Massive particles & unitarity cuts

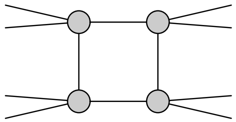
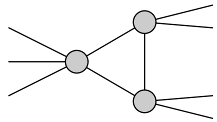
EDOARDO MIRABELLA

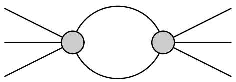
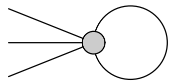
In collaboration with R. Britto

Outline

- Introduction
- On-Shell bubbles
- Single cut & Tadpoles
- Conclusions

Introduction

$\mathcal{A}_{\text{1-loop}} = \sum_i$  $d_i + \sum_j$  c_j

$\sum_k$  $b_k + \sum_m$  $a_m + \mathcal{R}$

The task: Computation of the coefficients $\&$ of the rational part

- Passarino-Veltman reduction, [Passarino, Veltman '79 ...]
- Singular limits of $\mathcal{A}_{\text{1-loop}}$ + on-shell formalism, [Bern, Dixon, Dunbar, Kosower '94,'95 ...]
  - Not only double but also multiple cuts
  - Works great in the massless case ...

    $\hookrightarrow$ $PP \to Z/W + 4$-jets computed [BLACKHAT; '11] $\rightsquigarrow$ (see Z. Bern' s talk)
  - Few issues in the massive case

    $\hookrightarrow$ Bypassed in numerical methods [Ossola, Papadopoulos, Pittau; '07] $\rightsquigarrow$ (see R. Pittau' s talk)
    [Mastrolia, *et al.*; '10]

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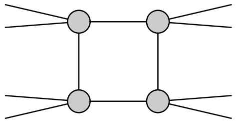
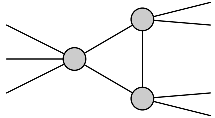
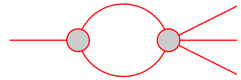
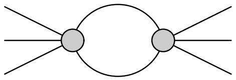
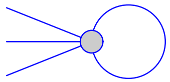
$\mathcal{A}_{\text{1-loop}} = \sum_i$  $d_i + \sum_j$  $c_j + \sum_l$  $\tilde{b}_l$

$\sum_k$  $b_k + \sum_m$  $a_m + \mathcal{R}$

The issues are related to tadpoles and on-shell bubbles

- Tree-level amplitudes become singular
- Double and single cut diverge,
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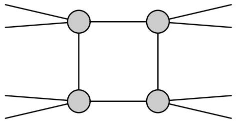
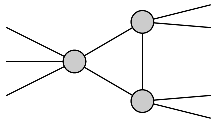
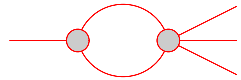
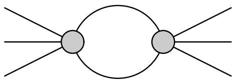
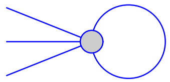
$\mathcal{A}_{\text{1-loop}} = \sum_i$  $d_i + \sum_j$  $c_j + \sum_l$  $\tilde{b}_l$

$\sum_k$  $b_k + \sum_m$  $a_m + \mathcal{R}$

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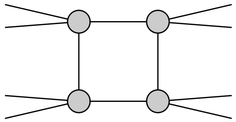
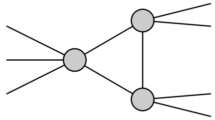
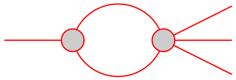
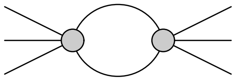
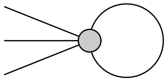
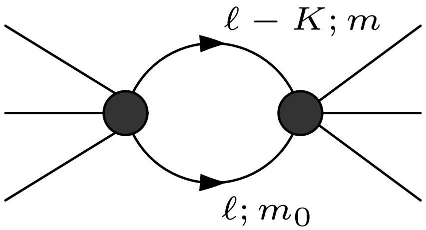
$\mathcal{A}_{\text{1-loop}} = \sum_i$  $d_i + \sum_j$  $c_j + \sum_l$  $\tilde{b}_l$

