h.c.}\right\}\,,\end{aligned}$$

which provides an interpretation for the (pseudo-)invariants Y_1, Y_2 Y_3, Z_1, $Z_2, \ldots, Z_7$.

The Higgs mass-eigenstate basis

The three physical neutral Higgs boson mass-eigenstates are determined by diagonalizing a 3×3 real symmetric squared-mass matrix that is defined in the Higgs basis.[‡] The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}. Under a U(2) transformation,

$$\theta_{12}\,,\ \theta_{13}\ \text{are invariant, and}\qquad e^{i\theta_{23}} \to (\det\, U)^{-1} e^{i\theta_{23}}\,.$$

One can express the mass eigenstate neutral Higgs directly in terms of the original shifted neutral fields, $\overline{\Phi}_a^0 \equiv \Phi_a^0 - v\widehat{v}_a/\sqrt{2}$:

$$h_k = \frac{1}{\sqrt{2}}\left[\overline{\Phi}_{\bar{a}}^{0\,\dagger}(q_{k1}\widehat{v}_a + q_{k2}\widehat{w}_a e^{-i\theta_{23}}) + (q_{k1}^*\widehat{v}_{\bar{a}}^* + q_{k2}^*\widehat{w}_{\bar{a}}^* e^{i\theta_{23}})\overline{\Phi}_a^0\right]\,,$$

for $k = 1, \ldots, 4$, where $h_4 = G^0$.

[‡]For details, see H.E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs-doublet model. II: The significance of $\tan\beta$," *Phys. Rev.* **D74**, 015018 (2006) [hep-ph/0602242].

The *invariant* quantities $q_{k\ell}$ are given by:

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}
4	i	0

The $q_{k\ell}$ are functions of the angles θ_{12} and θ_{13}, where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

Since $\widehat{w}_a e^{-i\theta_{23}}$ is a *proper* U(2)-vector, we see that the neutral mass-eigenstate fields are indeed invariant under basis transformations.[§] Inverting the previous result yields:

$$\Phi_a = \begin{pmatrix} G^+\widehat{v}_a + H^+\widehat{w}_a \\ \dfrac{v}{\sqrt{2}}\widehat{v}_a + \dfrac{1}{\sqrt{2}}\displaystyle\sum_{k=1}^{4}\left(q_{k1}\widehat{v}_a + q_{k2}e^{-i\theta_{23}}\widehat{w}_a\right) h_k \end{pmatrix} .$$

[§]Likewise, $e^{i\theta_{23}}H^+$ and its charge conjugate are U(2)-invariant fields.

The gauge boson–Higgs boson interactions

$$\mathscr{L}_{VVH} = \left(g m_W W_\mu^+ W^{\mu\,-} + \frac{g}{2c_W} m_Z Z_\mu Z^\mu\right) \mathrm{Re}(q_{k1}) h_k + e m_W A^\mu (W_\mu^+ G^- + W_\mu^- G^+)$$

$$- g m_Z s_W^2 Z^\mu (W_\mu^+ G^- + W_\mu^- G^+)\,,$$

$$\mathscr{L}_{VVHH} = \left[\tfrac{1}{4} g^2 W_\mu^+ W^{\mu\,-} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu\right] \mathrm{Re}(q_{j1}^* q_{k1} + q_{j2}^* q_{k2})\, h_j h_k$$

$$+ \left[\tfrac{1}{2} g^2 W_\mu^+ W^{\mu\,-} + e^2 A_\mu A^\mu + \frac{g^2}{c_W^2}\left(\tfrac{1}{2} - s_W^2\right)^2 Z_\mu Z^\mu + \frac{2ge}{c_W}\left(\tfrac{1}{2} - s_W^2\right) A_\mu Z^\mu\right] (G^+ G^- + H^+ H^-)$$

$$+ \left\{ \left(\tfrac{1}{2} e g A^\mu W_\mu^+ - \frac{g^2 s_W^2}{2c_W} Z^\mu W_\mu^+\right) (q_{k1} G^- + q_{k2}\, e^{-i\theta_{23}} H^-) h_k + \text{h.c.} \right\},$$

$$\mathscr{L}_{VHH} = \frac{g}{4c_W} \mathrm{Im}(q_{j1} q_{k1}^* + q_{j2} q_{k2}^*) Z^\mu h_j \overleftrightarrow{\partial}_\mu h_k - \tfrac{1}{2} g \left\{ i W_\mu^+ \left[q_{k1} G^- \overleftrightarrow{\partial}^\mu h_k + q_{k2} e^{-i\theta_{23}} H^- \overleftrightarrow{\partial}^\mu h_k \right] + \text{h.c.} \right\}$$

$$+ \left[ieA^\mu + \frac{ig}{c_W}\left(\tfrac{1}{2} - s_W^2\right) Z^\mu \right] (G^+ \overleftrightarrow{\partial}_\mu G^- + H^+ \overleftrightarrow{\partial}_\mu H^-)\,.$$