$\sum_k$  $b_k + \sum_m$  $a_m + \mathcal{R}$

Coefficient of the on-shell bubbles

Double cut in a nutshell



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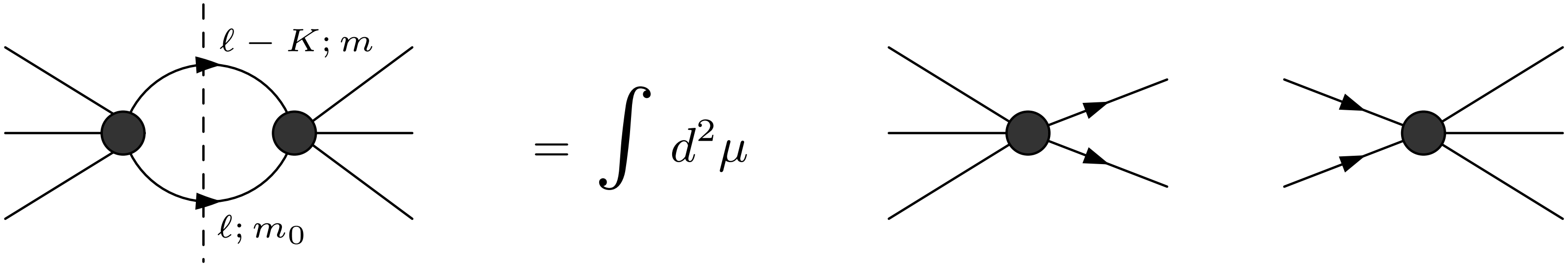
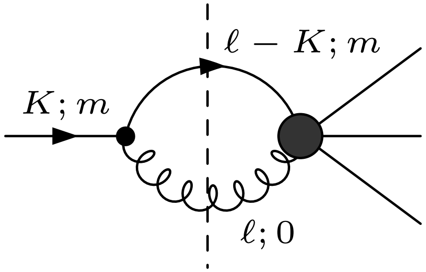


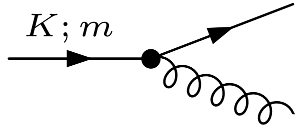
$$[d^2\mu \equiv d^4\ell \; \delta\left(\ell^2 - m_0^2\right) \delta\left((\ell - K)^2 - m^2\right)]$$

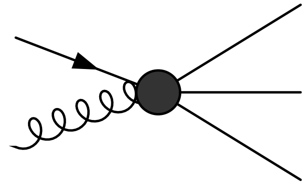
- Integral performed via:
  - Spinor integration $d^2\mu \rightsquigarrow \langle\lambda, d\lambda\rangle \; [\tilde{\lambda}, d\tilde{\lambda}]$ [Cachazo, Svrček, Witten; '04] [Britto *et al.*; '05]
  - Generalized Cauchy theorem $d^2\mu \rightsquigarrow dz \; d\bar{z}$ [Mastrolia; '09]
- Evaluate the $B_0(K^2, m_0^2, m^2)$ coefficients [Britto *et al.*; '05] . . .
- . . . And triangles & boxes coefficients [Britto, Feng; '08] [Britto, Feng, Mastrolia; '08]

Double cut & on-shell bubbles



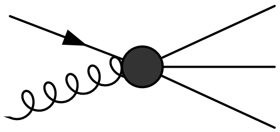
$= \int d^2\mu$





If massive external states are present:

- Double cut in their momentum enters . . .
- . . . And it is relevant for the coefficient of $B_0(m^2, 0, m^2)$
- Divergent since