The cubic and quartic Higgs couplings

$$\mathscr{L}_{3h} = -\tfrac{1}{2}v\, h_j h_k h_\ell \Big[q_{j1}q^*_{k1}\mathrm{Re}(q_{\ell 1})Z_1 + q_{j2}q^*_{k2}\,\mathrm{Re}(q_{\ell 1})(Z_3+Z_4) + \mathrm{Re}(q^*_{j1}q_{k2}q_{\ell 2}Z_5\, e^{-2i\theta_{23}})$$

$$+\mathrm{Re}\Big([2q_{j1}+q^*_{j1}]q^*_{k1}q_{\ell 2}Z_6\, e^{-i\theta_{23}}\Big) + \mathrm{Re}(q^*_{j2}q_{k2}q_{\ell 2}Z_7\, e^{-i\theta_{23}})\Big]$$

$$-v\, h_k G^+G^-\Big[\mathrm{Re}(q_{k1})Z_1 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}}Z_6)\Big] + v\, h_k H^+H^-\Big[\mathrm{Re}(q_{k1})Z_3 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}}Z_7)\Big]$$

$$-\tfrac{1}{2}v\, h_k\Big\{G^-H^+\, e^{i\theta_{23}}\, \Big[q^*_{k2}Z_4 + q_{k2}\, e^{-2i\theta_{23}}Z_5 + 2\mathrm{Re}(q_{k1})Z_6\, e^{-i\theta_{23}}\Big] + \mathrm{h.c.}\Big\}\,,$$

$$\mathscr{L}_{4h} = -\tfrac{1}{8}h_j h_k h_l h_m\Big[q_{j1}q_{k1}q^*_{\ell 1}q^*_{m1}Z_1 + q_{j2}q_{k2}q^*_{\ell 2}q^*_{m2}Z_2 + 2q_{j1}q^*_{k1}q_{\ell 2}q^*_{m2}(Z_3+Z_4)$$

$$+2\mathrm{Re}(q^*_{j1}q^*_{k1}q_{\ell 2}q_{m2}Z_5\, e^{-2i\theta_{23}}) + 4\mathrm{Re}(q_{j1}q^*_{k1}q^*_{\ell 1}q_{m2}Z_6\, e^{-i\theta_{23}}) + 4\mathrm{Re}(q^*_{j1}q_{k2}q_{\ell 2}q^*_{m2}Z_7\, e^{-i\theta_{23}})\Big]$$

$$-\tfrac{1}{2}h_j h_k G^+G^-\Big[q_{j1}q^*_{k1}Z_1 + q_{j2}q^*_{k2}Z_3 + 2\mathrm{Re}(q_{j1}q_{k2}Z_6\, e^{-i\theta_{23}})\Big]$$

$$-\tfrac{1}{2}h_j h_k H^+H^-\Big[q_{j2}q^*_{k2}Z_2 + q_{j1}q^*_{k1}Z_3 + 2\mathrm{Re}(q_{j1}q_{k2}Z_7\, e^{-i\theta_{23}})\Big]$$

$$-\tfrac{1}{2}h_j h_k\Big\{G^-H^+\, e^{i\theta_{23}}\, \Big[q_{j1}q^*_{k2}Z_4 + q^*_{j1}q_{k2}Z_5\, e^{-2i\theta_{23}} + q_{j1}q^*_{k1}Z_6\, e^{-i\theta_{23}} + q_{j2}q^*_{k2}Z_7\, e^{-i\theta_{23}}\Big] + \mathrm{h.c.}\Big\}$$

$$-\tfrac{1}{2}Z_1G^+G^-G^+G^- - \tfrac{1}{2}Z_2H^+H^-H^+H^- - (Z_3+Z_4)G^+G^-H^+H^-$$

$$-\tfrac{1}{2}(Z_5H^+H^+G^-G^- + Z^*_5H^-H^-G^+G^+) - G^+G^-(Z_6H^+G^- + Z^*_6H^-G^+) - H^+H^-(Z_7H^+G^- + Z^*_7H^-G^+)\,.$$

The Higgs-fermion Yukawa couplings

The Yukawa Lagrangian, in terms of the quark mass-eigenstate fields, is:

$$-\mathscr{L}_{\rm Y} = \overline{U}_L \Phi^{0\,*}_{\bar a} h^U_a U_R - \overline{D}_L K^\dagger \Phi^-_{\bar a} h^U_a U_R + \overline{U}_L K \Phi^+_a h^{D\,\dagger}_{\bar a} D_R + \overline{D}_L \Phi^0_a h^{D\,\dagger}_{\bar a} D_R + {\rm h.c.}\,,$$

where K is the CKM mixing matrix. The $h^{U,D}_a$ are 3×3 Yukawa coupling matrices. It is convenient to write:

$$h^Q_a = \kappa^Q \widehat{v}_a + \rho^Q \widehat{w}_a \quad \Longrightarrow \quad \kappa^Q \equiv \widehat{v}^*_{\bar a} h^Q_a \quad {\rm and} \quad \rho^Q \equiv \widehat{w}^*_{\bar a} h^Q_a\,, \quad (Q = U \ {\rm or}\ D)\,.$$

Under a U(2) transformation, κ^Q is invariant, whereas $\rho^Q \to (\det U)\rho^Q$.

By construction, κ^U and κ^D are proportional to the (real non-negative) diagonal quark mass matrices M_U and M_D, respectively, whereas the matrices ρ^U and ρ^D are independent complex 3×3 matrices. In particular,

$$M_U = \frac{v}{\sqrt{2}}\kappa^U = {\rm diag}(m_u\,,\, m_c\,,\, m_t)\,, \qquad M_D = \frac{v}{\sqrt{2}}\kappa^{D\,\dagger} = {\rm diag}(m_d\,,\, m_s\,,\, m_b)\,.$$

The fermion–Higgs boson interactions

The final form for the Yukawa couplings of the mass-eigenstate Higgs bosons and the Goldstone bosons to the quarks is [with $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$]:

$$-\mathscr{L}_Y = \frac{1}{v}\overline{D}\left\{M_D(q_{k1}P_R + q^*_{k1}P_L) + \frac{v}{\sqrt{2}}\left[q_{k2}\,[e^{i\theta_{23}}\rho^D]^\dagger P_R + q^*_{k2}\,e^{i\theta_{23}}\rho^D P_L\right]\right\}Dh_k$$

$$+\frac{1}{v}\overline{U}\left\{M_U(q_{k1}P_L + q^*_{k1}P_R) + \frac{v}{\sqrt{2}}\left[q^*_{k2}\,e^{i\theta_{23}}\rho^U P_R + q_{k2}\,[e^{i\theta_{23}}\rho^U]^\dagger P_L\right]\right\}Uh_k$$

$$+\left\{\overline{U}\left[K[\rho^D]^\dagger P_R - [\rho^U]^\dagger K P_L\right]DH^+ + \frac{\sqrt{2}}{v}\overline{U}\left[KM_DP_R - M_UKP_L\right]DG^+ + \text{h.c.}\right\}$$

Since $e^{i\theta_{23}}H^+$ and the h_k are invariant fields, $\mathscr{L}_Y$ depends only on invariant quantities: the matrices M_Q and $\rho^Q e^{i\theta_{23}}$ and the invariant angles θ_{12} and θ_{13}. The unphysical parameter $\tan\beta$ does *not* appear.

The couplings of the neutral Higgs bosons to quark pairs are generically flavor-nondiagonal and CP-violating (since the q_{k2} and the matrices $e^{i\theta_{23}}\rho^Q$ are not generally either pure real or pure imaginary).

Symmetries in the Higgs-fermion interactions

A general 2HDM exhibits CP-violating neutral Higgs boson couplings to fermions and tree-level FCNCs mediated by neutral Higgs boson exchanges. These effects can be removed by imposing a symmetry.

- Condition for CP-conserving neutral Higgs–fermion interactions:¶

$$Z_5(\rho^Q)^2\,,\; Z_6\rho^Q \;\text{ and }\; Z_7\rho^Q \;\text{ are real matrices } (Q=U,\, D \text{ and } E).$$

- Type-I and Type-II Higgs-fermion interactions

Type I: $\epsilon_{\bar{a}\bar{b}} h_a^D h_b^U = \epsilon_{ab} h_{\bar{a}}^{D\,\dagger} h_{\bar{b}}^{U\,\dagger} = 0$, i.e., $h_2^U = h_2^D = 0$ in some basis;
Type II: $\delta_{a\bar{b}}\, h_{\bar{a}}^{D\,\dagger} h_b^U = 0$, i.e., $h_1^U = h_2^D = 0$ in some basis,

which can be implemented with a $\mathbb{Z}_2$ symmetry (with appropriate choices for the transformations of the scalar and fermion fields), or with supersymmetry.

¶CP symmetry cannot be exact due to the unremovable phase in the CKM matrix that enters via the charged current interactions mediated by either $W^\pm$, $H^\pm$ or $G^\pm$ exchange.