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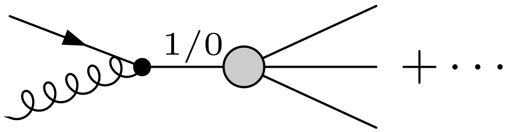
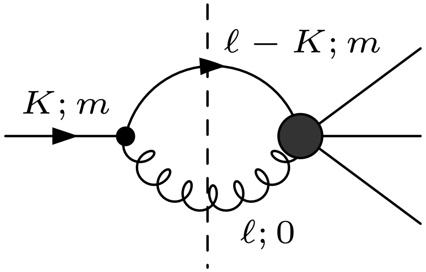
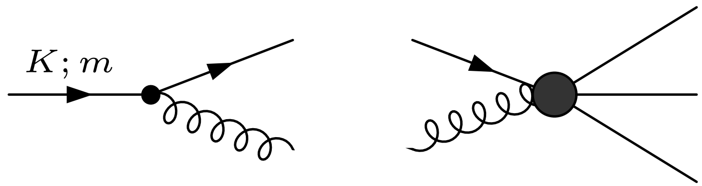


Problems in the 1-particle reducible diagrams (1PRD) correcting the external legs.

Double cut & on-shell bubbles



$$= \int d^2\mu$$



- Possible way out : [Ellis, Giele, Kunszt, Melnikov; '08]

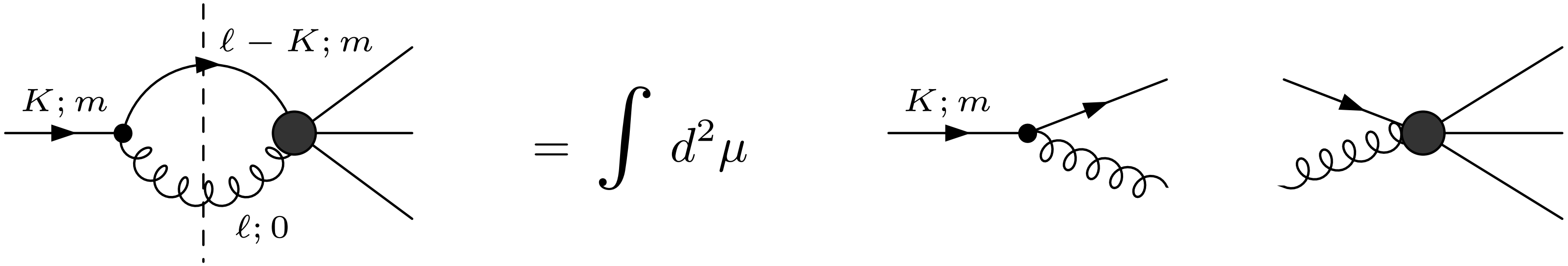
THE IDEA: $\left(\begin{array}{l} \text{1PRD are discarded} \\ \text{by the LSZ formula} \end{array} \right) \Longrightarrow \left(\begin{array}{l} \text{discard them} \\ \text{from scratch} \end{array} \right)$

  - 1-particle reducible contributions discarded
  - Gauge dependence introduced

    $\hookrightarrow$ Feynman gauge adopted
  - Polarization sum modified

    $\hookrightarrow$ $\sum_\lambda \epsilon^\mu_\lambda \epsilon^{*\,\nu}_\lambda = -g^{\mu\nu}$

Double cut & on-shell bubbles



$= \int d^2\mu$

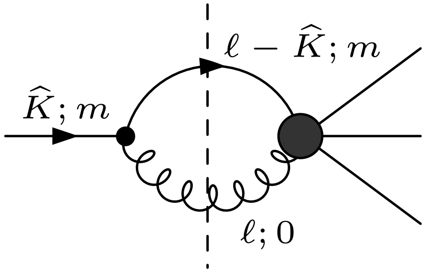
- Our method: [Britto, EM; '11]

  THE IDEA: $\left(\begin{array}{l} \text{1PRD enter the} \\ \text{double cut} \end{array} \right) \Longrightarrow \left(\begin{array}{l} \text{keep them as far} \\ \text{as possible!} \end{array} \right)$