Invariant expressions for the Type-I and Type-II conditions are given by:

$$\text{Type I:} \quad \kappa^D \rho^U = \rho^D \kappa^U \,, \qquad\qquad \text{Type II:} \quad \kappa^D \kappa^U + \rho^{D\,\dagger} \rho^U = 0 \,,$$

where in both cases, $\rho^Q \propto \kappa^Q = \sqrt{2} M_Q / v$ (for $Q = U$, D). Hence, in both cases there are no off-diagonal neutral Higgs–fermion couplings.

The existence of a special basis (up to a rephasing of the Higgs fields) in which $h_2^U = h_2^D = 0$ (Type-I) or $h_1^U = h_2^D = 0$ (Type-II) promotes $\tan\beta$ to a physical parameter (where $\tan\beta$ is defined to be the magnitude of the ratio of the neutral Higgs vacuum expectation values in the special basis).

For example, suppose we define the invariant parameters

$$\tan\beta_D \equiv \frac{v}{3\sqrt{2}} \left| \mathrm{Tr}\left(\rho^D M_D^{-1}\right) \right| \,, \qquad \tan\beta_U \equiv \frac{\sqrt{2}}{3v} \left| \mathrm{Tr}\left([\rho^U]^{-1} M_U\right) \right| \,.$$

Then, in a Type-II model these two quantities coincide and the physical parameter $\tan\beta = \tan\beta_D = \tan\beta_U$. Thus, in a model-independent analysis, measuring $\tan\beta_D$ and $\tan\beta_U$ can shed light on the symmetries of the Higgs Yukawa couplings.

The decoupling limit in the general 2HDM

In the decoupling limit, one of the two Higgs doublets of the 2HDM receives a very large mass and is therefore decoupled from the theory. This is achieved when $Y_2 \gg v^2$ and $|Z_i| \lesssim \mathcal{O}(1)$ [for all i]. The effective low energy theory is then a one-Higgs-doublet model, i.e. the SM Higgs sector.

We order the neutral scalar masses according to $m_1 < m_{2,3}$ and define the Higgs mixing angles accordingly. The conditions for the decoupling limit are:

$$|s_{12}| \lesssim \mathcal{O}\left(\frac{v^2}{m_2^2}\right) \ll 1\,, \qquad |s_{13}| \lesssim \mathcal{O}\left(\frac{v^2}{m_3^2}\right) \ll 1\,,$$
$$\mathrm{Im}(Z_5\, e^{-2i\theta_{23}}) \lesssim \mathcal{O}\left(\frac{v^2}{m_3^2}\right) \ll 1\,.$$

In the decoupling limit, $m_1 \ll m_2, m_3, m_{H^\pm}$. In particular, the properties of h_1 coincide with the SM-like Higgs boson with $m_1^2 = Z_1 v^2$ up to corrections of $\mathcal{O}(v^4/m_{2,3}^2)$, and $m_2 \simeq m_3 \simeq m_{H^\pm}$ with squared mass splittings of $\mathcal{O}(v^2)$.

Far from the decoupling limit, one typically finds that all Higgs bosons have a similar mass of $\mathcal{O}(v)$ and $none$ are SM-like.

In the decoupling limit of a general 2HDM, the CP-violating and flavor-changing neutral Higgs couplings of the SM-like Higgs state h_1 are suppressed by factors of $\mathcal{O}(v^2/m^2_{2,3})$. In contrast, the corresponding interactions of the heavy neutral Higgs bosons (h_2 and h_3) and the charged Higgs bosons ($H^\pm$) can exhibit both CP-violating and flavor non-diagonal couplings (proportional to the ρ^Q).

The decoupling limit is a generic feature of extended Higgs sectors.[‖]

- Thus, the observation of a SM-like Higgs boson does not rule out the possibility of an extended Higgs sector in the decoupling regime.
- A precision Higgs program can reveal small deviations from the decoupling limit, indicating the existence of a new heavy mass scale.

[‖]However, if some terms of the Higgs potential are absent, it is possible that no decoupling limit may exist. In this case, the only way to have very large Higgs masses is to have large Higgs self-couplings.

Lessons for future work

• Precision measurements of the properties of a SM-like Higgs may reveal small deviations, which can indicate the presence of a non-minimal Higgs sector and/or new physics beyond the Standard Model (characterized by a new mass scale $\gg m_Z$).

• Basis-independent methods provide a powerful technique for studying the theoretical structure of the two-Higgs doublet model.

• These methods provide insight into the conditions for CP-conservation (and violation), as well as other exact or approximate symmetries of the 2HDM that can distinguish between the two Higgs doublets.

• If nature suggest an elementary scalar sector with the structure of the 2HDM, then basis-independent techniques will be essential for performing a model-independent analysis to determine the ρ^Q $(Q = U, D)$ and eventually the Z_i. The tools for such an analysis have yet to be fully developed.