  - Regularize the divergences
  - Subtract 1PRD at the very end

    $\hookrightarrow$ Via the counterterm in the on-shell scheme

Double cut & on-shell bubbles



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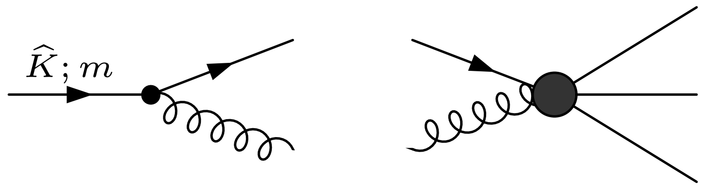


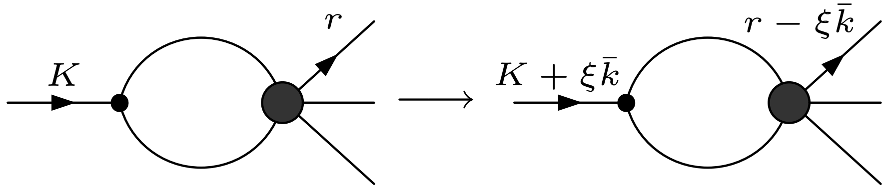
- Our method: [Britto, EM; '11]

1) Shift $K \to \widehat{K}$, $\quad r \to \hat{r}$

$\hookrightarrow$ By an amount $\pm\xi\bar{k}$

$\hookrightarrow$ $\hat{r}$ is on shell, $\widehat{K}$ not

$\hookrightarrow$ $1/0 \to 1/\xi$



2) Cut in the $\widehat{K}$ channel

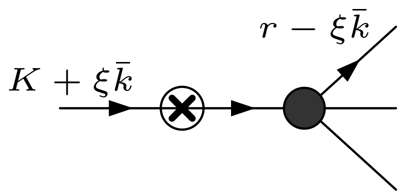
$\hookrightarrow$ Get the B_0 coefficient b

3) Expand $b \times B_0$ around $\xi = 0$

4) Add the counterterm

$\hookrightarrow$ Expand around $\xi = 0$

$\hookrightarrow$ Check $1/\xi$ and $\bar{k}$ cancellation



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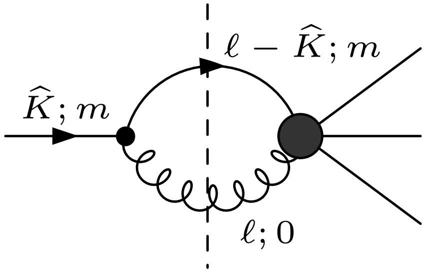

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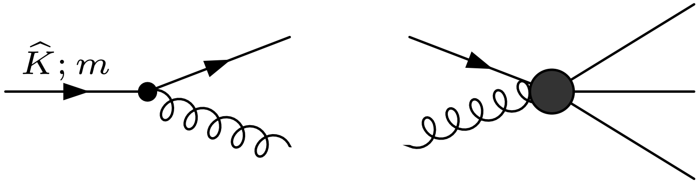
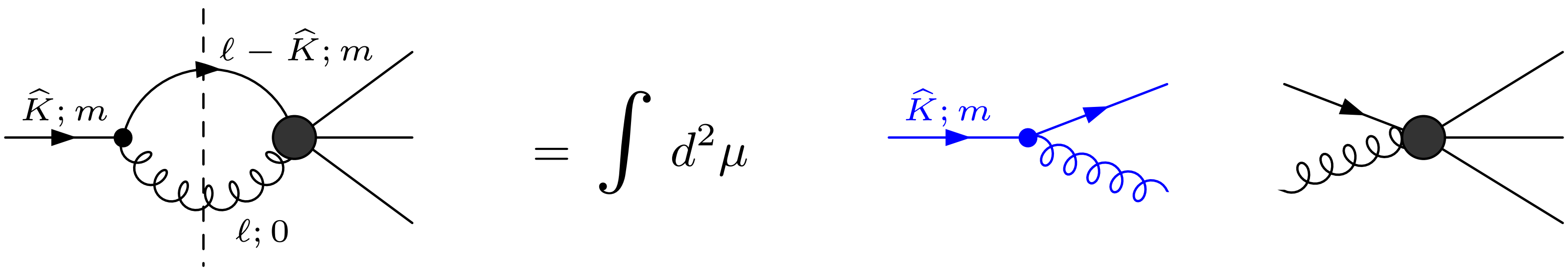


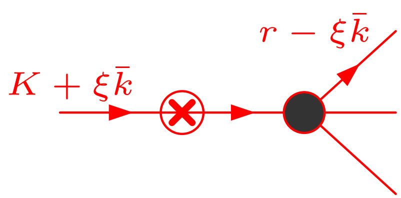
CAVEAT: Gauge dependence arises!

Double cut & on-shell bubbles



CAVEAT: Gauge dependence arises!

- Usually $\left\{ \begin{array}{l} \text{double cut} \\ \text{Ward identities} \end{array} \right.$ $\Rightarrow$ gauge terms vanish (on-shellness crucial)
- **But**: 3-point function is off shell ($\hat{K}^2 \neq m^2$)
- **Then**: residual q-dependence from $\epsilon^\mu_\pm$ ...
  $\hookrightarrow$ e.g. $\epsilon^\mu_-(p) = \sqrt{2}\Big(|p\rangle[q| + |q]\langle p|\Big)/[p,q]$
- ... Easy to handle
  - The only term surviving the double cut
  - Is cancelled modifying the counterterm
  - Can be avoided using $\sum_\lambda \epsilon^\mu_\lambda \epsilon^{*\,\nu}_\lambda = -g^{\mu\nu}$



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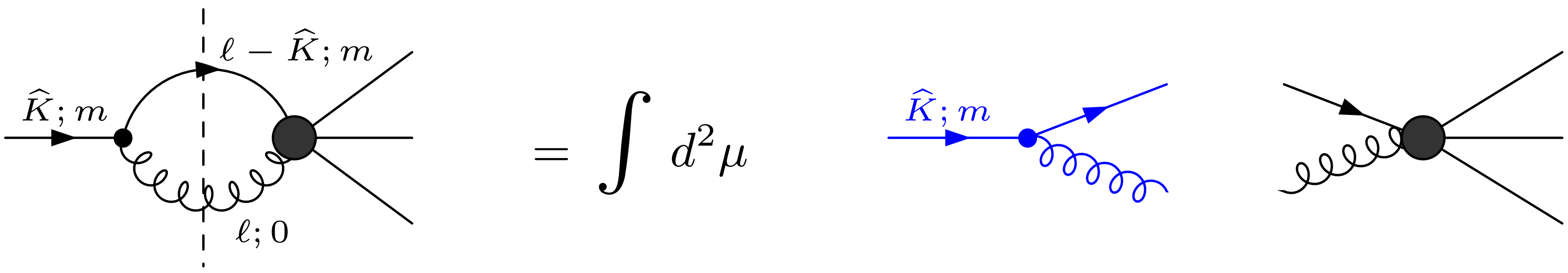
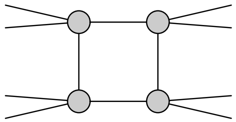
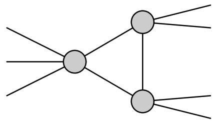
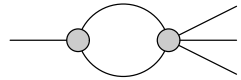
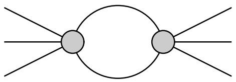
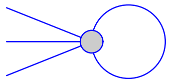


- To Summarize, our method:
  - \+ Regularizes the divergence.
  - \+ Uses the usual double cut.
  - \+ Puts on shell the "complicated" tree-level . . .
  - – . . . But the "easy" one is off shell.
  - – Introduces a gauge dependence . . .
  - \+ . . . Easy to handle!

It has been tested

– $H \to b\bar{b}$, $q\bar{q} \to t\bar{t}$, $gg \to t\bar{t}$.

$\mathcal{A}_{\text{1-loop}} = \sum_i$  $d_i + \sum_j$  $c_j + \sum_l$  $\tilde{b}_l$

$\sum_k$  $b_k + \sum_m$  $a_m + \mathcal{R}$

Coefficient of the tadpoles

Tadpole coefficients & unitarity – status

- Tadpole coefficients & unitarity:
  - Analytically under investigation . . .

    . . . But still merits (and needs) further study.
  - Numerically it works fine [Ossola, Papadopoulos, Pittau; '07] [Mastrolia *et al.*; '10]

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- Methods available
  - Kilgore' s proposal [Kilgore; '08]

    $\hookrightarrow$ Generalization of Forde' s method. [Forde; '07]
  - Auxiliary propagator method [Britto, Feng; '09]

    $\hookrightarrow$ Combination of the OPP and of the double cut
  - Spinor integration [Britto, EM; '10]

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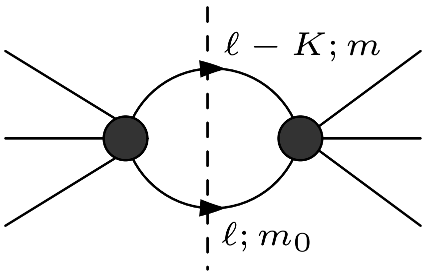
Double cut revisited



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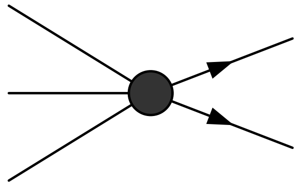

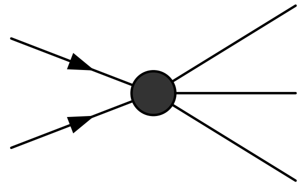

Double cut revisited

$$\Delta_2\left[\mathcal{I}\right] \equiv \int d^4\ell \ \left[\mathcal{I}\, D_0\, D_1\right] \ \delta^+\left(D_0\right) \ \delta^+\left(D_1\right)$$

Definitions $D_0 = \ell^2 - m_0^2;\ \ D_1 = (\ell - K)^2 - m^2;$

Double cut revisited

$$\Delta_2\,[\mathcal{I}] = \int dz d\bar{z}\, \frac{[\mathcal{I}\, D_0\, D_1]}{(1+z\bar{z})^2}$$

Definitions $D_0 = \ell^2 - m_0^2;\quad D_1 = (\ell - K)^2 - m^2;$

1) Reparametrizations

i) $\ell^\mu = \tilde{\ell}^\mu + \xi K^\mu$ [Anastasiou *et al.* '07]

ii) $\tilde{\ell}^\mu = t(p^\mu + \alpha q^\mu + z\epsilon_1^\mu + \bar{z}\epsilon_2^\mu)$ [Mastrolia '09]

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$$\Delta_2\,[\mathcal{I}] = \lim_{\Lambda\to\infty}\left[\int_0^{2\pi} d\beta\, \mathcal{F}(\Lambda,\beta)\right] - \pi \sum_{\text{poles } z_j} \text{Res}\{\mathcal{F}, z_j\}$$

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What about single cuts?

Single cut integration

$$\Delta_1 \left[\mathcal{I} \right] \equiv \int d^4\ell \;\; \left[\mathcal{I} \, D_0 \right] \;\; \delta^+ \left(D_0 \right)$$

Definitions $D_0 = \ell^2 - m_0^2;$

Single cut integration

$$\Delta_1 \left[\mathcal{I}\right] = \int dt \left\{ \lim_{\Lambda\to\infty} \int_0^{2\pi} d\beta \, \mathcal{G}(\Lambda,\beta) - \pi \sum_{\text{poles } z_j} \text{Res}\{\mathcal{G}, z_j\} \right\}$$

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1) Reparametrizations

i) $\ell^\mu = \tilde{\ell}^\mu + \xi K^\mu$ (K arbitrary)

ii) $\tilde{\ell}^\mu = t(p^\mu + \alpha q^\mu + z\epsilon_1^\mu + \bar{z}\epsilon_2^\mu)$

iii) Let $K^2 \to \infty$

2) Generalized Cauchy formula

i) $z = \Lambda e^{i\beta} \quad \bar{z} = \Lambda e^{-i\beta}$

ii) Both $\int \mathcal{G}$ and Res$\{\mathcal{G}\}$ enter

Single cut integration

$$\Delta_1 \left[\frac{1}{D_0} \right] = \lim_{\Lambda \to \infty} \frac{\pi K^2}{2} \Lambda^2 \left(\int dt\; t \right) + \mathcal{O}(\Lambda)$$

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Tadpole integral $\int d^4\ell \frac{1}{D_0} \longrightarrow \mathcal{I} = \frac{1}{D_0}$

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- Single cuts of a tadpole $\sim \Lambda^2$
- Bubble, triangle, box, behave differently

  e.g. for a bubble $\Delta_1[1/(D_0 D_1)] \sim \log(\Lambda^2)$
- STRATEGY: *Find the tadpole coefficient of* $\int \mathcal{I}$ *selecting the* $\Lambda^2 t-$*terms of* $\Delta_1[\mathcal{I}]$
- CAVEAT: spurious terms enter! *i.e.* $\int \mathcal{I} = 0 \not\Rightarrow \Delta_1[\mathcal{I}] = 0$

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The simplest example could be helpful

Single cut integration

$$\mathcal{I} = \frac{(2R_1 \cdot \ell)}{D_0 D_1},$$

Definitions $D_0 = \ell^2 - m_0^2;\ D_1 = (\ell - K_1)^2 - m_0^2;$

We want a(0), the coefficient of $A_0(m_0^2)$.

1) OPP decomposition ($K_1 \perp$ to n, ℓ_7, ℓ_8)

$$\mathcal{I}_{2,1} = \frac{1}{D_0}\ a(0) + \quad (\text{terms in } n,\ \ell_7,\ \ell_8)$$

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$$\Delta_1 \left[\mathcal{I}_{2,1}\right] = \Delta_1 \left[\frac{1}{D_0}\right] a(0) + \Delta_1 \left[(\text{terms in } n,\ \ell_7,\ \ell_8) \right]$$

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3) Compute the single cut

$$0 = \frac{q^\mu}{(q \cdot K_1)} \left[K_{1\mu} \;\; a(0) \qquad\qquad + R_{1\mu} + \;\; (\text{terms} \perp \text{to } K_1^\mu) \right]$$

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4) Decompose $R_1^\mu = \alpha_1 K_1^\mu + \alpha_n n^\mu + \alpha_7 \ell_7^\mu + \alpha_8 \ell_8^\mu$

$$0 = \frac{q^\mu}{(q \cdot K_1)} \left[K_{1\mu} \left(a(0) + \alpha_1 \right) \qquad + \quad (\text{terms} \perp \text{to } K_1^\mu) \right]$$

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$$a(0) = -\alpha_1 = -\frac{2K_1 \cdot R_1}{K_1^2}$$

Single cut integration

- To Summarize, the single spinor integration:
  - + Allows the computation of the tadpole coefficients . . .
  - + . . . And of the bubbles as well.
  - + Works when the Gram determinant vanishes.
  - – Involves spurious terms.
  - – Is not easy to automate.

Conclusions & Outlook

CONCLUSIONS

- Unitarity with massive particles is tricky
- Our method for the on-shell bubbles coefficients
  - Combines nicely unitarity & LSZ formula
  - Treats easily gauge dependence
- Single cut spinor integration
  - can compute tadpole coefficients (and not only!)
  - Requires to deal with the spurious terms
  - Ok when the Gram determinan vanishes

OUTLOOK

- Check the efficiency of our method for the on-shell bubbles
- Further study of the tadpoles via unitarity techniques